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Exploring Nonlinear Supply Chains: The Dynamics of Capacity Constraints

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While most supply chain models assume linearity, real production and distribution systems often operate in constrained contexts. This article aims to analyse the consequences of capacity limits in the order-up-to replenishment policy with minimum mean squared error forecasting under independently and identically distributed random demand. Our study shows that the impact of this nonlinearity is often significant and should not be ignored. In this regard, we introduce the concept of a *settling capacity*, which informs when our knowledge from a linear analysis is a reasonable approximation in a nonlinear context. If the available capacity is less than the settling capacity, the nonlinear effects can have a significant impact. We compare the Bullwhip Effect and Fill Rate in constrained contexts to well-established results for linear supply chains. We reveal the capacity limit acts as a production smoothing mechanism, at the expense of increasing inventory variability. We proceed to analyse the economic consequences of the capacity constraint and show that it can actually reduce costs. We provide an approximate solution for determining the optimal capacity depending on the demand, the unit costs, and the lead time.

Keywords: Bullwhip Effect; Capacity Planning; Fill Rate; Order-Up-To Policy; Supply Chain Dynamics.

1. Introduction

The Bullwhip Effect is a major source of supply chain inefficiencies due to its harmful consequences including storage, shortage, labour, obsolescence, and transport costs

(Lee et al. 1997). Its strategic importance has led to a large amount of research over the last two decades. In this regard, the amplification of the variability of orders has become a widely used metric to assess internal supply chain performance. At the same time, the Fill Rate is a key measure of customer satisfaction within production and distribution systems (Song 1998). It is popular in high volume industries as it measures the demand fulfilment experienced by the customer. That is, the Fill Rate measures the external supply chain performance. The major challenge for supply chains is to provide high customer service level at low operating cost. Thus, the complementary indicators, the Bullwhip Effect and the Fill Rate, are popular metrics in the analysis of supply chains.

Control engineering has been a useful technique in supply chain studies (Dejonckheere et al. 2003). Multiple works have used this methodology to analyse and improve the dynamic supply chain performance (e.g. John et al. 1994; Disney and Towill 2002; Warburton and Disney 2007; Jaipuria and Mahapatra 2014). This approach is based on the assumption of system linearity, where supply chains are represented by a set of linear difference or differential equations.

Linear supply chain models are based on some key assumptions in order to: (1) provide a meaning for the negative values of variables (e.g. inventories can become negative indicating backlogs or orders can become negative indicating free returns to suppliers); (2) avoid saturation (e.g. unconstrained capacities); and (3) consider the system has no process uncertainty (e.g. orders are always fulfilled completely, on time, and with no quality loss or over production). Nonetheless, these assumptions could be deemed unrealistic in many practical settings (Spiegler et al. 2012). Hence, our research objective is to understand the consequences of nonlinear features in production and distribution systems[1].

Due to the complexity involved in nonlinear modelling, this article focuses only on capacity constraints and its impact on supply chain dynamics via the following research questions (RQ):

RQ1. How do capacity constraints affect the well-established Bullwhip Effect and the Fill Rate supply chain measures?

RQ2. Can we use the same key performance metrics in both linear and constrained models?

RQ3. Is there an optimal capacity limit? If so, what does it depend on?

RQ4. What level of capacity would allow managers operating in constrained environments to safely use the results from a linear analysis?

With this aim, we employ simulation methods supported by statistical techniques. Sterman (2002) highlights the importance of computer-based simulation by arguing that a fundamental limit of human cognition is our inability to mentally simulate the dynamics of complex nonlinear systems. When linearity assumptions are removed, complex dynamic behaviours are revealed (Wang et al. 2014), and the nonlinear effects may even play the dominant role (Nagatani and Helbing 2004). In this context, mathematical analysis becomes so complex that resorting to simulation to gain insight has become an essential alternative. Nonetheless, we note that Spiegler et al. (2016a, 2016b) have recently studied the impact of supply chain nonlinearities by developing a methodology based on (1) simplifying the model and (2) applying advanced linearization techniques.

Our study is concerned with the classic order-up-to (OUT) replenishment policy (Chen et al. 2000) for three main reasons. First, it is a widely used replenishment method in both academic studies and in the industry (Disney et al. 2013). Second, it is optimal at minimising local inventory costs (Karlin 1960). Third, this policy is relatively simple and operates on a discrete time periodic basis, reducing the complexity of our analysis. In order to gain a thorough understanding of the nonlinear effects induced by capacity constraints, we study only a single supply chain echelon.

The structure of this paper is as follows. Section 2 reviews the literature and outlines our contribution. Section 3 summarises a linear supply chain, while section 4 highlights the key performance indicators and previously known results. Section 5 presents our nonlinear simulation model, and section 6 considers the dynamic consequences when facing a step input and explores both the operational and the financial impact of capacity constraints under stochastic demand. Finally, section 7 concludes by revisiting our research questions and suggesting a future research agenda.

2. Literature Review and Our Contribution

Capacity issues have been explored in the literature from different perspectives. Cachon et al. (1999) investigated the relationship between the ordering decisions of the retailer and the capacity choice of the supplier, exploring both manipulable and truth-inducing mechanisms for capacity allocation. Olhager et al. (2001) argued that managers need to determine appropriate capacity levels required to support their production plan. In this regard, two main capacity strategies were identified by Chou et al. (2007): reactive and conservative. Chou et al. (2007) found that although reactively adjusting the capacity according to a demand forecast was the dominant alternative, the conservative strategy – which entails making capacity decisions only after the organisation is running at full capacity as a consequence of an actual increase in demand – worked better in some circumstances. Cheng et al. (2012) proposed a production-inventory model for manufacturing systems with capacity constraints. Facing a seasonal demand, they find an optimal trade-off between inventory cost and capacity utilisation. In this field, the concept of 'demand-supply balancing' has emerged and acquired strategic importance. Coker and Helo (2016) have recently reviewed this notion and its key role in operations management, as well as successful practical methodologies for balancing demand and supply.

However, works which consider the implications of capacity limits on the dynamic supply chain performance are rather scarce. Of those that do exist, the questions we have raised have not been addressed, but these works do offer some insight and background to our problem.

Evans and Naim (1994) explored capacity constrained supply chains and noticed that while capacity limits tend to decrease inventory service levels, capacity limits generally led to improved dynamic performance. Using Multi-Attribute Utility Theory, they accounted for eight different indicators, concluding that the unconstrained system did not always produce the best response. This intriguing finding warrants further investigation. Since then, several works have assessed the impact of limited capacity on supply chain performance, often using discrete event simulation, which we now review.

Helo (2000) highlighted the damaging consequences of capacity constraints on supply chain agility. Suwanruji and Enns (2006) compared Materials Requirement Planning, Kanban, and the OUT policies, examining how capacity constraints affected their performance. They found that the relative performance ranking depended on the capacity limit. Studying a model with finite capacity, Wikner et al. (2007) showed that production planning and control systems should proactively regulate backorders. Wilson (2007) studied a short duration capacity loss due to transportation disruption highlighting a negative impact on the Fill Rate, although a dynamic improvement was sometimes generated.

Although these previous works did not mention the concept directly, the dynamic improvement induced by capacity constraints can be explained by a mitigation in the Bullwhip Effect. Cannella et al. (2008) showed that supply chain performance can be enhanced as capacity restrictions limit the ability to overproduce and have a dampening effect on order variability. They studied six different capacity levels revealing that – in the presence of information distortions – increasing the capacity can lead to higher costs. Chen and Lee (2012) and Li et al. (2014) also highlighted that a finite capacity had a smoothing effect on the order variance, as the receipts tend to be less variable than the original orders. Nevertheless, Nepal et al. (2012) and Hussain et al. (2016) concluded that capacity limits increased the variability of net stock, reducing Fill Rates. Yang et al. (2014) analysed inventory models with setup costs and concluded that the optimal policy depended on the capacity restriction.

In summary, the literature has shown that constraining capacity allows the supply chain to mitigate the Bullwhip phenomenon (increasing internal performance) but at the expense of reducing inventory service levels (decreasing external performance). This insight hints at the existence of an optimal, finite, capacity limit. However, little previous work has been directed towards this idea, leading us to pose the four aforementioned, insufficiently explored, research questions.

The contribution of this paper is to increase our understanding of nonlinear supply chains by analysing the relationship between the system performance and its capacity. We confirm the existence of an optimal capacity limit from a financial perspective and identify it, which does not appear to have been previously obtained. We introduce the concept of a *settling capacity* to indicate when the knowledge derived from a linear analysis can be applied in a constrained environment. We also question if the same metrics that have proven to be so useful in linear models, such as the convex weighted sum of the square root of the orders and inventory variances (see section 4.2), can still be used in a nonlinear setting.

Having briefly summarised the pertinent literature on the subject and highlighted our research aims, before considering the capacity constrained supply, we will next define the linear model in section 3 and review the known linear results in section 4.

3. Description of a Linear Supply Chain and its Main Assumptions

The sequence of events shown in figure 1 describes the discrete time operation of a supply chain (Disney et al. 2016). First, both product (material flow) and demand (information flow) are received from the upstream and downstream echelons, respectively. Then demand is satisfied and both the net stock and the work-in-progress (WIP) levels are updated. Finally, demand is forecasted and orders are placed. Figure 1 focuses on a single supply chain echelon, but multiple echelons can be linked together to form an entire supply chain. Notice we highlight the time lag between shipping and receiving the product. If this lead time is zero, the product will be received the moment after the order is placed (at the beginning of the next period) and that receipt will influence the order placed in the next period.

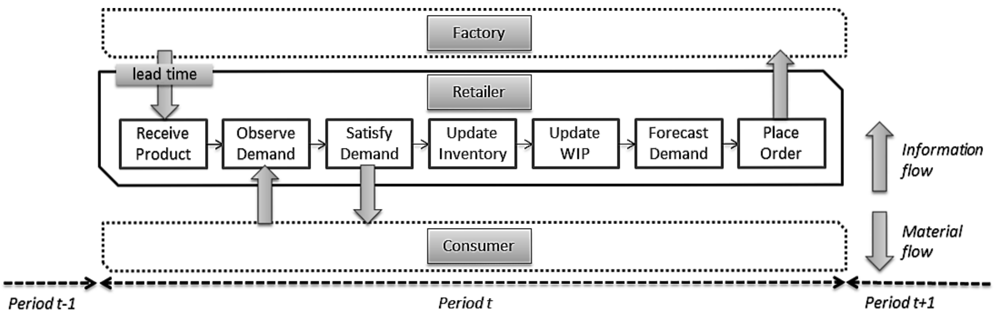


Figure 1. The sequence of events in the supply chain model.

A common replenishment algorithm is the OUT policy (Disney and Lambrecht 2008). This inventory model can be expressed in words as “let the production targets be equal to the sum of a forecast of demand ($\widehat{\mathrm{D}}_{\mathrm{t}}$), plus the discrepancy between target (TNS) and actual (NS_{t}) levels of net stock (on-hand inventory), plus the difference between the target ($\widehat{\mathrm{W}}_{\mathrm{t}}$), and actual ($\mathrm{W}_{\mathrm{t}}$) WIP (on-order inventory)". Formally, the order quantity at time t (O_{t}) is given by

$$O_t = \widehat{D}_t + (TNS - NS_t) + \left(\widehat{W}_t - W_t\right). \tag{1}$$

The receipts (R_{t}) are the orders that have been placed $\mathrm{T}_{\mathrm{P}} + 1$ periods ago, where T_{P} is the lead time,

$$R_t = O_{t-T_P-1}. \tag{2}$$

This linear model assumes that orders are always fulfilled after a fixed lead time (key assumption I). Both orders and receipts are real numbers. When orders are negative, products are returned to the supplier without cost (key assumption II). Moreover, orders are unconstrained. That is, unlimited production and distribution capacities are assumed (key assumption III).

The actual net stock (NS_{t}) is the accumulated difference between receipts and demand. The inventory balance equation is given by

$$NS_t = NS_{t-1} + R_t - D_t. \tag{3}$$

The inventory can be either positive or negative. The former refers to storage, while the latter refers to back orders. Hence, demand that is not fulfilled in a period is backlogged and will be fulfilled as soon as on-hand inventory becomes available (key assumption IV). In addition, unlimited storage capacity is assumed (key assumption V), and the model does not consider defective products, quality loss, or random yields (key assumption VI).

The WIP (W_{t}) represents the inventory in-transit between echelons and is the accumulated sum of the difference between the previous order and the current receipt.

With (2), W_t can also be modelled as the sum of the orders that have been placed but not yet received via

$$W_t = \sum_{i=1}^{T_P} O_{t-i} = W_{t-1} + O_{t-1} - R_t. \tag{4}$$

Note that when $T_P = 0$, items are received before the next order is generated, hence the WIP is zero.

Regarding the target levels, it is usual to consider that a constant safety stock, a TNS, and a variable target WIP exists (John et al. 1994). Under this approach, the TNS is a decision variable to be optimised, while the desired WIP ($\widehat{W}_t$) is a forecast of future consumption over the lead time given by

$$\widehat{W}_t = \widehat{D}_t \cdot T_P. \tag{5}$$

It is a frequent practice in inventory management literature (e.g. Schneeweiss 1974; Costas et al. 2015; Disney et al. 2016) to consider demand (D_t) to be an independent and identically distributed (i.i.d.) random variable following a normal distribution with a mean of μ_D and a standard deviation of σ_D ($D_t \sim N(\mu_D, \sigma_D)$). In this case, the minimum mean square error (MMSE) forecast of the demand for all future periods ($\widehat{D}_{t+x}$) is given by its conditional expectation (Disney et al. 2016):

$$\forall x, \widehat{D}_{t+x} = \mu_D. \tag{6}$$

Having described a linear supply chain model as a base case, we will now review its known performance in section 4 in preparation for our nonlinear supply chain study in sections 5 and 6.

4. Performance Indicators and Background in the Linear Supply Chain

In this section, we describe the main operational and financial indicators used to measure supply chain performance. In addition, we summarise the main known results for the linear model as a preliminary step to analyse the nonlinear system.

4.1. Operational Metrics

Wang and Disney (2016) provide a recent review of Bullwhip literature. They assert that this phenomenon is usually measured through the Bullwhip (BW) ratio defined as the variance of the orders divided by the variance of demand,

$$BW = \frac{\sigma_O^2}{\sigma_D^2}. \tag{7}$$

The net stock amplification (NSAmp) measure is the ratio of the variance of net stock to the variance of the demand,

$$NSAmp = \frac{\sigma_{NS}^2}{\sigma_D^2}. \tag{8}$$

Order variability mainly contributes to production costs, while inventory variability determines the echelon's ability to meet demand in a cost-effective manner. This is the key trade-off faced by supply chain managers (Disney et al. 2006), and the reason why both the BW and the NSAmp ratios are commonly used to measure supply chain performance.

Under i.i.d. demand, the OUT policy with MMSE forecast acts as a 'pass-on-orders' strategy, similar to the Kanban system often advocated by the lean production community (Disney et al. 2016). That is, the orders in each period are simply the current demand ($\mathrm{O_t} = \mathrm{D_t}$). Consequently, $\mathrm{BW} = 1$ and $\mathrm{NSAmp} = 1 + \mathrm{T_P}$.

The NSAmp ratio is commonly used as a proxy for the inventory service level in Bullwhip studies, as in linear systems inventory availability is proportional to

$\sqrt{\mathrm{NSAmp} \cdot \sigma_D{}^2}$ (Chen and Disney 2007). Furthermore, the Fill Rate, β, is decreasing in NSAmp. This is a popular metric in high-volume industries as it expresses the proportion of demand that is immediately fulfilled from net stock (Disney et al. 2015). The fulfilled demand (F_t) is calculated by

$$F_t = \begin{cases} 0 \; if \;\; D_t \leq 0; \\ 0 \; if \; R_t + NS_{t-1} \leq 0; \\ R_t + NS_{t-1} \; if \;\; 0 \leq R_t + NS_{t-1} \leq D_t; \\ D_t \; if \;\; R_t + NS_{t-1} > D_t. \end{cases} \tag{9}$$

Note that $R_t + NS_{t-1}$ is the inventory available after the deliveries have been received from the upstream node that can be used to fulfil current demand. Negative demand means that net returns from customers are larger than those delivered. Thus, β is the ratio of mean fulfilled demand to the mean of the demand that can be satisfied, $(D_t)^+$:

$$\beta = \frac{E(F_t)}{E((D_t)^+)}. \tag{10}$$

Here $\mathrm{E}(\cdot)$ is the expectation operator and $(x)^+ = \max(0, x)$ is the maximum operator. The calculation of β is complex as it is influenced by a number of variables (the lead time, safety stock, mean and standard deviation of the demand, demand correlation, and the correlation between demand and inventory) and is often only studied in linear systems (Sobel 2004; Disney et al. 2015).

4.2. Financial Metrics

Assuming the main goal of a firm is to maximise financial gain, many authors have used economic measures to assess supply chain performance. In the literature, inventory and order costs are often considered jointly in order to evaluate the performance of a replenishment strategy (Disney and Grubbström 2004).

The classic approach to modelling inventory costs (IC) is to assign a unit inventory holding cost (h) and a unit backlog cost (b) (Kahn 1987). Note that a backlog is incurred when the net stock is negative; otherwise, a holding cost is incurred. This inventory cost model can be summarised as

$$IC = h \cdot E((NS_t)^+) + b \cdot E((-NS_t)^+). \tag{11}$$

As the demand is assumed to be normally distributed, and a linear system exists, the inventory levels will also be normally distributed and the inventory costs will be governed by the TNS and minimised when

$$TNS^* = \sigma_{NS} \cdot \phi^{-\mathbf{1}}\left(\frac{b}{b+h}\right) \tag{12}$$

(Hosoda and Disney 2009). In (12), $\phi^{-\mathbf{1}}(\cdot)$ is the inverse of the cumulative distribution function of the standard normal distribution.

There is a wide range of costs functions that can be used to assign order costs (OC), see Disney et al. (2012) for a review. A common order cost function considers a certain guaranteed capacity (GC) is available in each period. If less capacity than GC is required, labour stands idle for a proportion of the period, hence an opportunity cost is incurred (n per unit). If more than GC is needed, labour works overtime at a higher unit cost (of p per unit). This model can be described by

$$OC = n \cdot E((GC - O_t)^+) + p \cdot E((O_t - GC)^+). \tag{13}$$

First order conditions reveal that to minimise (13) we should set GC to

$$GC^* = E(O_t) + \sigma_O \cdot \phi^{-\mathbf{1}}\left(\frac{p}{p+n}\right). \tag{14}$$

The minimised total costs (TC), i.e. the minimised sum of inventory and order costs, per period are linearly related to the standard deviation of orders and inventory (Kahn 1987; Disney et al. 2012) according to

$$\min(TC) = K_{NS} \cdot \sigma_{NS} + K_O \cdot \sigma_O, \tag{15}$$

where $\mathrm{K_{NS}} = \mathrm{g_1(b,h)} = \frac{\mathrm{b+h}}{\sqrt{2\pi}} \cdot \mathrm{e}^{-\mathrm{erf}^{-1}\left(\frac{2\mathrm{b}}{\mathrm{b+h}}-1\right)^2}$ and

$\mathrm{K_O} = \mathrm{g_2(n,p)} = \frac{\mathrm{n+p}}{\sqrt{2\pi}} \cdot \mathrm{e}^{-\mathrm{erf}^{-1}\left(1-\frac{2\mathrm{n}}{\mathrm{n+p}}\right)^2}$. Here, $\mathrm{erf}^{-1}(\cdot)$ is the inverse error function.

Thus, the objective function J can be formulated as a convex weighted sum of the square root of the BW and the NSAmp ratios by

$$J = K_{NSA} \cdot \sqrt{NSAmp} + K_{BW} \cdot \sqrt{BW}, \tag{16}$$

where $\mathrm{K_{NSA}} = \frac{\mathrm{K_{NS}}}{\mathrm{K_{NS}+K_O}}$ and $\mathrm{K_{BW}} = 1 - \mathrm{K_{NSA}}$ are used to express the weight (relative importance) of each indicator. If J is minimised, total costs are minimised in the linear model.

Having defined the performance measures commonly used in linear models, we will now define our capacitated supply chain model, before analysing its performance in section 6.

5. The Capacity Constrained Supply Chain: Simulation Model

In this section, we consider the impact of constrained capacity, one of the key nonlinearities highlighted in section 2. In this case, a decision variable must be added – the capacity limit (CL), defined as the maximum quantity of product that the firm can receive in each period[2]. To incorporate this nonlinearity into the OUT policy, the most common approach is to place a limit on the order quantity (Cannella et al. 2008), by replacing (1) with

$$O_t = \min\left\{\widehat{D}_t + (TNS - NS_t) + \left(\widehat{W}_t - W_t\right), CL\right\}. \tag{17}$$

With the aim of studying the impact of the capacity limit on the supply chain, we have developed a nonlinear simulation model, where (2) – (6) and (17) represent the system equations following the sequence of events in figure 1, (7) – (10) have been implemented as operational metrics, and (11), (13), (15) and (16) have been used as financial indicators.

We have employed MATLAB R2014b to construct a simulation model, whose parametric space is formed by ten factors that can be categorized into: (a) Three external parameters: mean and standard deviation of the demand, and backlog unit cost, (b) Four internal parameters: lead time; and holding, opportunity and over-time unit costs, and (c) Three decision variables: target net stock, guaranteed capacity, and the capacity limit.

Hence, the general problem can be defined as

$$[BW, NSAmp, \beta, J, TC] = f(\mu_D, \sigma_D, T_P, TNS, CL, GC, b, h, n, p) + \xi, \tag{18}$$

where ξ is the unexplained part of the response.

6. Analysis of the Capacity Constrained Supply Chain

Having defined our capacitated supply chain model, we now consider its dynamic performance in section 6.1, its operational (stochastic) performance in section 6.2, and its financial performance in section 6.3.

6.1. Dynamic Analysis of the Step Response

This section compares the step response of the capacity constrained supply chain to the linear supply chain in order to expose the differences in the two scenarios. This classic *rich picture* provides one with a firm understanding of dynamic behaviour (Disney and Towill 2003). Note that in this step demand case, the forecast $\widehat{\mathrm{D}}_{\mathrm{t}} = \mathrm{D}_{\mathrm{t}}$, as this exaggerates the differences in the dynamic performance, facilitating our discussion.

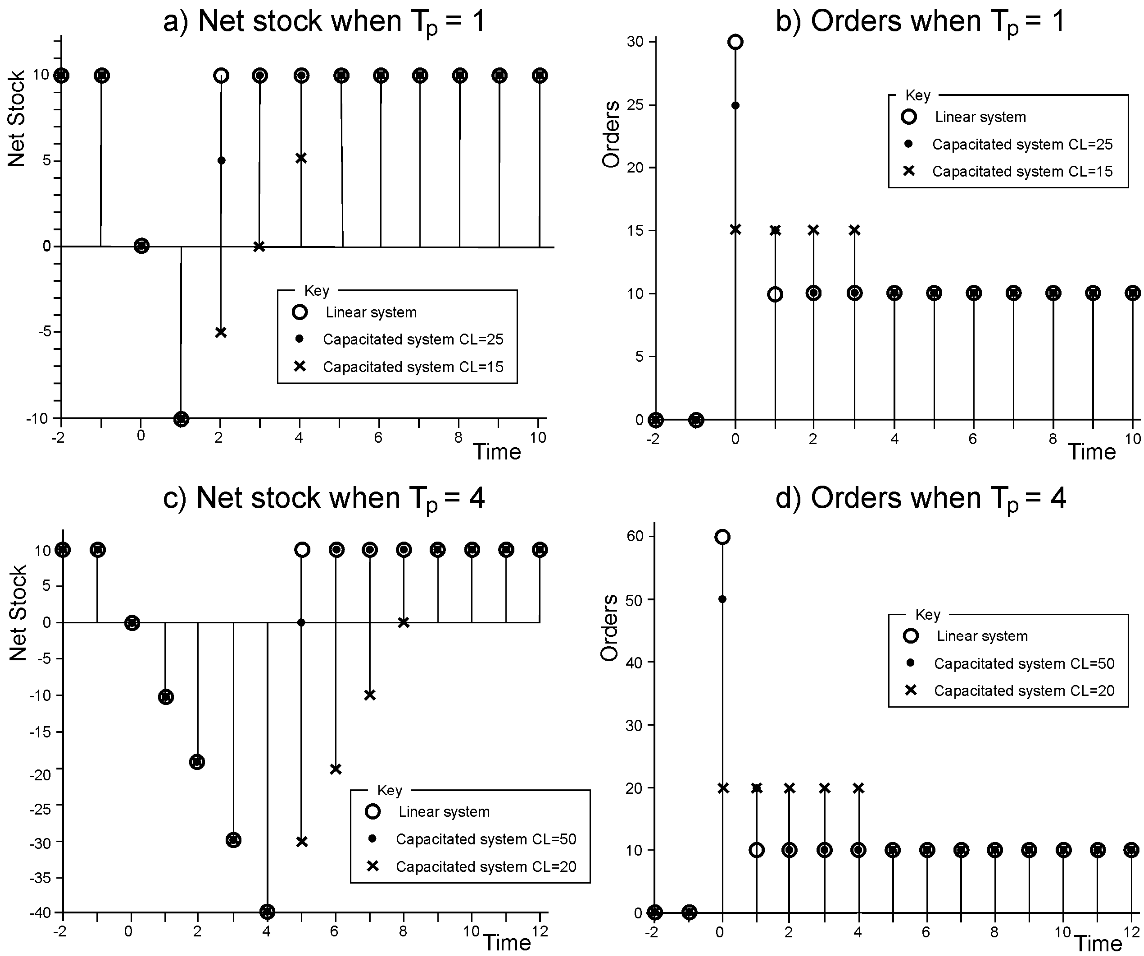


Figure 2. Step response (net stock and orders) for the linear and the capacitated system.

Figure 2 depicts the inventory and order response of both the linear and nonlinear systems when facing a 10-unit step in demand at time $t = 0$. The outputs have been obtained for $TNS = 10$, while the lead time has been set to $T_P = 1$ and $T_P = 4$. In each case, we have used two different values of the capacity limit in order to study the effect of this variable. The inventory and order response of the linear OUT policy quickly returns to steady state at the expense of generating the Bullwhip Effect. Notice for $T_P = 1$ the largest order is 30, while for $T_P = 4$ it is 60. For this reason, the capacity limit has been set to 15 and 25 when $T_P = 1$, and 20 and 50 when $T_P = 4$, in the nonlinear system.

It can be seen from figures 2b and 2d that the difference between the linear orders and the capacity limit at t=0 are added to the nonlinear orders in subsequent periods. Under these circumstances, we can imagine the BW ratio (and order costs) would be desirably reduced if demand was instead a random variable rather than a step input.

Figures 2a and 2c show the negative impact of capacity constraints on the inventory. The inventory level takes much longer to return to the steady state in the constrained supply chain. As the inventory level is lower in this transient phase, we expect the supply chain is more vulnerable to demand shocks and may suffer from reduced Fill Rates.

We conclude that the capacity constraint acts to reduce the BW ratio at the expense of increasing the NSAmp ratio and decreasing the Fill Rate. The more severe the capacity constraint, the more accentuated these effects become (however, $\mathrm{CL} > \mu_\mathrm{D}$ is required to ensure that the system is stable). This insight is consistent with prior research (Cannella et al. 2008, Nepal et al. 2012, Lin et al. 2014). In the following subsections, we aim to further explore the impact of the capacity constraints via the system response to a random demand and an economic analysis.

6.2. The Operational Impact of Capacity Constraints

This section analyses the consequences of the capacity limit via the Bullwhip Effect and the Fill Rate[3]. Note, since the cost parameters and the guaranteed capacity do not have an impact on these indicators, the simulation problem in (18) can be simplified to $[\mathrm{BW}, \mathrm{NSAmp}, \beta] = \mathrm{f}(\mu_\mathrm{D}, \sigma_\mathrm{D}, \mathrm{T_P}, \mathrm{TNS}, \mathrm{CL}) + \xi$.

In the linear system, for the OUT policy under i.i.d. demand with MMSE forecasting, $\mathrm{BW} = 1$ and $(\mathrm{NSAmp} - \mathrm{T_P}) = 1$ (Disney et al., 2016). That is, these indicators do not depend on the variables $(\mu_\mathrm{D}, \sigma, \mathrm{T_P}, \mathrm{TNS})$. In order to explore the behaviour of these indicators, we first wonder whether these relationships hold true in the nonlinear environment. Preliminary tests revealed the following hypothesis for the capacitated model:

Hypothesis 1. T_P *and* TNS *have no significant effect on* BW *and* $(NSAmp - T_P)$.

To investigate this premise, we used a full factorial design of experiments (DoE) with two levels per factor ($\mu_D = \{100, 200\}, \sigma_D = \{20, 80\}, T_P = \{1, 4\}, TNS = \{10, 50\}, CL = \{210, 300\}$). We chose this parameter set as it covers a wide range of supply chain settings (e.g. the coefficient of variation $CoV = \sigma_D/\mu_D$ varies between 10% and 80%, and the coefficient of capacity $CoC = CL/\mu_D$ varies between 1.05 and 3).

Statistically analyzing the 32 runs reveals the p-value associated with both variables (T_P and TNS) in the parameter estimates of a least-squares fit is significantly higher than 0.05 (even close to 1) for both outputs, see table 1. This means that the impact of T_P and TNS on BW and $(NSAmp - T_p)$ can be ignored.

This allows us to study BW and $(NSAmp - T_P)$ as a function of the remaining three factors (μ_D, σ, CL). Since both the standard deviation and the capacity limit can be related to the mean demand – by means of the coefficient of variation $(CoV = \sigma_D/\mu_D)$ and the coefficient of capacity $(CoC = CL/\mu_D)$ – further experimentation led us to consider if these could be combined to reduce the problem to two dimensions. We developed the following hypothesis:

Hypothesis 2. BW *and* $(NSAmp - T_P)$ *can be expressed as a function of CoC and CoV.*

To study this premise, we followed the same methodology we used to test Hypothesis 1. We laid out a full factorial DoE with three factors and three levels ($\mu_D = \{100, 200, 400\}$, $CoC = \{1.1, 1.2, 2\}$, $CoV = \{0.1, 0.2, 0.4\}$). In this case, the p-value of μ_D obtained for both indicators is greater than 0.05, see table 1, leading us to fail to reject H2 at 5% significance. This shows that the impact of μ_D is not significant when the ratios CoC and CoV are considered.

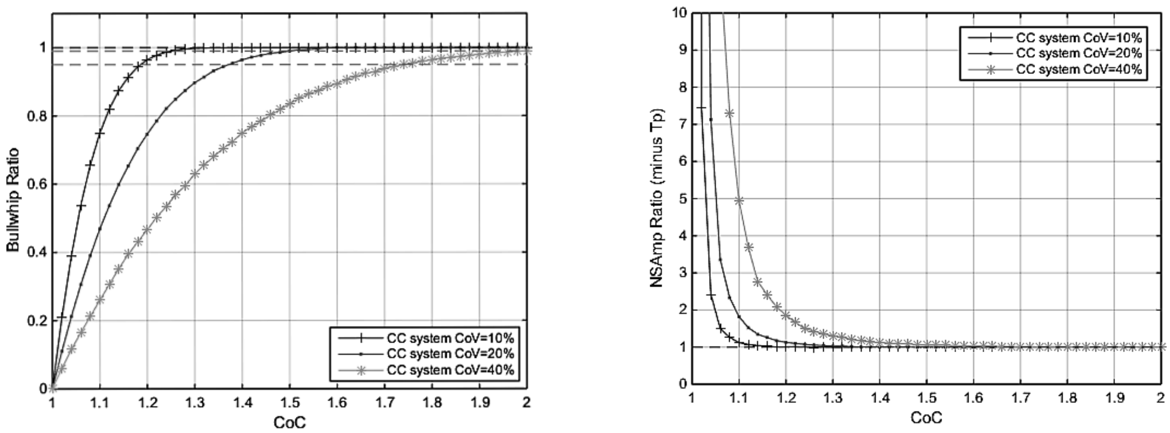
Contrast	**Factor**	**F (*BW*)**	**p-v (*BW*)**	**F (*NSAmp**)**	**p-v (*NSAmp**)**
Hypothesis 1	*Intercept*	5.677	0.025**	1.041	0.317
	μ_D	36.406	0.000**	7.669	0.010**
	σ_D	7.348	0.012**	6.215	0.019**
	T_P	0.000	**0.997**	0.001	**0.973**
	TNS	0.000	**0.994**	0.003	**0.956**
	CL	33.140	0.000**	7.632	0.010**
	Overall Model	15.379	0.000**	4.304	0.006**
Hypothesis 2	*Intercept*	1.676	0.208	16.060	0.001**
	μ_D	0.000	**0.985**	0.006	**0.938**
	CoC	43.173	0.000**	17.164	0.000**
	CoV	104.049	0.000**	16.862	0.000**
	Overall Model	49.074	0.000**	11.494	0.000**

Note (*): We refer to the difference between the NSAmp ratio and T_P, as in the linear model.

(**): The significant factors at the confidence level 95% are highlighted.

Table 1. Statistical analysis of the results of the DoEs performed to verify Hypotheses 1 and 2.

In line with RQ1, the results from H1 and H2 reveal that, in the capacity constrained supply chain, BW and $(\mathrm{NSAmp} - \mathrm{T_P})$ can be considered to be a function of CoC and CoV, i.e. $\mathrm{BW} = \mathrm{f_1(CoC, CoV)}$ and $(\mathrm{NSAmp} - \mathrm{T_P}) = \mathrm{f_2(CoC, CoV)}$. Thus, from this point on we investigate the nonlinear supply chain for different values of CoC and CoV. Figures 3a and 3b displays the BW and NSAmp as functions of both coefficients[4]. In each graph, the abscissa represents the CoC ($\mathrm{CoC} \geq 1$ is required for stability). We have plotted the curves for three different values of the CoV: 10%, 20%, and 40%, as this parameter is typically less than 50% in practice (Dejonckheere et al. 2003).



(a) BW ratio versus CoC and CoV, with dashed lines in BW={0.95, 0.99, 1}.

(b) ($\mathrm{NSAmp} - \mathrm{T_P}$) versus CoC and CoV, with dashed lines in ($\mathrm{NSAmp} - \mathrm{T_P}$)=1.

Figure 3. BW and NSAmp functions in the capacity constrained system.

Figure 3a reveals that the nonlinear model response tends to that of the linear model – that is, BW=1 and $(\mathrm{NSAmp} - \mathrm{T_P}) = 1$ – when the capacity limit becomes sufficiently large. If the capacity frequently constrains the orders, Bullwhip is reduced. Figure 3a also shows that the BW ratio approaches zero when $\mathrm{CL} = \mu_\mathrm{D}$ and increases exponentially in the CoC. Notice, larger CoV lead to greater reductions in Bullwhip. Figure 3b shows that the capacity limit reduces the order variance, but increases the inventory variability (the greater the CoV, the greater the increase in inventory variance), which threatens the inventory service level. The trade-off between the BW and the NSAmp ratios suggests that an optimal capacity level exists, which we will explore economically in section 6.3.

As seen, when the capacity does not constrain the system, the linear assumption can be maintained. Thus, in order to investigate RQ4, we introduce the concept of a *settling capacity,* defined as the least capacity that allows us to assume the behaviour of the nonlinear system can be predicted by the results from linear systems with a certain level of confidence (95% or 99%). Figure 3a suggests that the settling capacity is approximately linear in the CoV leading to the following expression for the settling capacity, CL_s,

$$CL_s(95\%) \approx \mu_D + 1.86 \cdot \sigma_D, \tag{19}$$

$$CL_s(99\%) \approx \mu_D + 2.41 \cdot \sigma_D, \tag{20}$$

for normal i.i.d. demand with MMSE forecasting[5]. If we use linear knowledge in a constrained supply chain, where the available capacity is lower than the settling capacity, we may incur significant approximation errors.

Having studied the internal performance, we now focus on the external impact of capacity constraints by evaluating the Fill Rate. Figure 4 shows the consequences of the capacity limits on the Fill Rate. The Fill Rate is highly influenced by T_P and TNS in the linear system and we have selected $T_P = 1$ and $TNS = 0$ in these graphs.

Completing the analysis of RQ1, figure 4 shows how capacity constraints reduce Fill Rates revealing that when the CoV increases, the Fill Rate decreases and becomes more sensitive to the capacity limit. It can be seen that when $CL > CL_s$ the linear assumption can be maintained. However, when $CL < CL_s$, the impact of the nonlinearity must be considered as the Fill Rate can be seriously affected.

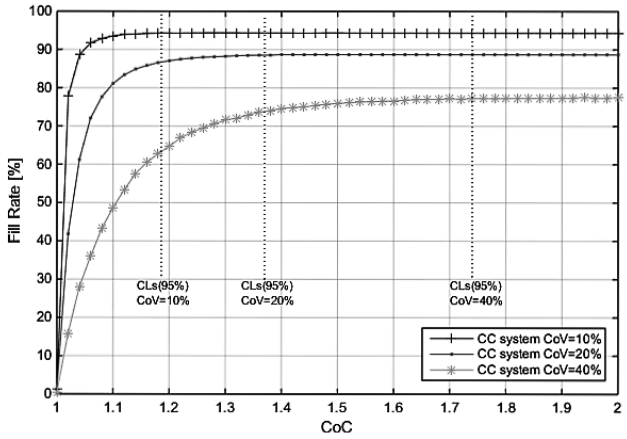


Figure 4. Fill Rate in the capacitated system for T_P=1 and TNS=0. The vertical dotted lines represent the settling capacity (95% level of confidence) for the three CoVs.

The impact of the capacity constraints on the Fill Rate is consistent with its impact on the NSAmp ratio. Nonetheless, we must statistically confirm whether (similar to the linear system) reducing the NSAmp ratio also leads to increased Fill Rates in this nonlinear system. Thus, the following hypothesis has been developed for RQ2:

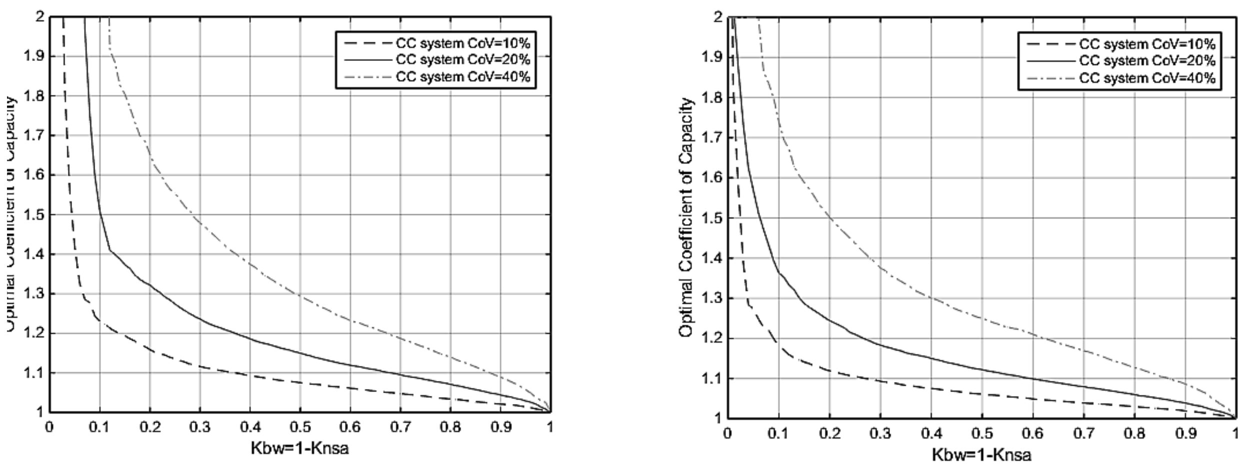
Hypothesis 3. There is a negative correlation between NSAmp and the Fill Rate.

To validate this hypothesis, as the normality of the continuous inventory variable cannot be verified, a Spearman correlation test was carried out. The p-value was found to be 0.000, hence the idea that the correlation is due to random sampling can be rejected: we can assume a correlation exists. Spearman's rho (-0.810) shows that this correlation is negative. That is, in the capacity constrained supply chain where backlogs are allowed, reducing the NSAmp ratio also increases the Fill Rate.

6.3. The Economic Impact of Capacity Constraints

This section explores the existence of an optimal capacity – the focus of RQ3. When using the optimal values for TNS and GC from (12) and (14), figures 5a (for $T_P = 1$) and 5b (for $T_P = 4$) represent the CoC that minimises the objective function J in the capacitated system over the whole range of permissible $K_{BW} = 1 - K_{NSA} \in [0,1]$ for the three different values of the CoV[6].

As expected, the BW ratio (J when $K_{BW} = 1$) is minimized when CoC = 1, i.e. $CL = \mu_D$. On the other hand, the NSAmp ratio ($K_{BW} = 0$) is minimised when $CL = \infty$. Between these points, the optimal CoC can be found, which is decreasing in K_{BW}, increasing in CoV, and decreasing in T_P, see figure 5. For example, if the supply chain demand follows a normal distribution with $\sigma_D = 100$, $\mu_D = 20$ (CoV $= 20\%$), and $K_{BW} = 0.3$ the optimal capacity limit is approximately is 123 (CoC ≈ 1.23 from figure 5a) if $T_P = 1$ and 118 (CoC ≈ 1.18 from figure 5b) when $T_P = 4$.



(a) For T_P=1. (b) For T_P=4.

Figure 5. Optimal CoC in terms of J in the *capacity* constrained system.

Under these circumstances, they key question is: Does minimising J lead to minimising the economic metric TC in the capacitated system? To understand this, we conducted another simulation study focused on supply chain costs. This leads to the following hypothesis, in line with RQ2, being revealed:

Hypothesis 4. Optimising J leads to minimising TC.

We have explored different scenarios based on sets of values of the relative weight of the BW ratio (K_{BW}, which depends on the four unit costs), the lead time (T_P), and the coefficient of variation (CoV, by varying the standard deviation of the demand while $\mu_D = 100$). With the aim of analysing this premise, we developed a full factorial DoE with three levels per factor ($K_{BW} = \{0.25, 0.5, 0.75\}, T_P = \{1,2,4\}, \mathrm{CoV} = \{10\%, 20\%, 40\%\}$)[7], which we assume adequately covers the parameter space.

We have simulated the resultant 27 different scenarios to capture both the CoC that minimises J and the CoC that minimises TC. With these data, a Wilcoxon signed-rank test (as normality is not necessary for the dependent variables) was performed to evaluate the difference between these paired observations. The p-value obtained was 0.000 in both cases, so we clearly reject the null hypothesis (equality of means). Therefore, we cannot assume that, as in the linear system, the CoC that minimises J also minimises TC. Nevertheless, the confidence interval (95%) for the difference between both values is (0.0062, 0.0187). Thus, even though we have not found the same relationship that exists in the linear system, the difference between both CoCs is rather small. Figure 6 represents the 27 scenarios in a scatter plot that represents both CoCs (note that the dash-dotted line shows the equality). This situation allows us to consider the optimal CoC from figure 5 to be a good approximation to minimise the sum of inventory and order costs depending on the demand, the unit costs, and the lead time.

As an example, figures 7a, 7c, and 7e plot J and TC obtained from simulation of three scenarios defined by: ($K_{BW} = 0.25,\ T_P = 1, \mathrm{CoV} = 20\%$), ($K_{BW} = 0.5, T_P = 2, \mathrm{CoV} = 40\%$), and ($K_{BW} = 0.75, T_P = 4, \mathrm{CoV} = 10\%$)[8]. As previously demonstrated, the CoC that minimises J tends to be slightly lower than the CoC that minimises TC. Notice that the corresponding TNS and GC have also been provided in figures 7b, 7d, and 7f.

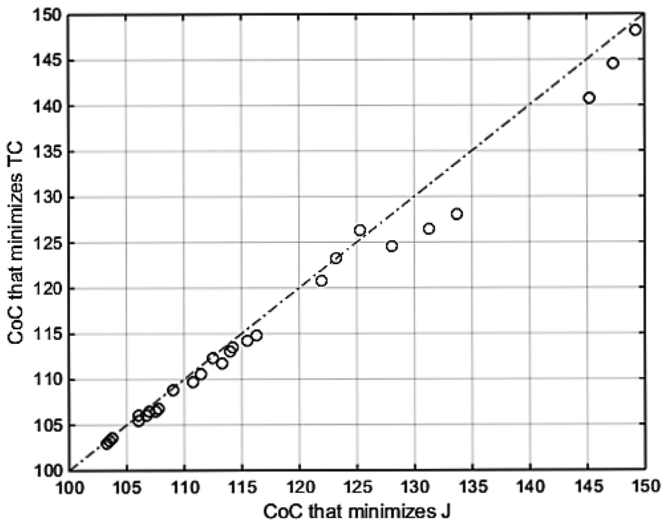
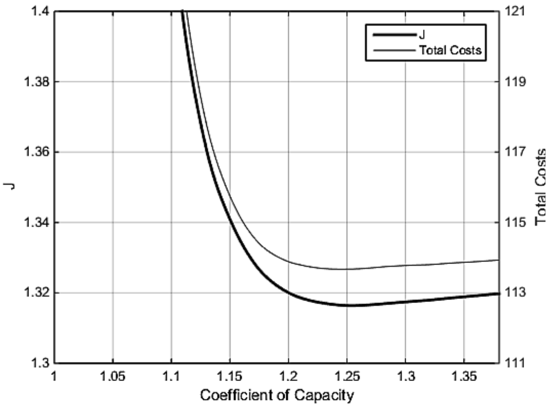


Figure 6. The relationship between the optimal CoC in terms of J and in terms of TC in the capacity constrained system.

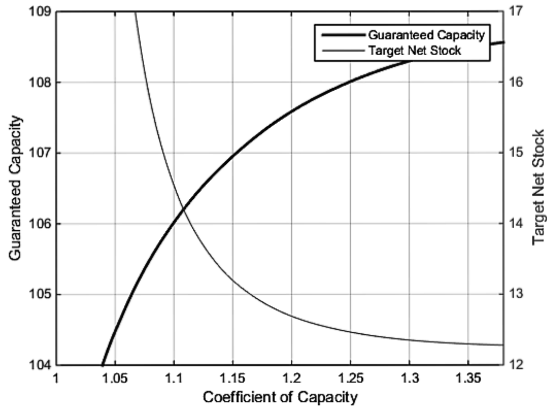
These results suggest that capacity limitations stop unnecessarily large orders being issued and this has some economic value. The decrease in the Fill Rate can be outweighed by the consequences of improving the operational performance of the supply chain. Furthermore, higher values of K_{BW} lead to greater percentage reductions in costs as the main advantage of constraining orders is to alleviate the Bullwhip phenomenon.

Importantly figures 7a, 7c, and 7e – together with the remaining scenarios – confirm the existence of an optimal capacity limit in the capacitated supply chain. That is, selecting an appropriate capacity reduces total costs and the unconstrained system is bettered (recall the performance of the linear system can be ascertained by letting $CL \rightarrow \infty$). For example, in figure 7e the costs in the linear supply chain are approximately 57, in the nonlinear system these can be reduced to approximately 42.5, a 25% decrease.

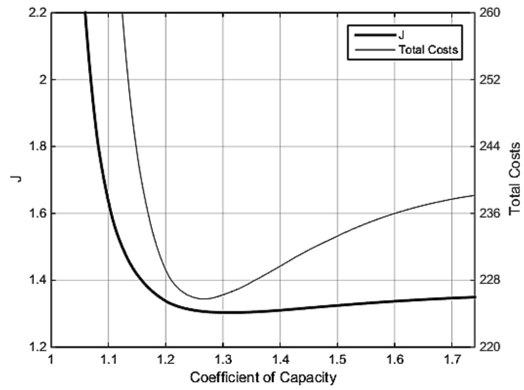
Ponte, B., Wang, X., de la Fuente, D., and Disney, S.M., (2017), "Exploring nonlinear supply chains: The dynamic impact of capacity constraints", International Journal of Production Research, 55 (14), 4053–4067. DOI: 10.1080/00207543.2016.1245884. ISSN: 0020-7543.



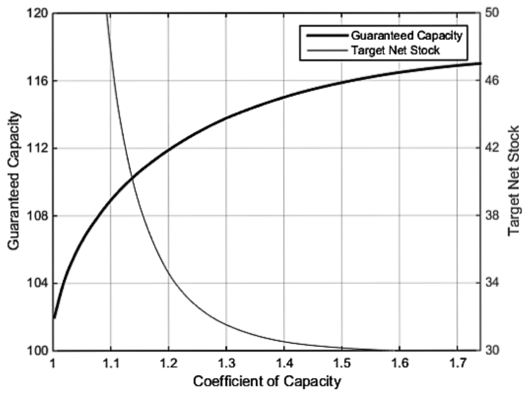
(a) J and TC for K_{BW}=0.25.



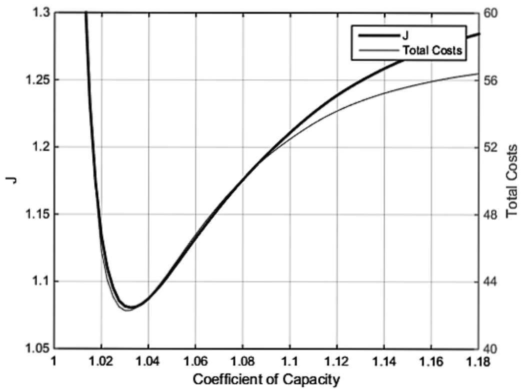
(b) TNS and GC for K_{BW}=0.25.



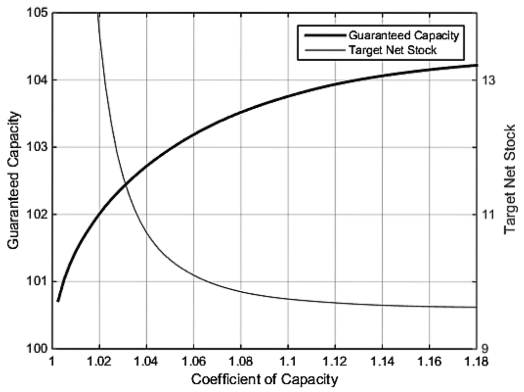
(c) J and TC for K_{BW}=0.5.



(d) TNS and GC for K_{BW}=0.5.



(e) J and TC for K_{BW}=0.75.



(f) TNS and GC for K_{BW}=0.75.

Figure 7. Economics of the capacitated system.

7. Conclusions and Managerial Implications

We have considered the impact of capacity constraints on supply chain performance, a problem largely untouched by the literature due to the complexity of the analysis involved. The supply chain we explored used the OUT replenishment policy and MMSE forecasting to satisfy an i.i.d. demand drawn from a normal distribution. For the four research questions we posed in the introduction, we found the following answers:

RQ1. Capacity constraints help to reduce the Bullwhip Effect as they have a smoothing effect on the orders. However, they increase inventory variability and reduce the achieved Fill Rate. In the supply chain scenario analysed, both the BW and (NSAmp – T_P) can be expressed as functions depending exclusively on the CoV and on the CoC; i.e. they are largely unaffected by the lead time, safety stock, and the mean demand. We have provided a graphical relationship for these dependencies.

RQ2. Similar to the linear supply chain, we have verified that reducing the NSAmp ratio leads to improving the Fill Rate in the capacitated system. The BW ratio has also proven to be a powerful indicator of the economic consequences of order variability. However, in contrast to a linear system, an optimisation based on a convex weighted sum of the standard deviation of order and inventory does not result in minimal costs in the capacity constrained system. Nonetheless, the difference between both the optimum linear system and the non-linear system has proven to be small.

RQ3. We have demonstrated that there is an optimal capacity limit that balances Bullwhip (capacity) and inventory costs. For random demand, the optimal capacity limit depends on the first and second moments of demand, a function of the inventory holding and backlog costs, and the normal and overtime production costs. We have provided an optimal approximation for the optimal coefficient of capacity that accounts for the lead time and the coefficient of variation.

RQ4. The estimated settling capacity gives an indication of when managers can safely use the results from a linear analysis with a certain degree of confidence.

Managerially, we have revealed some practical benefits to capacity constraints exist. Capacity limits can prevent unnecessarily large orders being issued, mitigate the Bullwhip phenomenon and reduce order costs. This reduction in order variability occurs at the expense of incrementing the net stock variance; consequently, decreasing the Fill Rate and increasing inventory costs. In this regard, our investigation revealed the existence of an optimal capacity in the capacitated system which may significantly outperform the linear system. Thus, supply chain managers can enjoy significant savings by viewing the capacity as a decision variable. We also provided an approximate solution for the optimal capacity depending upon the unit costs, the lead-time and the demand variability. This optimal capacity has shown to be decreasing in the relative importance of the variance of orders in comparison with the variance of inventory, increasing in the demand coefficient of variation and decreasing in the lead time.

Further research could be conducted to explore this nonlinear system since the results from linear models can be perilously inaccurate in some settings, perhaps with the nonlinear control theory techniques that have recently been applied to supply chains, see Spiegler et al. (2016a, 2016b), or with Markovian decision processes, see Li et al. (2016). Another line of future work could consider the impact of capacity constraints when the proportional OUT policy (Disney et al. 2016) is used to place replenishment orders. Finally, the study of other nonlinearities, such as non-negative orders (forbidden returns) and non-negative inventories (lost sales), may be worthy of further research. Our experience suggests that inventory restrictions lead to rich dynamic behaviours.

Notes

1. We use the term 'nonlinear supply chain models' to refer to supply chain models that include at least one nonlinearity. Herein we specifically studied the nonlinearity introduced by placing an upper limit on the order quantity.
2. Note the difference between the decisions variables CL and GC. CL is a capacity limit that cannot be surpassed by the order rate at any time (i.e. $\forall$t, CL$\geq O_t$). GC is the guaranteed capacity which can be exceeded by working overtime or by using a subcontractor at a higher cost. Naturally, CL≥GC.
3. Sections 6.2 and 6.3 are derived from simulating 201,000 time periods. The first 1,000 periods were discarded in order to avoid any possible impact of initial conditions. Stability of the response and consistency of the results were checked via Individual and Range control charts and Levene's test for equality of variance after running two repetitions of each test. The model has been validated by using known results from literature for the linear model (i.e. the nonlinear model when CL=∞).
4. In figures 3a, 3b, and 4 CoC increases in increments of 0.02.
5. To obtain (19) and (20), we used regression to approximate the relationship between BW and CoV (BW was chosen as we noticed this was the most sensitive of the three measures, see figures 3a, 3b, and 4). After testing several alternatives, we found that $BW \approx 1 - e^{\mu \cdot CoC_a}(1 + \sin(\varphi \cdot CoC_a)$, where $CoC_a = 1 - CoC$, and $\mu = \{-19.6271, -3.70803, -1.66262\}$ and $\varphi = \{8.76281, -2.42235, -1.32266\}$ when $CoV = \{10\%, 20\%, 40\%\}$, respectively. We then set $BW = \{0.95, 0.99\}$ and solved for CoC_a for each of the three CoV levels. Using regression again we then found the following linear relationships were a good approximation: $CoC(0.95) \approx 1 + 1.85905 \cdot CoV$, and $CoC(0.99) \approx 1 + 2.41286 \cdot CoV$. Finally, multiplying CoC throughout by μ_D provides (19) and (20).
6. The convex function J has been obtained in the capacity constrained system via simulation.
7. We have employed the following combinations of costs: for K_{BW}=0.25, {b=6, h=3, n=1, p=2}; for K_{BW}=0.5, {b=4, h=2, n=2, p=4}; and for K_{BW}=0.75, {b=2, h=1, n=3, p=6}.
8. Figures 6a, 6b, 6c, 6d, 6e, and 6f span from $\mathrm{CL} = \mu_D$ to $\mathrm{CL} = \mathrm{CL_s}(95\%)$. These graphs have been obtained via simulation, where CL increases in increments of 0.25. Each point plotted is the average of ten simulation runs.

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