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Abstract

In the Crawford-Sobel (uniform, quadratic utility) cheap-talk model, we consider a *simple* mediation scheme (a communication device) in which the informed agent reports one of the N possible elements of a partition to the mediator and then the mediator suggests one of the N actions to the uninformed decision-maker according to the probability distribution of the device. We show that no such *simple* mediated equilibrium can improve upon the unmediated N -partition Crawford-Sobel equilibrium when the preference divergence parameter (bias) is small.

Keywords: Cheap Talk, Mediated Equilibrium.

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1 INTRODUCTION

We consider the (uniform, quadratic utility) cheap-talk model of Crawford and Sobel (1982) (hereafter referred to as the CS model) to study the effect of mediation in comparison to unmediated cheap-talk communication. In this game, the strategic interaction and information-transmission between an uninformed decision maker (called the receiver) and an informed agent (called the sender) has been studied, and what can be achieved by unmediated cheap-talk communication is well established. CS have proved that any (Bayesian-Nash) equilibrium in this cheap-talk game is equivalent to a partition equilibrium where the informed agent reveals one of the finitely many elements of the partition in which the true state of nature lies. The number of elements, N , in the most informative partition equilibrium in the CS model depends on the value of the preference divergence parameter, b .

Krishna and Morgan (2004) introduced a new unmediated communication protocol which improved the welfare of the two players relative to the CS equilibrium. Krishna and Morgan also constructed an example of mediated communication in the CS model demonstrating the possibility of Pareto improvement.

Blume, Board and Kawamura (2007) consider the effect of adding noise to the sender's message in the Crawford-Sobel model. The noise can be interpreted as communication error. Also, one can think of their communication scheme as a special kind of a mediator who passes on messages from the sender to the receiver with some exogenous noise added on. Specifically, with some probability, the mediator passes on the sender's message to the receiver unchanged; otherwise, independent of the message sent by the sender, the mediator passes on a random message from some fixed error distribution. They show that for a sufficiently small amount of noise, it is possible to improve upon the N -partition CS equilibrium as long as $b \neq \frac{1}{2N^2}$ for any integer N and $b < \frac{1}{2}$.

Goltsman, Horner, Pavlov and Squintani (2009) subsequently analysed the general form of mediation in the cheap-talk framework of CS. They derive the optimal unconstrained mediated mechanism and an upper bound on the receiver's payoff that can be achieved by any mediated equilibrium. This upper bound is achieved by the construction of Blume *et al* (2007) when the level of noise is chosen appropriately.¹ This bound is also the same as the value of the expected payoff to the receiver that is achieved by the equilibrium of the modified CS game constructed by Krishna and Morgan when $b < \frac{1}{8}$.

Here, we should point out that in all the above three models, welfare-improving mediation or noise involves larger number of messages compared to the minimal number of messages required in the most informative CS equilibrium. None of the papers mentioned above addresses this issue of whether

¹Blume (2011) subsequently showed that this upper bound can also be implemented without any communication via a device or without any strategic mediator. Instead, it can be achieved by a strategy-correlated equilibrium of the game in which initially both players privately receive a signal from a correlation device and then the CS game is played.

welfare-improving (non-strategic) mediation must *necessarily* involve additional messages and more actions induced in equilibrium.

We, in this paper, consider mediation schemes² in which the informed agent reports one possible element of a partition to a mediator (a communication device) and then the mediator suggests an action to the uninformed decision-maker according to the probability distribution of the device. We ask the question whether it is sufficient to use *simple* schemes (involving the same number of messages, N , as in the most informative CS equilibrium) for mediation to be Pareto superior to the CS equilibrium.

In particular, we concentrate on a specific form of mediated equilibria, that we call *N -simple mediated equilibria*, in which the mediator is restricted to use the same number (N) of inputs and outputs as the number of elements of the N -partition CS equilibrium. This clearly imposes a restriction on the mediated mechanisms that one may consider, however this restriction is made in order to enable us to answer the above question. The mediator, associated with a specific probability distribution, can be interpreted as a communication scheme that the players mutually agree to use. In this scheme, the possible number of elements the sender is allowed to use and the receiver is expected to receive via the mediator is restricted to N , as in the most informative CS equilibrium. Note that b , and hence N , is commonly known to the players.

The main result of our paper (Theorem 1) is that the N -partition CS equilibrium cannot be improved upon by the corresponding N -simple mediated equilibrium when the preference divergence parameter b is small (less than $\frac{1}{2N^2}$). In other words, when b is small, mediation needs to use more messages (relative to the minimal number of messages that can be used in the best CS equilibrium) in order to improve upon the N -partition CS equilibrium. If mediation or noise is simply a randomisation over the messages that the sender would have reported in an original unmediated CS equilibrium, then it cannot improve information transmission, when the degree of preference misalignment is small.

Although Goltsman *et al* (2009) provided a necessary and sufficient condition for an incentive compatible mediation rule to be optimal and showed that two specific mediation rules proposed in the literature (that of Blume *et al* (2007) and by Krishna and Morgan (2004)) are indeed optimal for certain values of b , we do not know what the structure of other optimal mediation rules might be.

One might ask if there exists an optimal mediation rule which is also an N -simple mediated equilibrium. If the answer is yes, this could imply that a suitable randomisation over the messages used in the original unmediated CS equilibrium could improve information transmission and there would

²Ivanov (2010) considers the role of a strategic mediator in the CS framework. He shows that, for any bias b , there exists a strategic mediator who can help achieve the optimal payoffs obtained through a non-strategic mediator. Dessein (2002) considered delegation to an intermediary in the CS framework. However, the role of his “intermediary” is different from that of “mediation” in our context. Kovac and Mylovanov (2009) studied the relative performance of noisy or stochastic mechanisms and deterministic mechanisms in a very similar principal-agent setting.

be no need to use additional messages. However, we cannot deduce the minimal number of messages required in all such optimal mediation rules from Goltsman *et al* (2009).

Theorem 1, in this paper, thus advances our understanding of the effect of mediation in the CS model. It partially answers the question of whether the minimal size of the message space needs to be larger for mediation to Pareto dominate the CS equilibrium. Although we do not answer the question of whether or not an N -simple mediated equilibrium can always improve on the CS equilibrium for large b , we provide a suggestive example.

2 THE MODEL

2.1 Crawford-Sobel Game

Our set-up is identical to the uniform-quadratic utility CS Model, as presented in the literature. Informed readers may wish to skip this subsection.

There are 2 agents. The informed agent, called the sender (S), precisely knows the state of the world, θ , where $\theta \sim U[0, 1]$, and can send a message at no cost, based on his private information, to the other agent, called the receiver (R). The receiver, however doesn't know θ but must choose some decision y , based on the information contained in the signal. The receiver's payoff is $U^R(y, \theta) = -(y - \theta)^2$, and the sender's payoff is $U^S(y, \theta, b) = -(y - (\theta + b))^2$, where $b > 0$ is a parameter that measures the 'bias' in their preferences.

CS have shown that any equilibrium of this game is essentially equivalent to a partition equilibrium where only a finite number of actions are chosen in equilibrium and each action corresponds to an element of the partition. For $b < \frac{1}{2N(N-1)}$, where $N \geq 2$ is an integer, there is a partition equilibrium³ in which the state space is partitioned into N elements, characterised by $0 = a_0 < a_1 < a_2 < \dots < a_{N-1} < a_N = 1$, where $a_k = \frac{k}{N} + 2bk(k - N)$; in this equilibrium, S sends a message for each element $[a_{k-1}, a_k)$, and given this message, R takes the optimal action $y_k = \frac{a_{k-1} + a_k}{2}$. We call this the N -partition CS equilibrium. For $\frac{1}{2N(N+1)} \leq b < \frac{1}{2N(N-1)}$, the "best" equilibrium (the one that maximises the receiver's expected payoff, EU^R) is the N -partition CS equilibrium⁴. For such an equilibrium, the receiver's expected payoff is $EU^R = -\frac{1}{12N^2} - \frac{b^2(N^2-1)}{3}$, while the sender's expected payoff is $EU^S = EU^R - b^2$.

³For $\frac{1}{4} \leq b$, babbling is the only equilibrium.

⁴Note that for any $b < \frac{1}{2N(N+1)}$, an N -partition CS equilibrium does exist. However, it is not the "best". Chen, Kartik and Sobel (2008) provide a formal selection argument for the "best" equilibrium.

2.2 Mediated Equilibrium

Within the CS framework, we now consider mediation, a possible structure of which could be as follows: S sends a message based on his private information to the mediator; the mediator then chooses an action according to a commonly-known probability distribution and recommends it to R . We here consider a specific form of direct mediation (in the spirit of canonical mechanisms, as initiated and analysed extensively by Forges (1986) and Myerson (1982, 1986)) and formally define such a mediated talk below.

Definition 1 An $N \times M$ mediated talk is $[\{x_k\}_{k=0}^N, \{y_j\}_{j=1}^M, \{p_{kj}\}_{k=1, \dots, N; j=1, \dots, M}]$ where $0 = x_0 < x_1 < x_2 < \dots < x_{N-1} < x_N = 1$, each $y_j \in [0, 1]$ for $j \in \{1, 2, \dots, M\}$, each $p_{kj} \in [0, 1]$ for $k \in \{1, 2, \dots, N\}$, $j \in \{1, 2, \dots, M\}$ with $\sum_{j=1}^M p_{kj} = 1$.

In an $N \times M$ mediated talk, S reports one of the N possible elements, $[x_{k-1}, x_k)$, in which the true state θ may lie, to the mediator and given the report $\theta \in [x_{k-1}, x_k)$, the mediator then recommends to R one action, y_j , out of the M possible actions with probability p_{kj} .

Our mechanism⁵ (an $N \times M$ mediated talk) is said to be in equilibrium if it is incentive compatible for both players, that is, if (i) S has the incentive to be truthful to the mediator given the probabilities p_{kj} , and (ii) R has the incentive to obey the mediator's recommendation y_j , given the posterior probabilities on the state of nature.

In what follows, we will focus only on $N \times N$ direct⁶ mediated equilibria. We will call such an equilibrium an N -simple mediated equilibrium.

Let $f_k(\theta)$ denote the difference in expected utility from inducing the distributions $\{p_{k+1j}\}_{j=1}^N$ and $\{p_{kj}\}_{j=1}^N$ that a type θ sender would obtain. Formally, $f_k(\theta) = \sum_{j=1}^N (p_{kj} - p_{k+1j}) [y_j - (\theta + b)]^2$.

Definition 2 For any specific value of b , an $N \times N$ (or, N -simple) mediated equilibrium is an $N \times M$ (with $M = N$) mediated talk that satisfies

- (i) incentive compatibility for S : for all $k \in \{1, 2, \dots, N-1\}$, $f_k(x_k) = 0$ and $f'_k(\theta) > 0$.
- (ii) incentive compatibility for R : $y_j = \underset{y}{\operatorname{argmax}} - \sum_{k=1}^N q_{kj} \frac{1}{(x_k - x_{k-1})} \int_{x_{k-1}}^{x_k} (y - \theta)^2 d\theta$ for all $j \in \{1, 2, \dots, N\}$.

⁵This is the type of mechanism Krishna and Morgan (2004) also considered to construct an example in their paper. Myerson (website) used a "discrete" version of such a mechanism.

⁶We are using a suitable version of the revelation principle (Myerson 1982) here to characterise the set of N -simple mediated equilibria involving direct messages only, to cover all simple mediation schemes which can use any N inputs and any N outputs. As it turns out, considering only such direct mechanisms is not restrictive, as a revelation principle type result does hold in this context and can be proved using the methods constructed by Ray (2001).

$\{1, 2, \dots, N\}$, where q_{kj} , the posterior probability that $\theta \in [x_{k-1}, x_k)$, is given by $q_{kj} = \frac{(x_k - x_{k-1})p_{kj}}{\sum_{k=1}^N (x_k - x_{k-1})p_{kj}}$.

In (i), $f_k(x_k) = 0$ essentially corresponds to the ‘‘arbitrage condition’’ in the CS equilibrium. Incentive compatibility for the sender requires that the x_k type sender is indifferent between inducing $\{p_{k+1j}\}_{j=1}^N$ and $\{p_{kj}\}_{j=1}^N$. Also, a sender with type $\theta \in (x_k, x_{k+1})$ should prefer the distribution $\{p_{k+1j}\}_{j=1}^N$ over $\{p_{kj}\}_{j=1}^N$, implying that $f'_k(\theta) > 0$. Note that $f'_k(\theta) > 0$ at $\theta = x_k$ and since $f'_k(\theta)$ is constant, it has to be positive everywhere.

In (ii), incentive compatibility for R requires that when y_j has been recommended, R indeed chooses the action y_j because it maximises his expected utility given his posterior beliefs.

2.3 Characterisation

An N -simple mediated equilibrium can be characterised easily. Incentive compatibility for R , as in Definition 2(ii), requires for all $j = 1, \dots, N$, $\sum_{k=1}^N p_{kj} [(y_j - x_{k-1})^2 - (y_j - x_k)^2] = 0$.

This implies

$$y_j = \frac{1}{2} \left[\frac{(1 - \sum_{k \neq j} p_{jk})(x_j^2 - x_{j-1}^2) + \sum_{k \neq j} p_{kj}(x_k^2 - x_{k-1}^2)}{(1 - \sum_{k \neq j} p_{jk})(x_j - x_{j-1}) + \sum_{k \neq j} p_{kj}(x_k - x_{k-1})} \right]. \quad (1)$$

Incentive compatibility for S , as in Definition 2(i), requires $\sum_{j=1}^{N-1} (p_{k+1j} - p_{kj})(y_j - y_N) > 0$ and

$$\sum_{j=1}^N (p_{kj} - p_{k+1j}) [y_j - (x_k + b)]^2 = 0, \text{ for all } k = 1, \dots, N - 1.$$

The latter implies, for all $k = 1, \dots, N - 1$,

$$2(x_k + b) = \frac{\sum_{j=1}^{N-1} (p_{kj} - p_{k+1j})(y_j^2 - y_N^2)}{\sum_{j=1}^{N-1} (p_{kj} - p_{k+1j})(y_j - y_N)}, \text{ i.e.,}$$

$$2(x_k + b) = \frac{(1 - \sum_{j \neq k} p_{kj} - p_{k+1k})(y_k^2 - y_N^2) + (p_{k+1k} - 1) + \sum_{j \neq k+1} p_{k+1j}(y_{k+1}^2 - y_N^2) + \sum_{j \neq k, k+1} (p_{kj} - p_{k+1j})(y_j^2 - y_N^2)}{(1 - \sum_{j \neq k} p_{kj} - p_{k+1k})(y_k - y_N) + (p_{k+1k} - 1) + \sum_{j \neq k+1} p_{k+1j}(y_{k+1} - y_N) + \sum_{j \neq k, k+1} (p_{kj} - p_{k+1j})(y_j - y_N)}. \quad (2)$$

Thus, an N -simple mediated equilibrium is characterised by $[\{x_k\}_{k=0}^N, \{y_j\}_{j=1}^N, \{p_{kj}\}_{k=1, \dots, N; j=1, \dots, N}]$ satisfying equations (1) and (2) with the constraints that $\sum_{j=1}^{N-1} (p_{k+1j} - p_{kj})(y_j - y_N) > 0$, for all $k = 1, \dots, N - 1$.

3 RESULTS

To state and prove our results, we take $b < \frac{1}{2N(N-1)}$, for which the N -partition CS equilibrium exists. We first observe that the N -partition CS equilibrium is actually equivalent to a particular N -simple mediated equilibrium, namely, one with $x_k = a_k = \frac{k}{N} + 2bk(k-N)$, for all $k \in \{1, \dots, N\}$; $y_j = \frac{a_{j-1} + a_j}{2}$, for all $j \in \{1, \dots, N\}$ and $p_{kj} = 0$, for all $k, j \in \{1, \dots, N\}$, $k \neq j$.

Note that, in the class of simple mediated equilibria, $EU^S = EU^R - b^2$, that is, ex-ante, the sender's and receiver's welfare ranking agree. For any simple mediated equilibrium, it is also clear that

$$EU^R = - \sum_{k=1}^N [(1 - \sum_{j \neq k} p_{kj}) \int_{x_{k-1}}^{x_k} (y_k - \theta)^2 d\theta + \sum_{\substack{j=1 \\ j \neq k}}^N p_{kj} \int_{x_{k-1}}^{x_k} (y_j - \theta)^2 d\theta],$$

which implies

$$EU^R = -\frac{1}{3} \sum_{k=1}^N [(1 - \sum_{j \neq k} p_{kj}) \{(y_k - x_{k-1})^3 - (y_k - x_k)^3\} + \sum_{\substack{j=1 \\ j \neq k}}^N p_{kj} \{(y_j - x_{k-1})^3 - (y_j - x_k)^3\}].$$

We are interested in the question when can the N -partition CS equilibrium not be improved upon by an N -simple mediated equilibrium, that is, when is the N -partition CS equilibrium indeed the best among the set of N -simple mediated equilibria. Our main result answers the above question by solving the following constrained maximisation problem that we call the *final problem*.

Final problem: Maximise EU^R among the set of N -simple mediated equilibria (as characterised in the previous section).

Before we state and prove our main result, as a first step, we consider the following constrained maximisation problem that we call the *initial problem*:

Initial problem: Maximise EU^R subject to equations (1) and (2).

The difference between the final problem and the initial problem is simply the set of restrictions that $\sum_{j=1}^{N-1} (p_{k+1j} - p_{kj}) (y_j - y_N) > 0$, for all $k = 1, \dots, N-1$.

Our first lemma below proves that for $b < \frac{1}{2N^2}$, the initial problem has a ‘‘corner’’ solution.

Lemma 1 For $b < \frac{1}{2N^2}$, a solution of the initial problem is achieved at $p_{kj} = 0$, for all $k \neq j$; $k, j \in \{1, \dots, N\}$.

Lemma 1 also characterises the values of $\{x_k\}_{k=1}^{N-1}$ at this solution of the initial problem. They are indeed the CS values given by $x_k = x_k^{CS} = \frac{k}{N} + 2bk(k-N)$, for all $k \in \{1, \dots, N-1\}$. The formal proof of Lemma 1 has been relegated to the Appendix.

The following result shows that the N -partition CS equilibrium indeed provides a (local) maximiser of EU^R among the set of N -simple mediated equilibria. The result is an immediate consequence of Lemma 1 and thus the formal proof is postponed to the Appendix.

Corollary 1 For $b < \frac{1}{2N^2}$, a local maximum of the final problem is achieved at $p_{kj} = 0$, for all $k \neq j$; $k, j \in \{1, \dots, N\}$ and $x_k = x_k^{CS} = \frac{k}{N} + 2bk(k - N)$, for all $k \in \{1, \dots, N - 1\}$.

We are now ready to state our main result.

Theorem 1 For $b < \frac{1}{2N^2}$, no N -simple mediated equilibrium can improve upon the N -partition CS equilibrium.

Our theorem above proves that for $b < \frac{1}{2N^2}$, the N -partition CS equilibrium is actually a global maximum among the set of N -simple mediated equilibria.

We first note that the global maximum exists, by appealing to the Weierstrass Theorem, since the objective function is continuous and is defined over a compact set.

To prove the theorem, we first reconsider EU^R of any N -simple mediated talk and write the following expression:

$$-3EU^R = \sum_{k=1}^N [(1 - \sum_{j \neq k} p_{kj}) \{(y_k - x_{k-1})^3 - (y_k - x_k)^3\}] + \sum_{\substack{j=1 \\ j \neq k}}^N p_{kj} \{(y_j - x_{k-1})^3 - (y_j - x_k)^3\}.$$

We work with the above expression, involving the variables, $[\{x_k\}_{k=0}^N, \{y_j\}_{j=1}^N, \{p_{kj}\}_{k=1, \dots, N; j=1, \dots, N}]$, satisfying equation (1) from the previous section, namely, $y_j = \frac{1}{2} \left[\frac{(1 - \sum_{k \neq j} p_{jk})(x_j^2 - x_{j-1}^2) + \sum_{k \neq j} p_{kj}(x_k^2 - x_{k-1}^2)}{(1 - \sum_{k \neq j} p_{jk})(x_j - x_{j-1}) + \sum_{k \neq j} p_{kj}(x_k - x_{k-1})} \right]$.

We redefine the above expression as a function, f , explicitly as a function of the variables $\{x_k\}_{k=1}^{N-1}$ and $\{p_{kj}\}_{k \neq j}$ such that the domain satisfies equation (1). Let us thus write

$$\begin{aligned} f(x_1, \dots, x_{N-1}; p_{12}, p_{13}, \dots, p_{1N}; p_{21}, p_{23}, \dots, p_{2N}; \dots; p_{N1}, \dots, p_{NN-1}) \\ = \sum_{k=1}^N [(1 - \sum_{j \neq k} p_{kj}) \{(y_k - x_{k-1})^3 - (y_k - x_k)^3\}] + \sum_{\substack{j=1 \\ j \neq k}}^N p_{kj} \{(y_j - x_{k-1})^3 - (y_j - x_k)^3\}. \end{aligned}$$

We first observe the following result which characterises a property of the function, f .

Lemma 2 For any (x_1, \dots, x_{N-1}) , $\frac{\partial f}{\partial p_{kj}} > 0$, at $p_{kj} = 0$, for all $k \neq j$; $k, j \in \{1, \dots, N\}$.

Abusing notation, we treat f as a function of $\{p_{kj}\}_{j \neq k}$, for any fixed level of (x_1, \dots, x_{N-1}) , and also, as a function of $\{x_k\}_{k=1}^{N-1}$, for fixed values of p_{kj} , for all $k \neq j$. We then consider the minimisation of f with respect to p_{kj} , for all $k \neq j$, and prove the following lemma.

Lemma 3 For any (x_1, \dots, x_{N-1}) , f , as a function of $\{p_{kj}\}_{j \neq k}$, attains a (global) minimum at $p_{kj} = 0$, for all $k \neq j$; $k, j \in \{1, \dots, N\}$.

The next lemma indicates an important property of f , as a function of (x_1, \dots, x_{N-1}) , for a particular value of $\{p_{kj}\}_{j \neq k}$.

Lemma 4 *At $p_{kj} = 0$, for all $k \neq j$, f , as a function of $\{x_k\}_{k=1}^{N-1}$, is strictly convex.*

The proof of Theorem 1 is now a direct consequence of the above lemmata. We provide the formal argument in the Appendix.

Theorem 1 proves that for $b < \frac{1}{2N^2}$, a global maximum among the set of N -simple mediated equilibria must coincide with the N -partition CS equilibrium. Also, Lemma 3 suggests that, for $b > \frac{1}{2N^2}$, a global minimum of f is attained at $p_{kj} = 0$, for all $k \neq j$; however, it need not be at the values of $\{x_k\}_{k=1}^{N-1}$ given by the N -partition CS equilibrium.

4 REMARKS

4.1 Illustration

Our result can be illustrated easily using a simple example with $N = 2$. Recall that a 2-partition CS equilibrium exists for $b < \frac{1}{4}$ and for $\frac{1}{12} \leq b < \frac{1}{4}$, the best CS partition equilibrium involves two elements. Theorem 1 confirms that the 2-partition CS equilibrium can not be improved upon by any 2-simple mediated equilibrium when $b < \frac{1}{8}$. Our result also suggests that a 2-simple mediated equilibrium may improve upon the 2-partition CS equilibrium when b is large enough, that is, for $\frac{1}{8} \leq b < \frac{1}{4}$. We illustrate this comment for $b = \frac{1}{6}$. Here, the 2-partition CS equilibrium is characterised by $a = \frac{1}{6}$, $y_1 = \frac{1}{12}$, and $y_2 = \frac{7}{12}$ with utilities $EU^R = -\frac{7}{144} \simeq -0.0486$ and $EU^S = -\frac{11}{144} \simeq -0.0764$.

From the characterisation presented in Section 2.3, a 2-simple mediated equilibrium is given by⁷ $(x, y_1, y_2, p_{11}, p_{12}, p_{21}, p_{22})$, where, $x, y_1, y_2 \in (0, 1)$, $p_{11}, p_{12}, p_{21}, p_{22} \in [0, 1]$ and $p_{11} + p_{12} = 1$, $p_{21} + p_{22} = 1$. The incentive compatibility constraints for S and for R can all be combined into one equation given by

$$\frac{(1-p_{12})x^2 + p_{21}(1-x^2)}{4[(1-p_{12})x + p_{21}(1-x)]} + \frac{p_{12}x^2 + (1-p_{21})(1-x^2)}{4[p_{12}x + (1-p_{21})(1-x)]} - x = b.$$

Thus, a 2-simple mediated equilibrium in this set-up can be characterised by three variables (p_{12}, p_{21}, x) , where, $x \in (0, 1)$ and $p_{12}, p_{21} \in [0, 1]$, satisfying the above equation.

It is now easy to check that for $b = \frac{1}{6}$, $x = 0.2245201023$, $y_1 = 0.1745967377$, $y_2 = 0.6077768002$, $p_{12} = 0.03$, and $p_{21} = 0.04$ constitute a 2-simple mediated equilibrium with utilities $EU^R \simeq -0.0483$ and $EU^S \simeq -0.0760$ and can improve upon the corresponding 2-partition CS equilibrium.

The interpretation of the above example is as follows. One can see that the partitioning point x and the two decisions y_1 and y_2 of the 2-simple mediated equilibrium are all larger than the corresponding values of the 2-partition CS equilibrium. The fact that the lower interval is bigger and the higher

⁷We drop the subscript in x_1 for presentational simplicity.

element of the partition is smaller in size means that more information is being transmitted. This is possible because the mediator is allowed to randomize between y_1 and y_2 .

4.2 Goltsman *et al* (2009)

One might be interested in knowing how our theorem compares with the corresponding results in Goltsman *et al* (2009) and in particular, if there is a connection between our Theorem 1 and Theorem 2 of Goltsman *et al* (2009). In Theorem 2 of Goltsman *et al*, an optimal mediation rule is provided. One interesting corollary of this theorem is that for $b = \frac{1}{2N^2}$, this particular optimal mediation rule cannot improve upon the N -partition CS equilibrium. This implies that the N -partition CS equilibrium is optimal, irrespective of the number of messages that the players are allowed to use, when $b = \frac{1}{2N^2}$. However, since Theorem 2 of Goltsman *et al* is about a specific optimal mediation rule which uses more messages than our N -simple mediated equilibrium and in general, there might be a continuum of optimal mediation rules. We thus feel that this theorem is not useful in answering the question posed in our paper.

A more meaningful approach⁸ might be to ask if the technique used by Goltsman *et al* to prove their Theorem 2 can provide any insight or an alternative way of proving our result. Goltsman *et al* introduced a lemma (Lemma 2 in their paper) to derive a necessary and sufficient condition for an incentive compatible mediation rule to be optimal and to provide an upper bound on the objective function using an incentive compatible mediation rule. One might try to identify such a condition and an upper bound in the more constrained setting of N -simple mediation rules. If the characterisation of optimal mediation rules in Lemma 2 of Goltsman *et al* could be appropriately modified to derive a characterisation of optimal N -simple mediated equilibria, then this would provide another method of proof of our Theorem 1. We would like to point out that we do not derive such a characterisation in this paper and that such an alternative proof is not straightforward either.

It is also worth mentioning two recent papers on the connection between communication equilibrium and correlated equilibrium (Blume (2011), Forges and Vida (2012)) that are relevant to our work. Forges and Vida (2012) proved that (essentially) every communication equilibrium of any finite Bayesian game with two players can be implemented as a strategic form correlated equilibrium of a game, extended by a cheap-talk phase before the original Bayesian game. On the other hand, specific to the CS model, Blume (2011) constructed a strategy-correlated equilibrium, that sends messages to both players before the sender sends any message to the receiver, to achieve the best possible payoff from the mediated equilibrium of the CS model. Importantly, in his construction, unlike our work, neither player needs to send messages to the device. Following these new results, one may be interested

⁸We thank an anonymous referee for suggesting this approach.

to know whether our N -simple mediated equilibrium can be obtained as a correlated equilibrium in the sense of Forges and Vida (2012), or as a strategy-correlated equilibria, as in Blume (2011). In particular, one may further ask whether or not these new constructible correlated or strategy-correlated equilibria will involve only a few (N many) messages. Clearly, these are important questions for future research.

5 APPENDIX

We collect the proofs of all our results in this section.

Proof of Lemma 1. It suffices to show that, for $b < \frac{1}{2N^2}$, the N -simple mediated equilibrium corresponding to $p_{kj} = 0$, for all $k \neq j$; $k, j \in \{1, \dots, N\}$ solves the initial problem.

Let us first consider the Lagrangian:

$$\begin{aligned} \mathcal{L} = & - \sum_{k=1}^N [(1 - \sum_{j \neq k} p_{kj}) \{(y_k - x_{k-1})^3 - (y_k - x_k)^3\} + \sum_{j=1}^N p_{kj} \{(y_j - x_{k-1})^3 - (y_j - x_k)^3\}] \\ & - \sum_{k=1}^{N-1} \lambda_k \left[\frac{(1 - \sum_{j \neq k} p_{kj} - p_{k+1k})(y_k^2 - y_N^2) + (p_{k+1k} - 1 + \sum_{j \neq k+1} p_{k+1j})(y_{k+1}^2 - y_N^2) + \sum_{j \neq k, k+1} (p_{kj} - p_{k+1j})(y_j^2 - y_N^2)}{(1 - \sum_{j \neq k} p_{kj} - p_{k+1k})(y_k - y_N) + (p_{k+1k} - 1 + \sum_{j \neq k+1} p_{k+1j})(y_{k+1} - y_N) + \sum_{j \neq k, k+1} (p_{kj} - p_{k+1j})(y_j - y_N)} \right] \\ & + 2 \sum_{k=1}^{N-1} \lambda_k x_k + 2b \sum_{k=1}^{N-1} \lambda_k. \end{aligned}$$

To prove the result, we just need to show that at the proposed solution, there exist $\{\lambda_k\}_{k=1}^{N-1}$ such that $\frac{\partial \mathcal{L}}{\partial x_k} = 0$, for all $k = 1, \dots, N-1$ and $\frac{\partial \mathcal{L}}{\partial p_{kj}} < 0$, for all $k \neq j$, when $b < \frac{1}{2N^2}$.

First, at $p_{kj} = 0$, for all $k \neq j$; $k, j \in \{1, \dots, N\}$, it is easy to check that $\frac{\partial y_k}{\partial x_k} = \frac{\partial y_k}{\partial x_{k-1}} = \frac{1}{2}$, and $\frac{\partial y_k}{\partial x_j} = 0$, for all $j \neq k, k-1$. Also, $\frac{\partial y_j}{\partial p_{kj}} = \frac{(x_k - x_{k-1})(x_k + x_{k-1} - x_j - x_{j-1})}{2(x_j - x_{j-1})}$, for all $k \neq j$ and $\frac{\partial y_j}{\partial p_{kl}} = 0$, for all $l \neq j$, for all k .

Subsequently, it can be shown that $\frac{\partial \mathcal{L}}{\partial x_k} = -3[(y_k - x_k)^2 - (y_{k+1} - x_k)^2] + \lambda_k - \frac{\lambda_{k+1}}{2} - \frac{\lambda_{k-1}}{2}$, for all $k = 1, \dots, N-1$ (since λ_0 and λ_N are not defined, define them to be equal to zero).

Now, $\frac{\partial \mathcal{L}}{\partial x_k} = 0$ implies $12b(y_{k+1} - y_k) + 2\lambda_k - \lambda_{k+1} - \lambda_{k-1} = 0$, for all $k = 1, \dots, N-1$.

This gives us a system of $(N-1)$ equations in $(N-1)$ variables, $\lambda_1, \dots, \lambda_{N-1}$, which can be succinctly written in matrix form as

$$\begin{bmatrix} 2 & -1 & 0 & 0 & \cdot & \cdot & 0 \\ -1 & 2 & -1 & 0 & \cdot & \cdot & \cdot \\ 0 & -1 & 2 & -1 & 0 & \cdot & \cdot \\ 0 & 0 & -1 & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & 0 & -1 & 2 & -1 \\ 0 & \cdot & \cdot & 0 & 0 & -1 & 2 \end{bmatrix}_{(N-1) \times (N-1)} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \cdot \\ \cdot \\ \cdot \\ \lambda_{N-1} \end{bmatrix}_{(N-1) \times 1} = \begin{bmatrix} 12b(y_1 - y_2) \\ 12b(y_2 - y_3) \\ 12b(y_3 - y_4) \\ \cdot \\ \cdot \\ \cdot \\ 12b(y_{N-1} - y_N) \end{bmatrix}_{(N-1) \times 1}$$

The $(N-1) \times (N-1)$ matrix above is symmetric and tridiagonal, the ij -th element of the inverse of which is given by $\frac{1}{4N}(i+j-|j-i|)(2N-|j-i|-i-j)$ (using results by Hu and O'Connell (1996) and Yamani and Abdelmonem (1997)).

Thus, solving the equations, we get, $\lambda_k = -\frac{2bk(N-k)}{N}[3 - 2bN^2 + 4bkN]$ (which is < 0 , for all $k = 1, \dots, N - 1$).

We are now ready to show that, when $b < \frac{1}{2N^2}$, $\frac{\partial \mathcal{L}}{\partial p_{kj}} < 0$, for all $k \neq j$, at the proposed solution (the CS equilibrium values of x_k 's and y_k 's) and with the above values λ_k , for all $k = 1, \dots, N - 1$.

For all $k \neq j$, we have,

$$\frac{\partial \mathcal{L}}{\partial p_{kj}} = [(y_k - x_{k-1})^3 - (y_k - x_k)^3] - [(y_j - x_{k-1})^3 - (y_j - x_k)^3] \\ - \lambda_k \left[\frac{(y_j - y_k)(y_j - y_{k+1})}{(y_k - y_{k+1})} \right] - \lambda_{k-1} \left[\frac{(y_k - y_j)(y_j - y_{k-1})}{(y_{k-1} - y_k)} \right] - (\lambda_j + \lambda_{j-1}) \left[\frac{\partial y_j}{\partial p_{kj}} \right].$$

We first prove that $\frac{\partial \mathcal{L}}{\partial p_{kj}} < 0$, for $k = 1$, when $b < \frac{1}{2N^2}$. Here,

$$\frac{\partial \mathcal{L}}{\partial p_{1j}} = \frac{(1-j)^2(2bN^2+1)(2bN^2-2bjN-1)(12b^2N^4-12b^2N^3-12jb^2N^3-4b^2jN^2+8b^2j^2N^2-12bN^2+6bN+6bjN+3)}{N^3(2bN^2+2bN-4bjN-1)(2bN^2-4bN-1)}.$$

Clearly, (i) $(1-j)^2 > 0$; (ii) $(2bN^2+1) > 0$; and (iii) $N^3 > 0$. Also note that as $b < \frac{1}{2N(N-1)}$, we have $2bN^2-2bN-1 < 0$. It is now easy to check that (iv) $2bN^2-2bjN-1 < 0$; (v) $2bN^2+2bN-4bjN-1 < 0$; and (vi) $2bN^2-4bN-1 < 0$.

Finally, the factor,

$$\text{(vii) } 12b^2N^4 - 12b^2N^3 - 12jb^2N^3 - 4b^2jN^2 + 8b^2j^2N^2 - 12bN^2 + 6bN + 6bjN + 3 \\ = 3(2N^2b - 1)^2 + 6Nb(1+j)(2Nb + 1 - 2N^2b) + 4N^2b^2[(j-2)^2 - 7 + j^2],$$

which we need to show is > 0 , for all $j \geq 2$. Clearly, it is so, for all $j \geq 3$. For $j = 2$, the factor is equal to $3(2N^2b - 1)^2 + 18Nb(2Nb + 1 - 2N^2b) - 12N^2b^2$, which can be shown to be > 0 , whenever $b < \frac{1}{2N^2 - \frac{4N}{3}}$. Since $\frac{1}{2N^2 - \frac{4N}{3}} > \frac{1}{2N^2}$, we have that the factor is for > 0 , for all $j \geq 2$, when $b < \frac{1}{2N^2}$. Hence, $\frac{\partial \mathcal{L}}{\partial p_{1j}} < 0$, for all $j \geq 2$, when $b < \frac{1}{2N^2}$.

We now show that $\frac{\partial \mathcal{L}}{\partial p_{kj}} < 0$, for all $k > 1$, when $b < \frac{1}{2N^2}$. Substituting the values for the x_k 's, y_k 's and λ_k 's, we have,

$$\frac{\partial \mathcal{L}}{\partial p_{kj}} = \frac{(k-j)^2(2bN^2-1)(2bN^2+1)(2bN^2+2bN-2bjN-2bkN-1)}{N^3(2bN^2+2bN-4bjN-1)(2bN^2+4bN-4bkN-1)(2bN^2-4bkN-1)}A, \text{ where,} \\ A = 12b^2N^4 - 36b^2kN^3 - 12jb^2N^3 + 24b^2N^3 + 32b^2k^2N^2 + 8jb^2N^2k + 4b^2N^2 - 36b^2N^2k - 12b^2jN^2 \\ + 8b^2j^2N^2 - 12bN^2 + 18bkN + 6bjN - 12bN + 3.$$

Here, clearly, (i) $(k-j)^2 > 0$; (ii) $(2bN^2+1) > 0$; and (iii) $N^3 > 0$. Once again, as $b < \frac{1}{2N(N-1)}$, we have $2bN^2-2bN-1 < 0$. Thus, one can verify that (iv) $2bN^2+2bN-2bjN-2bkN-1 = 2bN^2-2bN-1+2bN(2-j-k) < 0$; (v) $2bN^2+2bN-4bjN-1 = 2bN^2-2bN-1+4bN(1-j) < 0$; (vi) $2bN^2+4bN-4bkN-1 = 2bN^2-2bN-1+2bN(3-2k) < 0$ as $k \geq 2$; and (vii) $2bN^2-4bkN-1 = 2bN^2-2bN-1+2bN(1-2k) < 0$.

Finally, note that the factor,

$$A = 12b^2N^4 - 36b^2kN^3 - 12jb^2N^3 + 24b^2N^3 + 32b^2k^2N^2 + 8jb^2N^2k + 4b^2N^2 - 36b^2N^2k - 12b^2jN^2 \\ + 8b^2j^2N^2 - 12bN^2 + 18bkN + 6bjN - 12bN + 3 \\ = 12N^4b^2 - 12N^3b^2(3k+j-2) + 4N^2b^2[8k^2+2jk+1-9k-3j+2j^2] - 12N^2b + 6Nb(3k+j-2) + 3 \\ = [12N^4b^2 - 12N^2b + 3] + (3k+j-2)[6Nb - 12N^3b^2] + 12N^2b^2(3k+j-2)$$

$$\begin{aligned}
& +4N^2b^2[8k^2 + 2jk + 1 - 9k - 3j + 2j^2] - 12N^2b^2(3k + j - 2) \\
& = 3(2bN^2 - 1)^2 + 6bN(3k + j - 2) [1 + 2bN - 2bN^2] + 4N^2b^2[(k + j - 3)^2 + 7k(k - 2) + 2(k - 1) + j^2], \\
& \text{is } > 0, \text{ as, } 1 + 2bN - 2bN^2 > 0 \text{ (as, } b < \frac{1}{2N(N-1)} \text{) and } k \geq 2. \\
& \text{Hence, } \frac{\partial \mathcal{L}}{\partial p_{kj}} < 0, \text{ for all } k \neq j, \text{ and for all } k > 1, \text{ when, } (2bN^2 - 1) < 0, \text{ i.e., when, } b < \frac{1}{2N^2}. \blacksquare
\end{aligned}$$

Proof of Corollary 1. Note that, in the initial problem, we have dropped the constraints that $\sum_{j=1}^{N-1} (p_{k+1j} - p_{kj}) (y_j - y_N) > 0$, for all $k = 1, \dots, N - 1$, be satisfied and found the optimal solution of this modified constrained maximization problem with a larger “feasible set”. We now add the constraints that $\sum_{j=1}^{N-1} (p_{k+1j} - p_{kj}) (y_j - y_N) > 0$, for all $k = 1, \dots, N - 1$ to the above problem. However, notice that the above optimal solution, namely, the CS N -partition equilibrium, does satisfy these constraints and hence will be the solution of the desired maximisation problem. \blacksquare

Proof of Lemma 2. $f(x_1, \dots, x_{N-1}; p_{12}, p_{13}, \dots, p_{1N}; p_{21}, p_{23}, \dots, p_{2N}; \dots; p_{N1}, \dots, p_{NN-1})$

$$\begin{aligned}
& = (1 - \sum_{j \neq 1} p_{1j}) [(y_1)^3 - (y_1 - x_1)^3] + \sum_{j \neq 1} p_{1j} [(y_j)^3 - (y_j - x_1)^3] \\
& + (1 - \sum_{j \neq 2} p_{2j}) [(y_2 - x_1)^3 - (y_2 - x_2)^3] + \sum_{j \neq 2} p_{2j} [(y_j - x_1)^3 - (y_j - x_2)^3] \\
& + (1 - \sum_{j \neq 3} p_{3j}) [(y_3 - x_2)^3 - (y_3 - x_3)^3] + \sum_{j \neq 3} p_{3j} [(y_j - x_2)^3 - (y_j - x_3)^3] \\
& + \dots + (1 - \sum_{j \neq k} p_{kj}) [(y_k - x_{k-1})^3 - (y_k - x_k)^3] + \sum_{j \neq k} p_{kj} [(y_j - x_{k-1})^3 - (y_j - x_k)^3] \\
& + \dots + (1 - \sum_{j \neq N} p_{Nj}) [(y_N - x_{N-1})^3 - (y_N - 1)^3] + \sum_{j \neq N} p_{Nj} [(y_j - x_{N-1})^3 - (y_j - 1)^3].
\end{aligned}$$

Note that $\frac{\partial y_i}{\partial p_{kj}} = 0$, for all $i \neq k, j$. Thus,

$$\begin{aligned}
\frac{\partial f}{\partial p_{kj}} & = p_{1k} [3(y_k)^2 - 3(y_k - x_1)^2] \frac{\partial y_k}{\partial p_{kj}} + p_{1j} [3(y_j)^2 - 3(y_j - x_1)^2] \frac{\partial y_j}{\partial p_{kj}} \\
& + p_{2k} [3(y_k - x_1)^2 - 3(y_k - x_2)^2] \frac{\partial y_k}{\partial p_{kj}} + p_{2j} [3(y_j - x_1)^2 - 3(y_j - x_2)^2] \frac{\partial y_j}{\partial p_{kj}} \\
& + \dots + (- [(y_k - x_{k-1})^3 - (y_k - x_k)^3]) + (1 - \sum_{j \neq k} p_{kj}) [3(y_k - x_{k-1})^2 - 3(y_k - x_k)^2] \frac{\partial y_k}{\partial p_{kj}} \\
& + [(y_j - x_{k-1})^3 - (y_j - x_k)^3] + p_{kj} [3(y_j - x_{k-1})^2 - 3(y_j - x_k)^2] \frac{\partial y_j}{\partial p_{kj}} \\
& + \dots + p_{Nk} [3(y_k - x_{N-1})^2 - 3(y_k - 1)^2] \frac{\partial y_k}{\partial p_{kj}} + p_{Nj} [3(y_j - x_{N-1})^2 - 3(y_j - 1)^2] \frac{\partial y_j}{\partial p_{kj}}.
\end{aligned}$$

Using the incentive compatibility condition for R (equation (1) in the paper), we have,

$$\frac{\partial f}{\partial p_{kj}} = - [(y_k - x_{k-1})^3 - (y_k - x_k)^3] + [(y_j - x_{k-1})^3 - (y_j - x_k)^3].$$

For $p_{kj} = 0$, for all $k \neq j$, $y_k = \frac{x_k + x_{k-1}}{2}$, $y_j = \frac{x_j + x_{j-1}}{2}$ and

$$\begin{aligned}
\frac{\partial f}{\partial p_{kj}} & = - \left[\left(\frac{x_k - x_{k-1}}{2} \right)^3 - \left(- \left(\frac{x_k - x_{k-1}}{2} \right) \right)^3 \right] + \left[\left(\frac{x_j + x_{j-1} - 2x_{k-1}}{2} \right)^3 - \left(\frac{x_j + x_{j-1} - 2x_k}{2} \right)^3 \right] \\
& = \frac{3}{4} (x_k - x_{k-1}) (x_k - x_j + x_{k-1} - x_{j-1})^2 > 0. \blacksquare
\end{aligned}$$

Proof of Lemma 3. We consider the first order conditions for minimisation of f with respect to p_{kj} , $k \neq j$, for any (x_1, \dots, x_{N-1}) .

For all $k, j \in \{1, \dots, N\}$ such that $k \neq j$, we have,

$$\begin{aligned} \frac{\partial f}{\partial p_{kj}} &= -[(y_k - x_{k-1})^3 - (y_k - x_k)^3] + [(y_j - x_{k-1})^3 - (y_j - x_k)^3] \\ &= 3(y_k - y_j)(x_k - x_{k-1})(x_k - y_k - y_j + x_{k-1}). \end{aligned}$$

Thus, $\frac{\partial f}{\partial p_{kj}} = 0 \implies x_k - y_k - y_j + x_{k-1} = 0$, or, $y_k + y_j = x_k + x_{k-1}$. This means that if $\frac{\partial f}{\partial p_{kj}} = 0$, then $\frac{\partial f}{\partial p_{kl}} \neq 0$, for all $l \neq j$, because, $y_l \neq y_j$. Also, if $\frac{\partial f}{\partial p_{kj}} = 0$, then $\frac{\partial f}{\partial p_{jk}} \neq 0$. This proves that, for a fixed (x_1, \dots, x_{N-1}) , there cannot be a minimum of f at a strictly interior point (i.e., $0 < p_{kj} < 1$, for all $k, j \in \{1, \dots, N\}$ such that $k \neq j$).

We now check that, for a fixed (x_1, \dots, x_{N-1}) , there is no other minimum of f at boundary points (i.e., where some of the p_{kj} 's are equal to 0 or 1) which is strictly lower than the value of f at $p_{kj} = 0$, for all $k \neq j$.

Case 1. Suppose that $p_{il} = 0$, for all $(i, l) \neq (k, j)$ and $0 < p_{kj} < 1$. Then, $y_k = \frac{x_k + x_{k-1}}{2}$ and $y_j = \frac{1}{2} \left[\frac{(x_j^2 - x_{j-1}^2) + p_{kj}(x_k^2 - x_{k-1}^2)}{(x_j - x_{j-1}) + p_{kj}(x_k - x_{k-1})} \right]$, in which case $\frac{\partial f}{\partial p_{kj}} = 0 \implies y_k + y_j = x_k + x_{k-1} \implies x_j + x_{j-1} = x_k + x_{k-1}$, which is not possible.

So, if $p_{il} = 0$, for all $(i, l) \neq (k, j)$, then $\frac{\partial f}{\partial p_{kj}} \neq 0$. In fact, by continuity of f , $\frac{\partial f}{\partial p_{kj}}$ must be > 0 (since, from Lemma 2, we have $\frac{\partial f}{\partial p_{kl}} > 0$, for $p_{kl} = 0$, for all $k \neq l$). This shows that $0 < p_{kj} \leq 1$ and $p_{il} = 0$, for all $(i, l) \neq (k, j)$ cannot be a minimum of f .

Case 2. Suppose that $0 < p_{kj} < 1$, for some $k \neq j$, $k \in \{k_1, k_2, \dots, k_l\}$, $l \geq 2$, where $\{k_1, k_2, \dots, k_l\} \subseteq \{1, 2, \dots, N\}$ and $p_{kj} = 0$, $k \neq j$, $k \notin \{k_1, k_2, \dots, k_l\}$ satisfy the first order conditions for minimisation of f . Note from above that if $0 < p_{kj} < 1$, for some $k \neq j$, then $\frac{\partial f}{\partial p_{kj}} = 0$ and $\frac{\partial f}{\partial p_{kl}} \neq 0$, for all $l \neq j, k$ implying that $p_{kl} = 0$, for all $l \neq j, k$.

Since $\frac{\partial f}{\partial p_{kj}} = -[(y_k - x_{k-1})^3 - (y_k - x_k)^3] + [(y_j - x_{k-1})^3 - (y_j - x_k)^3] = 0$, if $0 < p_{kj} < 1$, it is easy to check that the value of f (evaluated at the above p_{kj} 's) is the same as the value of f evaluated at $p_{kj} = 0$, for all $k \neq j$.

Case 3. Suppose now that $p_{kj} = 1$, for some $k \neq j$, $k \in \{k_1, k_2, \dots, k_l\}$ where $\{k_1, k_2, \dots, k_l\} \subset \{1, 2, \dots, N\}$ and $p_{kj} = 0$, $k \neq j$, $k \notin \{k_1, k_2, \dots, k_l\}$. We now check if such a configuration is compatible with a global minimum of f . Without loss of generality, let $p_{k_1 j_1} = 1$ and $p_{j_1 l} = 0$, for all $l \neq j_1 \in \{1, \dots, N\}$. Let us denote by Δ the following components in f , given by,

$$\begin{aligned} \Delta &= (1 - \sum_{l \neq k_1} p_{k_1 l}) [(y_{k_1} - x_{k_1-1})^3 - (y_{k_1} - x_{k_1})^3] + \sum_{l \neq k_1} p_{k_1 l} [(y_l - x_{k_1-1})^3 - (y_l - x_{k_1})^3] \\ &+ (1 - \sum_{l \neq j_1} p_{j_1 l}) [(y_{j_1} - x_{j_1-1})^3 - (y_{j_1} - x_{j_1})^3] + \sum_{l \neq j_1} p_{j_1 l} [(y_l - x_{j_1-1})^3 - (y_l - x_{j_1})^3]. \end{aligned}$$

Note that, in this particular case, Δ becomes

$$[(y_{j_1} - x_{k_1-1})^3 - (y_{j_1} - x_{k_1})^3] + [(y_{j_1} - x_{j_1-1})^3 - (y_{j_1} - x_{j_1})^3],$$

which will always be greater than the value of Δ evaluated at $p_{kj} = 0$, for all $k \neq j$, given by

$$[(y_{k_1} - x_{k_1-1})^3 - (y_{k_1} - x_{k_1})^3] + [(y_{j_1} - x_{j_1-1})^3 - (y_{j_1} - x_{j_1})^3].$$

Case 4. Suppose now that $p_{kj} = 1$, for some $k \neq j$ and $k \in \{k_1, k_2, \dots, k_l\}$ and that $0 < p_{kj} < 1$, for some $k \neq j$, $k \in \{k_{l+1}, k_{l+2}, \dots, k_M\}$, where $\{k_1, k_2, \dots, k_l, k_{l+1}, k_{l+2}, \dots, k_M\} \subset \{1, 2, \dots, N\}$ and $p_{kj} = 0$, for all $k \neq j$, $k \notin \{k_1, k_2, \dots, k_l, k_{l+1}, k_{l+2}, \dots, k_M\}$. Combining the arguments in Cases 2 and 3 above, it can be shown that this configuration cannot correspond to a global minimum of f .

Case 5. Finally, suppose for every k , $p_{kj} = 1$, for some $k \neq j$ and also, for every k , if $p_{kj} = 1$ then $p_{lj} = 0$, for all $l \neq k$. In this case, it is easy to check that the value of f is equal to the value of f when $p_{kj} = 0$, for all $k \neq j$. This is because such a configuration is equivalent to assigning a permutation of $\{y_1, \dots, y_N\}$ to each element of the partition. ■

Proof of Lemma 4. $f(x_1, \dots, x_{N-1}; p_{12}, p_{13}, \dots, p_{1N}; p_{21}, p_{23}, \dots, p_{2N}; \dots; p_{N1}, \dots, p_{NN-1})$ evaluated at $p_{kj} = 0$, for all $k \neq j$, becomes,

$$\begin{aligned} & [(y_1)^3 - (y_1 - x_1)^3] + [(y_2 - x_1)^3 - (y_2 - x_2)^3] \\ & + \dots + [(y_i - x_{i-1})^3 - (y_i - x_i)^3] \\ & + \dots + [(y_N - x_{N-1})^3 - (y_N - 1)^3] \\ & = \frac{x_1^3}{4} + \frac{(x_2 - x_1)^3}{4} + \frac{(x_3 - x_2)^3}{4} + \dots + \frac{(1 - x_{N-1})^3}{4}. \end{aligned}$$

Let $A = [a_{ij}]_{i,j=1 \dots N-1}$ denote the Hessian matrix for f , as a function of $\{x_k\}_{k=1}^{N-1}$, at $p_{kj} = 0$, for all $k \neq j$.

$$A = \frac{3}{2} \begin{bmatrix} x_2 & -(x_2 - x_1) & 0 & 0 & \cdot & \cdot & \cdot & 0 \\ -(x_2 - x_1) & (x_3 - x_1) & -(x_3 - x_2) & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & -(x_3 - x_2) & (x_4 - x_2) & -(x_4 - x_3) & 0 & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & 0 & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & 0 & 0 & \cdot & \cdot \\ 0 & \cdot & \cdot & \cdot & \cdot & 0 & \cdot & (x_N - x_{N-2}) \end{bmatrix}.$$

$(N-1) \times (N-1)$

Let A_1, A_2, \dots, A_{N-1} denote the principal minors of A .

Clearly, $|A_1| = x_2 > 0$. Also, $|A_2| = x_2(x_3 - x_1) - (x_2 - x_1)^2 > 0$ and $|A_3| = (x_4 - x_2)|A_2| - x_2(x_3 - x_2)^2 > 0$, because $|A_2| - (x_3 - x_2)|A_1| > 0$.

One can check that for $k \geq 3$,

$$|A_k| = a_{kk}|A_{k-1}| - a_{kk-1}a_{k-1k}|A_{k-2}| = (x_{k+1} - x_{k-1})|A_{k-1}| - (x_k - x_{k-1})^2|A_{k-2}|.$$

Note that $|A_k| > 0$, if $|A_{k-1}| - (x_k - x_{k-1})|A_{k-2}| > 0$.

We now prove by induction that $|A_{k-1}| - (x_k - x_{k-1})|A_{k-2}| > 0$, for $k \geq 3$.

We know that $|A_2| - (x_3 - x_2)|A_1| > 0$. Suppose that $|A_{k-1}| - (x_k - x_{k-1})|A_{k-2}| > 0$, for $k \geq 3$.

$$\begin{aligned} \text{Then, } |A_k| - (x_{k+1} - x_k)|A_{k-1}| &= (x_k - x_{k-1})|A_{k-1}| - (x_k - x_{k-1})^2|A_{k-2}| \\ &= (x_k - x_{k-1})[|A_{k-1}| - (x_k - x_{k-1})|A_{k-2}|] > 0 \text{ (by induction).} \end{aligned}$$

Therefore, $|A_k| > 0$, for all k . Hence, A is positive definite which implies that f , as a function of $\{x_k\}_{k=1}^{N-1}$, at $p_{kj} = 0$, for all $k \neq j$, is strictly convex. ■

Proof of Theorem 1. We need to prove that, for $b < \frac{1}{2N^2}$, the N -partition CS equilibrium is actually a global maximum among the set of N -simple mediated equilibria.

Lemma 3 shows that a global minimum of f exists at $p_{kj} = 0$, for all $k \neq j$. Also, by Lemma 4, f , as a function of $\{x_k\}_{k=1}^{N-1}$, is strictly convex, at $p_{kj} = 0$, for all $k \neq j$, and hence, has a unique global minimum in $\{x_k\}_{k=1}^{N-1}$.

In the final problem, EU^R can be viewed as a function that is equal to $-\frac{1}{3}f$, with a further restriction on the domain of the variables given by the incentive compatibility constraint for S .

From Corollary 1, the N -partition CS equilibrium, given by $x_k = \frac{k}{N} + 2bk(k - N)$ for all $k \in \{1, \dots, N\}$; $y_j = \frac{x_{j-1} + x_j}{2}$ for all $j \in \{1, \dots, N\}$ and $p_{kj} = 0$ for all $k, j \in \{1, \dots, N\}$, $k \neq j$, is a local maximum of EU^R , and hence a local minimum of f , for $b < \frac{1}{2N^2}$.

Thus, for $b < \frac{1}{2N^2}$, the values of $\{x_k\}_{k=1}^{N-1}$ in the N -partition CS equilibrium, given by, $x_k = \frac{k}{N} + 2bk(k - N)$ for all $k \in \{1, \dots, N\}$ is the unique global minimum of f in $\{x_k\}_{k=1}^{N-1}$, at $p_{kj} = 0$, for all $k \neq j$, and hence, for $b < \frac{1}{2N^2}$, the variables $x_k = \frac{k}{N} + 2bk(k - N)$ for all $k \in \{1, \dots, N\}$ and $p_{kj} = 0$ for all $k, j \in \{1, \dots, N\}$, $k \neq j$ actually provide a global minimum of f .

Hence, in the final problem, among the set of N -simple mediated equilibria, the N -partition CS equilibrium must attain the global maximum of EU^R , for $b < \frac{1}{2N^2}$. ■

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