Wheat or Strawberries? Intermediated Trade with Limited Contracting.

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Why do developing countries fail to specialize in products in which they appear to have a comparative advantage? We propose a model of agricultural trade with intermediation that explains how hold-up resulting from poor contracting environments can produce such an outcome. We use the model to explore the role of production subsidies, support prices, easing sanitary and phytosanitary (SPS) requirements, and the creation of local markets in resolving the hold-up problem. The model highlights the importance of infrastructure in aligning production outcomes with comparative advantage and sheds light on the pass-through of the world price to the producer.

There is a large literature which documents that labor productivity in developing countries is significantly lower than in developed ones, and that this is more so in agriculture than in manufacturing. For example, Caselli (2005) shows that aggregate productivity for countries at the 90th percentile of income relative to the countries at the 10th percentile is 22, while this ratio for agriculture is 45.¹ Lagakos and Waugh (2013) show that for staples such as maize, rice, and wheat the ratio is 146, 90, and 83 respectively, twice as high as for agriculture overall. Yet farmers in developing countries persist in producing staples like wheat, corn or maize, rather than exotic fruits and vegetables that are highly valued in urban areas or export markets.

We explain why this could occur using a model of agricultural trade with intermediation and a lack of contract enforcement. In this setting comparative advantage alone is far from sufficient for the efficient pattern of specialization to take hold. We show how challenges of exporting agricultural produce exacerbate intermediation issues in poor contracting environments. Difficulties with transport, storage, and meeting sanitary and phytosanitary (SPS) requirements associated with perishable produce all hamper the ability of developing countries to capitalize on their productivity advantages in the export market.

Using the model with intermediated trade we identify conditions which ensure that agricultural producers make production decisions according to comparative

¹ Such differences in agricultural productivity could arise from differences in efficiencies conditional on the set of products made and/or composition effects.

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advantage. We show that local markets for perishable agricultural products, price guarantees, production subsidies as well as infrastructure improvements that reduce transport costs help align comparative advantage and production.

The environment in a developing country is very different from that in a developed one. A number of factors limit the farmer’s ability to transport his produce to an urban or export location by himself: roads are poor, trucks are expensive, and credit markets are poorly developed. Hence a farmer must rely on intermediaries (traders) to access these markets. At the same time, traders are scarce, irregular in their arrivals, and unreliable in their promises as contracts are poorly enforced. Though improvements in the contracting environment can alleviate the challenges of farm-gate trade in a developing country, the required judicial and political reforms to do this are difficult and time consuming to implement. We therefore take as given the problematic contracting environment in less developed countries, and build a simple model that captures essential features of such an environment.

In our setup, farmers have the technology to produce a staple, which we call wheat, and an exotic perishable produce, which we call strawberries. These goods differ along four dimensions: the farmer’s ability to consume them, the farmer’s efficiency in producing them, the kind of market in which they are traded, and the degree of perishability. The first good, wheat, is a storable staple that a farmer can subsist on and which is sold in a competitive market. The second, strawberries, is a perishable non-staple which is traded only through intermediaries. Farmers can survive on their wheat if need arises, but not on strawberries. Not only are strawberries nutritionally inadequate, but they are perishable, and have to be sold quickly. This gives intermediaries bargaining power when markets are thin, and makes farmers reluctant to grow strawberries, which makes intermediaries reluctant to enter, resulting in the expected thin markets materializing.

Traders, unlike farmers, have access to a competitive market for strawberries. Entry into intermediation is free and traders incur a sunk cost of entry. Farmers and traders cannot contract on price ex-ante and traders arrive at the farm-gate according to a random process. The trader who offers the highest price to the farmer gets the strawberries. If there is no trader at the farmer’s doorstep, he exercises his outside option. In the baseline version of the model the farmers’ outside option is given exogenously. It can be positive if a farmer can sell his produce to say, a canning factory, or it could be zero if no such option is available. We then extend the model to endogenize the farmer’s reservation price through the introduction of a competitive local market for strawberries.

We first solve the baseline model with an exogenous outside option and characterize all the possible equilibria as a function of the primitive parameters. Of particular interest is the region in parameter space with multiple equilibria where the hold-up problem occurs. Hold-up occurs because traders cannot credibly

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2 Outcomes are unaffected by whether we assume the trader knows how many competitors he faces, or not, as will be explained later on.
promise a worthwhile price to farmers if they make strawberries. Ex-post, a trader who is the sole buyer has no incentive to pay the farmer more than his outside option. Anticipating this, the farmer chooses not to produce strawberries unless he believes that enough traders will enter to ensure competition. However, were more farmers to produce strawberries, more traders would enter and this would make the production of strawberries profitable. So depending on farmers and traders beliefs about one another's actions, two equilibria could occur. In the “good” equilibrium farmers specialize in strawberries, which they have a comparative advantage in, and there is intermediation. In the “bad” equilibrium, there is no intermediation and the staple is produced.

The analysis of the model suggests that even if the government is not able to resolve the core issue, namely the underlying lack of enforceable contracts, there are policies that ensure specialization according to comparative advantage. An export board that provides farmers with an outside option, or a production subsidy for strawberries, eliminates the “bad” equilibrium, with the production subsidy being a more efficient policy than the export board. Agricultural extension programs and easing of SPS restrictions can also eliminate the “bad” equilibria.

We then extend the baseline model to include competitive local markets, where farmers can sell strawberries but which require traders and farmers pay an access cost. Such local markets, provide an endogenous outside option to farmers. The magnitude of this outside option depends on the total cost of accessing these local markets for farmers and traders and the export price of strawberries. Therefore, a sufficiently low cost of accessing the local market provides an outside option high enough to eliminate the coordination failure. Similarly, agricultural extension programs (which raise productivity) and easing of SPS requirements (which effectively increase the export price) raise the farmer's pay-off in the local market and can also eliminate the “bad” equilibria.

Our model suggests that farmers near cities and trading centers, where they have easy access to local markets, are less likely to be subject to hold-up and are more likely to produce in line with comparative advantage. In remote locations, on the other hand, farm-gate trade is likely to be prevalent and farmers fearing the power of intermediaries may choose not to produce strawberries.

The extent to which farmers share in the gains from higher export prices also depends on their access to a local market. With local markets, the level of intermediation is independent of export price fluctuations, and the farm-gate price adjusts one for one in response to world price changes. In contrast, without local markets, pass-through is incomplete and depends on the level of intermediation at the farm-gate: the thinner the intermediary market, the lower is the extent of pass-through. Thus farmers, especially those in more remote locations and thin intermediary markets, may benefit little from higher world prices.

In the rest of the introduction we discuss the modeling assumptions we make, provide anecdotal evidence in support of the model and relate it to the wider literature.
A. Motivating the Modeling Assumptions

While agricultural trade is a complex phenomenon, the model we present in subsequent sections focuses on some of its essential aspects in order to retain tractability. In this section we turn to the existing empirical literature and case studies to motivate our modeling choices and discuss historical examples of the hold-up problem that our paper focuses on.

Fafchamps, Gabre-Madhin and Minten (2005) describe the environment in which agricultural farmers and traders interact in Africa. They document that there are a large number of small intermediaries who specialize in buying from producers and selling to wholesale traders or exporters. In our model we focus on these small itinerant traders.

Fafchamps and Hill (2005) document that farmers in Africa face a decision whether to sell their produce at the farm-gate, or to travel to the nearest organized market. Farmers are less likely to travel to the market and more likely to sell to an itinerant trader when the nearest market is far, or the cost of transportation is high. Similarly, Osborne (2005) finds that in poorer and more remote areas, traders have more market power than in markets that are located close to big trading centers. Consistent with their findings, in our model, farmers in remote areas tend to sell at the farm-gate at lower prices and farmers close to ports with access to markets sell at higher prices.

A number of historical examples of the hold-up issue that we focus on in the paper have been documented. One such account can be found in Kranton and Swamy (2008). They argue that the Opium Agency, initiated by the East India Company (EIC) in India, had a similar problem and recognized it. As the agency was the sole procurer of opium it had monopsony power so that its agents had incentives to behave opportunistically towards farmers. Such behavior would have resulted in farmers switching to other crops. In order to prevent this the Opium Agency expended significant resources monitoring their own agents.

The role that cooperatives played in establishing the dairy industry in India, e.g., the Amul cooperative, is another historical account that supports our model. In India prior to “Operation Flood,” milk was hard to come by in urban areas. Farmers were reluctant to produce milk because of the risk of spoilage and the lack of distribution channels. Dairy cooperatives that took hold in India during “Operation Flood” encouraged production by giving farmers a “fair” price for their milk. The success of the cooperatives was a key part of the “white revolution” when India went from being a milk deficient nation in the 1970s to being the world’s largest milk producer in 2011.\(^3\)

An important feature of our work is that we do not allow for the possibility of repeated interactions between farmers and traders. This is justified by the extent of uncertainty that prevails in developing countries. Weather variability, political uncertainty and disease, all of which make people focus on the short term so

\(^3\)See Delgado, Narrod and Tiongco (2003) for more on how the white revolution occurred in India.
that the future is highly discounted. Such considerations call for the use of static
models that exclude repeated interactions and relational contracting.

Long term relations and reputational concerns have received a lot of atten-
tion in recent years and have been shown to play an important role in facili-
tating contracts. Banerjee and Duflo (2000) focus on the role played by repeated in-
teractions in the software industry in India. Macchiavello and Morjaria (forthc-
oming) focus on the role of repeated interactions in the context of rose exports from
Kenya, while McMillan and Woodruff (1999) look at credit relations among firms
in Vietnam. Antrs and Foley (2015) show that prepayment for orders is more
common when relational capital is low, i.e., at the start of a relationship. Greif
(2005) uses historical examples to study how contractual problems were resolved
among Magrabi traders. All these are established markets. Here, as in Kranton
and Swamy (2008), we assume that relational contracts are not possible. Our
focus is on why certain markets do not come into being when there is no repeated
interaction, rather than on the operation of established markets with repeated
interactions.

B. Relation to Existing Work

Our work fits into recent literature dealing with the effects of productivity and
trade costs on patterns of specialization, see Costinot and Donaldson (2012),
Costinot, Donaldson and Smith (2016), Sotelo (2015). However, all of this work
abstracts from intermediation and the related frictions that arise. In contrast, we
focus on precisely such frictions and explore their interaction with trade costs in
defining (often suboptimal) patterns of specialization.

Our work is related to Antras and Costinot (2011) who introduce intermediation
into a two-good two-country Ricardian framework to analyze the implications of
globalization. They focus on the welfare effects of integration in the goods market
versus intermediary markets, and, in contrast to us, assume that contracts are
fully enforced and hold-up is not possible.

Our model highlights how intermediation in the absence of contracts can lead
to a hold-up problem and multiple equilibria and looks for appropriate policies
to resolve it. Other papers that have emphasized multiplicity of equilibria and
explored how policy intervention can be welfare improving include the big push
and unbalanced growth theories as in Rosenstein-Rodan (1943), Nurkse (1961),
Hirschman (1988), and Murphy, Shleifer and Vishny (1989). This line of work
has relied on demand externalities to generate multiple equilibria. In contrast, we
focus on the contracting imperfections which result in the coordination failures
that make farmers choose to produce low priced staples, despite seemingly more
lucrative options being available.

Hausmann and Rodrik (2003) and Hausmann, Hwang and Rodrik (2007) por-
tray development as a process of self-discovery. Our model does not rely on
the lack of self-knowledge to explain the shortage of investment in nontraditional
products. With contractual frictions farmers know about their options but choose
not to avail of them because nontraditional product markets are thin and hold-up is likely.

Our paper is also related to a literature that focuses on price transmission in agricultural trade. Fafchamps and Hill (2008) analyze transmission of the export coffee price to the farmer who sells at the farm-gate. They find evidence of incomplete pass-through from the international price to farm-gate prices in their data. They argue that the cause of this incomplete pass-through is the lack of information about world price movements on the part of the farmer.4 Our model provides an alternative explanation for incomplete pass-through that does not rely on information frictions. In our set up, it is farmers in remote locations without access to competitive local markets who suffer from the incomplete pass-through of the export prices. When farmers have the option to sell at a local market where price adjusts one-to-one with the export price, the pass-through of the world price into the farm-gate price is complete.

A good deal of attention has been paid recently to the role of infrastructure in facilitating market access and encouraging development. Casaburi, Glennerster and Suri (n.d.) provide evidence that rural roads reduce search costs and encourage intermediation. Using a regression discontinuity framework, they show that the price of the local crop, namely rice, falls with the construction of rural roads. This occurs due to competition from more efficient producers outside the region. Since in our model the intermediated good is not consumed locally, road improvements effectively reduce the cost of getting the export good to the market and increase producer price whether at the farm-gate or the local market.

We proceed as follows. Section 1 lays out the model and constructs the equilibrium. Section 2 explores its welfare properties and ranks policies that can eliminate the coordination failure. Section 3 incorporates a local market where farmers can sell their produce if no intermediary shows up at the farm-gate. Section 4 deals with the role of infrastructure, agricultural extension programs and SPS requirements in facilitating agricultural exports. It also discusses the pass-through of changes in world prices into domestic ones. Section 5 concludes.

I. The Model

The modelling framework builds on Burdett and Mortensen (1998) and Galeaninos and Kircher (2008). The economy consists of a continuum of farmers of measure one and a continuum of traders whose measure $\theta$ is determined endogenously in equilibrium. Each farmer is endowed with one unit of labour which produces a single unit of the staple or $\alpha$ units of the perishable good. Farmers can either consume the staple or sell it at a given price which is normalized to unity.

The perishable good is traded in the competitive world market to which farm-

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4Mitra et al. (2015) is another paper that looks at the role of information on prices using an experimental framework.
ers have no access without intermediaries. The trader buys the good from the farmer at price \( p \) and sells it to the exporter at the world price, \( P^w \), with the objective of maximizing his expected profit \( \pi^T(p) \). There are an infinite number of potential traders who can become actual traders by paying a sunk entry cost \( \kappa_F \). Each trader who pays the sunk cost meets a single farmer at random, and with probability \( P_k \) he meets \( k \) competitors at the farm-gate. The farmer sells his strawberries to the trader offering the highest price.

The farmer chooses how much of his one unit of labour to allocate to the production of the intermediated good \( l \in [0, 1] \), and how much to the production of the subsistence good \( 1 - l \) to maximize his profit, \( \pi^F \). He produces strawberries if he expects a sufficient number of intermediaries to enter, making producing them more profitable than producing wheat. The equilibrium that we will characterize consists of three objects: \( l, \theta, F(p) \), where \( F(p) \) is the distribution of prices that traders offer upon entry into intermediation. All agents choose their strategies simultaneously and take each other’s actions as given. The strategies are played and the outcomes are revealed. Since all farmers and traders are ex-ante identical and of measure zero, their individual actions do not affect the equilibrium outcome.

We assume that farmers and traders are risk neutral. This causes farmers to specialize in either the export or the staple good.\(^5\)

A. The Meeting Process

We assume that farmers and traders meet according to a Poisson process. This process arises naturally when \( T_F \) traders arbitrarily meet one out of \( N \) farmers producing for export, and is convenient in modeling coordination frictions that result when there are many small market participants.

\( P_k \), the probability that a trader who randomly arrives at a farm-gate meets \( k \) rivals, can be derived as follows. Let \( 1/N \) be the probability that a given trader visits a given farmer. Then, if \( T_F \) and \( N \) go to infinity, while their ratio is equal to \( \theta \), i.e.,

\[
\lim_{T_F \to \infty, N \to \infty} \frac{T_F}{N} = \theta,
\]

\[
P_k = \lim_{T_F \to \infty, N \to \infty} \binom{T_F - 1}{k} \left( \frac{1}{N} \right)^k \left( 1 - \frac{1}{N} \right)^{T_F - 1 - k} = \frac{\theta^k}{k!} e^{-\theta}.
\]

It can also be shown that as the number of traders and farmers goes to infinity, the probability that \( k \) traders arrive at a farmer’s gate (denoted by \( Q_k \)) is identical to \( P_k \), the probability that a trader at the farm-gate meets \( k \) rivals.

\(^5\)Adding risk aversion on the side of the farmers moves the economy away from the corner solution as might be expected.
B. The Trader’s Problem

The trader’s problem consists of two parts. For a given level of market intermediation, $\theta$, a potential trader needs to decide whether to enter the intermediation market or not. Upon entry he has to decide what price to offer to the farmer he visits. As usual, we need to solve this backwards. Consider the problem of optimally choosing the price to post given the distribution of prices of all other traders.

Let $p$ be the price that a trader offers to a farmer. The most important piece of information for the trader is the distribution of prices that other traders offer, $F(p)$. For an arbitrary $p$, the probability that a given trader makes the highest bid in a meeting with $k$ rivals is given by $[F(p)]^k$. The unconditional probability that a trader offering price $p$ is the highest bidder involves summing over the number of potential rivals and is given by

$$\sum_{k=0}^{\infty} P_k[F(p)]^k = \sum_{k=0}^{\infty} e^{-\theta} \frac{\theta^k}{k!} [F(p)]^k = e^{-\theta(1-F(p))}.$$ 

When $\alpha l^*$ is the equilibrium output of the intermediated good, the trader who wins offering price $p$ makes $(P^w - p)\alpha l^*$. Thus, the expected profit of a trader offering price $p$ is

$$\pi^T(p) = (P^w - p)\alpha l^* e^{-\theta(1-F(p))}.$$ 

Let $R$ be the farmer’s reservation price, i.e., the farmer’s outside option. It could be the price offered by a local canning factory, or it could be 0 if no such option is available. Let $p_{max}$ denote the upper bound of the support for the price distribution. Next we summarize the properties of the distribution of price offers in equilibrium.

PROPOSITION 1: In the unique equilibrium of the price game, traders mix over the interval $[R, p_{max}]$ according to

$$F(p) = \begin{cases} 
0 & \text{for } p < R \\
\frac{1}{\theta} \ln\left(\frac{P^w - R}{P^w - p}\right) & \text{for } R \leq p \leq p_{max}
\end{cases},$$

where $p_{max}$, the upper bound of the support, is given by

$$p_{max} = P^w(1-e^{-\theta}) + Re^{-\theta}.$$ 

The traders’ expected profits equal $(P^w - R)\alpha l^* e^{-\theta}$.

The proof is in the Appendix. Here we provide an intuitive explanation of the results. First, we pin down traders’ profits at the farmer’s reservation price $R$. 

$^6$If $p$ is a random draw from $F(p)$, then the distribution of the maximum of the $k$ random draws, i.e., $\{p_i\}_{i=1}^k$ is $[F(p)]^k$. 
Since farmers do not accept a price below $R$ no trader will offer such a price. When a trader offers $R$ a farmer accepts it only if no other trader shows up at his farm-gate. Therefore, at $R$ the trader’s profits are:

$$\pi^T(R) = (Pw - R) \alpha l^* e^{-\theta} = \pi^T.$$  

For $F(p)$ to be the equilibrium distribution of prices, traders’ profits at every $p$ in its support must be the same, so that traders are willing to randomize over them. In the appendix we show that $F(p)$ is a continuous distribution with no gaps in the support which starts at $R$. Hence at any $p$ traders’ profits must be equal to those at $R$:

$$\pi^T(p) = (Pw - p) \alpha l^* e^{-\theta(1-F(p))} = \pi^T.$$  

Following Burdett and Judd (1983) equation (2) yields $F(p)$ uniquely as in (1). Continuity of $F(.)$ and absence of mass points makes economic sense: a gap in the support would result in profits falling in price for such intervals, while a mass point would result in a jump up in profits at prices just below the mass point.

Note each trader could offer a particular price, with different traders choosing different prices so that $F(p)$ emerges, or each trader could be mixing over prices according to $F(p)$. How the distribution arises is irrelevant for the equilibrium outcomes.\(^7\)

The upper bound of the support, $p_{\text{max}}$, is a convex combination of the world price and the farmer’s reservation price.\(^8\) As the level of intermediation increases, $p_{\text{max}}$ increases, but as $e^{-\theta} < 1$ for any value of $\theta$, $p_{\text{max}}$ remains below $Pw$. The farmer’s reservation price, $R$ bounds the value of $p_{\text{max}}$ from below.  

Trader’s expected profits given in equation (2) are increasing in $Pw$, and decreasing in $R$ and $\theta$. Note that these are exactly the profits of a trader competing a la Bertrand who sets his price after observing the number of competitors he faces at the farm-gate. If he has no competitors at the farm-gate, he offers the farmer $R$ and makes profits of $\alpha l^* (Pw - R)$. This event occurs with probability $e^{-\theta}$. Otherwise, competition forces him to raise his price to $Pw$ and he makes zero profits.

Furthermore, whether traders observe the number of competitors before or after they make offers yields same equilibrium expected price, level of intermediation and the production decisions of farmers. For details see Krishna (2009). The advantage of the former assumption is that it gives rise to a continuous distribution of farm-gate prices with the upper end of the support strictly below the world price, which is consistent with documented price variations, e.g., see Fafchamps

\(^7\)All that matters to a trader is the probability that his price is the highest and not the particular strategies that other traders use.

\(^8\)To see this note that: $e^\theta = 1 + \theta + \frac{\theta^2}{2} + \frac{\theta^3}{3!} ... > 1$ for any $\theta > 0$ and thus, $0 < e^{-\theta} < 1$.  


and Hill (2008).

Now that we can evaluate the traders' expected profits prior to entry, we can consider their entry decision.

**PROPOSITION 2:** The free entry level of intermediation is

\[
\theta = \begin{cases} 
\ln \left( \frac{(P^w - R)\alpha l^*}{\kappa_F} \right) & \text{if } l^* \geq l_{\text{min}}, \\
0 & \text{if } l^* < l_{\text{min}} 
\end{cases}
\]

where

\[
l_{\text{min}} = \frac{\kappa_F}{\alpha(P^w - R)}.
\]

**PROOF:**

Entry of traders will continue until:

\[
\pi_T(p) = \kappa_F.
\]

Since profits are the same at every point in the support, we can solve for the level of intermediation by equating profits at the lower end of the support, \( R \), to the cost of entry, \( \kappa_F \):

\[
(P^w - R)\alpha l^* e^{-\theta} = \kappa_F.
\]

Solving equation (4) for \( \theta \) gives

\[
\theta = \ln \left( \frac{(P^w - R)\alpha l^*}{\kappa_F} \right).
\]

Thus, the equilibrium level of intermediation is increasing in the world price and the output of the export good. It is decreasing in the sunk cost and the farmer’s reservation price. Note that \( \theta > 0 \) if and only if

\[
\ln \left( \frac{(P^w - R)\alpha l^*}{\kappa_F} \right) > 0,
\]

or

\[
l^* > l_{\text{min}} = \frac{\kappa_F}{\alpha(P^w - R)}.
\]

\[
\square
\]

Proposition 2 implies that positive levels of intermediation prevail when the output of the export good, \( \alpha l^* \), exceeds a minimum level, denoted by \( \alpha l_{\text{min}} = \kappa_F/(P^w - R) \). In other words, if \( l^* > l_{\text{min}} \), intermediation occurs.

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9Under the alternative assumption a two point distribution of prices arises over R and the world price.
C. The Farmer’s Problem

We now describe the problem of a risk neutral farmer. Let \( G_k(p) = [F(p)]^k \) be the cumulative density function of the highest price offered when a farmer meets \( k \) traders. Each farmer has a linear utility function defined over the units of the staple good. A farmer who allocates \( l \) units of labor to the intermediated good and \( 1 - l \) units to the staple earns a profit

\[
\pi^F(l, p) = (\alpha p - 1) l + 1,
\]

when he sells the intermediated good at price \( p \). Since the farmer is risk neutral he maximizes expected profits, and since he only consumes the numeraire good, his indirect utility is the same as his income.

Let \( E(p) \) be the price farmers expect to obtain for the export good at the farm-gate. If \( \alpha E(p) - 1 > 0 \), the farmer will produce only the export good. When evaluating the expected price, farmers take the level of intermediation \( \theta \), the pricing strategy of traders \( F(\cdot) \), and the meeting process \( \{Q_k\}_{k=0}^\infty \) as given. Recall that \( Q_k \) was defined as the probability that \( k \) traders arrive at a farmer’s gate.

**Lemma 1:** The expected price at the farm-gate is given by

\[
E(p) = \sum_{k=0}^{\infty} Q_k \int_R p dG_k(p)
\]

\[
= P^w \left( 1 - e^{-\theta}(1 + \theta) \right) + R e^{-\theta}(1 + \theta).
\]

As \( 0 < e^{-\theta}(1 + \theta) < 1 \), the expected price is a convex combination of the world price \( P^w \) and the farmer’s reservation price \( R \).\(^{10}\)

The direct proof is in the Appendix. A useful intuition for the expression of the expected price comes from the observation outlined previously, that the expected price is the same whether traders make their bids before or after the number of competitors has been observed. If at most one intermediary shows up at the farm-gate, the farmer gets \( R \). This happens with probability \( e^{-\theta}(1 + \theta) \). If more than one intermediary shows up at the farm-gate the price is bid up to \( P^w \), which happens with probability \( 1 - e^{-\theta}(1 + \theta) \). Hence the expression in equation (5).

The expected producer price increases with the world price \( P^w \), and productivity of the export good \( \alpha \), and decreases with the cost of farm-gate intermediation \( \kappa_F \). Changes in \( \alpha \) and \( \kappa_F \) operate via \( \theta \): a higher \( \alpha \) and lower \( \kappa_F \) increase \( \theta \), raising the weight on \( P^w \). A higher \( P^w \) affects the expected price both directly, and through raising the level of intermediation.

\(^{10}\)To see this note that: \( e^\theta = 1 + \theta + \frac{\theta^2}{2!} + \frac{\theta^3}{3!} \ldots > 1 + \theta \) for any \( \theta > 0 \) so \( \frac{1}{1+\theta} < e^{-\theta} \) or \( 1 > e^{-\theta}(1 + \theta) \).
A higher value of $R$, on one hand, increases the farmer’s outside option and therefore the expected farm-gate price. On the other hand, it decreases the level of intermediation, which pushes the expected farm-gate price down. The direct effect of an increase in $R$ dominates and the expected price rises:

$$\frac{\partial E(p|\theta(l = 1))}{\partial R} = e^{-\theta} > 0.$$  

A risk neutral farmer allocates labor between production of the export good and the staple depending on which is more profitable. Hence, the farmers’ labor supply as a function of the expected price is given by

$$l(E(p)) = \begin{cases} 1 & \text{if } \alpha E(p) > 1 \\ [0, 1] & \text{if } \alpha E(p) = 1 \\ 0 & \text{if } \alpha E(p) < 1. \end{cases}$$

Note that if $\alpha R \geq 1$ so is $\alpha E(p)$, and farmers specialize in the export good even if no intermediaries enter, i.e., $\theta = 0$. When $\alpha R < 1$, whether or not farmers specialize in the export good depends on the prevailing level of intermediation. Define $\theta_{\text{min}}$ as the minimum level of intermediation necessary to induce farmers to produce the export good. As $\theta$ rises and $E(p)$ exceeds $1/\alpha$, farmers specialize in the production of the export good and $l(.) = 1$. We can write the farmers’ choice of labor as the best response function to the prevailing level of intermediation:

$$l(\theta) = \begin{cases} 1 & \text{when } \alpha R \geq 1 \\ 1 & \text{for } \theta > \theta_{\text{min}} \\ [0, 1] & \text{if } \theta = \theta_{\text{min}} \\ 0 & \text{for } \theta < \theta_{\text{min}} \end{cases} \text{ when } \alpha R < 1.$$  

Formally, $\theta_{\text{min}}$ is the solution to $\alpha E(p) = 1$. It is easy to see that $\theta_{\text{min}}$ does not depend on $\kappa_F$ and decreases as $\alpha$, $P^w$ or $R$ rise.

### D. Equilibrium

We first discuss the possible equilibrium configurations in terms of the traders’ and farmers’ best response functions. We then describe the relationship between the primitive parameters and the equilibrium configurations that they imply.

In equilibrium, each farmer chooses what to produce, and each active trader chooses what price to offer, so as to maximize their respective profits. All potential traders are indifferent between becoming active or not, and the decisions of these agents are mutually consistent.

The equilibrium consisting of three objects: $\theta, F(p), l(\theta)$, is pinned down by equations (3), and (1), and (7). Depending on the values of the primitive parameters there are four possible equilibrium configurations, all depicted in Figures 1a-1d. Figures 1a and 1b show the equilibrium outcomes when $R < 1/\alpha$ and
farmers specialize in the staple unless there is enough intermediation. In Figures 1c and 1d \( R > 1/\alpha \), so that farmers will produce the intermediated good regardless of the intermediation level.

In Figure 1a there is a unique equilibrium with complete specialization in the staple good. \( \theta_{\text{min}} > \theta(1) \) so that the best response functions \( l(\theta) \) and \( \theta(l) \) have only one intersection at the origin. Even if all farmers specialized in strawberries, intermediation would not be profitable. This occurs when \( P_w \) and \( \alpha \) are low (e.g., agriculture is inefficient) and/or \( \kappa_F \), the cost of entry for traders, is high.

Figure 1b depicts a case with multiple equilibria. The farmer’s best response function, \( l(\theta) \), and traders’ free entry condition, \( \theta(l) \), intersect three times resulting in three equilibria. There are two complete specialization equilibria where farmers produce either strawberries or the staple, and an equilibrium where farmers are indifferent between producing either one. Which of the equilibria occurs depends on entry of intermediaries. When intermediaries enter, \( \alpha E(p) > 1 \), all farmers produce strawberries. If intermediaries do not enter, then all farmers produce the staple. When \( \theta = \theta_{\text{min}} \) and \( \alpha E(p) = 1 \), farmers are indifferent between making either wheat or strawberries. This last equilibrium is unstable: small perturbations will move the economy to one of the two stable equilibria. For this reason in the rest of the paper we focus on the two stable equilibria.\(^{11}\)

When \( R > 1/\alpha \), it is profitable to make strawberries even if there are no intermediaries, so \( l \equiv 1 \). In this case, there are two possibilities. Either intermediaries enter in equilibrium or they do not. If intermediation is profitable when intermediaries have no competition, i.e., their profits of \( \alpha(P_w - R) \) exceed the intermediation cost \( \kappa_F \), then intermediation must occur in equilibrium. This case is depicted in Figure 1c. Figure 1d depicts the case where \( \alpha(P_w - R) < \kappa_F \) so that intermediation is inherently unprofitable and does not occur. This case occurs when \( R \) and/or \( \kappa_F \) are high or \( \alpha \) and/or \( P_w \) are low.

Next we outline the conditions on the primitive parameters that give rise to each of the equilibrium types. Figure 2 depicts the possible equilibrium outcomes for different values of the primitive parameters \( R \) and \( \kappa_F \) given \( \alpha \) and \( P_w \).

When \( R < 1/\alpha \) and \( \kappa_F \) is relatively low, whether farmers produce the export good or not depends on their beliefs about the prevailing level of intermediation. In this case, multiple equilibria in the sense of Figure 1b are endemic. In Figure 2 we call this region \( M \) for Multiplicity. When \( \kappa_F \) becomes so high that traders find it unprofitable to enter, none of the export good will be made. In Figure 2 this region is labeled \( W \) for Wheat production, which corresponds to the situation in

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\(^{11}\)While we limit the discussion of the unstable equilibrium, it is worth noting that this equilibrium may involve pure or mixed strategies on the part of the farmer. To see this, note that trader profits are linear in \( l \), so the free entry level of intermediation depends on the average output of the export good per farmer. Let \( \bar{l} \) be the average level of output of strawberries per farmer such that \( \theta(\bar{l}) = \theta_{\text{min}} \), shown in Figure 1b. It is easy to see that whether all farmers produce \( l = \bar{l} \), or this fraction of farmers make strawberries and the rest make wheat, the average output per farmer remains \( \bar{l} \). More generally, whatever is the output of the individual farmer, as long as the average output per farmer is \( \bar{l} \), the best-response level of intermediation \( \theta(\bar{l}) = \theta_{\text{min}} \) implies \( \alpha E(p) = 1 \), and farmers are indifferent between making either of the two goods.
Figure 1a. The boundary between region $M$ and $W$ is defined by the condition $E(p|\theta) = 1/\alpha$ as this is when farmers are indifferent between making the staple or the export good.\footnote{More precisely, the boundary between the regions is given by a locus of $R$ and $\kappa_F$ that satisfy the $P^{w}(1 - \frac{\kappa_F}{\alpha(\frac{P^{w}-R}{\kappa_F} + 1 + \ln \frac{\kappa_F}{\alpha(\frac{P^{w}-R}{\kappa_F})})} + R \left( \frac{\kappa_F}{\alpha(\frac{P^{w}-R}{\kappa_F})} \right)) = \frac{1}{\alpha}$ obtained by substituting $\theta(1) = \ln \left( \frac{\alpha(P^{w}-R)}{\kappa_F} \right)$ into the expression for $E(p|\theta)$ in equation (5).}

When $R$ is above $1/\alpha$, the farmer’s outside option for strawberries is high enough so that making strawberries becomes his dominant strategy. When $R > P^{w} - \kappa_F/\alpha$, intermediation is unprofitable and no traders enter, so strawberries are sold to the canning factory which pays $R$. In Figure 2 this corresponds to the region labeled $S - NF$ for Strawberries and No Farm-gate Intermediation. Only in the triangular area, labeled $S - F$ for Strawberries and Farm-gate Intermediation, is there a unique equilibrium where strawberries are produced and sold to intermediaries.

This case is depicted in Figure 1c. The two regions are separated by the boundary condition for intermediary entry: $R = P^{w} - \kappa_F/\alpha$.

II. Efficiency Properties of the Equilibrium and Policy Implications

We now examine the efficiency properties of the equilibrium. Our main result is that efficiency obtains when primitive parameters rule out multiplicity of equilibria. When multiple equilibria are endemic, only the equilibrium where farmers specialize in the export good is constrained efficient. In other words, the source of the inefficiency lies in the coordination failure.

In this section we will solve the social planner’s problem and describe the constrained efficient allocation in detail.

A. Constrained Efficient Allocation

A social planner chooses the level of intermediation and the labor that farmers allocate to the production of the export good to maximize the aggregate output subject to the constraint that traders and farmers meet randomly. This is a standard notion of constrained efficiency in models with search frictions, see for example Shi (2001) or Shimer (2005).

The planner maximizes welfare, which is the value of output net of the intermediation costs, by choosing the socially optimal level of intermediation $\theta^{P}$ and labor allocation $l^{P}$:

$$\max_{l^{P},\theta^{P}} \left[ P^{w}(1 - e^{-\theta^{P}}) + Re^{-\theta^{P}} \right] \alpha l^{P} - \kappa_F \theta^{P} I[l^{P} > 0] + (1 - l^{P})$$

where $I[l^{P} > 0]$ is the indicator function taking the value 1 when $l^{P} > 0$ and 0.
otherwise. The first order condition
\[
\alpha l^P e^{-\theta^P} (P^w - R) - \kappa_F = 0,
\]
implies that for any given level of \(l^P\) the socially optimal level of intermediation is
\[
\theta^P(l^P) = \max \left\{ 0, \ln \left( \frac{\alpha l^P(P^w - R)}{\kappa_F} \right) \right\}.
\]
Notice that this expression coincides with the trader’s best response function for a given level of $l$, indicating that the level of intermediation is not distorted.

Given $\theta^P$, the maximized welfare in equation (8) is linear in $l^P$ so the social planner will set $l^P$ equal to either 0 or 1. He will assign all farmers to produce the export good and set $l = 1$ when

$$P^w(1 - e^{-\theta^P(1)}) + Re^{-\theta^P(1)} \alpha - \kappa_F \theta^P(1) \geq 1.$$

The left-hand side of the equation (10) corresponds to the level of welfare when $l^P = 1$, and the right-hand side corresponds to the level of welfare that arises when only the staple is produced, $l^P = 0$. At first sight this condition looks different from the farmers’ choice of labor, namely choose $l = 1$ when

$$\alpha E(p) \geq 1.$$

However, using equation (9), the social planner’s rule for labor allocation can be
shown to be equivalent to that of the farmers’ in equation (11).\textsuperscript{13}

So, the decentralized equilibrium is potentially inefficient only in the region with multiple equilibria, i.e., region $M$ in Figure 2. Here the decentralized allocation could result in wheat being made, while the social planner would prescribe specialization in strawberries. The reason for the potential inefficiency is the coordination failure. In this section we will explore policies that can resolve this coordination failure and ensure that the decentralized allocation coincides with the constrained efficient one.

It’s worth pointing out that even the constrained efficient allocation suffers from a search inefficiency typically present in search and matching models: a farmer may have no traders matched with him or a trader may lose to a competitor. Such events involve waste relative to the operation of frictionless markets.

\subsection*{B. Eliminating the Coordination Failure}

We have shown that the only distortion in the competitive allocation is due to the coordination failure that generates the “bad” equilibrium in the region with multiplicity, i.e., region $M$ in Figure 2. The natural question to ask is how policy can eliminate this inefficient outcome. If we can ensure that it is the dominant strategy for the farmer to produce the export good, then the inefficiency is eliminated and the economy attains the constrained optimum. One way to do this is to offer a production subsidy. Another is to offer a price support as is often done by export boards. We show below both policies can eliminate multiplicity. The production subsidy however, dominates the price support in terms of welfare, as the latter distorts the level of intermediation.

\section*{A Production Subsidy}

Suppose the economy is in the region $M$ and the government offers a per unit subsidy slightly above $1/\alpha - R$, say $s = 1/\alpha - R + \epsilon$. Then farmers will specialize in the export good no matter what the level of intermediation is. Even with no traders, farmers’ expected income from making the export good exceeds that from making the staple: $\alpha(1/\alpha - R + \epsilon) > 1$. Thus, in Figure 1b, such a subsidy ensures $l(\theta) = 1$ independent of $\theta$. Knowing that farmers will produce the export good, traders will enter, and the “bad” equilibrium is eliminated.

\textsuperscript{13}Consider equation (11):

$$\alpha \left[ P^w (1 - e^{-\theta}(1 + \theta)) + R e^{-\theta}(1 + \theta) \right] \geq 1.$$  

It can be rearranged as

$$\alpha \left[ P^w (1 - e^{-\theta}) + R e^{-\theta} \right] - \alpha (P^w - R) e^{-\theta} \geq 1.$$  

As $\alpha (P^w - R) e^{-\theta}$ can be replaced by $\kappa F$ due to free entry, equation (10) coincides with equation (11). The observation that the welfare in a constrained efficient allocation coincides with the expected revenue of the farmers is more general and is going to be used again when we introduce local markets.
In other regions the production subsidy is either ineffective or welfare decreasing. In region $W$, for example, farmers have a comparative advantage in the staple and specialize in it. As a result, subsidizing production of strawberries reduces overall welfare. If the economy is in the $S - F$ or $S - NF$ regions, farmers always specialize in strawberries and the production subsidy will just be a transfer between the government and the farmers with no real effects on the producer price or production decisions.

PROPOSITION 3: A per unit production subsidy greater than $1/\alpha - R$ can eliminate the inefficient equilibrium and raise welfare if the economy is in region $M$. It will lower welfare if the economy is in region $W$, and have no effect otherwise.

Export Board

Suppose the export board which incurs no costs pays the farmer $R$ for strawberries and sells them to an importer or a canning factory for $R^0$. What is the effect of such an export board on welfare? Such a board effectively offers insurance to farmers who do not meet a trader. When the board offers $R$ above $1/\alpha$, it ensures that farmers specialize in strawberries regardless of the level of intermediation. In region $M$ such an export board eliminates the multiplicity of equilibria. However, in contrast to a production subsidy, a higher $R$ reduces the level of intermediation. A lower level of intermediation means that a smaller share of the produced export good is sold at the world price $P^w$ by intermediaries and is instead sold at $R^0$ by the export board. Thus with $R^0 < P^w$, even if all farmers make strawberries, welfare is below what it would be with a production subsidy.

Figure 1b can be used to illustrate the effect of an export board raising $R$ starting with $R^0$ in region $M$. An increase in $R$ raises the expected price as shown in equation (6). This makes the farmer require a lower level of intermediation before switching over to strawberries so that $\theta_{\min}$ moves to the left. Once $R$ reaches $1/\alpha$, farmers are willing to make strawberries even without intermediation so that $l(\theta) = 1$ for all $\theta$, and the coordination failure is eliminated. In addition, at every $l$ there is less intermediation so that the $\theta(l)$ curve shifts up. As a result, the equilibrium level of intermediation when strawberries are made falls below the level of intermediation implied by $R^0$ which introduces an additional distortion into the competitive allocation.

PROPOSITION 4: In region $M$ both per unit production subsidy or an export board that sets $R$ greater than $1/\alpha$ can resolve the coordination failure. However, welfare is higher when the production subsidy is used.

This makes perfect sense as the only distortion in the competitive allocation is the coordination failure. A subsidy to production of more than $1/\alpha - R$ fixes this, leaving intermediation unaffected, while a price support that eliminates multiplicity reduces intermediation, introducing an additional distortion.
Given that the board’s choice of $R$ distorts the level of intermediation, it is worth exploring how welfare changes with its choice of $R$. We show that as long as the export good is produced welfare is maximized when the export board sets $R$ exactly equal to $R^0$. In other words, an export board that pays farmer $R$ while selling the produce at $R^0 \neq R$ distorts the level of intermediation in the competitive allocation. This makes the case for boards that act in the farmer’s interests once the coordination failure is resolved by setting $R = R^0$.

To see this let $R^0$ be the price that the board receives from selling strawberries. As the board uses no resources in its operation, welfare is the value of output less all costs and is given by:

$$W = \alpha \left( P^w (1 - e^{-\theta(R)}) + R^0 e^{-\theta(R)} \right) - \kappa F \theta(R).$$

Note that the share of output that is not sold to the intermediaries is sold at $R^0$, while the level of intermediation $\theta$ is a function of $R$, the price set by the export board. Hence, an increase in $R$ affects welfare only via its effect on $\theta$. A higher $R$ reduces revenue by $\alpha \left[(P^w - R^0) e^{-\theta} \left(\partial \theta / \partial R\right)\right]$ but also reduces costs by $\kappa F \left(\partial \theta / \partial R\right)$. Substituting for $e^{-\theta}$ from the free entry condition the net effect is

$$\frac{\partial W}{\partial R} = \kappa F \left( \frac{P^w - R^0}{P^w - R} - 1 \right) \frac{\partial \theta}{\partial R}.$$

Note that

$$\frac{\partial W}{\partial R} = 0 \text{ if } R = R^0.$$

Thus, welfare is maximized at $R = R^0$. An increase in $R$ reduces intermediation and raises the share of the output disposed of at $R^0$ rather than $P^w$, which has a negative effect on welfare. It also reduces the resources expended on entry. At $R = R^0$, these two just wash out.

### III. Local Markets

An important aspect of agricultural trade in less developed countries is that farm-gate trade coexists with organized markets. In this section we introduce competitive local markets where farmers and traders can exchange the export good at a single market clearing price. Introducing such local markets provides a way of endogenizing the farmers’ reservation price.

Local markets provide a fallback for farmers who could not sell their produce at the farm-gate. Without loss of generality in this section we assume that the exogenous $R$ described in the baseline model is 0, so farmers without a match at the farm-gate go to the local market where they can sell their produce at the market clearing price, $P^M$. To access the local market each farmer pays the cost $\tau_M$ which captures his travel and time cost of getting there.
In contrast, traders face a choice between going to the farm-gate, the local market or abstaining from intermediation altogether. As in the baseline model, traders who go to the farm-gate pay the sunk cost \( \kappa_F \), while traders who go to the local market pay a sunk cost \( \kappa_M \). We continue to assume that each trader has a capacity of \( \alpha \) units, and so can deal with only one farmer’s supply. While \( \theta \) continues to be the measure of traders at the farm-gate\(^{14}\), \( \theta^M \) denotes the measure of traders going to the local market. The local market is perfectly competitive and farmers and traders take the price of strawberries as given. The equilibrium price \( P^M \) equates the supply of strawberries (by farmers) to the demand (by traders).

\[
R(P^M) = P^M - \frac{\tau^M}{\alpha}
\]

\[S^M(P^M) \]

\[P^M(P^M) \]

**Figure 3. Equilibrium in the local market**

Supply in the local market comes from farmers. If \( P^M < \frac{\tau_M}{\alpha} \) a farmer would make a loss from selling to the local market, so supply is zero. When \( P^M \geq \frac{\tau_M}{\alpha} \), the local market provides a positive outside option to the farmer, and unmatched farmers sell their produce in the local market. In this manner, the farmer’s pay-off in the local market pins down his reservation price at the farm-gate:

\[
R(P^M) = \max \left\{ P^M - \frac{\tau^M}{\alpha}, 0 \right\}.
\]

Note that even when \( P^M > \frac{\tau_M}{\alpha} \), farmers prefer to be matched at the farm-gate where there is a possibility of making more than what they would get at the local market. As a result, only the output that is not sold at the farm-gate, i.e., \( \alpha e^{-\theta(R(P^M))} \), ends up at the local market.

From equation (3), a higher reservation price discourages trader entry at the farm-gate. Since the reservation price \( R(P^M) \) increases with \( P^M \) a rise in \( P^M \)

\(^{14}\)Recall that the measure of farmers is normalized to 1 so that the measure of traders at the farm-gate is also the ratio of traders at the farm-gate to farmers, i.e., the thickness of this market. The measure of traders in the local market is denoted as \( \theta^M \) while the ratio of traders to farmers in the local markets is unity in equilibrium as shown below.
results in a fall in $\theta$. A fall in $\theta$ in turn increases the number of farmers who remain unmatched and sell in the local market. As a result, supply in the local market rises with $P^M$. The supply function is:

$$S^M(P^M) = \begin{cases} \alpha e^{-\theta R(P^M)} & \text{if } P^M \geq \frac{\tau_M}{\alpha} \\ 0 & \text{if } P^M < \frac{\tau_M}{\alpha} \end{cases}$$

and is depicted in Figure 3. Note that when $P^M = \frac{\tau_M}{\alpha}$, $R(P^M) = 0$ and $\theta = \ln(\alpha P^w/\kappa_F)$, so that supply is $\kappa_F/P^w$.

Demand in the local market depends on trader entry in response to the prevailing local market price, $P^M$, and is a step function as depicted in Figure 3:

$$D^M(P^M) = \begin{cases} 0 & \text{if } P^M > P^w - \frac{\kappa_M}{\alpha} \\ [0, \infty) & \text{if } P^M = P^w - \frac{\kappa_M}{\alpha} \\ \infty & \text{if } P^M < P^w - \frac{\kappa_M}{\alpha} \end{cases}$$

When price in the local market, $P^M$, is above $P^w - \kappa_M/\alpha$, no trader will go there as he is sure to make a loss. So the demand for strawberries is zero in this case. If $P^M$ is less than $P^w - \kappa_M/\alpha$, traders make positive expected profits and a flood of entry into the local market is unleashed. In this case, the demand for strawberries is unbounded. When $P^M$ is exactly equal to $P^w - \kappa_M/\alpha$, expected profits from going to the local market are zero so that traders are indifferent between going to the local market, the farm-gate, or staying out of intermediation altogether. Hence, at this price demand is perfectly elastic.

In Figure 3, the equilibrium in the local market is where supply and demand intersect. Supply from farmers is zero when $P^M < \frac{\tau_M}{\alpha}$, and demand from traders is zero when $P^M > P^w - \frac{\kappa_M}{\alpha}$. As a result, positive quantities are traded in the local market only when $P^w \geq (\kappa_M + \tau_M)/\alpha$. In words, local markets are viable if the total cost of accessing them for intermediaries and farmers together does not exceed the per-unit world price of the export good. In the real world such costs are related to the quality of roads and infrastructure. Since the demand for the export good is perfectly elastic at $P^M = P^w - \kappa_M/\alpha$, the equilibrium price when local markets are viable has to be

$$P^{M*} = P^w - \frac{\kappa_M}{\alpha}. \quad (13)$$

When $P^w < (\kappa_M + \tau_M)/\alpha$ local markets are not viable, farmers have no alternative to the farm-gate trade, and we are back in the baseline case. This gives the farmer’s endogenous reservation price in equilibrium to be

$$R^* = \max \left\{ 0, P^w - \left( \frac{\kappa_M + \tau_M}{\alpha} \right) \right\}. \quad (14)$$
Note that the farmer’s reservation price depends only on the sum of the costs of accessing the local market to the farmer and to the trader, and not their distribution over the two agents. The higher is this total cost, the lower is the farmer’s outside option. Once the total cost of accessing the local market becomes so high that the farmer’s pay-off to selling there turns negative, his reservation price goes to zero.\(^{15}\) Note also that as there is a continuum of agents, \(P^M\) and \(R\) are not random.

With \(R^*\) given in equation (14) and with \(l = 1\), equation (3) implies that the equilibrium level of intermediation at the farm-gate is

\[
\theta^* = \max \left\{ 0, \ln \left( \frac{\kappa}{P^w - R^*} \right) \right\}
\]

\[
= \max \left\{ 0, \ln \left( \frac{\alpha [P^w - \max\{0, P^w - (\kappa_M + \tau_M)\}]}{\kappa_F} \right) \right\}
\]

\[
= \max \left\{ 0, \ln \left( \min\{\alpha P^w, \kappa_M + \tau_M\} \right) \right\}.
\]

When local markets are viable, \(\theta^* = \ln ((\kappa_M + \tau_M)/\kappa_F)\), so that the level of intermediation does not depend on the world price or agricultural productivity but only on the ratio of the total cost of accessing the local market relative to the farm-gate.

Similarly the expected price at the farm-gate depends on whether the local market is viable or not. If \(P^w \geq \frac{\kappa_M + \tau_M}{\alpha}\), then local markets are viable, \(\theta^* = \ln \left( \frac{\kappa_M + \tau_M}{\kappa_F} \right)\), and the expected price the farmer obtains is

\[
E_L(p) = P^w - \frac{\kappa_M + \tau_M}{\alpha} (1 + \theta^*) e^{-\theta^*}
\]

\[
= P^w - \frac{\kappa_F}{\alpha} \left( 1 + \ln \left( \frac{\kappa_M + \tau_M}{\kappa_F} \right) \right).
\]

If \(\frac{\kappa_F}{\alpha} < P^w < \frac{\kappa_M + \tau_M}{\alpha}\), then local markets are not viable and the endogenous reservation price is zero, \(\theta^* = \ln(\frac{\alpha P^w}{\kappa_F})\), and the expected price is

\[
E_L(p) = P^w (1 - (1 + \theta^*) e^{-\theta^*})
\]

\[
= P^w - \frac{\kappa_F}{\alpha} (1 + \ln(\frac{\alpha P^w}{\kappa_F})).
\]

Of course, if \(P^w \leq \frac{\kappa_F}{\alpha}\), then \(\theta^* = 0\), and the expected price is equal to the farmer’s outside option \(R^* = 0\). Thus, the expected price in the relevant scenarios is given

\(^{15}\) This suggests that the price at the farm-gate is not a linear function of distance but has a has a kink in it where the regime changes. This seems to be a testable implication of the model.
by:

$$E_L(p) = \begin{cases} 
  P^w - \frac{\kappa_F}{\alpha} \left( 1 + \ln \left( \frac{\kappa_M + \tau_M}{\kappa_F} \right) \right) & \text{if } P^w \geq \frac{\kappa_M + \tau_M}{\kappa_F} \\
  P^w - \frac{\kappa_F}{\alpha} (1 + \ln \frac{\alpha P^w}{\kappa_F}) & \text{if } \frac{\kappa_F}{\alpha} < P^w < \frac{\kappa_M + \tau_M}{\kappa_F} \\
  0 & \text{if } P^w \leq \frac{\kappa_F}{\alpha}
\end{cases}$$

When local markets are viable, changes in $\alpha$ and $P^w$ have only a direct effect on the expected price since the level of intermediation stays intact. Without local markets besides the direct effect of a change in productivity or the world price on the expected price, there is also a change in the level of intermediation.

As in the baseline case, the farmer’s decision to produce strawberries depends on the expected price, and via it, on the level of intermediation. His best response function is given in equation (7), where instead of the exogenous outside option $R$ we have $R^*$ from equation (14) and $\theta_{min}$ is the solution to $\alpha E_L(p) = 1$.

Finally, we can pin down $\theta^{M*}$, the measure of traders in the local market. In equilibrium, in order for the supply to equal demand the measure of traders and farmers who go to the local market must be equalized so that the ratio of their measures is unity. This poses no problem as at $P^{M*}$ traders are indifferent between their options (entering the local market, farm-gate trade or abstaining from intermediation) and their measure in the local market adjusts to equal the measure of unmatched farmers, $e^{-\theta^*}$. Thus,

$$\theta^{M*} = e^{-\theta^*}.$$  

Next we proceed to describe the types of equilibria and their properties in the augmented model.

A. Equilibrium

An equilibrium consists of the objects: $\theta^*$ defined in equation (15), $F(p)$, $l(\theta)$ defined as in the baseline model, $P^{M*}$ defined in equation (13), the farmer’s reservation price $R^*$ given by equation (14), and the measure of traders in the local market $\theta^{M*}$ given by equation (17). As in the baseline model, both multiple and unique equilibria are possible. Figure 4 shows the possible equilibrium types as a function of the primitive parameters $\kappa_M + \tau_M$ and $\kappa_F$, for fixed values of $P^w$ and $\alpha$. The topmost horizontal line $\kappa_M + \tau_M = \alpha P^w$ separates the region where local markets are viable from the region where they are not. For values of $\kappa_M + \tau_M$ greater than $\alpha P^w$ we are back in the baseline setup with $R = 0$ and we do not focus on this region here.\(^\text{16}\) The interesting cases lie below the line where local markets are viable.

\(^\text{16}\)More specifically, when $\kappa_M + \tau_M > \alpha P^w$, $R = 0$ and only the $M$ and $W$ regions are possible as depicted in Figure 4. Note that local markets only exist in the $M$ region when $\alpha P^w - 1 \leq \kappa_M + \tau_M \leq \alpha P^w.$
In the baseline case, producing the export good becomes the dominant strategy for farmers when $R$ exceeds $1/\alpha$. With local markets, the analogous condition is that the farmer’s pay-off at the local market exceeds that of making the staple: $\alpha P^w - (\kappa_M + \tau_M) > 1$. In Figure 4, the line $\kappa_M + \tau_M = \alpha P^w - 1$ demarcates the region where producing strawberries is the dominant strategy from the region where strawberries are produced only if the intermediation level is high enough.

The level of intermediation, in turn, depends on the ratio between the total cost of accessing the local market, $\kappa_M + \tau_M$, and the cost of farm-gate intermediation, $\kappa_F$. With access to local markets, the export good can reach the world market in two ways: via the local market or through an intermediary at the farm-gate. When $\kappa_M + \tau_M < \kappa_F$, the technology of farm-gate intermediation is dominated by the technology of local markets, as the latter both costs less and provides certain results. Thus, in Figure 4 farm-gate intermediation is not sustainable below the $45^\circ$ line. Above the $45^\circ$ line, whether intermediaries enter and farm-gate trade exists or not, depends on whether or not farmers make strawberries.

The regions labeled $S - FL$ and $S - L$ lie below the $\kappa_M + \tau_M = \alpha P^w - 1$ line. Here the farmer’s decision to make strawberries is independent of the level of intermediation, because the pay-off at the local market is higher than the pay-off from making the staple. In region $S - FL$ which lies above the $45^\circ$ line both farm-gate trade and a local market coexist. Below the $45^\circ$ line the local market is more efficient than farm-gate trade, so in region $S - L$ strawberries are traded exclusively at the local market.
Above the $\kappa_M + \tau_M = \alpha P^w - 1$ line, the farmer’s payoff from selling at the local market is less than what he would obtain from growing wheat. Thus, the local market option no longer ensures strawberry production. Whether strawberries are made or not depends on the level of farm-gate intermediation. In Figure 4 regions $M$ and $W$ lie both above the line $\kappa_M + \tau_M = \alpha P^w - 1$ and above 45° line. In region $M$ multiple equilibria are endemic. As in the baseline model, in the “good” equilibrium farm-gate trade coexists with local markets, and in the “bad” equilibrium farmers produce wheat and intermediaries do not enter. As $\kappa_F$ rises, it reduces farm-gate intermediation until we switch to the region $W$ where only wheat is made. The switch occurs when farmers are indifferent between wheat and strawberries, i.e., $\alpha E_L(p) = 1$ with intermediation at the free entry level. This condition can be rewritten as\(^{17}\)

$$\kappa_M + \tau_M = \kappa_F \left( \frac{\alpha P^w - \kappa_F - 1}{\kappa_F} \right).$$

IV. Policy Implications with Local Markets

As in the baseline case, with local markets the equilibria in region $M$, where farmers specialize in the production of the staple, are inefficient. In this section we discuss policies that can eliminate this inefficiency in the presence of the local markets.

A. The Role of Infrastructure

There are critical differences in outcomes due to local markets. First, with local markets, lower costs of accessing them can eliminate the inefficient equilibria that occur in region $M$. This is because cheaper access to local markets boosts the farmer’s outside option, making production of strawberries the dominant strategy regardless of the level of intermediation. In contrast, a lower cost of accessing the farm-gate, $\kappa_F$, cannot eliminate the coordination failure that causes farmers to specialize in wheat in region $M$. The intuition is easy to see from Figure 4. Starting at a point in region $M$, reducing $\kappa_M + \tau_M$ below $\alpha P^w - 1$ moves the economy from region $M$ into regions where farmers specialize in strawberries and the equilibrium is constrained efficient. Reducing $\kappa_F$ would keep the economy in region $M$.

Second, the effects on welfare of lower farm-gate and local market entry costs differ, and their relative strengths depend on the level of intermediation prevailing at the farm-gate. When intermediation is extensive, little is sold in local markets so that reductions in costs of accessing them have a limited impact on welfare. On the other hand, when there is little intermediation, most of the output gets

\(^{17}\)Substituting $\theta^*$ from equation (15) into the expression for the expected price given in equation (16) and rearranging the terms yields the result.
Sold in local markets and reductions in the costs of accessing them for farmers or traders raise welfare considerably.

To see this more formally, consider welfare with a viable local market. Each unit of the export good is ultimately sold at $P^w$, irrespective of whether the farmer makes the sale at the farm-gate or the local market, so the revenue from producing the export good is $\alpha P^w$. The cost of farm-gate intermediation is $\kappa_F \theta^*$. With probability $e^{-\theta^*}$, farmers do not meet an intermediary at the farm-gate and sell the produce at a local market, so the total cost for traders and farmers of accessing the local market is $(\kappa_M + \tau_M)e^{-\theta^*}$. Thus, welfare is given by:

$$W_L = \alpha P^w - \kappa_F \theta^* - (\kappa_M + \tau_M)e^{-\theta^*}. \quad (18)$$

A unit fall in $\kappa_M$ or $\tau_M$ raises welfare by $e^{-\theta^*}$ while a unit fall in $\kappa_F$ raises welfare by $\theta^*$. Since $e^{-\theta^*}$ is decreasing in $\theta^*$, reductions in $\kappa_M$ or $\tau_M$ are more efficacious in terms of raising welfare when $\theta^*$ is low enough, which occurs when $\kappa_F$ is high relative to $\kappa_M + \tau_M$ and farm-gate intermediation is thin.\(^{19}\)

**B. Agricultural Extension Programs**

In many countries, including developed ones, considerable effort is devoted to agricultural extension programs. These programs are usually directed towards raising productivity in agriculture by introducing farmers to best practice techniques. In our model, we think of agricultural extension programs as raising $\alpha$, the productivity of the farmer in strawberries. Both in the baseline model and in the local markets extension, an increase in $\alpha$ can eliminate an inefficient equilibrium and increase welfare. These beneficial effects are more pronounced in the presence of local markets. This should be expected as all output is sold at the world price rather than some going for a lower reservation price $R$.

Figure 5 describes the effect of a higher $\alpha$ in the baseline model. The line $R = 1/\alpha$ shifts downward as $\alpha$ increases, and shrinks the $M$ region where multiple equilibria occur. A higher $\alpha$ also increases the level of intermediation and rotates the downward sloping line, where intermediation is just viable, to the right. Finally, with a higher $\alpha$, farmers specialize in strawberries at a lower farm-

\(^{18}\) As in the baseline case welfare is equivalent to the expected earnings of the farmer as there are no consumers and entry into intermediation is free. Formally,

$$W_L = \alpha E_L(p) = \alpha \left[ P^w (1 - (1 + \theta^*)e^{-\theta^*}) + R^* (1 + \theta^*)e^{-\theta^*} \right]$$

$$= \alpha P^w - \alpha (P^w - R^*) e^{-\theta^*} - \alpha (P^w - R^*) \theta^* e^{-\theta^*}$$

$$= \alpha P^w - \kappa_F \theta^* - (\kappa_M + \tau_M) e^{-\theta^*}$$

where we take advantage of the fact that in equilibrium $\alpha (P^w - R^*) = \kappa_M + T_M$ and $\alpha (P^w - R^*) e^{-\theta^*} = \kappa_F$. The expression makes sense as with local markets all of the good ultimately gets sold in the world market.

\(^{19}\) More precisely, $\frac{\partial W_L}{\partial (\kappa_M)} = -e^{-\theta}$ $\geq \frac{\partial W_L}{\partial (\kappa_F)} = -\theta$ when $\theta < 0.567$. As $\theta = \ln \left( \frac{\kappa_M + \tau_M}{\kappa_F} \right)$, this is equivalent to $(\kappa_M + \tau_M) < \kappa_F \exp(0.567) = \kappa_F (1.76)$. This makes sense as $\theta$ is low when $\kappa_F$ is high.
gate price, which shifts the $\alpha E(p) = 1$ condition separating the region $M$ and $W$ to the right. All of these expand the region where strawberries are made.\textsuperscript{20}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure5.png}
\caption{Effect of a 50 percent increase in $\alpha$ on the equilibrium types without local markets.}
\end{figure}

Analogously, Figure 6 shows the effect of a higher $\alpha$ when local markets offer a positive pay-off. In this case the inefficient equilibria (region $M$) occur at higher costs of accessing the local market, $\kappa_M + \tau_M$. A higher $\alpha$ raises the farmer’s pay-off at the local market which shifts up the line $\kappa_M + \tau_M = \alpha P^w - 1$ below which farmers specialize in strawberries regardless of the intermediation level. It also shifts up the line $\kappa_M + \tau_M = \alpha P^w$ below which local markets are viable. So the regions S-FL and S-F, where farmers prefer to produce strawberries, expand. Note that, with local markets, changes in $P^w$ have the same effect as changes in $\alpha$ as only the product of the two matters.

The change in welfare due to a rise in agricultural productivity is also greater in the presence of local markets. In the baseline case, welfare in the constrained efficient allocation is given by:

\begin{equation}
W = \alpha P^w(1 - e^{-\theta}) - \kappa_F \theta,
\end{equation}

which follows from equation (12) and setting $R = R^0 = 0$. When $\alpha$ increases, it causes a direct rise in welfare, as well as an indirect effect through an increase in intermediation, which raises the share of output sold at $P^w$. Higher levels of

\textsuperscript{20}Figure 7 shows that a higher world price similarly contracts the M region.
intermediation also involve higher costs of intermediation which have a negative effect on welfare. Simple algebra shows that the change in welfare is given by

\[(20) \quad \frac{\partial W}{\partial \alpha} = P^w - \frac{\kappa_f}{\alpha}.\]

With local markets, the level of intermediation is not affected and increases in productivity are valued at the world price by society. To see this, differentiate welfare in equation (18)

\[(21) \quad \frac{\partial W_L}{\partial \alpha} = P^w.\]

As \(\frac{\partial W_L}{\partial \alpha}\) in equation (21) exceeds \(\frac{\partial W}{\partial \alpha}\) in equation (20), it follows that local markets enhance society’s (and the farmer’s) ability to benefit from productivity improvements\(^{21}\). There are thus clear synergies between the creation of local markets and productivity enhancing activities: the former helps farmers obtain more from the latter.

---

\(^{21}\)Since expected profits of traders are zero with or without local markets, social welfare is equal to \(\alpha E(p)\) in the baseline model and \(\alpha E_L(p)\) in the extension with local markets.
C. SPS Requirements

So far we have treated $P^w$ as a competitive price determined in the world market. However in developing countries exporters face low effective world prices due to risks associated with SPS requirements. While fees for fulfilling the SPS requirements are not large, the risks of failing them are high. For example, Ferrier, Petersen and Landes (2012) estimate that in the case of Alphonso mangoes from India the direct costs of production and SPS inspection fees account for about 30 percent of the final price. This cost estimate, however, ignores the risk of failing the SPS inspection, which reduces the effective price that the exporters face. In this regard, Roy and Thorat (2008), who focus on exports of grapes from India, document that failure rates in meeting SPS requirements for new exports are as high as 80 percent. All of these suggest that the effective price that exporters face in the developing countries is significantly lower than the quoted market price.

We can think of the effective price that exporters face, $P^w_{SPS}$, as a function of four things: the probability that the SPS requirements are met in the inspection $\lambda$, the sunk SPS inspection fee $\psi$, the scrap value of the goods for export if they fail the inspection $S$, and the price of the good after the SPS inspection has been passed $P^w$. $P^w_{SPS}$ can be written as

\begin{equation}
P^w_{SPS} = P^w \lambda - \psi + (1 - \lambda)S.
\end{equation}

From equation (22) it is easy to see that higher effective price $P^w_{SPS}$ works like an increase in the world price $P^w$. In our model when the exogenous outside option
is zero, a higher $P^w$ has the same effect on welfare as a higher $\alpha$, both with and without local markets. This can be seen from (18) and (19) as only the product $\alpha P^w$ matters. As a result, a sufficiently high $P^w$ ensures that farmers specialize in the export good and that local markets are viable, expanding the region where strawberries are made.

Since a higher effective price expands the regions where strawberries are made, our model suggests that SPS requirements could prove to be a significant barrier to exports (and specialization according to comparative advantage). A number of policies could make passing SPS requirements less risky for the exporters in developing countries. One such policy is verifying the SPS requirements at the point of origin. If exports are inspected at the destination, products that do not meet the SPS requirements are destroyed, so $S = 0$. If they are inspected and certified in the country of origin, then $S$ may be positive. This suggests that ensuring SPS requirements are satisfied domestically increases the effective price of exports. Similarly, policies that raise $\lambda$ and reduce $\psi$ increase $P^w_{SPS}$, the effective export price.

Furthermore, Roy and Thorat (2008) document that SPS inspection failure rates among Indian grape exporters fell to 1 percent within a year of exporting, suggesting that $\lambda$ rises as exporters gain experience in meeting the SPS requirements. To the extent that exporters learn from one another, exporting may raise $\lambda$ not just for a particular exporter, but also for other actual or potential exporters. For example, new exporters could hire knowledgeable workers from experienced exporters as they prepare to enter. For such reasons pioneers may not internalize the benefits of a higher $\lambda$ in the future. As a result, even products that are viable in the long run may not be produced without interventions of some kind. Partnerships with multinationals who have experience in meeting such requirements, providing insurance for early exporters, or direct government involvement are possible remedies.

\section*{D. Pass-through of International Prices}

Pass-through of world prices to the producer also differs with local market access. With local markets, the level of intermediation is independent of export price fluctuations, so the farm-gate price adjusts one for one in response to world price changes. This follows from equation (16). Intuitively, a rise in $P^w$ is accompanied by an equivalent rise in the local market price, which serves as the farmer’s reservation price. Thus, traders’ profits remain the same and the change in the world price is entirely passed into the farm-gate price. In other words, from equation (16), when local markets are viable,

$$\frac{\partial E_L(p)}{\partial P^w} = 1.$$  

In contrast, when farmers have no access to local markets their outside option
stays constant, trader profits rise, as does the measure of intermediaries. In
this way some of the increase in $P^w$ pays for the higher level of intermediation,
and the pass-through of the world price into the producer price is incomplete.
Differentiating equation (5) with respect to $P^w$ and taking advantage of the fact
that $\frac{\partial \theta}{\partial P^w} = \frac{1}{P^w - R}$,
$$\frac{\partial E(p)}{\partial P^w} = 1 - e^{-\theta} < 1 \text{ for } \theta > 0.$$  
In other words, without local markets the pass-through depends on the prevailing
level of intermediation: the thinner the market, the lower is the pass-through.

This is consistent with Fafchamps and Hill (2008) who find that the pass-
through of the changes in world commodity prices to producer prices is only
partial. Our model suggests why this happens, and suggests that it should be the
case in remote locations where intermediary markets are thin, and local markets
are not viable due to high cost of travel for farmers and traders.

As with other policies, local markets enhance the extent of welfare gains from
increases in the world price. This is easy to see because welfare in the constrained
efficient allocation, with or without local markets, is just the farmer’s revenue.
Our model thus helps answer why farmers, especially in poorly connected loca-
tions, seem to gain so little from higher world prices of their agricultural exports.

V. Conclusion

In this paper we develop a model of intermediation in the absence of binding
contracts and apply it to a Ricardian setting. This provides a new reason for
why developing countries specialize producing staples, despite the availability of
what seem to be more lucrative options. The inability of farmers and traders
to contract on price makes farmers subject to hold-up. The anticipation of such
hold-up in turn prevents farmers from exploring seemingly lucrative options.

A new role for marketing boards and for local markets emerges. Both can
provide farmers an outside option that discourages opportunistic behavior by
intermediaries. Local markets prove to be particularly effective policies in this
regard. Their presence also enhances efficiency, as the entire output finds its
way to the world market despite intermediation frictions, and allows farmers to
capture more of the gains arising from productivity improvements and higher
world prices. On the other hand, SPS requirements are shown to be particularly
damaging as they make the inefficiencies associated with hold-up more likely.
A case for intervention arises when there is learning by doing in meeting SPS
requirements that spills over to other producers. Incomplete pass-through of
export prices to producers at the farm-gate, a pattern observed especially in
remote areas, emerges naturally in this setting as well.
REFERENCES


A1. Mathematical Appendix

PROOF OF PROPOSITION 1:

We show that the support of the price distribution $F(\cdot)$ starts at $R$, has no gaps
and the distribution function is continuous, i.e., the density function has no mass
points. Since no farmer will accept a price below $R$, the support of $F(\cdot)$ cannot
include any such points. Suppose the support of $F(\cdot)$ starts at $p > R$. Then a
trader who offers a price in the interval $[R, p]$ will only win if there are no other
traders, i.e., with probability $P_0 = e^{-\theta}$. His expected profit is:

$$\pi^T(p) = \alpha(P^w - p)l^* e^{-\theta},$$

which is decreasing in $p$. Thus, the trader would be better off charging $R$, or any
price in $[R, p)$ than offering $p$ which contradicts the assumption that $p$ is in the
support of the mixed strategy equilibrium.

Next, we establish that there are no gaps or atoms in the support of the dis-
tribution. Let us first rule out gaps in the support of the distribution. Suppose
there is a gap in the support of $F(\cdot)$: no one bids in the interval $(p', p'')$. If there
is no mass point at $p''$, then a trader who posts a price $p^* \in (p', p'')$ will be better
off than bidding $p''$, as the probability of winning does not decrease, but the profit
margin rises. Hence, there are no gaps in the support unless there is a mass point
at $p''$. Such a mass point would cause a jump down in profits at prices just below
$p''$, and validate the gap in support of the posited price distribution. Can we
rule out such atoms at $p''$? Yes, we can. If there is an atom at $p''$, then bidding
$p'' + \epsilon$ causes a discrete jump in trader’s profits as he increases the offer price only
marginally, but this increases his probability of winning discretely.

The same argument rules out atoms at any $\hat{p}$ in the interior of the support of the
distribution or at $R$: bidding $p = \hat{p} + \epsilon$ causes a discrete jump in trader’s profits
as he increases the offer price only marginally, but this increases his probability
of winning discretely. In equilibrium all prices in the support must yield the
same profits, hence such mass points cannot occur. They cannot even occur at
the upper end of the support. As will be confirmed later, the upper end of the
distribution support is given by $p_{\text{max}} < P^w$. If there were a mass point at $p_{\text{max}}$,
raising $p$ slightly above $p_{\text{max}}$ must raise profits which rules out a mass point at
$p_{\text{max}}$.

Next we can use the property of the equality of payoffs at every point of the
support to obtain the explicit expression for the cumulative distribution of bids,
$F(p)$. Equating the expected profits at an arbitrary price $p$ and expected profits
at the lower end of the support $R$, i.e., setting $\pi^T(p) = \pi^T(R)$, we can uniquely
solve for the bidding function of the trader as a function of world price $P^w$, market
thickness $\theta$, and the farmer’s outside option $R$:

$$
\alpha(Pw - p)e^{-\theta(1 - F(p))} = \alpha(Pw - R)e^{-\theta} \\
F(p) = \frac{1}{\theta} \ln \left( \frac{Pw - R}{Pw - p} \right).
$$

At the upper end of the support the cumulative density function equals unity. Hence solving

$$
F(p_{\text{max}}) = 1 = \frac{1}{\theta} \ln \left( \frac{Pw - R}{Pw - p_{\text{max}}} \right)
$$

for $p_{\text{max}}$ yields the expression for the upper end of the support:

$$
p_{\text{max}} = Pw(1 - e^{-\theta}) + e^{-\theta} R.
$$

\[ \square \]

PROOF OF LEMMA 1: By definition, the expected value of the price the farmer gets is

$$
E(p) = \sum_{k=0}^{\infty} Q_k E_k(p) \\
= Q_0 R + \sum_{k=1}^{\infty} Q_k \left[ \int_{R}^{p_{\text{max}}} pg_k(p) dp \right],
$$

where

$$
Q_k = \frac{\theta^k}{k!} e^{-\theta},
$$

$$
G_k(p) = [F(p)]^k,
$$

and

$$
g_k(p) = k[F(p)]^{k-1} f(p).
$$

First we obtain the expected price when $k$ traders show up and then take the expectations over all possible realizations of $k$. The expected price conditional on
the number of traders is

\[ E_k(p) = \int_R^{p_{\text{max}}} p g_k(p) dp \]

\[ = \frac{k}{\theta^k} \int_R^{p_{\text{max}}} \left[ \ln \left( \frac{P^w - R}{P^w - p} \right) \right]^{k-1} \frac{p}{P^w - p} dp \text{ for } k \geq 1 \]

We start by solving for the indefinite integral, a key part of \( E_k(p) \)

\[(A1) \int \left[ \ln \left( \frac{P^w - R}{P^w - p} \right) \right]^{k-1} \frac{p}{P^w - p} dp.

To do so we change variables. Let

\[ x = \ln \left( \frac{P^w - R}{P^w - p} \right) , \]

so that \( p \) in terms of \( x \) is \( p = P^w - e^{-x}(P^w - R) \) and the corresponding differential \( dp = e^{-x}(P^w - R)dx \). Using the change of variables, the integral (A1) becomes

\[ \int x^{k-1} \left( e^x \frac{P^w}{(P^w - R)} - 1 \right) e^{-x}(P^w - R)dx \]

\[ = P^w x^k + (P^w - R)(k - 1)e^{-x} \left( \sum_{j=0}^{k-1} \frac{x^j}{j!} \right) . \]

Substituting for \( x \) in terms of \( p \) gives

\[ \int_R^{p_{\text{max}}} \left[ \ln \left( \frac{P^w - R}{P^w - p} \right) \right]^{k-1} \frac{p}{P^w - p} dp = A2 + A3 \]

where

\[(A2) \] \[ = \frac{P^w}{k} [\ln \left( \frac{P^w - R}{P^w - p_{\text{max}}} \right)]^k |_{R}^{p_{\text{max}}} \]

\[ = \frac{P^w}{k} [\ln \left( \frac{P^w - R}{P^w - p_{\text{max}}} \right)]^k - \frac{P^w}{k} [\ln \left( \frac{P^w - R}{P^w - R} \right)]^k \]

\[ = \frac{P^w}{k} \theta^k , \]
and

\[(A3) = (P^w - R)(k - 1)! \left[ \frac{P^w - p}{P^w - R} \sum_{j=0}^{k-1} \frac{\ln \left( \frac{P^w - R}{P^w - p} \right)^j}{j!} \right] \right|_{P_{\text{max}}}^{P}\]

\[= (k - 1)! \{(P^w - R)e^{-\theta} \left[ \sum_{j=0}^{k-1} \frac{\ln \left( e^\theta \right)^j}{j!} \right] - (P^w - R) \left[ 1 - \sum_{j=0}^{k-1} \frac{\ln \left( \frac{P^w - R}{P^w - p} \right)^j}{j!} \right] \}\]

\[= (P^w - R)(k - 1)! \{e^{-\theta} \sum_{j=0}^{k-1} \frac{\theta^j}{j!} - 1 \} \].

In evaluating the integrals we use the fact that

\[P^w - p_{\text{max}} = e^{-\theta} (P^w - R)\].

Next we find \(E_k(p)\) for \(k > 1\) for the given integration limits.

\[E_{k \geq 1}(p|\theta) = \int_{R}^{P_{\text{max}}} \frac{p}{P^w - p} \left[ \ln \left( \frac{P^w - R}{P^w - p} \right) \right]^{k-1} dp\]

\[= \frac{k}{\theta k} [A2 + A3] = P^w + (P^w - R) \frac{k!}{\theta k} \left( e^{-\theta} \sum_{j=0}^{k-1} \frac{\theta^j}{j!} - 1 \right)\]

Hence the expected price conditional on at least one trader showing up is as follows:

\[\sum_{k=1}^{\infty} Q_k E_k(p) = \sum_{k=1}^{\infty} \frac{\theta^k}{k!} e^{-\theta} \left[ P^w + (P^w - R) \frac{k!}{\theta k} \left( e^{-\theta} \sum_{j=0}^{k-1} \frac{\theta^j}{j!} - 1 \right) \right]\]

\[= e^{-\theta} P^w (e^\theta - 1) + (P^w - R) e^{-\theta} (-\theta)\]

where we use the fact that

\[\sum_{k=1}^{\infty} \left( e^{-\theta} \sum_{j=0}^{k-1} \frac{\theta^j}{j!} - 1 \right) = -\theta.\]

This can be verified as follows:

\[\sum_{k=1}^{\infty} \left( e^{-\theta} \sum_{j=0}^{k-1} \frac{\theta^j}{j!} - 1 \right) = \sum_{k=1}^{\infty} \left( e^{-\theta} \left( \sum_{j=0}^{\infty} \frac{\theta^j}{j!} - \sum_{j=k}^{\infty} \frac{\theta^j}{j!} \right) - 1 \right)\]
\[
= \sum_{k=1}^{\infty} \left( e^{-\theta} \left( e^\theta - \sum_{j=k}^{\infty} \frac{\theta^j}{j!} \right) - 1 \right)
\]
\[
= -e^{-\theta} \sum_{k=1}^{\infty} \sum_{j=k}^{\infty} \frac{\theta^j}{j!}
\]
\[
= -e^{-\theta} \sum_{j=1}^{\infty} \sum_{k=1}^{j} \frac{\theta^j}{j!}
\]

The change in the order of summation in the last line can be verified by writing out the expression before and after the change, and noting that the first term in the former corresponds to the last term in the latter. Thus,

\[
\sum_{k=1}^{\infty} \left( e^{-\theta} \sum_{j=0}^{k-1} \frac{\theta^j}{j!} - 1 \right) = -e^{-\theta} \sum_{j=1}^{\infty} \frac{\theta^j}{j!} \left( \sum_{k=1}^{j} 1 \right)
\]
\[
= -e^{-\theta} \sum_{j=1}^{\infty} \frac{\theta^j}{(j-1)!}
\]
\[
= -e^{-\theta} \theta \sum_{j=1}^{\infty} \frac{\theta^{j-1}}{(j-1)!}
\]
\[
= -\theta.
\]

Finally, the first moment of price is

\[
E(p) = Q_0 R + \sum_{k=1}^{\infty} Q_k E_k(p)
\]
\[
= P^w - e^{-\theta} (P^w - R)(1 + \theta).
\]

\[\square\]