Small Open Economy DSGE Model

with a

Banking Industry

Modelled on

the Financial Intermediation Approach –

Evaluating the Performance on UK Data

By

Ivona Videnova

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Cardiff Business School,
Cardiff University

Thesis Supervisors:
Prof. Patrick Minford (Primary)
Dr. David Meenagh (Secondary)

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DEDICATION

I would like to dedicate my work to my family who never stopped believing in my abilities, who were always there to pick me up when I was feeling down, who loved and supported me unconditionally through the biggest emotional rollercoaster of my life.

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ABSTRACT

This thesis proposes a new open economy DSGE RBC model with financial intermediation. The objective is to provide a general equilibrium model that can simultaneously account for the behaviour of output and interest rate spreads by solely focusing on the real side of the economy. The standard open economy model is extended to incorporate banking industry, modelled on the production approach, and foreign debt elastic interest rate, which removes the model’s nonsationary features. The model’s ability to replicate the data is tested using indirect inference method on both stationary and nonstationary UK data. The same algorithm is used to estimate the parameter values using both types of time series data. This thesis provides the first estimates for the labour share in loan production and elasticity of foreign interest rates with respect to foreign debt obligations for UK economy. The model was retested using the parameter estimates. The results indicate that the proposed framework is able to account for the joint behaviour of output, the interest rate spread and the interest on foreign debt but it was rejected on other endogenous variables.
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CHAPTER 1

INTRODUCTION

Over the years, business cycle literature has evolved tremendously. The canonical work of Kydland and Prescott (1982) – “Time to Build and Aggregate Fluctuations”, spurred an entire branch of macroeconomics. Despite scepticism by some researchers, the race to identify drivers of the cyclical variability of economic aggregates led to the development of various niches in this field. In the early stages the proposed models focused on the dynamics of the endogenous variables within a closed economy set up. Some of the augmentations considered were multiple sectors, government authority, central bank and monetary policy, information asymmetry, heterogeneous agents, adjustment costs, and labour market frictions. Each of these modifications led to an improvement in the ability of real business cycle models to conform with the stylised facts present in the data. A general consensus began emerging – although the technological shock is an important factor of business cycle dynamics, it is insufficient to explain the volatility and comovements of macroeconomic variables.

The closed economy models can highlight important features in an economy. However, in the case of small countries relative to the rest of the world, they provide
only part of the story. This has led to the extension of the real business cycle model to include foreign factors, first proposed by Mendoza (1991). In addition to the various features that could be modelled in the closed economy case, the open economy option allows for the analysis of the current account, capital account, trade balance, exchange rate regimes, international financial markets, sovereign debt and the impact of world prices and interest rates. The inclusion of each of these features provides a broader view over the economy. In addition, these factors can be modelled as another source of disturbance that would improve the model’s ability to match the data. As empirical evidence suggests two of the major sources of disturbance in relatively small countries are world prices and interest rates.

Open economy models could be considered universal in a sense that they could be used to analyse both developed and emerging markets, provided that the economy is small enough so that it could not influence the rest of the world. Alternatively, better modelling choice are international business cycle models. Since most countries fall within the first category, open economy models are more widely applicable. Although a lot has been done in this area, these models have their limitations and thus there are areas to investigate. Once such area is the effect of the introduction of financial intermediary modelled on the production approach. This feature has been analysed in a closed economy case and the results are promising. Thus, it would be interesting to see how the inclusion of a banking firm as a sec-
ond sector would affect the dynamic properties of the open economy model. More precisely what would be the effect of technological shocks in the financial industry on the rest of the economy and is the model able to account for the joint behaviour of output and interest rates?

To answer this question a small open economy was constructed in the manner of Meenagh et al. (2005) which was augmented by an external debt elastic interest rate as suggested by Schmitt-Grohe and Uribe (2003) and extended to include a second sector that acts as a financial intermediary between the representative agent and the goods producing sector.

In order to highlight the way this thesis would fit in the existing body of research, a review of the relevant literature is presented. A detailed description of the proposed dynamic stochastic general equilibrium model is also provided. The assumptions made, functional forms used, the manner in which different segments of the model are combined as well as the limitations of the proposed framework will be discussed. The model is calibrated on UK data and the dynamic responses of the endogenous variables to a temporary one percent increase in the exogenous shocks will be discussed. Particular attention is drawn to the impact of a positive technological shock in the financial industry. The analysis suggests that a temporary increase in productivity of the bank would increase loan production and investment and raise output. Both interest rates on deposits and loans would
increase but the effect on the former is larger. This would result in a drop in the interest rate spread, defined as the difference between the loan and deposit rates. These results conform with economic intuition. Furthermore, the countercyclical properties of the interest rate spread are confirmed by empirical evidence as can be seen from figure 1.1.

In order to determine whether the model can generate the patterns observed in the time series data, it is tested using indirect inference method. By fitting an auxiliary model on simulated, implied by the model, and actual UK data, a Wald statistic can be constructed. Based on the similarity of the actual and artificial data, the Wald test could reject or not reject the null hypothesis that the model is
a good representation of the true data generating process. Using indirect inference, the joint behaviour of output, interest rate spread and foreign debt interest rate, indicated by the model and by the historical data is compared. The test results based on initial parameter values reject the model and the outcome is the same regardless of the type of time series data that is used.

The indirect inference approach is also used to estimate the parameters of the model. This is one of the main contributions of the thesis since two of the parameters have never been estimated using UK data. These are the labour share of output in the loan production function and the elasticity of the interest rate on foreign debt with respect to deviations of foreign debt from its steady state value. The estimated values using stationary (nonstationary) data are 0.077 (0.0437) for the labour share in the financial intermediary sector and 0.0049 (0.0097) for the interest elasticity.

The best estimates for the parameter values are used to perform a robustness check by using different endogenous variables in the auxiliary model. The results from this exercise are mixed but they represent a promising beginning. The model is not rejected under the null when variables on interest rates are included. However, one of the statistics demonstrates a strong rejection, namely when capital is used in the model, which raises a question regarding one of the assumptions. Possible solutions to this issues were proposed but these are left for future research.
The thesis is divided as follows. Chapter 2 presents the relevant literature. It traces the evolution of RBC models from their infancy in section one through to the development of open economy models in section two. Section three focuses on closing methods used to induce stationarity in this type of models. The last section discusses the literature regarding the production approach used to model the financial intermediary. Chapter 3 presents the model, the choice of initial parameter values, methods that could be used to filter the data and the impulse response functions generated by the model. Chapter 4 begins by describing the indirect inference method and provides the empirical results from the test using initial and estimated parameter values using filtered data. A robustness check is also provided. Chapter 5 repeats the same analysis from Chapter 4 using nonstationary data. Chapter 6 concludes and provides possible avenues for future research.
2.1 The Backbone of RBC Models

In 1982 Kydland and Prescott published their canonical work ‘Time to build aggregate fluctuations’. The research was based on Solow (1956, 1957) who built a theory of economic growth driven by an exogenous technological change but did not include the consumer side of the problem. This was done by Cass (1965) and Koopmans (1965) who based their works on Ramsey (1928); however, the frameworks had no fluctuations. Kydland and Prescott (1982) developed a model that simultaneously was able to mimic several properties of the data: the cyclical variation, the comovements between GDP and the other variables, and the autocovariance in real output. The non-time-separability of the utility function assumed could not be rejected by Altug (1989). The necessity to account for the uncertainty in the state of technology has also been stressed by Prescott (1986a).
Given that the original model does not take into account government policies, neither fiscal nor monetary, it performs well. The TFP shock was extracted using the Solow residual described in Solow (1957). This potentially could create bias in the results since the TFP residual is prone to overestimate the variance due to high persistence.

Prescott (1986a) has argued that a more appropriate way to estimate the TFP shock is to approximate it to a random walk with drift plus a serially uncorrelated measurement error. In this way the measurement takes into account the high persistence of productivity shocks.

Despite the novelty, the model has limitations. The assumptions that there are no government, money, and/or frictions in the labour or capital markets prevents the analysis of any government intervention to stabilise the economy and this may lead to counterfactual results. One of the strong opponents of the RBC framework has been Summers (1986) who stated that the model is based on irrelevant structures and ill-defined concepts that lack scientific support. This harsh criticism is not without merits. Summers (1986) has questioned the accuracy of the parameters and the fact that prices are not included. In response, Prescott (1986b) has stated that in the perfectly competitive equilibrium framework everything is regarded in terms of relative prices. The last remark made by Summers (1986) concerns the exchange rate mechanism. In small economies, in particular,
partial breakdowns inevitably result in cyclical volatility of the nominal and real economy. This is why the model should be created in a way to take into account the country’s interaction with the rest of the world. This could be achieved by either using an open economy RBC model or an international RBC model. This criticism is particularly valid in the case of the UK. Although the UK is a developed country with strong service sectors, particularly education and finance, its output is very small compared to the rest of the world. In 2014 the fraction of the world’s GDP that could be attributed to the UK was 4.82%, TrendingEconomics (2016). This statistic, although impressive in the real world, in a stylised framework represents a very small fraction. Given the importance of foreign trade and international financial markets, restricting the model to a closed economy would inevitably affect the ability of the model to conform with the data.

Summers was not the only one who expressed scepticism regarding the assumption and performance of the original model. The heart of the RBC literature, the technological shock, has been subject to scrutiny by many economists. The source, nature and size of the exogenous disturbances have been questioned numerous times.

Although Mankiw (1989) acknowledged that without the TFP shock, the model would not be able to reproduce the slight procyclical movement of the wage rate, he argued that there is little evidence of some important adverse technologi-
cal shock. Summers (1986) concurred the above mentioned argument and stressed that the forces driving the cyclical fluctuations are not well defined, and lack micro evidence. However, an argument can be made that there is not sufficient evidence to the contrary. Hartley, Hoover and Salyer (1997) have also expressed their scepticism and stated that the TFP is whatever it has to be to make the real business cycle model conform with the stylised facts observed in the data. The conjecture was reiterated by the work of Cogley and Nason (1995) and McCallum (1988). The latter questioned not only the nature and source of the volatility but also its size, arguing that if it reflects the state of knowledge the aggregate volatility should be small due to the existence of multiple independent shocks across different industries that are not all positively correlated. The only exceptions to this criticism are shocks to the oil prices which affect the cost of production in every industry. This was concurred by Mankiw (1989) who argued that it is necessary to have only a few industries that are subject to large disturbances.

The variance critique was addressed by the development of multi-sectoral models. This framework allows economists to investigate the comovements between production in various industries over the business cycle. An early example of a multi-sector model based on Kydland and Prescott’s (1982) work was published by Long and Plosser (1983). They investigated the output persistence and comovements in different sectors that are subjected to different independent and
identically distributed random errors. This assumption guaranteed that the results would not be biased by serially correlated exogenous variables. Long and Plosser (1983) found evidence that supported the baseline RBC model but admitted that the TFP shocks are not the sole reason for the cyclical behaviour of the real variables in the economy.

Another cause for concern regarding the plausibility of RBC models is the procedure used to derive the TFP variance. McCallum (1988) argued that the method used by Prescott (1986) ignores the possibility of adjustment costs, which if present would overstate variance.

An investigation into the above mentioned proposition was done by Eichenbaum (1991) who performed sensitivity analysis to small perturbations in the theoretical model. Some of the main findings are that RBC models are very sensitive to labour hoarding behaviour which diminishes the ability of the TFP to account for up to 60% of the fluctuations. However, these results may be inaccurate due to a questionable assumption, i.e. that employers hoard relatively more workers during recession. Labour hoarding assumptions are subject to debate. An argument could be made that it is equally likely that labour hoarding is procyclical since in the expansion side of the cycle firms would be able to afford the extra amount of workers as a precaution in case additional labour is demanded; and in a recession there would be a large pool of workers seeking employment, thus, negating the
rationale for hoarding them during such times.

Concerns regarding measurement error were published in a work by Griliches (1995). Ignoring factors such as human capital and research and development in the labour and capital statistics, or the possibility of increasing returns to scale could lead to biased results. The success of the original RBC model was also questioned by Cogley and Nason (1995) who used several benchmark models to simulate data and evaluate the autocorrelation of GNP and the impulse response functions. Their results demonstrate a weak endogenous propagation mechanism which demonstrates the need to extend the original model in a way that addresses this issue.

Another implausible aspect of the original RBC framework is the way the labour market has been set up. Ignoring unemployment leads to a strong positive correlation between labour hours and real wage rate which contradicts the stylised facts and estimates provided by Gali (1999). There is also inconsistency in the values of the elasticity of labour supply between micro and macro studies. Mankiw (1989) has asserted that the intertemporal substitutability of leisure is far too weak to get real business cycle models to work. The variance of total labour hours based on simulated data is significantly below the one suggested by the data. In the original paper, this discrepancy was attributed to measurement error. However, Kydland and Prescott (1982) allowed for the possibility that the model is too
simplistic. Similarly, the variance of current consumption is understated by the model but not as much as the variance in employment. The baseline RBC model considers variation in the total hours worked to be entirely driven by the variation in the hours worked and ignores any changes that may be a consequence of agents’ movements from employment state to unemployment state. Thus, the problem may lie in the assumption that there are changes only in the intensive margin, whilst possible variations in the extensive margin are ignored.

Other counterfactual results are the fluctuations of the hours worked relative to changes in productivity (the wage rate). According to the data the variation in the former compared to the latter is much larger than the model predictions. The standard RBC models also depend on a large elasticity of labour supply which is contradicted by the empirical evidence from micro studies. One famous paper that investigates the implications of changes in the extensive margin is the Indivisible Labour of Hansen (1985). The analysis is based on a simple one-sector stochastic growth model which employs a lottery principle to whether or not a person is employed. The elasticity of the intertemporal substitution of leisure for the representative household is infinite and it does not depend on the elasticity of substitution implied by the preferences of the individual agents in the economy. One of the main findings is that the variance in total hours generated using the indivisible labour model is almost twice as large as the one obtained from the base-
line RBC model which considers only the intensive margin. Both models were not able to replicate the ratio between labour hours and real wage rate. The benchmark model is less volatile while the one proposed by Hansen overstates it. The result seems plausible since both models represent opposite extremes of the real world. The explanation was reaffirmed by Kydland and Prescott (1990b). The research suggests that an optimal choice is to model for both the intensive and the extensive margins which would reflect the state of the real world and thus provide better conformity with the data. Hansen and Wright (1992) have also addressed the issues regarding the labour market statistics in the original model. An argument was made that the inclusion of non-separable preferences would increase the labour supply elasticity. Regardless of the applied changes, little improvement was made with regards to the correlation coefficient. The authors have also proposed that the introduction of a shock to government expenditure would affect the labour hours to productivity due to the effect on the labour supply curve. Similar results would be achieved if household production was introduced. Having a similar goal, Kydland (1984) postulated two types of labour based on their effectiveness in production and found that this modification increases the variability of hours relative to output. Thus, in the presence of shocks that shift the labour supply and demand curves, the strong counterfactual results regarding the correlation between hours worked and the return to labour input are removed. Examples of such shocks are
tax rate changes, innovations to the money supply, changes in the labour force, changes in the utilisation rate, and shocks to government spending.

An example that incorporates one of the above mentioned shocks is a paper by Kydland and Prescott (1988). They extended the canonical RBC model by introducing a variable workweek of capital which tremendously improved the properties of the simulated data with respect to the standard deviation of output and the hours worked as well as the smaller variance of the technological disturbance required to generate the deviations in output. Similarly, McCallum (1988) and King and Rebelo (2000) found that the introduction of either variable capital utilisation or heterogeneous workers improved the models’ performance with respect to volatility, especially in the case of small, nonnegative changes in technology.

Numerous papers have been written on the importance of explicitly modelling government within the RBC framework. Hartley, Hoover and Salyer (1997) argued that it would not be possible to obtain a full account of the cyclical variability in the real economy without considering institutional differences. McCallum (1988) has also noted the importance of the public sector since public consumption could possibly result in deviations from the Pareto optimum, unless the government is assumed to behave as a benevolent planner whose preferences exactly match the one of the representative agent. Similar argument can be made regarding the exclusions of taxes which inevitably distort allocations by creating a wedge. These
ideas were confirmed by Christiano and Eichenbaum’s (1992) model which incorporated shocks to government spending. The results showed improvements but the assumption that government and private consumption are perfect substitutes is too unrealistic. King, Plosser and Rebelo (1988b) have also introduced government interventions via a proportionate tax rate on output. Within their framework if the government objective is to keep a balanced budget, it would require a decrease in tax rates during an economic expansion, provided that there is no change in planned expenditure. This would lead to an amplified effect of the positive TFP shock.

Even Kydland and Prescott (1990b) have acknowledged that not all fluctuations can be attributed to the technological shocks and that monetary shocks are some of most likely culprits for a significant fraction of aggregate fluctuations.

One way to integrate money within the RBC framework is via a cash-in-advance constraint. The origins of this branch in economics can be traced to Lucas (1982). Interesting papers on the matter are Svensson (1985), Lucas and Stokey (1987), and Cooley and Hansen (1989). The last paper focuses on the business cycle implications of the cash-in-advance constraint. This method highlights the impact of the inflation tax on the real economy by focusing on consumption, but assuming that leisure and investment are credit goods. The authors argued that the influence of money on real activity is most likely due to changes in the
private sectors’ information set about the decision rules of the monetary authority. King, Plosser and Rebelo (1988a) have stressed that an insight into the character of the real fluctuations is needed before the importance of the role of money in real activity be evaluated.

In contrast to Cooley and Hansen (1989), the model proposed by King and Plosser (1984) treats money as a passive variable in a sense that there is no active management of the money supply by the monetary authorities. The way they have introduced it is based on the so called reverse causation, implying that the monetary policy is not simply a method that the central bank uses to mitigate deviations in output but it is a major cause in the cyclical behaviour of variables. The authors extended the original model by introducing the concepts of inside and outside money. Lagos (2006) has provided the following two definitions:

**Definition 1** Outside money is money that is either of a fiat nature or backed by some asset that is not in zero net supply within the private sector of the economy. Thus, outside money is a net asset for the private sector...

**Definition 2** Inside money is an asset representing ... any form of private credit that circulates as a medium of exchange. Since it is one private agent’s liability and at the same time some other agent’s asset, inside money is in zero net supply within the private sector. The qualifier inside is short for (backed by debt from) inside the private sector.
King and Plosser’s (1984) model features two sectors – goods and financial industries. Similar to the research focus of this thesis, the financial sector’s output acts as an intermediate good used in the production of output. Other features of the model are non-interest-bearing government-supplied fiat currency which is not a perfect substitute for transaction services, and a market for one-period nominal bonds. If there is a bank regulation, the return on deposits would not be equal to market rates. There are also some strong assumptions. First, the authors assumed a full depreciation rate, deterministic production of transaction services and fixed proportions of labour services to transaction activities for both households and firms. These strong assumptions are not present in the research presented in this thesis. In the proposed model the banking industry is subjected to technological shocks and labour in the two sectors is not in a fixed proportion but determined by the demand for labour by the firm and the financial intermediary. Another very strong assumption that raises scepticism is the restriction on the transaction services to be proportional to the stock of deposits, ensuring that inside money is positively correlated with output. Similar harsh restriction exists in the DSGE model presented here. It is assumed that both loans and deposits are held for one period after which they have to be repaid (interest and principal) and that all deposits are used in the production of loans.

King and Plosser (1984) considered two scenarios – with and without govern-
ment regulation. One of the crucial findings is that the correlation between money and the business cycle is mostly due to the variability of deposits and the real side of the economy. Therefore changes in outside money would mainly result in fluctuations in the inflation rate and leave the real side of the economy unaltered. Similar findings were presented by Kydland and Prescott (1990a) who argued that the monetary base had little effect on the real business cycle.

An interesting question is whether King and Plosser’s (1984) conclusions would hold in the case of unconventional monetary policy such as quantitative easing (QE). Joyce et al. (2012) provide an excellent starting point in the analysis of the properties of QE and its implications. The authors have highlighted the differences between conventional monetary policy and QE and credit easing. The methods by which major central banks have implemented QE has also been outlined. The theoretical transmission mechanism as well as relevant empirical studies have been discussed. Empirical studies can be broadly divided into those that focus on the impact on financial markets and on the economy in general. Examples of studies that fall into the former category using US data are Gagnon et al. (2011), D’Amico and King (2010), Krishnamurthy and Vissing-Jorgensen (2011), Neely (2011), and Hamilton and Wu (2011). They all agree that QE had an effect on long term interest rates. However, there is a significant difference between the actual results. Furthermore, Wright (2011) have found that the effects had small persistence.
Similar studies have been done using UK data, for example, Meier (2009) and Joyce et al. (2011). These papers’ findings also confirm the positive impact of QE.

Most of the above-mentioned papers use event studies to determine the effect of QE and the reported results are highly dependent on the window size. Joyce and Tong (2012) have found that the reaction time after an announcement of upcoming QE varies, thus, it is very difficult to determine the optimal window size.

There are also many studies that attempt to gauge the impact of QE on the wider economy. The data that is mostly analysed is on US, UK, Euro area, and Japan. Examples of such studies include Baumeister and Benati (2010), Kapetanios et al. (2012), Chung (2012), Fahr et al. (2010), and Chen (2012). Various models and techniques have been employed, such as, structural VARs, DSGE models, EDO models, and GARCH models among others. The estimated impact of QE varies from one paper to another. Some papers report that the results are equivalent to a 3% drop in interest rates while others – are equivalent to 0.5% at the most. It has also been argued that there is an impact on the real side of the economy; however, the effects are delayed.

Despite the fact that most of the papers report positive results of the effectiveness of QE and credit easing, the actual values vary a lot. Furthermore, the observed recovery is slow and fragile. This raises the question of whether QE is the optimal policy choice, or whether its effects could be improved if combined
with another type of intervention, or if the sluggish improvement is a result of a less aggressive QE than is necessary. It is possible that stronger effects would be observed in the medium and long term. The effects could be both positive, in terms of easing recovery and boosting economic growth, and negative, i.e. rapid inflation.

It could be argued that since this unconventional monetary policy is a relatively recent event in the UK, it could not be appropriately represented in a model that would be tested using a data set that covers more than 35 years. That is why this, the investigation of QE would be left for future research.

The rest of this chapter will consider mainly three papers that serve as the building blocks of the DSGE model which is the focus of this research. Whilst the preceding literature provides the various base frameworks that could be utilised, the following papers are the inspiration for this research. As Bernard of Chartres said:

...we see more and farther than our predecessors, not because we have keener vision or greater height, but because we are lifted up and borne aloft on their gigantic stature...

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2.2 Small Open Economy

The models highlighted in this section are not an exhaustive list of the small open economy. It is rather a collection of papers that present interesting features. It is important to reiterate that the research focus of this thesis are the cyclical properties of open economy models with fully flexible prices and wages and perfectly competitive markets. Thus, examples of papers which do not fall in this category, regardless of their significance in the macroeconomic field, will not be considered.

One of the first papers published in this niche is Mendoza (1991). Based on Canadian data, the author has explored the ability of the model to mimic some of the stylised facts such as the positive correlation between investment and savings, and the countercyclical behaviour of the trade balance and found that the model conforms with the data. Correia et al. (1995) have argued that this type of model is consistent with the cyclical fluctuations in Portugal. A significant discovery by both authors was that the functional form of the representative agent is paramount for the results generated by the model.

More recent work in this area was done by Meenagh et al. (2005), Davidson et al. (2010) and Onishchenko (2011). The authors have adopted a similar approach to modelling the economy and used indirect inference to evaluate it but the evaluation was done using different data sets. The first two are based on UK data
whilst the third tests the model on Ukraine. The results suggest that the model is able to match the dynamics of output, effective exchange rates and interest rates regardless of the data set.

Arfa (2010) evaluated an open economy DSGE model with a second shock reflecting the energy imports and found that the latter contributed little to the cyclical fluctuations of variables in the French economy. The model performed relatively well in its ability to capture stylised facts in the data except in the case of trade balance. Similar work was done by Bruno and Portier (1995) who also considered the French economy and the impact of oil on the variability of the endogenous variables. They assumed that oil and capital are imperfect substitutes and used a more rigorous way to evaluate the economy. They compare the impulse response functions from simulated data and those obtained from VAR estimates and found that the theoretical model overestimates the magnitude of the technological shock.

Another empirical study using French data was presented by Feve and Langot (1996). Using GMM estimation, the authors found that the model type that best fits the French data is an open economy one with search and bargaining in the labour market. Assuming that there is a common European business cycle, at least for the continental Europe, this model should also fit data sets from other countries as well.
Small open economy models are suitable not only for developed but also for emerging countries. An example of a study that focuses on interest rates in emerging markets is the paper by Neumeyer and Perri (2005). The authors have considered five countries: Argentina, Brazil, Korea, Mexico, and Philippines where the interest rates can be characterised as highly volatile, countercyclical and leading the cycle. They have evaluated two ways of modelling the interest rates: one as completely independent from the fundamental shocks and another as a variable that is highly dependent on these shocks. In the second case the real interest rate has two components - an international rate and a country risk component which is affected by fundamental shocks and at the same time amplifies the effects of those shocks. The results suggest that the second modelling choice can account for the stylised facts. The analysis also implies that default risk can account for 27% of output volatility.

Uribe and Yue’s (2003) paper has similar a focus to the one described above, namely country spread and business cycles in emerging markets. They have presented a small open economy model that contains ‘gestation lags in the production of capital, external habit formation, a working-capital constraint that requires firms to hold non-interest-bearing liquid assets in an amount proportional to their wage bill, and an information structure according to which, in each period, output and absorption decisions are made before that period’s international financial con-
ditions are revealed’, Uribe and Yue (2003, p. 4). The main goal is to determine whether the spreads drive business cycles or vice versa and whether the effect of US interest rates is direct or through their effect on country spreads. Based on their analysis, Uribe and Yue (2003) have concluded that 20% of the volatility in emerging markets’ aggregate variables is due to US interest rate shocks relative to 12% due to country spread shocks which also account for 60% of the volatility of the spreads. Furthermore, an increase in US interest rates leads to a delayed overshooting in country spreads. Also, the effect of US interest rate shocks on the domestic economy is mainly due to its effect on country spreads. The aggregate volatility increases if country spreads react to business conditions.

Another paper that focuses on emerging markets has been published by Lizarazo (2013). The author incorporates risk averse international investors within a small open economy model with endogenous default risk. The goal is to determine whether investors’ preferences characterised by decreasing absolute risk aversion (DARA) could account for the stylised facts regarding sovereign bond spreads and capital inflows in emerging countries. The analysis has demonstrated that DARA preferences result in the dependence of default risk, capital flows, and bond prices on not only fundamentals but also international investors’ wealth and risk aversion. Lizarazo (2013) has argued that as a general rule the wealthier or less risk averse the investor is, the lower the credit constraint of the emerging country will
be, and vice versa. The DARA preferences also result in a higher equilibrium level of debt and more volatile spreads, defined as the rate on sovereign debt over the risk free rate. Qualitative results are also positive as the model is able to account for the negative correlation between investors’ performance, measured by their wealth, and the interest rate spreads.

One study that also uses a relatively large number of countries, seventeen to be precise, and utilises a small open economy model has been presented by Miyamoto and Nguyen (2014). The authors have argued that in a business cycle model with debt adjustment costs, common shocks can account for almost 50% of output fluctuations over the last 100 years. Within this framework common shocks capture the effect of worldwide shocks and shocks coming from large countries, and they have an impact on both developed and emerging economies, the effect on developed economies being greater. The authors have indicated that the effect of common shocks is dependent on the degree of openness of the country; the more open an economy is, the greater the impact of these shocks.

An interesting paper of an open economy model is the one by Balsam and Eckstein (2001). The authors incorporate a second sector of non-traded goods to account for the significantly higher volatility in consumption relative to output of the Israeli economy when compared to other countries. One of the main findings is that almost any volatility of consumption can be achieved by changing the three
parameters of the CES utility function and the share of non-traded goods in government expenditures, holding the production side parameters constant. However, Stockman and Tesar (1995) have found that accounting for nontraded goods is an important but insufficient adjustment to enable a small open economy to fully replicate the data. Based on a study that includes seven developed economies, the authors have concluded that technology shocks alone are insufficient. They have proposed taste shocks as a possible addition to the model. The results suggest that the inclusion of the second type of shocks significantly improves the performance of the model.

Another paper that features both traded and non-traded goods is the one by Uribe (2002). Using data from Argentina, the author has shown that after the announcement of currency peg total consumption, the real effective exchange rate, and the consumption of non-traded goods gradually increase. The author refers to this phenomenon as the price-consumption regularity. Uribe (2002) has argued that standard open economy models are unable to account for this regularity. The author refers to this inability as the price-consumption puzzle of currency pegs. To address this issue in the existing literature, Uribe (2002) has suggested three methods: an uncertain duration of currency pegs, borrowing constraints, and habit formation in consumption. The main focus of his paper is the last one – habit formation. Although habit formation resolves the puzzle, the quantitative effects in
terms of the size of the consumption booms and the terms-of-trade appreciation are too small to match the data. Another paper that focuses on the Argentinian economy has been presented by Rozada et al. (2004). The authors have estimated the elasticity of substitution of non-tradable goods relative to tradable goods to be between 0.40 and 0.48. A paper that provides estimates for parameters also using data on Argentina, as well as on Mexico has been published by Garcia-Cicco et.al. (2010). Using a large data set (from 1900 to 2005) and a simple open economy model, the authors have provided estimates for the parameters defining the stochastic process of the productivity shock and the parameter governing the degree of capital adjustment costs. Based on the results Garcia-Cicco et.al. (2010) have concluded that the model, driven only by productivity shock, performs poorly at explaining business cycles in Argentina and Mexico. However, when they added country-premium shocks, preference shocks, and domestic spending shocks, the model results improved tremendously. Therefore, introducing financial imperfections, should contribute significantly to the overall ability of small open economy models to account for business cycle facts in emerging markets.

A two-sector small open economy model that features uncertainty in the duration of a currency peg has been presented by Mendoza and Uribe (1999). Their model, calibrated to reflect Mexico’s data, is able to account for the following regularities of exchange-rate-based stabilisation: large real appreciations, large ex-
ternal deficits, and recessions that predate currency collapses. Mendoza and Uribe (2000) also discuss the implications of exchange rate management on the business cycle dynamics. The paper demonstrates that within the framework of incomplete insurance markets, the risk of collapse of the exchange rate regimes generates large distortions on wealth and relative prices. The welfare cost is substantially larger than the one generated due to a lack of credibility in the perfect foresight models.

Another model that focuses on exchange-rate-based stabilisation has been presented by Lahiri (2001). The author has developed a small open economy model with cash-in-advance constraint and endogenous labour supply. This specification is able to generate results consistent with exchange-rate-based stabilisation without resorting to either imperfect credibility or to price rigidities. The model generates ‘consumption and output boom in response to a permanent cut in the rate of devaluation, . . . cumulative current account deficit, . . . a sustained real exchange rate appreciation, . . . . . an increases in labour supply’, Lahiri (2001, pp. 1174-1175).

An extension of the standard open economy model that focuses on capital utilisation rates and habit formation was done by Letendre (2004). The author compared several models that include one or both features mentioned above and discovered that the model that performs best is the one that incorporates both. This conclusion can present a next step in the investigation of the proposed model
in the next chapter in the search for a better fit.

Information frictions and uncertainty could also be employed to improve the small open economy model’s ability to fit the data. One paper that explores the idea is Boz et al. (2011). The authors have introduced imperfect information by assuming that the representative household is not fully informed about the trend-cycle decomposition of the productivity shock. Instead they estimate the components using Kalman filter. The authors have used data on Mexico to estimate the model parameters using GMM estimation. It can be concluded that based on the values of the second moments from actual and simulated data, the uncertainty enables the model to match features of the business cycles of emerging markets.

One very fascinating paper containing innovative ideas regarding open economy models is Jaimovich and Rebelo’s (2008) paper on news in business cycles. The model generates robust results regarding the comovements with respect to news about future TFP, investment specific technical changes and as a result of sudden stops shocks. To generate the results, the authors assumed one or more of the following: short-run wealth effects on the labour supply, adjustment costs, and debt elastic interest rates. Another paper that explicitly models the effect of news is Durdu et al. (2013). The authors have modelled an economy where default risk, interest rates and debt are affected by news about future fundamentals. Durdu
et al. (2013) have argued that there is precision difference in the news regarding developed and developing countries which results in a higher volatility of emerging markets’ aggregates in a response to news shocks. The model generates results that conform with the stylised facts regarding the relationship between default rates and the precision of news and also allows for default episodes in good times.

Many papers have presented model specifications that focus on the determinants of default risk. However, few have tried to account for the frequency of default of emerging countries on their foreign debt obligations. An example of the latter is D’Erasmo (2007). The goal of the research is to present a model that can account for two stylised facts: emerging economies default on average three times within a period of 100 years and have an average debt-to-GDP ratio of approximately 0.58. Some of the specific model features are: different political phases of the government, variable timing of renegotiation, bargaining over recovery rates, private information, endogenous periods of exclusion, and credit terms influenced by sovereign ratings. The model is able to account for the following stylised facts of emerging countries: a higher volatility of consumption relative to output, countercyclical interest rates and current account, and a large fraction of the debt to output ratio. However, the model generates low average interest rate in equilibrium which is inconsistent with the data. Another paper that utilises a bargaining game in the debt renegotiation after a default has been published by
Bi (2008). The main finding suggests that delays in negotiating debt restructuring could occur in equilibrium and that the outcome is efficient.

Uribe (2007) has presented a paper that aims to determine the implications of foreign debt collateral constraints on the dynamics of a small open economy and whether lending practices based on aggregate rather than individual factors leads to overborrowing, i.e. the excessive borrowing in a boom phase. Within a theoretical framework, the author considers two cases: first, the effect on equilibrium dynamics and level of debt when the constraint is based on aggregate indicators such as GDP, level of debt, and trade deficit among others; and a second case when the constraint is based on the individual borrower or investment project. Within this framework two crucial assumptions are employed. First the shadow price of funds equals the world interest rate. Second, when the debt ceiling binds, it happens for all agents simultaneously. The main finding is that the aggregate borrowing constraint does not result in higher levels of debt than the case when there are individual borrowing limits. This result holds whether the credit constraint is constant or depends on the asset prices. Uribe (2007) has also demonstrated the importance of the two assumptions mentioned above by presenting two cases when either one does not hold, i.e. heterogeneous agents and debt elastic interest rate. In both cases the equilibrium level of debt is higher when an aggregate credit ceiling is imposed.
Korinek (2010) has analysed optimal borrowing decisions in emerging market economies with collateral-dependent borrowing constraints with particular attention to the implications of accumulation of too much dollar denominated debt. The model has demonstrated that excessive dollar borrowing and the resulting amplification effects lead to a more rapid decline in aggregate demand and exchange rates depreciate. The author has provided an example of a policy that may mitigate the dollar effect - a proportionate tax on dollar debt in the form of reserve requirement. Bianchi (2011) has extended Korinek’s (2010) model. The author has presented a quantitative analysis of the externality that occurs in models with collateral constraints when debt is denominated in tradable goods but leveraged on non-tradable output. Possible policies limiting the effect have also been discussed.

Kim and Zhang (2012) have proposed a model with decentralised borrowing, where the privet sector decides how much to borrow, as opposed to centralised (government) borrowing. The decision of whether to default or not on foreign obligations remains with the government. The results indicate that the decentralised case increases default risk and the cost of borrowing. This model can generate underborrowing in equilibrium even with overborrowing incentives depending on the default penalties.

A business cycle model with collateral constraint can also be used to account for
the financial downturns and the following recessions in emerging markets as a result of Sudden Stops. This idea has been presented by Mendoza (2010). The constraint is binding only when the leverage ratio is high enough. The underlying rationale is that leverage increases during the boom part of the cycle and eventually triggers the collateral constraint resulting in a deflation that lowers the level of credit and collateral. Due to the reduced availability of working capital output, investment and equity prices also fall. This in turn further tightens the constraint, resulting in an amplification of the reaction of macroeconomic variables. Therefore, within this framework large shocks are not necessary to induce economic recessions. This conclusion deals with one of the main criticisms of early RBC models, namely that they depend on very large shocks to replicate business cycle dynamics.

Small open economy models are a useful tool in the investigation of the effects of world prices and interest rates on the domestic economy. An example of a paper that looks into this subject matter is Kose (2002). The author has presented a multi-sector small open economy model that is subjected to a world price shock. The variance decomposition analysis suggests that world price shocks, namely capital and intermediate goods prices and interest rates, are a significant determinant of the cyclical behaviour of developing countries. The reported values indicate that between 79% and 97% of the total variation in domestic variables is due to the world price shocks. The model can also account for the volatility and comovement
of sectoral outputs. Guimaraes (2011) has also argued that world interest rate shocks are more important than technological shocks in the determination of the level of debt which triggers default.

The effect of debt interest rate volatility on the business cycles of emerging markets has been investigated by Fernandez-Villaverde et al. (2011). Based on data from Argentina, Brazil, Ecuador, and Venezuela, the authors’ analysis suggests that the interest rate volatility, at which emerging economies borrow from international markets, changes over time. The authors have argued that changes in the volatility would induce a precautionary reduction of foreign debt to minimise the volatility of expected marginal utility. This in turn would result in a lower level of output, consumption and labour hours.

A preliminary draft by Fernandez et al. (2016) demonstrated the results from the authors’ recent analysis of the effect of world prices on individual economies. Although the model presented is not a micro-founded RBC open economy model, the empirical results indicate important factors that should be considered when one constructs an open economy model. The analysis is based on 138 countries, both developed and developing, and focuses on commodity prices, namely agricultural, metal, and fuel. Fernandez et al. (2016) have found that jointly these prices can account for approximately 30% of aggregate fluctuations in the individual countries. This evidence suggests that world prices are a key factor of business
cycle fluctuations in a large number of countries. Therefore, open economy models are a better choice relative to the closed alternatives for a majority of the countries.

A paper that incorporates commodity markets in a small open economy model has been presented by Fernandez et al. (2015). They have developed a multi-country structure that consists of several small open economies. The model contains the following distinct features: a country-specific commodity sector, domestic and foreign goods are imperfect substitutes, investment goods are produced using home and foreign goods, and commodity prices are assumed to be a function of idiosyncratic shock and latent common factor. The common factor captures the propensity of commodity prices to move together. The result from the analysis indicates that commodity price shocks and the subsequent spillover effect on the country risk premium can account for 42% of output variance.

Kim and Kose's (2000) paper is not only an interesting representative of small open economy models, but also is a factor in the modelling choice discussed in the next section of this thesis. The authors have evaluated the implications of fixed and endogenous discount factor. They have argued that the two functional forms have little implication for the overall dynamics of the model but have acknowledged that the endogenous discount factor induces stationarity which makes the analysis of the model easier. A similar example was given by Schmitt-Grohe and Uribe (2016). The authors present a formulation of the discount factor which
is essentially endogenous since it depends on variables that represent a cross sectional average of the two endogenous variables. Kim and Kose (2000) presented evidence that the endogenous discount factor model induces an impatience effect and implausible dynamics by permanent productivity shocks which led to the conclusion that the use of the fixed discount factor is preferred in studies, which aim to understand business cycle dynamics in open economies. This conclusion has significant implications for the choice of a closing method discussed in the next section.

An important conclusion that can be drawn based on the papers discussed above is that the success of open economy models is not dependent on the data set used since each of these models have used data on different and seemingly unrelated countries: Canada, Portugal, the UK, France, Mexico, Israel, Argentina, Brazil, Korea, Philippines and Ukraine, to name a few. The analysis also suggests that the inclusions of real frictions, additional sectors and shocks, significantly improves the ability of small open economy models to fit the data. This combined with the fact that some of the crucial features that characterise the UK economy and other countries of the same size cannot be explored in a closed economy setup, requires the explicit modelling of the rest of the world. In this way, the variables such as relative prices, real exchange rates, the export and import industries, and foreign debt could be taken into account. The open economy model that serves as a base
frame for the proposed model in this thesis is the one described by Meenagh et al. (2005). Although not all features will be used due to issues with the compatibility of the equations with features that will be introduced from other RBC frameworks, the model is a great starting point for this research.

2.3 Closing the Economy - External Debt Elastic Interest Rate

This section will focus on a specific feature present in small open economy models, namely the nonstationarity properties of the model. This issue was briefly mentioned in the previous section. As discussed stationarity of the model is necessary since the approximation solutions usually used to solve these type of models require a steady state value. A very eloquent description of the associate problems of nonstationarity is presented by Schmitt-Grohe and Uribe (2003). A brief summary of their discussion is provided here.

Most open economy models assume that the home country representative agent can acquire foreign debt at an exogenously determined risk-free rate of interest. This results in a dependency of the steady state values on the initial net foreign asset position. This allows temporary shocks to have long run effects on the state of economy. The random walk property of equilibrium dynamics results in infinitely large variance in some of the variables in the model. Any attempt to remove the random walk component results in a distortion of the low-frequency properties of the model. Schmitt-Grohe and Uribe (2003,2016) discuss several
methods that could be used to induce stationarity in the model. The objective is to evaluate the possible implications of these features on business cycle frequency data. The proposed adjustments are external debt elastic interest rate, internal debt elastic interest rate, portfolio adjustment costs, complete asset markets, external (endogenous) discount factor, internal (endogenous) discount factor, and the perpetual youth model.

2.3.1 External debt elastic interest rate

Most small open economy models assume that the interest rate that the domestic agents face in the international bonds market is exogenously determined. The underlying rationale of this assumption is that since the home country is small in relation to the rest of the world it is not able to influence the international markets. Therefore, the cost of borrowing is exogenously determined. This assumption ignores the fact that in reality the cost of borrowing depends on the level of debt that has been accumulated. In contrast, an external debt elastic interest rate (EDEIR) assumes that the interest rate at which the home country representative agent borrows from abroad is an increasing and convex function of the net level of debt.

\[
r_t^f = \tilde{r}^f + \varphi \left( e^{(d_t^f - \bar{d})} - 1 \right)
\]

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The intuition of this specification is straightforward. The larger the level of debt relative to its steady state value, the higher the premium, which the home country residents have to pay on top of the constant world interest rate. A constant world interest rate is assumed to simplify the analysis. The increase in the premium induces an increase in savings, reduces consumption and debt growth. Schmitt-Grohe and Uribe (2016) have argued that this formulation is appealing from both a theoretical and empirical standpoint. From a theoretical perspective, this is a simple method to capture financial frictions. The authors have also provided micro-foundations within the framework of imperfect enforcement of international debt contracts. In terms of the empirical evidence in support of the EDEIR, Schmitt-Grohe and Uribe (2016) have argued that, especially in the case of emerging markets data, the debt sensitive interest rate is more a suitable specification and that it plays a significant role in explaining the cyclical behaviour of the trade balance. The authors have calibrated a small open economy model with EDEIR and presented evidence that this specification is able to match the second moments and the behaviour of the impulse response functions with respect to Canadian data. Thus this stationarity inducing feature is suitable for both emerging and developed small open economies.
2.3.2 Internal debt elastic interest rate

The internal debt elastic interest rate (IDEIR) model is almost identical to the one with EDEIR. The only difference is that in the case of IDEIR the interest rate premium is a function of the household’s individual debt position while in the EDEIR case the premium is a function of the cross sectional average level of debt. In the model with an EDEIR the \( d^f \) parameter is equal to the steady state value of \( d^f_t \). However, in the model with an IDEIR, for the same value of \( d^f \) the steady state of \( d^f_t \) is lower in the deterministic case. Therefore, it can be concluded that in the IDEIR case the agents borrow less in the steady state due to the fact that they internalise the fact that their position increases the premium. However, this results in a steady state value for \( d^f_t \) that generates a negative premium in the deterministic case and this is not an appealing property.

2.3.3 Portfolio adjustment costs

The third way to induce stationarity in the model is to assume that there are convex costs associated with a level of debt which is different from the long-term level. The small open economy model with portfolio adjustment costs (PAC) differs from the one with EDEIR in two ways. First, there is an additional term in the household’s budget constraint that reflects the presence of adjustment costs. Second, the interest rate on foreign debt is exogenously determined. Since the
adjustment costs are assumed to be convex, the interest rate rises with the level of debt.

2.3.4 Complete asset markets

The standard small open economy model features incomplete asset markets, i.e. the asset return is not state-contingent. If the representative agents could choose from different assets it would allow them to smooth consumption not only over time but also across different states. The complete assets markets (CAM) assumption would bring realism to the model and it is sufficient to induce stationarity in the equilibrium dynamics. This is achieved through the diversification of risk and the resulting constant marginal utility of consumption, thus eliminating the random walk that is present in open economy models in the absence of any stationarity inducing features. Schmitt-Grohe and Uribe (2016) have compared the quantitative prediction of a small open economy model with CAM and with EDEIR. The results are very similar. The authors have argued that introducing smoothing across states contributes little towards the overall ability of the model to conform with business cycle facts.

2.3.5 External (endogenous) discount factor

This stationarity inducing feature assumes that the subjective discount factor depends on ‘endogenous variables which are taken as exogenous by the individual
households’ Schmitt-Grohe and Uribe (2016, p.150). Thus, the discount factor is considered external (EDF). The authors have assumed that the EDF depends on the cross-sectional averages of consumption and labour.

\[ \theta_{t+1} = \beta (c_t, h_t) \theta_t \]

\[ \beta (c_t, h_t) = \left( 1 + c_t - \frac{h_t^\omega}{\omega} \right)^{-\psi_1} \]

The parameter \( \psi_1 \) affects both: the stationarity of the model and the steady state value of the trade-balance-to-output ratio. This could be a potential problem since if the value of the parameter calibrated to match a specific trade-balance-to-output ratio is too high, it may not induce stationarity. That is why the authors have proposed a second specification.

\[ \beta (c_t, h_t) = \left( \psi_2 + c_t - \frac{h_t^\omega}{\omega} \right)^{-\psi_1} \]

In the above equation \( \psi_2 \) affects the steady state value and \( \psi_1 \) can be used to induce stationarity. However, this approach would imply that there could be a set of values that satisfy the stationarity criterion but that lead to different results in a calibration exercise.

2.3.6 Internal (endogenous) discount factor

The internal (endogenous) discount factor (IDF) is almost identical to the EDF. The only difference is that in the case of IDF, the endogenous discount
factor depends on the individual levels of consumption and effort, whilst in the EDF case it depends on the average level. Despite the difference, the steady states of the IDF and EDF economies are identical as well as the levels of consumption, labour and foreign debt.

2.3.7 The perpetual youth model

The perpetual youth model (PYM) developed by Blanchard (1985) can also be used as a method to induce model stationarity. This is achieved due to the assumption of finite probability of death. Cardia (1991) has argued that this method creates a wedge between the world interest rate and the subjective discount factor which in turn induces stationarity. The method that Schmitt-Grohe and Uribe (2016) employ is very similar to the one proposed by Cardia (1991) with one notable difference – the preference specification. One of the main assumptions in this model is that the debt accumulated by agents that are now deceased would not be repaid. The fraction of unpaid debts every period is assumed to be very small and deterministic. There is a constant premium over the interest rate that serves as compensation for the possibility that some debts would not be repaid. This method of inducing stationarity is much more complicated than the other examples presented in this section. At the same time, it does not provide better qualitative results when compared to other methods; thus it is not an appealing choice.
2.3.8 Motivation behind the choice of ‘closing’ method - EDEIR

Schmitt-Grohe and Uribe’s (2016) main argument is that all stationarity inducing features generate virtually identical second moment predictions and impulse response functions to a technological shock. Only in the case of complete asset markets is the variance of consumption slightly lower. In contrast, Seoane (2015) has found that the choice of a ‘closing’ method affects the results when the model is calibrated to match the dynamics of emerging markets. Another paper that raises concerns has been presented by Lubik (2007). The author has argued that in the case of additively separable risk premium and internalisation, a stable equilibrium exists only if the parameter values meet certain conditions. On the other hand, if there is no internalisation, i.e. in the EDEIR case, there is a unique equilibrium under all possible parameter values. This is confirmed by the analysis in this thesis since the model satisfies the Blanchard-Kahn conditions proposed by Blanchard and Kahn (1980).

The model presented in the next chapter uses the external debt elastic interest rate specification to induce stationarity. Although in terms of performance, all of the above features present almost identical results in the case of developed countries, the economic intuition behind this assumption is very appealing when considered in a model that focuses on interest rates. It generates predictions that fit the date for both developed and emerging markets. EDEIR is a much simpler
method than PYM and it is also preferred to the CAM which generates smaller consumption variability relative to every other method. If the deterministic case of the model is considered, the EDEIR does not imply a negative interest rate premium that is present when IDEIR is used. Relative to the EDEIR method, in the model with EDF, the stationarity property is more dependent on the choice of parameter values. EDEIR would make the empirical testing of the model easier relative to IDF, since some of the first order conditions of IDF that are part of the system of equations describing the economy contain a variable that represents the expected discount value of the utility from the next period onward. Data for such variable is not available and it would have to be extracted from other equations in the model, which could be challenging.

2.3.8.1 Limitations of the EDEIR assumption

The EDEIR method acknowledges the relationship between a country’s debt level and the cost of borrowing on the international markets. It can be argued that this specification is supported by recent history since countries with higher debt to GDP ratio were penalised by the market and this was demonstrated by higher borrowing costs. Although, the interest rate is an increasing function of the level of debt, the model specification could be perceived as too simplistic since it ignores any risk.

There are various models that have been developed which explore the rela-
tionship between the debt level and interest rates in great detail. For example, there are models that focus on the sovereign debt and the factors that determine bond yields. Arghyrou and Kontonikas (2012) have identified three main factors that affect the interest rate on government bonds: international risk, credit risk and liquidity risk. The authors have performed an empirical investigation of the response of 10-year government bond yield spread to its past value, the real effective exchange rate, and a stock market volatility index, over two distinct periods – the pre-crisis and the crisis periods. The crisis period analysis also includes a contagion variable. The authors have reported a much stronger dependence of interest rate spreads on international risk and macro fundamentals during the crisis period relative to the pre-crisis one. Attinasi et al. (2009) have also investigated euro area government bond yield spreads relative to Germany. The authors have found that the yield spreads capture the effect of credit risk, liquidity risk, higher international risk aversion, higher expected budget deficit and debt level relative to Germany. Other papers that focus on the determinants of government bonds yield spreads are: Codogno et al. (2003), Geyer et al. (2004), Berrios et al. (2009), and Longstaff et al. (2011) who paid particular attention to international risk factors; Favero et al. (2010) have explored the liquidity effects; and Bernoth et al. (2004) have investigated country specific factors, namely debt level, deficit and debt-service ratio. An interesting paper has been presented by Acharya et al.
(2014). The authors have investigated the link between bank bailouts and the sovereign debt crisis. Using data on credit default swaps, they have demonstrated the feedback between the financial and government credit risks and argued that bank bailouts lead to an increase in sovereign credit risk which in turn is transferred into future taxation and inflation risk. An alternative approach to analysing sovereign debt crisis has been presented by Arghyrou and Tsoukalas (2011). The authors have adopted a method mostly used in currency crises literature to investigate the Greek sovereign debt crisis. They have found that the main contributing factors are the deterioration of macroeconomic fundamentals and the shift in expectations regarding EMU commitment and fiscal guarantees.

This list of papers mentioned in this subsection is far from exhaustive but it provides a good overview of the factors that need to be considered when one models the relationship between interest rates and the level of debt. Although the above mentioned papers provide valuable insight regarding the relationship, such detailed exposition of the links between the variables is outside the scope of this research. That is why the model presented in the next chapter adopts the EDEIR feature as described by Schmitt-Grohe and Uribe (2003) and leaves these insights as something to be considered in future extensions of the model.
2.4 Financial Intermediation

The role of financial intermediaries and their impact on the economy have been a popular topic of discussion for some time. However, “. . . the crisis, of course, has precipitated an uptick in the pace of this research and offered many new issues to study” Gertler and Kiyotaki (2010, p. 48).

Early examples of general equilibrium models that include financial frictions are Bernanke and Gertler (1989), Kiyotaki and Moore (1997) and Bernanke et al. (1999). It could be argued that earlier literature focused more on the demand side by introducing constraints on non-financial borrowers. For example, Carlstrom and Fuerst (1997) have introduced financial frictions by incorporating agency problem and endogenous agency costs. Kiyotaki and Moore (1997) have presented a model with collateral constraints. Kawrk (2002) has incorporated heterogeneous firms and adjustment costs of investment decisions to new information.

More recent literature has mainly focused on the supply side of credit. Most of them investigated models that feature credit constraint imposed on financial institutions. Christiano et al. (2010) have incorporated liquidity constraints on financial intermediaries within a framework with multiple assets. Gertler and Karadi (2011) have investigated the implications of endogenously determined balance sheet constraints. Goodfriend and McCallum (2007) and Benk et al. (2010) have augmented a standard DSGE model with a costly production of loans. A
model that incorporates bank fragility has been presented by Angeloni and Faia (2013). A similar aspect was considered by Collard et al. (2016). In their model, bank fragility arises from deposit insurance and excessive risk taking.

All of the above papers have their merits and provide fascinating insights into the way financial intermediaries affect the economy. Each of them is a testimony that supports the conclusion by Adrian and Shin (2010b, p.29) that “fluctuations in financial conditions have a far-reaching impact on the workings of the real economy”.

However, the method that is adopted in this thesis is the production function approach in a similar fashion to the one presented by Benk et al. (2010). In contrast to some of the papers cited above which assume a New Keynesian type of economy, the method does not depend on any ad hoc assumptions. The method is adopted from microeconomic theory of bank behaviour and has an appealing economic intuition.

Given the vast literature on financial intermediation, the following review presents only papers that introduce financial intermediation via loan production function within a New Classical framework.
2.4.1 The production approach to financial intermediation as a form of inside money

The final distinct feature of the proposed model is the inclusion of financial intermediary using the bank production function approach. The origins of this approach can be traced back to the work of Hicks (1935) and Sealey and Lindley (1977). Hicks (1935) argued that one of the reasons agents hold money is because there are costs associated with transferring assets from one form to another. Based on the same idea, Sealey and Lindley (1977) have recognised that the standard portfolio choice model used to describe banking behaviour was unable to capture key distinctions between banking firms and financial markets, namely the costs and constraints of producing loans. The authors’ work also deviates from other papers that use the theory of the firm to account for the behaviour of banks. Most research at the time used various measures of output, including total assets, earning assets, total deposits, number of accounts, and/or income. Sealey and Lindley (1977) have argued that this inadequate separation of inputs and outputs regarding the financial firm is a result of the lack of understanding by most authors about the difference between technical and economic output. Sealey and Lindley (1977, pp. 1252–1253) have stated that:

**Definition 3** *Technical production is a process of transformation directed by individuals and this is considered desirable by some group of people.*

51
Based on the definition of technical production, the output of the financial firm is a set of services that the firm provides to its clients regardless of whether they are savers or borrowers. In contrast:

**Definition 4** *Economic production is the process of creating output that is more valued than the factors of production used and this value must be reflected by the market prices.*

Based on the definition of economic production, only services that generate income and result in the acquisition of assets can fall into this category. Therefore, all economic production is technical production, but only some of the technical output can be classified as an economic one. This conclusion contradicts the work of Pesek (1970) and Towey (1974) who considered deposits as a measure of output.

Sealey and Lindley’s (1977) idea has been adopted in both partial and general equilibrium models. A partial equilibrium model regarding banking costs was published by Berk and Green (2004). The adjustments that the authors made enabled the analysis to account for some of the empirical regularities regarding mutual funds data without resorting to the assumption that investors are irrational.

Examples of a costly production of credit as a means to account for the behaviour of the banking industry within a general equilibrium model are Gillman and Kejak (2004, 2008), Benk et al. (2005, 2010), Braun and Gillman (2006), and Gillman (2011) to name a few. In all these papers, the authors have created an
economy that features a banking industry and monetary policy introduced via a cash-in-advance constraint. This specification allows the analysis of the effects of both inside and outside money as well as their interaction which dates back to the work of Gurley and Shaw (1960). Before the analysis of the recent literature could proceed, an overview of Gurley and Shaw’s (1960) analysis on inside and outside money should be presented.

“Money in a Theory of Finance” by Gurley and Shaw (1960) became a very influential work in monetary economics. In their book, the authors have presented several theoretical models in an attempt to present a unified theory of money and finance that includes banking theory. Within a neoclassical framework, the authors have discussed at length the money neutrality proposition, the rationale for monetary policy, and the effect of financial intermediaries on the demand for money. In their analysis there is a clear distinction between inside money, i.e. claims that private agents have on each other, and outside money, i.e. claims that the private sector has on the government. Gurley and Shaw (1960) have asserted that a one-off increase in the growth rate of money supply would have no impact on the real side of the economy in a framework that considers only outside money. However, if both inside and outside money are considered, a change in the nominal money will have real effects and this change could be in either inside or outside money. However, Patinkin (1965) has argued that there is a link between inside
and outside money; therefore, if changes in outside money result in equal changes in inside money, the money neutrality proposition holds.

Within Gurley and Shaw’s (1960) framework, the government can determine the general price level by affecting the quantity of any financial asset in circulation. Furthermore, the introduction of financial intermediaries affects the interest elasticity of money demand since more assets backed by private credit, which are considered substitutes for money by the authors, can be used as a medium of exchange. Money demand is also affected by financial innovation which reduces the need to hold cash balance. These factors should be considered in the design of monetary policy.

One of the first papers that investigates the impact of inside and outside money on the real economy within a real business cycle framework is one by King and Plosser (1984). As mentioned earlier in the review, King and Plosser’s (1984) model features a financial industry sector based on the production approach. Within their framework, the final good is produced using three inputs: capital labour and transaction services. The transaction services are supplied by the financial intermediary and are produced using a constant returns to scale production function with labour and capital. Although the transactions services are assumed to be proportionate to deposits, the latter is not explicitly included in the production function. This specification differs from other notable papers in this niche as it
will become evident from the review of subsequent papers.

Another paper that incorporates a banking firm based on the production approach in general equilibrium model that considers both – inside and outside money – is Gillman and Kejak (2004). The authors have introduced a financial firm that uses an AK type of production function in a cash-in-advance economy. The adjustment generates an inflation tax because the financial intermediary has to set aside funds to meet reserve requirements. Extended versions of the model which include deposits and exchange credit can generate demand for money functions that are similar to the demands for monetary aggregates such as the monetary base, M1 and M2. The model generates statistics that are consistent with the velocities and ratios of the monetary aggregates. However, the authors have acknowledged that an AK production function is too simplistic and that the inclusion of labour would provide better results.

Based on Gillman and Kejak’s (2004) work, Benk et al. (2005) have presented a model featuring the banking production approach in a stochastic environment. Using this method, the authors have extended a standard business cycle model with cash-in-advance constraint to include the costly production of credit. One of the main assumptions of the model is that a fraction of the consumption goods is bought on credit, and that money and exchange credit are substitutes. The production function of credit exhibits an increasing marginal cost and is subjected
to a productivity shock in a similar fashion to the standard Cobb-Douglas production used to describe the behaviour of the goods producer. There are several forms of disturbances that affect the economy, namely monetary, productivity in goods sector, and productivity in the credit sector shocks. The authors have calibrated the model using US data. One of the main findings of the analysis is that the estimated series for the credit shock is able to capture the effect of banking regulation on the industry. Due to its effect on output and the resulting comovements, Benk et al. (2005) have proposed that the inclusion of credit shocks could significantly improve a model’s ability to account for business cycle fluctuations.

Goodfriend (2004) has presented a model that incorporates broad money demand, loan production, asset pricing, and arbitrage between banking and asset markets. The specification of loan production function differs from the one proposed by, for example, Benk et al. (2010). While the authors of the latter have assumed that credit is produced using effective labour and deposits, Goodfriend (2004) has proposed that it is a function of management effort and loan collateral. At first glance, this assumption appears to conform with economic intuition since the size of a loan is affected by the value of a collateral. However, the loans issued by a financial institution are effectively the deposits made by the public and not the collateral on which the bank has a claim in the case of a default. Despite this questionable assumption, the model provides an interesting insight regarding the
effect of broad liquidity on the transmission of monetary policy. The author has argued that interest rate targeting provides an automatic accommodation of any changes in the demand for narrow liquidity - currency and bank reserves, but it ignores the effects of broad liquidity. According to Goodfriend (2004), an effective interest rate policy should take into account the manner in which supply and demand for bank deposits affect market rates and the link between those rates and the interbank rate.

Goodfriend and McCallum (2007) have extended the work of Goodfriend (2004) by quantifying the results and trying to determine their relevance in policy design. The authors have presented an analytical framework that includes both a banking sector and transaction-facilitating money. The particular specifications generate a number of distinct interest rates: a short-term interbank interest rate; collateralised loan rate; uncollateralised loan rate; one-period government bond rate; net marginal product of capital; and shadow nominal pure intertemporal rate, which the authors have referred to as the “benchmark” rate. Goodfriend and McCallum (2007) have found that ignoring costly production of credit can lead to a four percent point difference in the target value. Furthermore, the use of a Taylor rule that ignores the effect of broad liquidity would suggest an interest rate cut that is too low to offset a negative productivity shock to the banking industry.
Another paper that describes the interaction between financial intermediaries and monetary policy is the one by Adrian and Shin (2010). The model differs from the other papers presented in this section in that it is not a general equilibrium model that incorporates a costly production of credit. However, the model demonstrates an interesting perspective on financial intermediation and thus could be an interesting future line of research. Within Adrian and Shin’s (2010) framework, financial intermediaries play a major role in the determination of the price of risk and in this way affect the business cycle. The authors have argued that monetary policy can affect the balance sheets of financial intermediaries which in turn determines the risk appetite. For example, lowering the target rate would increase the value of assets, thus raising the net worth of the financial institution which would lead to an expansion in the asset portfolio, i.e. new lending, and additional leverage. Therefore, as the authors have argued, monetary policy can affect real decision via the risk-taking channel. However, Adrian and Shin’s (2010) paper does not provide any qualitative or quantitative results regarding the effect on real macroeconomic variables such as aggregate investment, consumption or output. Therefore, to determine the validity of some of their arguments, their findings should be investigated further within a general equilibrium model.

Braun and Gillman (2006) presented a general equilibrium model that features a banking sector which provides two services: intertemporal credit which facilitates
saving and investment, and exchange credit which facilitates transactions and is a substitute for money. The model features cash-in-advance constraint and two sectors: the goods sector and the banking industry which uses capital and labour as opposed to only labour as demonstrated in the costly credit by Gillman (1999) and only capital as in the AK by Gillman and Kejak (2004). Braun and Gillman (2006) have compared two cases for the market structure of the banking industry – monopolistic and perfect competition. The model has been calibrated to fit data on Japan. The results from the analysis suggest that the bank recapitalisation improves profitability without affecting the size of the industry while the deposit guarantees leave the former unaltered and improve the latter.

One paper that uses human and physical capital in the production function of the banking firm is the one by Gillman and Kejak (2008). Within this endogenous growth framework, credit is produced using effective labour, i.e. labour indexed by human capital, physical capital and deposits. The model is calibrated to fit the stylised facts of the US economy. The results indicate that within a cash-in-advance setup with endogenous growth and a banking sector modelled on the financial intermediation approach, the model can simultaneously account for the three negative inflation effects on output, investment and interest rates.

An identical formulation of the production function has been used by Gillman et al. (2006). In their model, the financial intermediary has two functions: costly
credit production and asset management which is assumed to be costless. Credit is produced via a production function that uses labour, human and physical capital, and deposits as inputs. The financial intermediary accepts deposits, the quantity of which is determined by the representative agent. It also buys government bonds and distributes money. Both money and credit can be used to purchase goods. The theoretical model indicates that an increase of financial development, governed by the parameters of the credit production function, can have negative effect on economic growth. This counterintuitive result, however, is supported by the empirical analysis.

The effects on US GDP growth rate volatility using models that feature costly credit production have been investigated by Benk et al. (2007). Both endogenous and exogenous growth models have been considered. The setup includes productivity, money supply and credit shocks. The authors have decomposed the variance by the type of shock, spectral frequency and subperiod. An interesting finding is that the productivity shock has the smallest impact on growth rate volatility when the entire period is considered; thus, supporting the argument that only productivity shock is insufficient to account for business cycle dynamics. The results also indicate that the credit shock accounts for almost half of the volatility in both endogenous and exogenous growth models when the entire sample is used. Its effect is larger at short run and business cycle frequencies relative to the impact
at long run frequency. Since the analysis covers the period from 1919 to 2004, an
interesting analysis would be to investigate the credit shock contribution to the
volatility in an extended sample to include the more recent events.

Onishchenko (2012) has proposed a dynamic stochastic general equilibrium
model which incorporates an investment bank that operates as a loan producer to
the government. In this framework the banking sector modelled on the produc-
tion approach acts as an intermediary between the representative agent and the
government. This assumption is in contrast to the assumptions made in the other
papers presented in this section in which the financial sector acts as an interme-
diary between the consumer and the goods producer. In the model presented by
Onishchenko (2012) the representative agent invests directly into physical capital
and indirectly into sovereign debt through the financial intermediary which un-
derwrites the newly issued government bonds. The goods producer does not have
access to the financial intermediary and rents capital directly from the represen-
tative agent. The model is calibrated on US data. The analysis indicates that the
banking productivity shock is a major contributor to the aggregate fluctuations.

Benk et al. (2010) have used a banking industry model based on the production
approach for exchange credit to account for the US velocity. There are three factors
of production: labour, human capital, and deposits. The authors have identified
three shocks that affect the velocity of money – shock to the money supply, bank
productivity shock, and shock in the goods production. The model was calibrated for the US and UK economies and the effects of these shocks on both the level and volatility of the velocity have been quantified. The results indicate that the size of the effect of each of these differ depending on the economic stability of the period. For example, in periods of economic distress there is a greater need for monetary policy intervention, thus the effects of the money supply growth rate shock and credit shock account for a much larger proportion of the velocity volatility.

Gillman (2011) has utilised the production function approach of the banking industry and presented a model that focuses on intertemporal credit. In this framework, outside money is ignored and the focus is on the implications of inside money on real variables. The model assumes that the financial intermediary accepts deposits which, together with labour, are the two factors of production. The author has provided a detailed aggregate demand and supply analysis as well as the effect of bank technology on the equilibrium level of endogenous variables. The calibrated model is used to demonstrate the effect of bank crises on the level of output, the size of the interest rate spread, defined as the difference between the loan and deposit rates, and the labour market. Despite its simplicity, the model provides a good insight into the workings of an economy.

Gillman’s (2011) specification of the production function is preferred to the one presented by Goodfriend (2004) and Goodfriend and McCallum (2007) since,
as discussed, the value of the collateral affects the size of the loan but the resources used are the deposits that the financial intermediary has accepted. The formulation is also more appealing than the one presented by Kind and Plosser (1984) because in the latter transaction services are produced with labour and capital without any explicit connection to the stock of deposits. Gillman’s (2011) loan production function is tractable and easy to extend, as it is evident in the papers by Benk et al. (2010) and Gillman and Kejak (2008). The first paper adds human capital and the second augments the production function to include both physical and human capital. It would also be relatively easy to include the collateral constraint proposed by Goodfriend (2004). This could be done either by including it in the production function or as a constraint to the amount of lending. Thus, the loan production function as described by Gillman (2011) provides a great starting point. That is why this is the specification adopted in this thesis.

Furthermore, similar to Gillman (2011), the proposed model in the next chapter focuses on inside money alone. There are several reasons for this decision. First, as argued by King and Plosser (1984), the correlation between money and the business cycle is mostly due to the variability of deposits and real variables. Second, the model in this thesis is tested using UK data. Since the UK has had many regime changes regarding the supply of money and price determination, the data series would have had to be divided into several small samples. This would
negatively affect the power of the test and the significance of the results regarding
the structure of the banking industry. Last but not least, this assumption signifi-
cantly simplifies the model without generating any structures that would be an
obstacle to extend it in those lines in the future.

2.5 Conclusion

This chapter outlined the framework within which the proposed model will fall.
First, an overview of the RBC framework was given. Secondly, some examples of
open economy models were outlined. Thirdly, the issue regarding the nonstation-
arity of open economy models was addressed. Last but not least, a brief summary
of relevant articles that contributed to the development of banking production
approach to financial intermediation was given. The next chapter will provide the
detailed description of the proposed macroeconomic framework. It is based on the
open economy in Meenagh et al. (2005), adjusted by the introduction of an exter-
nal debt elastic interest rate as explained by Schmitt-Grohe and Uribe (2003), and
augmented with a financial intermediation sector described by Gillman (2011).
CHAPTER 3

MODEL

3.1 Introduction

The focus of this chapter is the proposed new DSGE model. It incorporates features from three different DSGE models and by design it could be considered an extension to each one of them. The main contribution here is the unique combination of open economy dynamics, relative prices, the elasticity of the foreign interest rate to the level of debt, and a second sector that represents a financial intermediary which produces loans and acts as an intermediate good to final goods production. The chapter is divided as follows. First, a brief description of the main sectors will be given and is supplemented by a diagrammatic representation. Secondly, the data used and the choice of initial parameter values will be explained. Last but not least, the impulse response functions generated using single temporary shocks will be considered as an indicator of the model’s internal dynamics.

3.2 Model Description

The following set of equations describe the characteristics of a small open economy populated with infinitely lived agents with identical preferences. It is
assumed that the two countries, domestic and the rest of the world, are identical. However, the home country is assumed to be relatively small and thus it cannot influence the variables in the foreign country.

The representative agent faces two optimisation problems: optimal allocation of consumption expenditure between domestic and imported goods; and lifetime utility maximisation problem subject to a budget constraint. There are two sectors in the home country: a goods production sector of which output is a homogenous good used for consumption, exogenous government spending, exports and capital investment; and financial sector which channels funds from the consumer to the goods producing firm. Both industries are owned by the consumer and they earn
zero economic profit. Cross-country trade occurs only in final goods. The home country representative agent can also borrow directly from abroad.

Figure 3.1 is a diagrammatic representation of the model. It outlines the different sectors and the channels through which they interact every period. Each variable is in real per capita terms and it is adjusted by the respective relative price level where necessary. The representative consumer purchases domestic goods $c^h_t$ at a price $p^h_t$ from the goods sector and $c^f_t$ at a price $p^f_t$ (which is equal to the real effective exchange rate $Q_t$) from the rest of the world. The agent receives labour income from the two sectors in the home country $p^h_t w_t$ (where $w_t$ is the producer real wage) in exchange for supplying labour to the goods and loans producers, denoted as $l_{gt}$ and $l_{bt}$ respectively. He also receives his deposit, made in period $(t - 1)$, plus the interest due $d_t (1 + r^d_t)$ and borrows from abroad $d^f_{t+1}$. The total income, including borrowing is either spent on consumption goods $c_t$, or used to repay debt $d^f_t (1 + r^f_t)$ accumulated in the previous period (both principal and interest due), or deposited in the bank $d_{t+1}$. The financial intermediary uses those deposits and labour to produce loans $q_{t+1}$ which are lent out to the final goods producer. The costs of production are covered by the repayment of the loans made in the last period plus the interest income, $q_t (1 + r^q_t)$. The final goods producer borrows from the lending institution and uses the funds to invest in capital goods which together with labour are used in the production of output. The output is
then sold at the relative domestic price level.

The relationships described above will become clearer once the full model set up is described. The rest of this section is divided as follows. First the consumer’s optimisation problems will be outlined. Secondly, the equations characterising the representative firm will be discussed. Thirdly, the behaviour of the financial intermediary as an agent that channels funds from the consumer to the goods producer will be described. Last but not least, the market clearing conditions, the equations depicting the interactions with the rest of the world, and the functions characterising the behaviour of the endogenous variables with exogenous dynamics will be outlined.

3.2.1 The consumer’s problems

The representative agent faces two optimisation problems: an intratemporal problem of optimal consumption basket and an intertemporal utility maximisation problem.

3.2.1.1 The choice of optimal consumption basket

Both the domestic and foreign country produce a single tradable good \(c_i^t\) for \(i = h, f\). It is assumed that the final consumption good is a composite good of domestic and foreign consumption goods which are combined using an Armington aggregator, a linear and homogenous function with constant elasticity of
substitution properties, Armington (1969). The consumption aggregator is:

\[ c_t = \left[ \chi \left( c_t^h \right)^{-\theta} + (1 - \chi) \eta_t^f \left( c_t^f \right)^{-\phi} \right]^{-\frac{1}{\theta}} \]  (3.1)

In the above equation \( c_t \) denotes the final consumption good, \( c_t^h \) denotes the fraction of the final consumption good that is produced domestically, and \( c_t^f \) is the level of the consumption of foreign goods. The elasticity of substitution is \( \sigma = \frac{1}{1+\theta} \) and \( \chi \) and \( (1 - \chi) \) are the weights of domestic and foreign goods in the consumption function. It is assumed that only consumption goods are imported, therefore \( c_t^f = im_t \), where \( im_t \) denotes the amount of imports. The AR(1) process \( \eta_t^f \) is a shock to the demand for foreign goods and can be viewed as preference error.

\[ \eta_t^f = \left( \eta_{t-1}^f \right)^{\sigma^f} + \varepsilon_t^f \]  (3.2)

Let \( P_t \) be the general price level in the home country, \( P_t^h \) - the price level of domestically produced goods, and \( P_t^f \) - the price of imported goods (the general price level in the rest of the world) in terms of home currency. Then the expenditure constraint can be written as:

\[ P_t c_t = P_t^h c_t^h + P_t^f c_t^f \]

Normalising the expenditure constraint by \( P_t \) results in:

\[ c_t = \frac{P_t^h}{P_t} c_t^h + \frac{P_t^f}{P_t} c_t^f \]

Let \( P_t \) be the numeraire and define the relative prices as follow:

\[ p_t^h = \frac{P_t^h}{P_t} \quad \text{and} \quad p_t^f = \frac{P_t^f}{P_t} \]
The expenditure constraint can be expressed as:

\[ c_t = p_t^h c_t^h + p_t^f c_t^f \]

Since \( p_t^f \) is defined as the general price level in the foreign country (the rest of the world) expressed in domestic currency, relative to the general price level in the home country, it can be argued that it is equal to the real effective exchange rate, \( Q_t \). Therefore, the expenditure constraint can be written as:

\[ c_t = p_t^h c_t^h + Q_t c_t^f \] (3.3)

The representative agent uses the consumption aggregator eq. (3.1) and expenditure constraint eq. (3.3) to obtain the Marshallian demands for \( c_t^h \) and \( c_t^f \) as functions of the composite consumption \( c_t \), and the real effective exchange rate \( Q_t \) by performing the following steps. First, the optimisation problem will be set up using the Lagrange function. Secondly, the conditional factor demands \( \frac{c_t^f}{c_t} \) will be obtained. Thirdly, the solution for \( c_t^h \) as a function of \( c_t, p_t^h, \) and \( Q_t \) will be found. Last but not least, the relationship between the relative domestic price level and the real effective exchange rate will be found.

The consumer’s optimal consumption basket problem is to maximise consumption expressed by equation (3.1) subject to the expenditure constrained (3.3). The problem can be written using the Lagrangian function \( \Xi \), where \( \xi \) is the Lagrange
multiplier:

\[
\max_{\{c_t^h, c_t^l\}_{t=0}^{\infty}} \Xi \left( c_t^h, c_t^l \right) = \left[ \chi \left( c_t^h \right)^{-\theta} + (1 - \chi) \eta_t^l \left( c_t^l \right)^{-\theta} \right]^{-\frac{1}{\theta}} + \xi \left( c_t - p_t^h c_t^h - Q_t c_t^l \right)
\]

The representative agent chooses allocations \( \{c_t^h, c_t^l\}_{t=0}^{\infty} \) such that given prices \( \{p_t^h, Q_t\} \), the value of the function \( \Xi \left( c_t^h, c_t^l \right) \) is maximised. The first order conditions are:

\[
c_t^l : -\frac{1}{\theta} \left[ \chi \left( c_t^h \right)^{-\theta} + (1 - \chi) \eta_t^l \left( c_t^l \right)^{-\theta} \right]^{-\frac{1}{\theta}-1} \left( 1 - \chi \right) \eta_t^l \left( -\theta \right) \left( c_t^l \right)^{-\theta-1} = \xi p_t^h (3.5)
\]

\[
c_t^h : -\frac{1}{\theta} \left[ \chi \left( c_t^h \right)^{-\theta} + (1 - \chi) \eta_t^l \left( c_t^l \right)^{-\theta} \right]^{-\frac{1}{\theta}-1} \chi \left( c_t^h \right)^{-\theta-1} = \xi p_t^h (3.5)
\]

\[
\xi : c_t = p_t^h c_t^h + Q_t c_t^l
\]

The above expressions can be simplified using the Armington aggregator which can be written as:

\[
\left( c_t^l \right)^{-\theta} = \left[ \chi \left( c_t^h \right)^{-\theta} + (1 - \chi) \eta_t^l \left( c_t^l \right)^{-\theta} \right]
\]

Raising both sides to the power \( \left( -\frac{1}{\theta} - 1 \right) \), yields:

\[
\left( c_t^l \right)^{-\frac{1}{\theta}-1} = \left[ \chi \left( c_t^h \right)^{-\theta} + (1 - \chi) \eta_t^l \left( c_t^l \right)^{-\theta} \right]^{-\frac{1}{\theta}}
\]

\[
\left( c_t \right)^{1+\theta} = \left[ \chi \left( c_t^h \right)^{-\theta} + (1 - \chi) \eta_t^l \left( c_t^l \right)^{-\theta} \right]^{-\frac{1}{\theta}-1}
\]

Using the above equation to substitute out \( \left[ \chi \left( c_t^h \right)^{-\theta} + (1 - \chi) \eta_t^l \left( c_t^l \right)^{-\theta} \right]^{-\frac{1}{\theta}-1} \) and cancelling factors off leads to the following simplified system of equations:

\[
\left( c_t \right)^{1+\theta} \left( 1 - \chi \right) \eta_t^l \left( c_t^l \right)^{-\theta-1} - \xi Q_t = 0
\]

\[
\left( c_t \right)^{1+\theta} \chi \left( c_t^h \right)^{-\theta-1} - \xi p_t^h = 0
\]

\[
c_t - p_t^h c_t^h + Q_t c_t^l = 0
\]

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Expressing the first two equations in terms of the inverse of the shares of domestic and foreign goods in the consumption basket leads to:

\[
\left(\frac{c_t}{c^d_t}\right)^{1+\theta} \left(1 - \chi \right) \eta^f_t = \xi Q_t \tag{3.7}
\]

\[
\left(\frac{c^h_t}{c^d_t}\right)^{1+\theta} \chi = \xi p^h_t \tag{3.8}
\]

\[
c_t - p^h_t c^h_t + Q_t c^f_t = 0 \tag{3.9}
\]

Equations (3.7) and (3.8) can be equated by substituting out $\xi$, and simplified such that the relative demand of foreign to domestic consumption goods is a function of their respective weights in the consumption basket, the relative prices and the preference shock:

\[
\left(\frac{c_t}{c^d_t}\right)^{1+\theta} \left(1 - \chi \right) \eta^f_t \frac{1}{Q_t} = \xi = \left(\frac{c^h_t}{c^d_t}\right)^{1+\theta} \chi \frac{1}{p^h_t}
\]

\[
\frac{c^f_t}{c^d_t} = \frac{\left(1 - \chi \right)}{\chi} \left(\eta^f_t \frac{p^h_t}{Q_t}\right)^{\frac{1}{1+\theta}} \tag{3.10}
\]

Equation (3.10) is the marginal rate of substitution between domestic and foreign goods $\left(MRS_{c^d_t, c^f_t}\right)$. It demonstrates that, ceteris paribus, the higher the relative domestic price level $p^h_t$, the lower the real effective exchange rate $Q_t$, the lower the weight to domestic consumption in the basket $\chi$, and the higher the preference shock $\eta^f_t$ - the higher the share of foreign goods relative to domestically produced goods $\frac{c^f_t}{c^d_t}$ will be in the composite consumption basket $c_t$. Using the $MRS_{c^d_t, c^f_t}$ the
The relative factor demands can be substituted out using equation (10) in (13) and (14).

\begin{align*}
\frac{c_t}{c_t^h} &= \left[ \chi + (1 - \chi) \eta_t^f \left( \frac{c_t^f}{c_t^h} \right)^{-\theta} \right]^{-\frac{1}{\theta}} \\
\frac{c_t}{c_t^h} &= p_t^h + Q_t \left( \frac{c_t^f}{c_t^h} \right) \\
\end{align*}

The relative factor demands can be substituted out using equation (10) in (13) and (14).
The above can be simplified as follows:

\[(13) : \frac{c_t}{c_t^l} = \left[ \chi + (1 - \chi) \eta_l^f \left( \frac{1 - \chi}{\chi} \right)^{\frac{1}{1+\sigma}} \left( \eta_l^c \right)^{\frac{1}{1+\sigma}} \left( \frac{p_l^h}{Q_l} \right)^{\frac{1}{1+\sigma}} \right]^{-\frac{1}{\sigma}} \]

\[(14) : \frac{c_t}{c_t^l} = p_l^h \left[ 1 + \frac{Q_l}{p_l^h} \left( \frac{1 - \chi}{\chi} \right)^{\frac{1}{1+\sigma}} \left( \eta_l^c \right)^{\frac{1}{1+\sigma}} \left( \frac{p_l^h}{Q_l} \right)^{\frac{1}{1+\sigma}} \right] \]

To simplify illustration, let \( A \equiv \left[ 1 + \left( \frac{1 - \chi}{\chi} \right)^{\frac{1}{1+\sigma}} \left( \eta_l^c \right)^{\frac{1}{1+\sigma}} \left( \frac{p_l^h}{Q_l} \right)^{\frac{1}{1+\sigma}} \right] \),

\[(13) : \frac{c_t}{c_t^l} = \chi^{-\frac{1}{\sigma}} A^{\frac{1}{\sigma}} \]

\[(14) : \frac{c_t}{c_t^l} = p_l^h A \]

Therefore,

\[p_l^h A = \chi^{-\frac{1}{\sigma}} A^{\frac{1}{\sigma}}\]

\[p_l^h = \chi^{-\frac{1}{\sigma}} A^{-\frac{1+\sigma}{\sigma}}\]

\[A^{-\frac{1+\sigma}{\sigma}} = p_l^h \chi^{\frac{1}{\sigma}}\]

\[A = (p_l^h)^{-\frac{\sigma}{1+\sigma}} \chi^{-\frac{1}{1+\sigma}}\]
\[
\frac{c_t}{c_t^h} = \frac{p_t^h}{p_t^h} \left[ 1 + \left( \frac{1 - \chi}{\chi} \right)^{\frac{1}{\frac{1}{\gamma}} - \frac{1}{\gamma}} \left( \eta_t^h \right)^{\frac{1}{\frac{1}{\gamma}} - \frac{1}{\gamma}} \left( \frac{p_t^h}{Q_t} \right)^{\frac{1}{\frac{1}{\gamma}} - \frac{1}{\gamma}} \right] = p_t^h \frac{A^h}{A^h} = p_t^h \left( p_t^h \right)^{-\frac{1}{\frac{1}{\gamma}} - \frac{1}{\gamma}} \chi^{-\frac{1}{\frac{1}{\gamma}} - \frac{1}{\gamma}} = \left( p_t^h \right)^{\frac{1}{\frac{1}{\gamma}}} \chi^{-\frac{1}{\frac{1}{\gamma}}} = \left( p_t^h \right)^{\frac{1}{\frac{1}{\gamma}}} \chi^{-\frac{1}{\frac{1}{\gamma}}}
\]

\[
\frac{c_t}{c_t^h} = \left( p_t^h \right)^{\frac{1}{\frac{1}{\gamma}}} \chi^{-\frac{1}{\frac{1}{\gamma}}}
\]

(3.15)

Using the expenditure constraint and dividing both sides by \( c_t^h \), a second equation for \( \frac{c_t}{c_t^h} \) can be obtained.

\[
c_t = p_t^h c_t^h + Q_t c_t^f
\]

\[
\frac{c_t}{c_t^h} = p_t^h + Q_t \frac{c_t^f}{c_t^h}
\]

(3.16)

Substituting equations (3.10) and (3.15) into (3.16) generates the solution for \( p_t^h \) in terms of \( Q_t \), the exogenous variable \( \eta_t^f \), and parameters.

\[
\left( p_t^h \right)^{\frac{1}{\frac{1}{\gamma}}} \chi^{-\frac{1}{\frac{1}{\gamma}}} = p_t^h + Q_t \left( \frac{1 - \chi}{\chi} \right)^{\frac{1}{\frac{1}{\gamma}}} \left( \eta_t^f \right)^{\frac{1}{\frac{1}{\gamma}}} \left( \frac{p_t^h}{Q_t} \right)^{\frac{1}{\frac{1}{\gamma}}}
\]

\[
\chi^{-\frac{1}{\frac{1}{\gamma}}} = \left( p_t^h \right)^{\frac{1}{\frac{1}{\gamma}}} + Q_t \left( \frac{1 - \chi}{\chi} \right)^{\frac{1}{\frac{1}{\gamma}}} \left( \eta_t^f \right)^{\frac{1}{\frac{1}{\gamma}}} \left( \frac{1}{Q_t} \right)^{\frac{1}{\frac{1}{\gamma}}}
\]

\[
\chi^{-\frac{1}{\frac{1}{\gamma}}} = \left( p_t^h \right)^{\frac{1}{\frac{1}{\gamma}}} + \left( \frac{1 - \chi}{\chi} \right)^{\frac{1}{\frac{1}{\gamma}}} \left( \eta_t^f \right)^{\frac{1}{\frac{1}{\gamma}}} (Q_t)^{\frac{1}{\frac{1}{\gamma}}}
\]

\[
\left( p_t^h \right)^{\frac{1}{\frac{1}{\gamma}}} = \chi^{-\frac{1}{\frac{1}{\gamma}}} - \left( \frac{1 - \chi}{\chi} \right)^{\frac{1}{\frac{1}{\gamma}}} \left( \eta_t^f \right)^{\frac{1}{\frac{1}{\gamma}}} (Q_t)^{\frac{1}{\frac{1}{\gamma}}}
\]

\[
p_t^h = \left( \chi^{-\frac{1}{\frac{1}{\gamma}}} - \left( \frac{1 - \chi}{\chi} \right)^{\frac{1}{\frac{1}{\gamma}}} \left( \eta_t^f \right)^{\frac{1}{\frac{1}{\gamma}}} (Q_t)^{\frac{1}{\frac{1}{\gamma}}} \right)^{\frac{1}{\frac{1}{\gamma}}}
\]

(3.17)

Substituting the above solution for \( p_t^h \) into the inverse of equation (3.15) generates
the Marshallian demand for $c^h_t$.

\[
\frac{c^h_t}{c_t} = (p^h_t)^{-\frac{1}{\tau + \sigma}} \chi^{\frac{1}{\tau + \sigma}}
\]

\[
\frac{c^h_t}{c_t} = \left( \chi^{-\frac{1}{\tau + \sigma}} - \left( \frac{1 - \chi}{\chi} \right)^{\frac{1}{\tau + \sigma}} \left( \eta^f_t \right)^{\frac{1}{\tau + \sigma}} \left( Q_t \right)^{\frac{\theta}{\tau + \sigma}} \right)^{\frac{1 + \theta}{\theta}} - \frac{1}{\theta} \chi^{\frac{1}{\tau + \sigma}}
\]

\[
\frac{c^h_t}{c_t} = \chi^{-\frac{1}{\tau + \sigma}} \left( Q_t \right)^{\frac{\theta}{\tau + \sigma}} \chi^{-\frac{\theta}{\tau + \sigma}}
\]

\[
\frac{c^h_t}{c_t} = \left( \chi^{-\frac{1}{\tau + \sigma}} \left( 1 - (1 - \chi)^{\frac{1}{\tau + \sigma}} \left( \eta^f_t \right)^{\frac{1}{\tau + \sigma}} \left( Q_t \right)^{\frac{\theta}{\tau + \sigma}} \right) \chi^{-\frac{\theta}{\tau + \sigma}} \right)^{-\frac{1}{\theta}}
\]

\[
\frac{c^h_t}{c_t} = \left( \chi^{-1} \left( 1 - (1 - \chi)^{\frac{1}{\tau + \sigma}} \left( \eta^f_t \right)^{\frac{1}{\tau + \sigma}} \left( Q_t \right)^{\frac{\theta}{\tau + \sigma}} \right) \right)^{-\frac{1}{\theta}}
\]

\[
c^h_t = \left( \chi^{-1} \left( 1 - (1 - \chi)^{\frac{1}{\tau + \sigma}} \left( \eta^f_t \right)^{\frac{1}{\tau + \sigma}} \left( Q_t \right)^{\frac{\theta}{\tau + \sigma}} \right) \right)^{-\frac{1}{\theta}} c_t
\]

(3.18)

The solution for $c^f_t$ can be obtained by using equations (3.10) and (3.15)

\[
\frac{c^f_t}{c^f_t} = \left( \frac{1 - \chi}{\chi} \right)^{\frac{1}{\tau + \sigma}} \left( \eta^f_t \right)^{\frac{1}{\tau + \sigma}} \left( p^h_t \right)^{\frac{1}{\tau + \sigma}}
\]

\[
\frac{c_t}{c^f_t} = (p^h_t)^{\frac{1}{\tau + \sigma}} \chi^{\frac{1}{\tau + \sigma}}
\]

\[
c^h_t = c^f_t \left( \frac{1 - \chi}{\chi} \right)^{-\frac{1}{\tau + \sigma}} \left( \eta^f_t \right)^{-\frac{1}{\tau + \sigma}} \left( p^h_t \right)^{-\frac{1}{\tau + \sigma}}
\]

\[
c^h_t = (p^h_t)^{-\frac{1}{\tau + \sigma}} \chi^{\frac{1}{\tau + \sigma}} c_t
\]

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Equating the two expressions for $c_h^t$ and simplifying generates the solution for $c_f^t$

$$c_f^t \left( \frac{1 - \chi}{\chi} \right)^{-\frac{1}{1+\beta}} \left( \eta_f^t \right)^{-\frac{1}{1+\beta}} \left( \frac{p_f^h}{Q_t} \right)^{-\frac{1}{1+\beta}} = (p_f^h)^{-\frac{1}{1+\beta}} \chi^{\frac{1}{1+\beta}} c_t$$

$$c_f^t \left( \frac{1 - \chi}{\chi} \right)^{-\frac{1}{1+\beta}} \left( \eta_f^t \right)^{-\frac{1}{1+\beta}} \left( \frac{1}{Q_t} \right)^{-\frac{1}{1+\beta}} = \chi^{\frac{1}{1+\beta}} c_t$$

$$c_f^t = (1 - \chi)^{\frac{1}{1+\beta}} \left( \eta_f^t \right)^{\frac{1}{1+\beta}} (Q_t)^{-\frac{1}{1+\beta}} c_t \quad (3.19)$$

Therefore, the Marshallian demands for domestic and foreign goods in terms of aggregate consumption and relative prices and the relationship between the home country relative price level and the real effective exchange rate are:

$$p_f^h = \left( \chi^{-\frac{1}{1+\beta}} - \left( \frac{1 - \chi}{\chi} \right)^{-\frac{1}{1+\beta}} \eta_f^t \left( Q_t \right)^{\frac{\theta}{1+\beta}} \right)^{\frac{1+\theta}{\beta}}$$

$$c_h^t = \left( \chi^{-1} \left( 1 - (1 - \chi)^{\frac{1}{1+\beta}} \eta_f^t \left( Q_t \right)^{\frac{\theta}{1+\beta}} \right) \right)^{-\frac{1}{\beta}} c_t$$

$$c_f^t = (1 - \chi)^{\frac{1}{1+\beta}} \left( \eta_f^t \right)^{\frac{1}{1+\beta}} (Q_t)^{-\frac{1}{1+\beta}} c_t \quad (3.19)$$

Equations (3.17) and (3.19) are equilibrium conditions.

3.2.1.2 The representative agent’s lifetime utility maximisation problem

The consumer’s preferences over consumption and leisure are described by the following utility function:

$$U(c_t, 1 - n_t) = \omega(1 - \rho_1)^{-\frac{1}{\beta}} \eta_t^c \epsilon_t^{(1-\rho_1)}(1 - \omega)(1 - \rho_2)^{-\frac{1}{\beta}} \eta_t^n \epsilon_t^{(1-\rho_2)} \quad (3.20)$$

The assumed functional form is standard in open economy RBC literature and depicts the trade-off between leisure time and consumption. The parameter $\omega$
is the preference bias for consumption, $\rho_2$ is governing the labour(leisure) supply
elasticity, and $\rho_1$ is the parameter of relative risk aversion. The AR(1) process $\eta_t^n$ represents a shock to the labour supply schedule. Here time is normalised to one, $l_t$ is leisure time and $n_t$ is time spent in employment.

$$n_t + l_t = 1 \quad (3.21)$$

In this economy there are two sectors that use labour as an input factor - the
goods producing industry and the financial intermediation sector. Let $l_{gt}$ be the share of employment used in the goods sector and $l_{bd}$ - the share in the financial intermediary industry.

$$n_t = l_{gt} + l_{bd} \quad (3.22)$$

It is important to point out that in this framework there is perfect labour mobil-
ity. This would ensure factor price equalisation in the labour market in a similar fashion as described by Samuelson (1948) with regards to the international mar-
kets. Hence, the representative agent earns the same real wage rate $w_t^c$ in both industries. Let $W_t$ be the nominal wage rate. It follows that, the consumer real wage is $w_t^c \equiv \frac{W_t}{P_t}$. Multiplying and dividing by the domestic price level $P_t^h$, and denoting the producer real wage as $w_t \equiv \frac{W_t}{P_t^h}$ leads to the following expression for the consumer real wage.

$$w_t^c = \frac{w_t}{P_t^h} = \frac{W_t P_t^h}{P_t P_t^h} = \frac{W_t P_t^h}{P_t^h P_t} = w_t p_t^h$$

$$w_t^c = w_t p_t^h \quad (3.23)$$
Equation (3.23) shows the relationship between the producer and consumer real wage, and \( p_h^t \) is the wedge resulting from the assumption that domestically produced goods are valued at the home price level and not the general price level, Minford (2015).

The income earned in the two industries \( (p_h^t w_t) \) is either spent on consumption, or deposited in the financial intermediary, or used to cover foreign debt obligations. Let \( d_{t+1} \) be the amount of deposits made in period \( t \) that will mature at the beginning of period \( t+1 \). At the beginning of period \( t+1 \), the consumer will receive its deposit plus interest \( r_{t+1}^d \). In this set up \( d_{t+1} \) is an endogenous state variable. Similarly, \( d_{t+1}^f \) are the debt obligations acquired in period \( t \) that have to be repaid at the beginning of period \( t+1 \) together with the incurred interest charge. In this model environment only the consumer is allowed to borrow from the international capital markets. Therefore, \( d_{t+1}^f \) represents the home country’s foreign debt liabilities. Given the assumed time convention and that there is no default risk, \( d_{t+1}, r_{t+1}^d, d_{t+1}^f, \) and \( r_{t+1}^f \) are known in period \( t \) and there is no expectations operator in front of them.

Therefore, at any period \( t \) the total funds available to the representative agent are a combination of labour earnings, interest income on savings and deposits made in the previous period, and the amount of funds borrowed from the rest of the world. The disposable funds are either spent on consumption goods, or
reinvested in the financial intermediary in the form of new deposits, or used to repay foreign debt obligations which have matured. All variables are in real terms, relative to the general price level. The consumer’s budget constraint can be written as:

\[ c_t + d_{t+1} + (1 + r_f^t)d_f^t Q_t = p_i^b w_t n_t + d_{t+1}^f Q_t + (1 + r_d^t)d_t \]  

(3.24)

Given equations (3.20), (3.21), and (3.24), the representative agent’s optimisation problem can be described as:

\[
\max_{\{c_t, n_t\}^\infty_{t=0}} U(c_t, 1 - n_t) = \omega(1 - \rho_1)^{-\eta_t^c(1-\rho_1)} + (1 - \omega)(1 - \rho_2)^{-\eta_t^n(1-\rho_2)}
\]

s.t.

\[ n_t + l_t = 1 \]

\[ c_t + d_{t+1} + (1 + r_f^t)d_f^t Q_t = p_i^b w_t n_t + d_{t+1}^f Q_t + (1 + r_d^t)d_t \]

The consumer faces a constrained optimisation problem: to choose optimal allocations \( \{c_t, n_t, d_{t+1}, d_f^t\} \) such that given prices \( \{Q_t, p_i^b\} \), his utility, as described by equation (3.20), is maximised. Let \( \lambda_t \) be the shadow price of consumption and \( \beta \) - the time preference discount factor; then the representative agent’s intertemporal optimisation problem can be written using the Lagrange function \( L(c_t, n_t, d_{t+1}, d_f^t) \), as:

\[
\max_{\{c_t, n_t, d_{t+1}, d_f^t\}^\infty_{t=0}} L = E_0 \sum_{t=0}^\infty \beta^t \left\{ \omega(1 - \rho_1)^{-\eta_t^c(1-\rho_1)} + (1 - \omega)(1 - \rho_2)^{-\eta_t^n(1-\rho_2)} + \lambda_t \left[ p_i^b w_t n_t + d_{t+1}^f Q_t + (1 + r_d^t)d_t - c_t - d_{t+1} - (1 + r_f^t)d_f^t Q_t \right] \right\}
\]

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The first order condition are:

\[ c_t : \beta^t \omega (1 - \rho_1)^{-1} \eta^c_t (1 - \rho_1) c_t^{-\rho_1} - \lambda_t \beta^t = 0 \]

\[ n_t : \beta^t (1 - \omega)(1 - \rho_2)^{-1} \eta^n_t (1 - \rho_2)(1 - n_t)^{-\rho_2}(-1) + \lambda_t \beta^t p_t^b w_t = 0 \]

\[ d_{t+1} : \lambda_t \beta^t (-1) + \lambda_{t+1} \beta^{t+1}(1 + r_{t+1}^d) = 0 \]

\[ d_{t+1}^f : \lambda_t \beta^f Q_t + \lambda_{t+1} \beta^{t+1}(1 + r_{t+1}^f)(-1)Q_{t+1} = 0 \]

\[ \lambda_t : p_t^b w_t n_t + d_{t+1}^f Q_t + (1 + r_t^d)d_t - c_t - d_{t+1} - (1 + r_t^f)d_t^f Q_t = 0 \]

\[ c_t : \omega \eta^c_t c_t^{-\rho_1} = \lambda_t \] (3.25)

\[ n_t : (1 - \omega) \eta^n_t (1 - n_t)^{-\rho_2} = \lambda_t p_t^b w_t \] (3.26)

\[ d_{t+1} : \beta(1 + r_{t+1}^d) = \frac{\lambda_t}{\lambda_{t+1}} \] (3.27)

\[ d_{t+1}^f : \beta(1 + r_{t+1}^f) \frac{Q_{t+1}}{Q_t} = \frac{\lambda_t}{\lambda_{t+1}} \] (3.28)

\[ \lambda_t : p_t^b w_t n_t + d_{t+1}^f Q_t + (1 + r_t^d)d_t - c_t - d_{t+1} - (1 + r_t^f)d_t^f Q_t = c_t \] (3.29)

Using equations (3.25) and (3.26), the shadow price of consumption, \( \lambda_t \), can be substituted out.

\[ (1 - \omega) \eta^c_t (1 - n_t)^{-\rho_2} = \omega \eta^c_t c_t^{-\rho_1} p_t^b w_t \]

\[ \frac{(1 - \omega) \eta^n_t (1 - n_t)^{-\rho_2}}{\omega \eta^c_t c_t^{-\rho_1}} = p_t^b w_t \] (3.30)

\[ \frac{c_t^{\rho_1}}{(1 - n_t)^{\rho_2}} = \frac{\omega \eta^c_t}{(1 - \omega) \eta^n_t} p_t^b w_t \] (3.31)

Using equations (3.25) and (3.27) and adjusting for the time period results in the
following relationship:

\[
\beta(1 + r^d_{t+1}) = \frac{\omega \eta^c_t c_t^{-\rho_1}}{E_t \left[ \eta^c_{t+1} c_{t+1}^{-\rho_1} \right]}
\]

\[
\beta(1 + r^d_{t+1}) = \frac{\eta^c_t c_t^{-\rho_1}}{E_t \left[ \eta^c_{t+1} c_{t+1}^{-\rho_1} \right]}
\] (3.32)

Using equations (3.27) and (3.28) leads to:

\[
(1 + r^d_{t+1}) = (1 + r^f_{t+1}) \frac{Q_{t+1}}{Q_t}
\] (3.33)

Equations (3.31), (3.32), and (3.33) are equilibrium conditions and part of the model listing. Equation (3.31) represents the marginal rate of substitution between consumption and leisure. It illustrates that, ceteris paribus, the higher the real producer wage rate, the higher the relative domestic price level, the higher the preference weight, the higher the current consumption preference shock, the lower the labour supply shock, the higher the risk aversion parameter, and the lower the intertemporal elasticity of substitution between current and future leisure, the higher the current consumption will be. The logic behind all these conclusions is quite straightforward, apart from one - the impact of the relative domestic price level. According to the expression for \( MRS_{c_t,n_t} \), an increase in the relative price \( p^h_t \) would lead to an increase in current consumption. Although this may seem counter intuitive at first, once investigated it becomes clear. Since \( p^h_t = \frac{p^h_t}{P_t} \) is a relative price, an increase would imply that either the domestic prices \( P^h_t \) have risen relative to those in the rest of the world, or the general price level has decreased.
Consider the two cases:

- Scenario one - consider an increase in $P_t^h$. Given fixed expenditure as described by equation (3.3), the representative agent would substitute home produced goods $c_t^h$ for imported goods $c_t^f$. This would affect the ratio of the two in the composite consumption basket $c_t$. This in turn would affect the weights used to calculate the general price level in the home country creating an income effect which would lead to an increase in the current consumption basket, for a given expenditure constraint. Therefore, an increase in $p_t^h$ leads to a substitution and income effect in the intratemporal optimisation problem which lead to an overall increase of current consumption as described by the intertemporal optimisation problem.

- Scenario two - consider a decrease in the general price level $P_t$. This would imply that for a certain level of income, ceteris paribus, the same level of expenditure on current consumption $(P_t c_t)$ would be achieved by an increase in the quantities of goods consumed.

The relationship depicted by (3.32) is the Euler equation. It states that the higher the interest rate on deposits or the larger the discount factor (implying that agents’ utility associated with future consumption approaches the one from consuming equal quantity in the present) , the larger the ratio of the next period consumption will be compared to the one in the current period.
The last equation from the consumer side of the economy that enters the final model listing is equation (3.33). It depicts the relationship between domestic and foreign (world) interest rates, adjusted by the expected change in the real effective exchange rate. This is the uncovered interest parity (UIP) condition, adjusted by the respective inflation rates (i.e. real UIP), and illustrates that any increase in foreign rates or expected future effective exchange rates would lead to a rise in current domestic interest rates. The last statement concludes the description of the consumer side of the economy. Next, the optimising behaviour of the goods producer will be discussed.

3.2.2 The domestic good producer sector

In this model, it is assumed that there is a single goods producer. The output of this sector is used for consumption (both private and public), exports, and capital investment. The technology used is described by a standard Cobb-Douglas production function, first estimated by Cobb and Douglas (1928):

\[ y_t = A_g z_{gt} l_{gt}^{a} k_t^{1-a} \]  

(3.34)

In the above equation a standard notation is used: \( y_t \) is real output, \( k_t \) - capital input, \( l_{gt} \) - labour input in the goods sector, \( z_{gt} \) - total factor productivity shock which is an AR(1) process, and \( A_g \) and \( \alpha \) are parameters. The representative firm
owns the capital which is accumulated according to the following law of motion:

\[ k_{t+1} = i_{kt} + (1 - \delta) k_t \]  
(3.35)

\( i_{kt} \) is gross investment and \( \delta \) is the depreciation rate. Let \( i_t \) denotes net investment; therefore by definition:

\[ i_t = i_{kt} - \delta k_t = k_{t+1} - k_t \]  
(3.36)

By definition, \( q_{t+1}^d \) is a static variable and \( q_t^d \) is the amount of loans borrowed from the financial intermediary in period \( t - 1 \) which has to be returned in period \( t \) plus interest (\( r_q^d \)). The goods producer borrows from the financial intermediary in order to invest in capital. Following Gillman (2011), it is assumed that new investment is entirely financed by new loans from the financial intermediary, i.e. the firm uses only debt financing. Incorporating this assumption into equation(3.36) leads to:

\[ i_t = i_{kt} - \delta k_t = k_{t+1} - k_t = q_{t+1}^d - q_t^d \]

Assuming that this holds every period, it follows that at time \( t = 0 \):

\[ i_0 = k_1 - k_0 = q_1^d - q_0^d \]

\[ i_0 = k_1 - 0 = q_1^d - 0 \]

Therefore:

\[ k_1 = q_1^d \]
Since $k_1 = q_{1t}^d$, than $k_2 = q_{2t}^d$; but if $k_2 = q_{2t}^d$ than $k_3 = q_{3t}^d$ and so on. Using forward substitution, it can easily be seen that

$$k_t = q_{jt}^d$$

(3.37)

for every period $t \in [0; +\infty)$. The current stock of capital equals the stock of outstanding loans. Another important assumption is that since the representative firm sells its output at the domestic price level, it evaluates its cost of production at the domestic price level as well. At any period $t$, the representative form receives revenue $y_t$ from selling the homogenous good, borrows from the bank $q_{dt+1}$, and uses these funds to cover the cost of labour $w_t q_l^g l_t$, invest in new capital $i_{kt}$, and repay the loans from the previous period $(1 + r^q) q_{jt}^d$. Therefore, the goods producer net cash flow function at time $t$ denoted by $\Pi_{gt}$ can be defined as:

$$\Pi_{gt} = y_t - w_t q_l^g l_t - i_t - \eta_t^k k_t + q_{jt+1}^d - (1 + r^q_t) q_{jt}^d$$

(3.38)

$\eta_t^q$ and $\eta_t^k$ are error terms and are defined as AR(1) processes which account for omitted labour and capital taxes within this model framework. Given equations (3.34), (3.35), (3.37), and (3.38), the goods producer’s optimisation problem can be defined as, choosing allocations $\{k_{t+1}, l_t, y_t, i_t, q_{jt+1}^d\}$, as such that given prices
The net cash flow function is maximised:

$$\max_{\{k_{t+1}; g_t, i_t, q^d_t\}_{t=0}^{\infty}} \Pi_{g_t} = y_t - w_t \eta_t^q l_{gt} - i_{kt} - \eta_t^k k_t + q^d_{t+1} - (1 + r^q_t) q^d_t$$

s.t.

\[
y_t = A_g z^{\alpha} g_t k_t^{1-\alpha} \\
k_{t+1} = i_{kt} + (1 - \delta) k_t \\
k_t = q^d_t
\]

The problem can be simplified by substituting the \(y_t\) and \(i_{kt}\) using the production function definition and the law of motion in the objective function and using that \(k_t = q^d_t\):

$$\Pi_{g_t} = y_t - w_t \eta_t^q l_{gt} - i_{kt} - \eta_t^k k_t + q^d_{t+1} - (1 + r^q_t) q^d_t$$

$$= y_t - w_t \eta_t^q l_{gt} - k_{t+1} + (1 - \delta) k_t - \eta_t^k k_t + q^d_{t+1} - (1 + r^q_t) q^d_t$$

$$= y_t - w_t \eta_t^q l_{gt} - k_{t+1} + (1 - \delta) k_t - \eta_t^k k_t + k_{t+1} - (1 + r^q_t) k_t$$

$$= y_t - w_t \eta_t^q l_{gt} - k_{t+1} + (1 - \delta) k_t - \eta_t^k k_t + k_{t+1} - (1 + r^q_t) k_t$$

$$= A_g z^{\alpha} g_t k_t^{1-\alpha} - w_t \eta_t^q l_{gt} - (\delta + \eta_t^k + r^q_t)$$

(3.39)

From equation (3.39) it is evident that the firm’s intertemporal optimisation problem has become an intratemporal one. The goods producer chooses the factor of inputs so that the expected cash flow stream is maximised:

$$\max_{\{k_t; g_t\}_{t=0}^{\infty}} \Pi_{g_t} = E_0 \sum_{t=0}^{\infty} \beta^t \left[ A_g z^{\alpha} g_t k_t^{1-\alpha} - \eta_t^q w_t l_{gt} - (r^q_t + \delta + \eta_t^k) k_t \right]$$

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The first order conditions are:

\[ l_t : \alpha A_g z_{gt} l_t^{\alpha-1} k_t^{1-\alpha} - \eta_t^q w_t = 0 \]

\[ k_t : (1 - \alpha) A_g z_{gt} l_t^{\alpha} k_t^{-\alpha} - (r_t^q + \delta + \eta_t^k) = 0 \]

\[ l_t : \alpha A_g z_{gt} \left( \frac{l_{gt}}{k_t} \right)^{\alpha-1} = \eta_t^q w_t \quad (3.40) \]

\[ k_t : (1 - \alpha) A_g z_{gt} \left( \frac{l_{gt}}{k_t} \right)^{\alpha} = (r_t^q + \delta + \eta_t^k) \quad (3.41) \]

The above equilibrium condition (3.41) states that the marginal product of capital \( (MP_k) \) equals to the marginal cost of capital \( (MC_k) \) represented by the sum of the interest rate on loans, the rate at which capital depreciates, and the effective tax rate. Let the required return on capital be defined as \( r_t^k \). Then, the relationship between the return on capital and the cost of borrowing from the financial intermediary is described by:

\[ r_t^k = \delta + r_t^q + \eta_t^k \]

Similarly, equation (3.40) states that at the optimum, \( (MP_l) = (MC_l) \). It can be seen that the error term for the omitted labour tax acts as the wedge between the \( MP_l \) and the real producer wage rate. The equations from the goods producer’s optimisation problem which enter the model listing are the production function (3.34), the capital accumulation equation (3.35), and the two optimality conditions - (3.40) and (3.41). The next section will describe the banking sector.
as the industry that channels funds from savers, the surplus units, to borrowers, the deficit units.

3.2.3 The financial intermediation sector

In this framework there is a second sector in the domestic economy - the financial intermediary. It is a simple but effective model of the banking firm which focuses on the implications of a financial intermediary on the real side of the economy. There is no risk of default (either on interest or on principal repayments), no reserve requirements or central bank regulation, and no off-balance sheet activities. The bank earns zero economic profit and it is entirely owned by the representative agent. Given that the agents in this model operate in a risk-free environment, it could be argued that the financial sector acts only as a link between savers (consumers) and borrowers (firms). It could be argued that this is either because there is no direct way for the representative agent to directly lend to the goods producing firm or that it would be too costly to do so. If the latter is the case, the rationale for the existence of the banking firm is to reduce transaction costs, which is one of the benefits of the banking industry in real life.

The financial intermediary accepts deposits from the representative agent and utilises them together with labour to produce loans lent to the goods producer. Since this is an open economy DSGE model, relative prices play a major role. As discussed earlier, the consumer variables are expressed in terms of the general
price level, the numeraire. The variables in the goods sector are relative to the
domestic price level since this is the price at which the firm sells its output. Thus,
the variables describing the financial industry can be expressed in terms of either
the general price level or the domestic price level. Given that the representative
agent owns the bank, without loss of generality, it is assumed that the financial
intermediary evaluates its profit relative to the general price level and all variables
are expressed in that way. This would imply that the demand for loans $q_{t+1}^d$ is
equal to the supply of loans $q_{t+1}$, divided by the relative domestic price level. To
illustrate this, let the nominal value of loans be $L_{t+1}$ which is the same for both
$q_{t+1}^d$ and $q_{t+1}$. Therefore:

$$q_{t+1}^d = \frac{L_{t+1}}{P_t^d} = \frac{L_{t+1}}{P_t^d} \frac{P_t}{P_t^d} = \frac{L_{t+1}}{P_t^d} \frac{1}{P_t^d} = q_{t+1} \frac{1}{P_t}$$

It is also important to reiterate the timing convention. $q_{t+1}$ ($d_{t+1}$) are the loans(deposits)
made(accepted) in period $t$ which will mature at the beginning of period $t+1$.
Therefore $q_{t+1}$ ($d_{t+1}$) is a state variable in period $(t+1)$, which value will have
been decided in period $t$. It is also assumed that the financial intermediary uses
constant returns to scale production function to generate loans $q_{t+1}$, using labour
input $l_{t+1}$ and deposits $d_{t+1}$:

$$q_{t+1} = A_b z_{t+1} l_{t+1}^{\gamma} d_{t+1}^{1-\gamma}$$

$A_b$ and $\gamma$ are parameters and $z_{t+1}$ is an AR(1) process representing the state of
technology and reflects any shocks to the banking industry that affect lending other than labour supply and the availability of deposits. Since the financial industry uses all deposits to create loans and there are no reserve requirements, it follows that:

\[ q_{t+1} = d_{t+1} \]  

Equation (3.44) is the bank’s balance sheet constraint. At any period \( t \), the value of the financial intermediary’s cash flow function \( \Pi_{bt} \) is equal to the cash inflow: the repayment of loans made in period \( (t - 1) \) plus the new deposits, less the cash outflow: issuing new loans, wage bill, and repayment of old liabilities that are due, i.e. deposits accepted in period \( (t - 1) \):

\[ \Pi_{bt} = (1 + r^q_t)q_t - (1 + r^d_t)d_t - p^h_t \eta^b_t w_t l_{bt} - q_{t+1} + d_{t+1} \]  

(3.45)

The banking sector faces the following constrained optimisation problem - choosing allocations \( \{q_{t+1}, d_{t+1}, l_{bt}, \} \) so that given prices \( \{r^q_t, r^d_t, p^h_t, w_t, \} \), production constraint (3.43), and balance sheet constraint (3.44), the net cash flow function (3.45), is maximised:

\[
\max_{\{q_{t+1}, d_{t+1}, l_{bt}, \}_{t=0}^{\infty}} \Pi_{gt} = (1 + r^q_t)q_t - (1 + r^d_t)d_t - p^h_t \eta^b_t w_t l_{bt} - q_{t+1} + d_{t+1}
\]

s.t.

\[ q_{t+1} = A_b z_{bt} l_{bt} d_{t+1}^{1-\gamma} \]

\[ q_{t+1} = d_{t+1} \]
Using the balance sheet constraint, the problem can be simplified by substituting out the deposit variable and cancelling terms.

\[
\max_{\{q_{t+1}, l_{bt}\}_{t=0}^{\infty}} \Pi_{bt} = \left( r^q_t - r^d_t \right) q_t - p^h_t \eta^h_t w_t l_{bt}
\]

s.t.

\[1 = A_b z_{bt} l_{bt}^{\gamma} q_{t+1}^{-\gamma} \]

The financial intermediary’s optimisation problem can be expressed using the Lagrange function \( \Lambda_{bt} \), where \( \mu_t \) is the lagrange multiplier:

\[
\max_{\{q_{t+1}, l_{bt}\}_{t=0}^{\infty}} \Lambda_{bt} = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \left( r^q_t - r^d_t \right) q_t - p^h_t \eta^h_t w_t l_{bt} + \mu_t \left( A_b z_{bt} l_{bt}^{\gamma} q_{t+1}^{-\gamma} - 1 \right) \right\}
\]

The intertemporal optimisation problem is to maximise the discounted stream of future net cash flow, subject to the adjusted production function. Since the representative agent owns the financial intermediary, the same discount factor is used in the above optimisation problem as in the consumer’s one. The exogenous variable \( \eta^h_t \) is an AR(1) process and it depicts the omitted income tax on labour in the financial industry. The first order conditions are:

\[q_{t+1} : \mu_t \beta^t \left( -\gamma \right) A_b z_{bt} l_{bt}^{\gamma} q_{t+1}^{-\gamma} - 1 + \beta^{t+1} \left( r^q_{t+1} - r^d_{t+1} \right) = 0 \]

\[l_{bt} : \beta^t \left( -p^h_t \right) \eta^h_t w_t + \beta^t \mu_t \gamma A_b z_{bt} l_{bt}^{\gamma-1} q_{t+1}^{-\gamma} = 0 \]

\[\mu_t : A_b z_{bt} l_{bt}^{\gamma} q_{t+1}^{-\gamma} = 1 \]
\[ q_t : \beta (r^q_{t+1} - r^d_{t+1}) = \mu_t \gamma A_b z_{bt} l^\gamma_{bt} q_{t+1}^{-\gamma-1} \quad (3.46) \]

\[ l_{bt} : \quad p_t^b \eta^b_t w_t = \mu_t \gamma A_b z_{bt} l^\gamma_{bt} q_{t+1}^{-\gamma} \quad (3.47) \]

\[ \mu_t : \quad A_b z_{bt} l^\gamma_{bt} q_{t+1}^{-\gamma} = 1 \quad (3.48) \]

Using equations (3.46) and (3.47), it can be seen that the discounted domestic interest rate differential \( \beta (r^q_{t+1} - r^d_{t+1}) \), i.e. the marginal benefit of a unit of loans production, relative to the marginal labour cost of producing that unit, i.e. \( p_t^b \eta^b_t w_t \), equals the amount of labour per unit of loans \( \frac{l_{bt}}{q_{t+1}} \)

\[ \frac{\beta (r^q_{t+1} - r^d_{t+1})}{p_t^b \eta^b_t w_t} = \frac{\mu_t \gamma A_b z_{bt} l^\gamma_{bt} q_{t+1}^{-\gamma-1}}{\mu_t \gamma A_b z_{bt} l^\gamma_{bt} q_{t+1}^{-\gamma}} \quad (3.49) \]

\[ \frac{(r^q_{t+1} - r^d_{t+1})}{p_t^b \eta^b_t w_t} = \frac{l_{bt}}{\beta q_{t+1}} \quad (3.50) \]

\[ (r^q_{t+1} - r^d_{t+1}) = \frac{p_t^b \eta^b_t w_t l_{bt}}{\beta q_{t+1}} \quad (3.51) \]

The first order condition for the shadow cost of loans \( \mu_t \), equation (3.48) can be simplified as follows:

\[ A_b z_{bt} l^\gamma_{bt} q_{t+1}^{-\gamma} = 1 \]

\[ \left( \frac{l_{bt}}{q_{t+1}} \right)^\gamma = \frac{1}{A_b z_{bt}} \]

\[ l_{bt} = \left( \frac{1}{A_b z_{bt}} \right)^{\frac{1}{\gamma}} \quad (3.52) \]

Let \( sp_t \) be the interest rate spread between loan and deposit rates. Using equations
(3.51) and (3.52), the following equalities can be obtained:

\[ s p_t = r_t^q - r_t^d \]  
\[ r_{t+1}^q - r_{t+1}^d = p_t^h \eta_t^h w_t \frac{l_t}{\beta q_{t+1}} \]  
\[ r_{t+1}^q - r_{t+1}^d = p_t^h \eta_t^h w_t \frac{1}{\beta} \left( \frac{1}{A_h z_t} \right)^\frac{1}{2} \]

Equations (3.54) and (3.55) imply that an increase in the real wage, effective income tax, labour input relative to the total production of loans, or a negative bank productivity shock, would result in an increase in the interest rate spread. The rationale behind these effects is straightforward. A positive shock to the error term \( \eta_t^h \) would imply an increase in income tax from employment in the financial sector; thus increasing the overall wage bill of the bank. The same logic can be used in the case of an increase in the consumer's real wage, \( p_t^h w_t \) or the demand for labour. In all cases there will be an increase in the wage bill, \( p_t^h w_t l_t \), which would result in, ceteris paribus, a rise in loan rates and the spread in order to compensate for the increase in the cost of production. A fall in the loan output would reduce the supply of loans. In order for the loan market to be in equilibrium, the price of loans must rise, i.e. an increase in the loan interest rate which, ceteris paribus, would widen the spread. Last but not least, a negative bank productivity shock would imply that for the same amount of inputs, there would be a lower amount of loans produced. Following the logic in the previous scenario, this would increase the domestic borrowing cost and the spread.
This concludes the exposition of the financial intermediary sector. The equilibrium conditions that would be used in the model listing are the adjusted production function (3.48), the spread definition (3.53) and relationship between the interest rate differential and the bank’s productivity (3.55). Next, the equations that describe the ways in which the home country is affected by the rest of the world and the market clearing conditions will be explained.

3.2.4 International and market clearing conditions

There are two ways in which the home country is affected by the rest of the world. One is through trade in consumption goods and the second one is via international financial markets. Let $\text{cap}_t$ be the capital account and $\text{cur}_t$ be the current account variables. By definition, the capital account shows the net liability level and the current account captures the trade balance less interest income debits, i.e. the payments due to interest rate charges as a result of accumulated debt.

\begin{align*}
\text{cap}_t &= \left( d_{t+1}^f - d_t^f \right) Q_t \\
\text{cur}_t &= (e_t - Q_{t}r_t) - r_{t}^f d_{t}^f Q_t
\end{align*}
In this model it is assumed that the balance of payments $bp_t$ holds every period and it can be described by the following equation:

$$
bp_t \equiv 0$$

$$bp_t = cap_t + cur_t$$

$$0 = cap_t + cur_t$$

Using the definitions for $cap_t$ and $cur_t$, the balance of payments equation becomes:

$$0 = (ex_t - Q_t im_t) - r^f_t d^f_t Q_t$$

$$\left(d^f_{t+1} - d^f_t\right) Q_t = r^f_t d^f_t Q_t - (ex_t - Q_t im_t)$$

$$\left(d^f_{t+1} - d^f_t\right) = r^f_t d^f_t - \left(\frac{ex_t}{Q_t} - im_t\right)$$

$$d^f_{t+1} = \left(1 + r^f_t\right) d^f_t - \left(\frac{ex_t}{Q_t} - im_t\right)$$  (3.56)

It implies that the home country accumulates new foreign debt in order to cover any interest payments due and the trade balance deficit. The interest rate on foreign debt is assumed to be an endogenous variable that depends on the average world interest rate, $r^w$, and the deviation of the foreign debt, $d^f_t$, from its steady state value $\overline{d}^f$. It is assumed that there is a perfect capital mobility which ensures that in the long run the average rate of interest in the rest of the world equals the steady state values of the home country and foreign debt interest rates; therefore, $r^w = \overline{r}^f$, where $\overline{r}^f$ is the steady state value of $r^f_t$. To induce stationarity, the external debt elastic interest rate is assumed to have the following definition as in
Schmitt-Grohe and Uribe (2003):

\[ r_t^f = \tilde{r}^f + \varphi \left( e^{(d_t^f - \tilde{d})} - 1 \right) \]  (3.57)

The home country debt premium is:

\[ \text{prem}_t = \varphi \left( e^{(d_t^f - \tilde{d})} - 1 \right) \]

In the definitions above, \( \varphi \) is a parameter and \( \tilde{d}^f \) is the steady state value of \( d_t^f \).

The home country market clearing condition is:

\[ y_t = c_t + i_{kt} + g_t + e_{xt} - im_t \]  (3.58)

\[ y_t = c_t + (k_{t+1} - (1 - \delta) k_t) + g_t + e_{xt} - im_t \]  (3.59)

where \( g_t \) denotes government expenditure, assumed to be exogenous:

\[ g_t = (g_t)^\sigma G_t^\epsilon \]  (3.60)

Recall the solution for consumption of foreign goods in the consumer’s intratemporal optimisation problem. According to equation (3.19):

\[ c_t^f = (1 - \chi)^{\frac{1}{1+\sigma}} \left( \eta_t^f \right)^{\frac{1}{1+\sigma}} \left( Q_t \right)^{-\frac{1}{1+\sigma}} c_t \]

In this model trade occurs only in final goods, therefore \( c_t^f = im_t \). If \( \sigma \equiv \frac{1}{1+\delta} \) is defined as the elasticity of substitution between domestic and foreign goods, equation (3.19) can be expressed as:

\[ im_t = (1 - \chi)^{\frac{1}{1+\sigma}} \left( \eta_t^f \right)^{\frac{1}{1+\sigma}} \left( Q_t \right)^{-\frac{1}{1+\sigma}} c_t \]

\[ im_t = (1 - \chi)^{\sigma} \left( \eta_t^f \right)^{\sigma} \left( Q_t \right)^{-\sigma} c_t \]  (3.61)
Since the two countries, despite having different sizes are symmetric in preferences, the export equation of the home country will be equal to the import equation of the rest of the world with respect to the home country production. Thus, exports are equal to:

\[
ex_t = (1 - \chi^F) \frac{1}{1 + \sigma^F} (\eta_t^F)^{\frac{1}{1 + \sigma^F}} (Q_t^F)^{-\frac{1}{1 + \sigma^F}} c_t^F
\]

\[
ex_t = (1 - \chi^F)^{\sigma^F} (\eta_t^F)^{\sigma^F} (Q_t^F)^{-\sigma^F} c_t^F
\]

\[
ex_t = (1 - \chi^F)^{\sigma^F} (\eta_t^F)^{\sigma^F} (Q_t)^{\sigma^F} c_t^F
\]

(3.62)

To arrive at equation (3.62), the following properties are used:

\[
Q_t^F = \frac{P_t}{P_t^F} = \left( \frac{P_t}{P_t^F} \right)^{-1} = (Q_t)^{-1}
\]

The parameters in the import/export equations, \(\sigma\) and \(\sigma^F\), are the elasticities of substitution, and \(\chi\) and \(\chi^F\) are the home bias parameters. The consumption level in the foreign country \(c_t^F\), the world demand for goods and services, is assumed to be endogenous variable with exogenous dynamics as defined by the following equation:

\[
c_t^F = (c_{t-1}^F)^{\mu^F} + \varepsilon_t^{c^F}
\]

(3.63)

There is an additional set of nine equations for the exogenous variables: the TFP in the goods sector - \(z_{gt}\), TFP in the banking sector - \(z_{bt}\); foreign demand shocks - \(\eta_t^F\) for the home country and the symmetrical \(\eta_t^F\) for the foreign country; preference shock for current consumption - \(\eta_t^c\); labour supply shock - \(\eta_t^n\); error terms used to
3.2.5 The system of equations describing the model environment

The model framework can be described by the following set of first order conditions, behavioural equations, definitions and market clearing conditions: (3.17), (3.21), (3.31), (3.32), (3.33), (3.34), (3.35), (3.37), (3.40), (3.41), (3.42), (3.32), (3.31), (3.32), (3.33), (3.34), (3.35), (3.37), (3.40), (3.41), (3.42),

\[
\begin{align*}
\eta_f &= \eta_f(\eta_f - 1)^{\nu_f} + \varepsilon_f \\
\eta_k &= \eta_k(\eta_k - 1)^{\nu_k} + \varepsilon_k \\
\eta_g &= \eta_g(\eta_g - 1)^{\nu_g} + \varepsilon_g \\
\eta_l &= \eta_l(\eta_l - 1)^{\nu_l} + \varepsilon_l \\
\eta_f &= (\eta_f - 1)^{\nu_f} + \varepsilon_f \\
\eta_k &= (\eta_k - 1)^{\nu_k} + \varepsilon_k \\
\eta_g &= (\eta_g - 1)^{\nu_g} + \varepsilon_g \\
\eta_l &= (\eta_l - 1)^{\nu_l} + \varepsilon_l \\
\lambda &= (\lambda - 1)^{\nu} + \varepsilon_l \\
\gamma &= (\gamma - 1)^{\nu} + \varepsilon_l \\
\delta &= (\delta - 1)^{\nu} + \varepsilon_l \\
\nu &= (\nu - 1)^{\nu} + \varepsilon_l \\
\varepsilon &= (\varepsilon - 1)^{\nu} + \varepsilon_l \\
\eta_f &= (\eta_f - 1)^{\nu_f} + \varepsilon_f \\
\eta_k &= (\eta_k - 1)^{\nu_k} + \varepsilon_k \\
\eta_g &= (\eta_g - 1)^{\nu_g} + \varepsilon_g \\
\eta_l &= (\eta_l - 1)^{\nu_l} + \varepsilon_l \\
\lambda &= (\lambda - 1)^{\nu} + \varepsilon_l \\
\gamma &= (\gamma - 1)^{\nu} + \varepsilon_l \\
\delta &= (\delta - 1)^{\nu} + \varepsilon_l \\
\nu &= (\nu - 1)^{\nu} + \varepsilon_l \\
\varepsilon &= (\varepsilon - 1)^{\nu} + \varepsilon_l \\
\end{align*}
\]
\[
\begin{align*}
    p_t^h &= \left(\chi^{-\sigma} - ((1 - \chi)/\chi)^\sigma \left(\eta_t \right)^\sigma (Q_t)^{1-\sigma}\right)^{1/\sigma} \\
    n_t &= l_{gt} + l_{bt} \\
    \frac{c_t^p}{(1 - n_t)^{\mu_2}} &= \frac{\omega \eta_t}{(1 - \omega) \eta_t^\mu p_t^h w_t} \\
    \beta(1 + r_{t+1}^d) &= \frac{\eta_t^p c_t^{p_t}}{E_t \left[\eta_{t+1} c_{t+1}^{p_{t+1}}\right]} \\
    (1 + r_{t+1}^d) &= (1 + r_{t+1}^f) \frac{Q_{t+1}}{Q_t} \\
    y_t &= A_g z_{gt} l_{gt}^\alpha k_{t}^{1-\alpha} \\
    k_{t+1} &= i_{kt} + (1 - \delta) k_t \\
    k_t &= q_t^d \\
    \eta_t^g w_t &= \alpha A_g z_{gt} \left(\frac{l_{gt}}{k_t}\right)^{\alpha - 1} \\
    \left(r_t^g + \delta + \eta_t^k\right) &= (1 - \alpha) A_g z_{gt} \left(\frac{l_{gt}}{k_t}\right)^\alpha \\
    q_{t+1}^d &= q_{t+1}^d \\
    p_t^b &= \frac{p_t^h \eta_t^b w_t}{\beta} \frac{1}{A_b z_{bt}} \left(\frac{1}{A_b z_{bt}}\right)^{1/\gamma} \\
    d_{t+1}^f &= \left(1 + r_{t+1}^f\right) d_t^f - \left(\frac{e x_t}{Q_t} - i m_t\right) \\
    y_t &= c_t + i_{kt} + g_t + e x_t - i m_t \\
    i m_t &= (1 - \chi)^\sigma \left(\eta_t^f\right)^\sigma (Q_t)^{-\sigma} c_t \\
    g_t &= (g_t)^{g^G} + \varepsilon_t^G \\
    e x_t &= (1 - \chi^F)^{100} \left(\eta_t^F\right)^{\sigma^F} (Q_t)^{\sigma^F} c_t^F \\
    c_t^F &= (c_{t-1}^F)^{\sigma^F} + \varepsilon_t^F
\end{align*}
\]
and the equations for the exogenous AR(1) processes, from (3.64), to (3.72)

\[
\begin{align*}
z_{gt} &= (z_{gt-1})^{\rho_{zg}} + \varepsilon_{t}^{zg} \\
z_{bt} &= (z_{bt-1})^{\rho_{zb}} + \varepsilon_{t}^{zb} \\
\eta_{it}^{f} &= (\eta_{it-1}^{f})^{\rho_{f}} + \varepsilon_{t}^{f} \\
\eta_{it}^{c} &= (\eta_{it-1}^{c})^{\rho_{c}} + \varepsilon_{t}^{c} \\
\eta_{it}^{n} &= (\eta_{it-1}^{n})^{\rho_{n}} + \varepsilon_{t}^{n} \\
\eta_{it}^{g} &= (\eta_{it-1}^{g})^{\rho_{g}} + \varepsilon_{t}^{g} \\
\eta_{it}^{k} &= (\eta_{it-1}^{k})^{\rho_{k}} + \varepsilon_{t}^{k} \\
\eta_{it}^{b} &= (\eta_{it-1}^{b})^{\rho_{b}} + \varepsilon_{t}^{b} \\
\eta_{it}^{F} &= (\eta_{it-1}^{F})^{\rho_{F}} + \varepsilon_{t}^{F}
\end{align*}
\]

These equations are log-linearised manually around the steady state values. Please refer to Appendix (Log-Linearisation) for details. In the final model listing some of the variables are substituted out. These are the following: \( p_{it}^{b} \), using equation (3.17), \( i_{kt} \) using (3.35), \( q_{it}^{d} \) using (3.37), and \( q_{t+1} \) using (3.37) and (3.42) and they are not solved for. DYNARE and MATLAB are used to obtain the solution for the DSGE model and the subsequent testing and estimation. Please refer to the Appendix (Programs) for details regarding the software and toolboxes used.
3.3 Model Limitations

Although the model specification captures multiple aspects of the economy, it is not without its limitations. It could be argued that some of the assumptions are too simplistic and lack realism. The following questions could be raised: "Why is the representative agent the only one who has access to the international financial markets?"; "Why should new investment be entirely financed by loans?"; "Why is the central bank regulation omitted?"; "Why are risk factors not included?"; and "Why are monetary and fiscal policies ignored?".

A simple answer to all those questions is that the inclusion of these additional features to the one already incorporated would present both analytical and computational difficulties. At the early stage of this research, a model was created which contained cash-in-advance constraint, government budget constraint stating that any deficit would be covered by the issuance of new debt, and a reserve requirement. This led to a complex derivation of the model’s first order and equilibrium conditions. Moreover, the resulting system of equations describing the economy did not have a unique stable equilibrium. This was validated by a computational exercise which confirmed that the eigenvalues calculated at the steady state did not satisfy the Blanchard-Kahn conditions. Various adjustments were made to the functional forms and elements were removed one at a time. However, any attempt to pinpoint the source of instability was futile.
That is why a decision was made to employ a steady bottom-up approach in the search for optimal model specification. The proposed model in this thesis is tractable and has a unique stable equilibrium. There is no closed form solution but the steady state can be obtained numerically.

In terms of the specific assumptions employed the following reasoning was applied. Due to the specific method used to describe the financial intermediary, namely the loan production function, any extension that results in different asset classes would tremendously complicate the analysis. The same argument is valid if the change is on the liability side of the bank’s balance sheet. This is why the financial intermediary does not have a direct access to the international financial markets.

The model would benefit from the inclusion of a monetary authority. This could easily be achieved via a cash-in-advance constraint in the manner of Benk et al. (2005, 2010) and Gillman and Kejak (2004, 2008). This would allow for an investigation of the impact of monetary shocks on the cyclical properties of the model. However, this would inevitably complicate the analysis and that is why it was not researched on this occasion.

Including a constant reserve requirement would simply create a wedge between the deposit and loan rates which would increase the interest rate spread by a constant fraction without altering the dynamic properties. Any changes in
the regulation, including changes in the reserve requirement, are captured by the TFP shock in the banking industry. This argument is supported by the empirical findings presented in the paper by Benk et al. (2005).

Modelling the behaviour of the government would allow for qualitative and quantitative evaluation of the effect of fiscal policy. However, allowing the government to issue debt would result in a portfolio choice for both the representative agent and the financial intermediary. This would further require the inclusion of different risk return structures. Again, this would complicate the analysis and should be considered in an independent study. Moreover, within the present setup the effects of taxation and government spending are captured by the exogenous variables: $G$ in the market clearing condition and the three error terms: one in the profit function of the bank and two more in the profit function of the goods producer. Therefore, it could be argued that the effect of some government decisions is captured by the model, with the limitation that the private sector is not aware of the government decision and considers this an exogenous disturbance.

Last but not least, one could question the assumption that new investment is entirely finance by loans. Again any relaxation of this assumption would present a modelling difficulty. Since reinvestment of profits is presumed to be costless, the representative firm would always find it optimal to choose the reinvestment of profits over costly borrowing from the bank. Thus, any relaxation in this manner
would require an explicit modelling of the costs of investment projects. Furthermore, there should be a constraint imposing a limit on the amount of available capital for reinvestment and a form of incentive that would create the need for investing in projects that would not be possible without external financing. A similar argument could be made for raising capital via equity. Although the intuition behind these propositions is straightforward the practical implementation in terms of behaviour equations and constraints presents a challenge which would require further consideration. That is why this assumption, albeit simplistic, was made.

All of the above suggestions could result in better cyclical properties of the model and it would be an interesting research to try to quantify their impact on the dynamic properties. However, this would require a working benchmark model so that the results could be compared. The model proposed in this chapter could be used as such a benchmark. Future work would entail comparing the result based on the proposed model here and the results from this model augmented by one of the above specifications. Once the contribution of each of these factors have been analysed alone, an attempt would be made to combine two or more features. This gradual build-up is considered a prudent and tractable approach, but due to its size it is something that would be achieved over time and thus falls outside the scope of the present research.
3.4 Calibration

One way to evaluate the model’s performance is to calibrate it, as described by Kydland and Prescott (1982). The chosen set of parameter values is based on either actual data or estimates from other empirical studies. The dynamic properties of the model are presented using impulse response functions.

3.4.1 Data and parameter choice

The model is calibrated on UK quarterly data. The sample period is Q3 1978 to Q3 2013. The full data set used, the reference codes, the description and any adjustments made have been detailed in the Appendix (Data). The sample size was determined by data availability at the time it was collected. It contains 141 observations which should be sufficient for any statistical inference. The actual time series data had to be transformed in per capita terms and where necessary adjusted for inflation. All variables are normalised by total population except the interest rates and relative prices. This data set was used to calculate the steady state values for all variables which are used as weights in the log-linearised system of equations. There is also an additional set of data series used solely for the purpose of gauging parameter values in the calibration stage, assuming that they contain better information regarding the ‘true’ values. However, due to a short data range these were not used in the estimation.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
<td>0.985</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Labour share in goods production</td>
<td>0.6154</td>
</tr>
<tr>
<td>$\rho_1$</td>
<td>Relative risk aversion coefficient</td>
<td>1.2</td>
</tr>
<tr>
<td>$\rho_2$</td>
<td>Elasticity of labour supply</td>
<td>1</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Capital depreciation rate</td>
<td>0.01299</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>Interest elasticity of foreign debt</td>
<td>0.000742</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Labour share in loan production</td>
<td>0.066</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Home bias in consumption</td>
<td>0.7</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Import demand elasticity</td>
<td>1</td>
</tr>
<tr>
<td>$\sigma^F$</td>
<td>Import demand elasticity (foreign country)</td>
<td>1.2</td>
</tr>
</tbody>
</table>

Table 3.1: Initial Parameter Values
Table 3.1 contains the initial set of parameter values used to evaluate the model’s performance. The subjective discount factor $\beta$ is calculated using data on interest rates. The steady state value of 0.015 indicates an approximate value of 0.985, using that $\beta = \frac{1}{1+r}$. The labour share in the goods sector, the parameter in the production function - $\alpha$, is set to 0.6154. The value was obtained using annual data on Total Compensation of Employees relative to Total Gross Value Added. The value appears to be low when compared with other empirical studies. However, since this is the initial stage of the analysis, the value will stay as it is. In later chapters, this parameter would be estimated. The relative risk aversion coefficient and the labour supply elasticity are set to $\rho_1 =1.2$ and $\rho_2 =1$ respectively. The values were obtained from Meenagh et. al. (2005). The depreciation rate is set to 0.01299. The statistic was calculated using data on Consumption of Fixed Capital Assets (ONS code - CIHA) and Net Capital Stock (ONS code - MLR3). These are annual data series. The formula is as follows. Let the $\delta^a$ be the annual value of $\delta$:

$$\frac{CIHA}{CIHA + MLR3} = \frac{\delta^a k}{\delta^a k + (1 - \delta^a)k} = \frac{\delta^a k}{k} = \delta^a$$

There are two methods - the straight line method and the diminishing balance one, to extract the quarterly value from the annual one:

$$\delta = \begin{cases} 
\frac{\delta^a}{4} \\
1 - (1 - \delta^a)^{1/4}
\end{cases}$$
The diminishing balance method was used (the second line) and the result is 0.01299. The interest elasticity of foreign debt denoted as \( \varphi \), is the parameter that determines the stationarity of the model and the speed of convergence to the steady state. Given that the model is solved in DYNARE, stationarity is a must in order that Blanchard-Khan conditions are satisfied. The parameter value was obtained using the data and the log-linearised equation for the external debt elastic interest rate. The value was calculated in MATLAB and the result was 0.00104. Although using this statistic would likely generate better results, a decision was made to use 0.000742 as suggested by Schmitt-Grohe and Uribe (2003) The reason behind this decision is as follows. The parameter measures an effective 'penalty' on the interest rate that a country would incur for acquiring additional debt. The difference is 0.000298 which is approximately 0.03% higher increase in the interest rate charged to the UK relative to the US. In the current low interest level environment, and taking into account that the risk profiles of the two countries are relatively the same, it is presumed that the value provided by Schmitt-Grohe and Uribe (2003) is the more accurate one. The labour share in loan production \( \gamma \) is set to 0.066. Although it may appear small, if we consider the value of the banking industry assets relative to the labour cost it seems reasonable. The value is calculated using data from Workbased Compensation of Employees: Financial and Insurance Activities (ONS code R2VQ) and Monthly Amounts Outstanding of Monetary
Financial Institutions (including MFI-owned Specialist Mortgage Lenders) Sterling Loans to Private Non-Financial Corporations and Household Sector. Home bias consumption as well as the import demand elasticities were obtained from Meenagh et. al. (2005).

3.4.2 Filtering the data

The focus of the next section is to review the response of the model’s variables at a business cycle frequency to a temporary shock. Therefore, the cyclical component should be extracted from the raw time series. There are several techniques used in the literature to remove the trend component from the original data. Some of the most popular methods are first order difference, linear (or log-linear) detrending, quadratic (or log-quadratic) detrending, two band pass filters presented by Baxter and King (1999) and Christiano and Fitzgerald (2003), and the HP filter by Hodrick and Prescott (1997).

First differencing, promoted by Box and Jenkins (1970), is a very simple method of removing the trend in raw data. It is based on the following assumptions: the trend component in the series is a random walk with no drift, the cyclical component is stationary, the two components are uncorrelated and the series contains a unit root which is due entirely to the trend component Canova (1998). Let $Y_t$ be the observed time series and $Y_t^c$ and $Y_t^s$ be the cyclical and secular components respectively. The method can be described by the following
The main advantages of first differencing are that the technique is very straightforward to apply and that it removes the unit root component from the data. However, this method introduces phase shift, i.e., it changes the timing relationships between series, Baxter and King (1999). The authors have determined that first order difference filter puts emphasis on higher frequencies whilst down-weighting lower frequencies. Larsson and Vasi (2012) have found that this type of filter produces significantly different results relative to the HP and band-pass filters, thus making it an undesirable choice.

Linear or log-linear detrending employs fitting a linear trend which then is removed from the data.

\[
Y_t = a + b \times t + \epsilon_t \\
Y_{ts} = \hat{a} + \hat{b} \times t \\
Y_{tc} = Y_t - Y_{ts}
\]

This method does not result in a phase shift but it is not able to remove unit roots. Given that many macroeconomic time series are characterised by unit roots this
method is also not appropriate.

The quadratic detrending method is very similar to the linear detrending. The only difference is that the trend variable also enters into the equation with a quadratic term.

\[ Y_t = a + b \times t + c \times t^2 + \epsilon_t \]

A quadratic trend can accommodate a rate of growth that is changing over time and it does not result in a phase shift. Even though it possesses desirable qualities, the quadratic detrending filter is less flexible relative to the HP filter since it does not allow the researcher to adjust the smoothness of the trend curvature and thus it is less flexible than some of the other alternatives.

The BK band pass filter suggested by Baxter and King (1999) is a two-sided time-invariant moving average. By adopting Burns and Mitchell’s (1946) definition of a business cycle, the filter passes cycles of time series with 6 quarters being the shortest cycle length and 32 quarters being the longest cycle length, thus removing higher and lower frequencies. The authors have argued that the exact band pass filter is of infinite order. That is why they have developed a set of criteria to best approximate the filter. Thus, the BK filter meets the following requirements. The filter extracts certain periodicity and does not alter the properties of the noise component. It does not introduce a phase shift. The technique employs a quadratic loss function that minimises the differences between the ‘best’ and
approximated filters. The filter generates stationary time series. The obtained cyclical component is unrelated to the length of the sample. However, since the filter is a moving average, there is a direct trade-off. Longer moving averages provide better approximations; however, this would imply that more observations would have to be removed from the subsequent analysis. Baxter and King (1999) have recommended removing three years of past and three years of future data for both annual and quarterly time series. This could be problematic in the case of a short sample size of quarterly data.

The CF band pass filter developed by Christiano and Fitzgerald (2003) is very similar to the BK band pass filter since it is also an approximation of the ‘best’ band pass filter which is a moving average of infinite order. However, there are some differences. Whilst the BK filter is a symmetric filter, the CF filter is not since it employs weights in the objective function. The approximation error of weights decreases as the sample size increases. The CF filter introduces a phase shift which is not a desirable quality and thus it could be considered inferior compared to the BK filter. However, it poses on major advantage over the band pass filter presented by the Baxter and King (1999), namely it removes data points only from the beginning of the sample and thus it allows the use of the most recent data in the subsequent analysis. One big criticism of the CF filter has been made by Smith (2016). The author has argued that if the data possesses a stochastic trend, the
CF filter results in spurious periodicity and the filtered cycles are characterised by a higher amplitude and longer duration.

In contrast to the band pass filter the HP filter proposed by Hodrick and Prescott (1997) is a smoothing procedure that aims to estimate the trend component by minimising the following function:

$$\min_{\{Y_{ts}\}_{t=1}^{T}} \left\{ \sum_{t=1}^{T} (Y_t - Y_{ts})^2 + \lambda \sum_{t=2}^{T-1} ((Y_{t+1}^{s} - Y_t^{s}) - (Y_t^{s} - Y_{t-1}^{s}))^2 \right\}$$

The first part minimises the difference between the time series and its trend component (which is its cyclical component). The second part minimises the second-order difference of the trend component. Lambda is the smoothing parameter. By changing the parameter value, the researcher is allowed to decide how much of the variability of the date is attributed to the cycle and how much is due to changes in the trend. Although there is a general consensus regarding the value of lambda when quarterly data is used, i.e. 1600, there is a considerable disagreement in the case of annual data. Backus and Kehoe (1992) used a value of 100 which is also the most widely used value in the literature. Baxter and King (1999) argued that it should be 10 since this is the value that generated the cyclical component closest to the one produced by a high pass filter. Ravn and Uhlig (2002) argued that it should be set at 6.25. Despite the lack of agreement regarding the value of lambda when annual data is used, it is not a concern when choosing the filter that is used in this thesis since the raw data gathered is quarterly and there is a general con-
sensus in the literature regarding the parameter value. The HP is very similar to a high pass filter and produces almost identical results on quarterly data, Baxter and King (1999). It is symmetric and does not introduce a phase shift. One paper that generates scepticism in the use of the HP filter is King and Rebelo (1993). The authors have argued that the cyclical component extracted using the HP filter is likely to capture only a subset of the time series variation and thus alters the measures of persistence, variability and comovements. However, Pedersen (2001) have argued that the HP filter is less distorting than any of the approximate high pass filters.

There have been many studies comparing the properties of some of the most popular filters as well as their effectiveness and the results are not unanimous. Canova (1998) has argued that different filters generate different results and thus the choice of a filter has a significant impact on the final results. Bjornland (2000) has reached to the same conclusion. Estrella (2007) argued that the HP, BK and exponential smoothing filters produce similar results when applied to an integrated data process and that a frequency domain filter shows the best results for frequency extraction. However, the HP and exponential smoothing filters are by far better when the objective is signal extraction. Guay and St-Amant (1997) concluded that the HP and BK filters perform well “when the spectrum of the original series has a peak at business-cycle frequencies ... [but] when the spectrum is dominated by low
frequencies, the filters provide a distorted business cycle”. Baxter and King (1999) have argued that with the exception of the first order difference filter, the HP filter, the high pass filter and the band pass filter produce largely similar results. Schmitt-Grohe and Uribe (2016) have compared the results from filtering data using quadratic detrending and the HP filter and have concluded that the results are largely the same. Larsson and Vasi (2012) have investigated the properties of HP, BK and CF filters and concluded that they generate similar cycles using quarterly data and the difference only occurs when annual data is employed.

Given that the HP filter developed by Hodrick and Prescott (1997) produces similar results to the band pass filters, does not introduce a phase shift, adequately removes unit roots, is straightforward to apply using Matlab software and does not result in a loss of any data points, it is the preferred method for filtering the data used in this research.

3.4.3 Residuals

Using the set of initial parameter values and filtered data, based on the method described by Hodrick and Prescott (1997), the residuals in each equation containing error terms or TFP shocks were obtained via the Solow residual method. The residuals autocorrelation coefficients are presented in table 3.2.

The corresponding graphs of the goods sector and the banking sector are 3.2 and 3.3. It can be seen from the graphs that in the goods producer sector the
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Equation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho^{ec}$</td>
<td>Euler equation</td>
<td>0.7872</td>
</tr>
<tr>
<td>$\rho^{nf}$</td>
<td>Imports equation</td>
<td>0.7202</td>
</tr>
<tr>
<td>$\rho^{m}$</td>
<td>MRS $n_t$ and $c_t$</td>
<td>0.7846</td>
</tr>
<tr>
<td>$\rho^{zg}$</td>
<td>Production function GS</td>
<td>0.6193</td>
</tr>
<tr>
<td>$\rho^{zb}$</td>
<td>Production function FS</td>
<td>0.7914</td>
</tr>
<tr>
<td>$\rho^{yg}$</td>
<td>MPL equation</td>
<td>0.6125</td>
</tr>
<tr>
<td>$\rho^{yk}$</td>
<td>MPK equation</td>
<td>0.5244</td>
</tr>
<tr>
<td>$\rho^{yb}$</td>
<td>Spread equation</td>
<td>0.5617</td>
</tr>
<tr>
<td>$\rho^{F}$</td>
<td>Exports equation</td>
<td>0.7023</td>
</tr>
<tr>
<td>$\rho^{cF}$</td>
<td>Foreign consumption demand</td>
<td>0.8500</td>
</tr>
<tr>
<td>$\rho^{g}$</td>
<td>Government spending</td>
<td>0.3192</td>
</tr>
</tbody>
</table>

Table 3.2: Residuals Autocorrelation Coef Based on Initial Parameter Values and HP data
residual leads the fluctuations in output which conforms with the idea that exogenous technological progress is a key determinant of the real economy. However, the volatility of the TFP shock is lower than the one in output, implying that there are other factors that contribute to the cyclical properties of the data. This is expected in the case of an open economy since the home country performance is highly dependent of the state of the rest of the world.

The graph of the residual in the production function could be interpreted in
several ways. It is evident that the TFP shock accounts for little if any of the variation in capital. There are several possible explanations.

- It could be the case that since this is an open economy which has access to international capital markets, the main impact on the cyclicity comes from outside the economy.

- It is possible that one or more of the model’s assumptions is too restrictive to reproduce accurate results:
  
  - One of the assumptions was that all deposits are used in the production of loans and that according to the balance sheet constraint deposits must equal loans
  
  - Another restriction is that all new capital is bought on credit, which has led to the conclusion that the stock of capital equals the outstanding amount of loans
  
  - It is also possible that the fixed one period maturity data is too short.

- Last but not least, it is possible that the method in which capital is calculated, i.e. based on the capital accumulation equation and investment and output data is inaccurate

The answer to these conjectures may be revealed once the model is subjected to harsher scrutiny.
Figure 3.3: Solow Residual in the FS
The residuals are fitted to an AR(1) process to estimate the parameter values. The parameter estimates are shown in table 3.2. Using the fitted data and the actual residuals, the distribution for each innovation in the error terms was found. The standard deviations were calculated using these distributions and used to simulate the data.

Given that this is a DSGE model, two variables enter the system of equations with an expectations operator in front of them. These are the expected consumption and the expected real effective exchange rate. To obtain a fitted value for them a VAR model was used on filtered data. The corresponding fitted values and actual series are shown in figures 3.4 and 3.5.

3.4.4 Impulse response functions

The impulse response functions (IRFs) will be used to present the internal dynamics of the model. Since the financial intermediation sector’s performance is one of the main interests in this thesis, only the IRFs from a single shock in the goods sector and the banking industry will be discussed. The rest can be found in Appendix (Impulse response Functions: Stationary Data). As expected a positive TFP shock in the goods sector increases output, consumption, capital and labour demand. All diagrams in figures 3.6 and 3.7 seem to be within reason. However, in terms of output propagation the graph lacks the hump shape and the effect lasts for only ten quarters. The TFP shock in the goods sector increases the spread
Figure 3.4: Actual and Expected Consumption - HP data
Figure 3.5: Actual and Expected REER - HP Data
which conforms with what was expected. It illustrates that an increase in the productivity of the firm would increase the demand for capital and the demand for loans. In order for the loan market to be in equilibrium, the interest rate would rise and this would increase the spread.

In the case of the TFP shock in the financial sector, the results also seem to be promising. An increase in the productivity of the banking industry implies that for the same level of inputs, there will be a higher level of output. Since
Figure 3.7: IRF (initial calibration): One off Shock in GS (fig.2)
Figure 3.8: IRF (initial calibration): One off Shock to FS (fig.1)
the amount of deposits is jointly determined by the demand of the intermediary and the supply by the representative agent, the bank raises the interest rates on deposits to increase savings. This would lead to lower consumption in the current period. However, given the upward sloping supply curve in the deposit market, the increase in the stock of deposits would be lower than the one desired by the financial intermediary. Therefore, there will be a reduction in the demand for labour in the banking industry. Given the increase in interest rates on deposits, the bank would raise the interest rate on loans. However, as the financial intermediary has become more efficient, the increase would not be as much as the one in the market for deposits. Therefore, the interest rate spread will decrease.

3.5 Conclusion

This chapter presented a new DSGE model. The model’s framework was explained as well as the various assumption that it rests upon and the reasoning behind this. The model was calibrated using parameter values from other economic studies or derived from the data. Although the parameters will be estimated in subsequent chapters, the derivations were necessary in order to present this model with a fair start in the competition for the discovery of the 'true' data generating process. This would also allow to narrow the search criteria later on. Using the initial parameter values, and standard deviations of the residuals, the model was solved and the simulated data was used to generate the impulse response functions.
Figure 3.9: IRF (initial calibration): One off Shock in FS (fig.2)
Although the IRFs present a promising beginning, the residual from the production function in the banking industry raises concerns regarding the model’s properties and underlying assumptions.
CHAPTER 4

TESTING AND ESTIMATION ON STATIONARY DATA

4.1 Introduction

The focus of this chapter is the testing and estimation of the model presented in chapter 3. The chapter is divided as follows. First the methodology used to test and estimate the model will be discussed. Secondly, the parameters used in the initial calibration will be tested. Then, using indirect inference and a VAR as an auxiliary model, the parameter values will be estimated. The last section will utilise the estimated values and use them to test alternative auxiliary models in order to determine the robustness of the results.

4.2 Methodology - Indirect Inference Approach

Since before the development of DSGE models, various methods of evaluating macroeconomic theory have existed, and evolved and new methods have been developed. The competition is not only to find the 'best' model but also the most accurate method of evaluating it. For a detailed comparison between some of the main methods used, please refer to Ruge-Murcia (2007) and Le et al. (2015b). Although, calibration is a valid method for evaluating a specific framework, and
it has been used for more than 30 years, the approach has been criticised by many since it is not as rigorous as econometric testing. Given that the proposed model in this thesis is a new one that has never been tested, and that some of the parameter values have never been estimated on UK data, an econometric evaluation is essential. The rest of this section presents the methodology used to test and estimate the parameter values using both stationary and nonstationary data. The chosen method is Indirect Inference and its origins and description will be provided in the following sections.

4.2.1 Indirect Inference method

The beginning of the indirect inference popularisation can be traced back to the works of Gourieroux et al. (1993), Gourieroux and Monfort (1997), Gouriéroux et al. (2000), Smith (1993) and Gregory and Smith (1991). Gourieroux et al. (1993) presented a method that is based on what they call ‘incorrect’ criterion for the estimation of an auxiliary model, but which generates a consistent estimator once subsequently applied to the simulated data. It is a general inference method which is asymptotically normal. The authors provide several applications including stochastic differential equations, and macroeconomic, microeconomic and finance models. Smith (1993) demonstrates two methods of indirect inference that use VAR auxiliary model and try to match the parameters obtained from the actual and simulated data. The first one is the simulated quasi-maximum likelihood
(SQML) method and the second one is the extended method of simulated moments (EMSM) method. For a detailed description of the SQML method, see Meenagh et al. (2009). The following is a brief summary of the methodology presented there.

The SQML method uses maximum likelihood (ML estimator) on an auxiliary model using both actual and simulated data, generated from an underlying model. The underlying model is presumed to be the ‘true’ data generating process. The SQML estimator is the one that minimises the difference between the two estimates from the procedure mentioned above. The auxiliary model can be any number of time series models, e.g. VAR, VARMA, PVAR, VARIMA etc.

The following illustration is provided in Meenagh et al. (2009). For convenience it is replicated here. Let \( y_t \) be a \( m \times 1 \) vector with actual data, \( x_t(\theta) \) be a \( m \times 1 \) vector with simulated data, and \( \theta \) be a \( k \times 1 \) vector of parameters. The auxiliary model has a probability density function \( f(y_t, \alpha) \), where \( \alpha \) is a \( q \times 1 \) vector of parameters of the auxiliary model and \( k \gg q \). The assumption is that there is a value of \( \theta \), e.g. \( \theta_\alpha \), for which the following two distributions are the same \( f(x_t(\theta_\alpha), a) = f(y_t, \alpha) \). The likelihood function for the auxiliary model for the actual data \( \{ y_t \}_{t=1}^T \) is:

\[
L_T(y_t, \alpha) = \sum_{t=0}^{T} \log f(y_t, \alpha)
\]

The ML estimator of \( \alpha \) is:

\[
a_T = \arg \max_{\alpha} L_T(y_t, \alpha)
\]
Similarly, the likelihood function of the auxiliary model for the simulated data is:

$$L_s(x_t(\theta), \alpha) = \sum_{t=0}^{S} \log f(x_t(\theta_0), \alpha)$$

And the ML estimator is

$$a_S = \arg \max_{\alpha} L_s(x_t(\theta), \alpha)$$

Therefore the SQML estimator for $\theta$ is:

$$\theta_{T,S} = \arg \max_{\theta} L_s(y_t, a_S(\theta))$$

This generates the value of $\theta$ that produces a value of $\alpha$ which maximises the likelihood function using observed data.

The principle for EMSM estimation is exactly the same as the one for SQML. The only difference is that in the EMSM method the estimator is based on generalised method of moments (GMM). Using a Monte Carlo study, Smith (1993) found that the SQML method is slightly less efficient than EMSM. However, the author has noted that the SQML estimator has a smaller mean squared error in relatively small samples and therefore it is the preferred method when used to evaluate macroeconomic models. The above discussion provides two examples of indirect inference.

To summarise, an indirect inference approach uses an auxiliary model fitted on actual and simulated data and measure the distance between the two, by employing a function of the coefficients from the auxiliary model on both types of data, which
could be a score, test statistic or IRF. Recent examples that use the indirect inference approach are Minford (2015), Davidson et al. (2010), Le et al. (2011, 2013, 2015a), Onishchenko (2011), Raoukka (2013), and Meenagh (2015a,b). These papers present the application of the method to different DSGE model and the used data sets vary between the UK, to the US, China, Ukraine and Greece. Despite some small differences, the approach is essentially the same.

4.2.2 Application procedure.

The steps undertaken to perform an indirect inference estimation using a directed Wald is as follows:

- A global optimisation simulated annealing algorithm is used to search for parameter values within a predefined set of upper and lower bounds. The bounds are set using economic data and intuition to prevent the algorithm from searching in areas which would either not find an optimal set or produce illogical results.

- For every given set of coefficients, the residuals from the model’s equations are calculated.

- These residuals are then fitted to equations that most likely represents their time series properties. In the case of stationary data a simple AR(1) is used. In the case of nonstationary data, the residuals are assumed to be either
trend stationary with a drift or are characterised by a unit root.

- Using the residuals, the innovation series are obtained, the i.i.d shocks. These series represent the structural shocks of the model.

- The innovations are then used to calculate the bootstrap simulations, under the null hypothesis that the model is correct; in this and in the subsequent chapter the number of bootstraps is set to 1000, i.e. this step creates 1000 ’artificial’ data sets.

- The simulated series are then fitted to an auxiliary model. In the case of stationary data, a vector autoregressive (VAR) model is used and in the case of nonstationary data a vector error correction (VEC) model is used. These models are widely used in the testing and estimation of DSGE models since the reduced form of a DSGE model can be represented as a VAR.

- Collect the coefficients from each simulation (the constant term is not included) to construct a distribution containing the sampling properties of the coefficients of the auxiliary model.

- Collect the variances as well. The inclusion of variances in the calculation of the Wald increases the probability of rejection since more criteria is imposed on the model.

- Calculate the Wald statistic for each simulation. The Wald statistics repre-
sent a function of the parameter estimates (coefficients and variances) from
the auxiliary model used on simulated data.

- Calculate the Wald statistic using the same auxiliary model and actual data

- Compare the Wald from actual data to the distribution of the Wald using
  simulated data

- Check if the Wald in the actual data lies within the 95% confidence interval.

- To achieve that, construct the Transformed Mahalanobis Distance (TMD).
  This takes the Wald, which is a $\chi^2$ chi-squared statistic, and converts it to a
  normal distribution which would allow an easier interpretation of the results,
  then calculate the t-statistic. The 95th percentile is 1.645

- If the value of TMD is less than 1.645 - the test does not reject the null
  hypothesis, i.e. that the model is a close approximation to the true data
  generating process. If the answer is no, the model is rejected.

- Collect the TMD value and the parameter set that generated the simulated
  data.

- Choose another set of parameter values and repeat these steps from the
  beginning
The intuition behind this method is as follows. A theoretical model is used to generate time series data, called ‘artificial’ data. This data is uniquely defined by the equations of the model and the chosen parameter set. Actual time series data is also collected. The procedure checks how close the two data sets are. The best estimates of the parameter values are those that create artificial data which most resembles the actual data.

This procedure can be used for both purposes - to test and to estimate parameters of the DSGE model. If the procedure is used for testing purposes the steps are performed only once on a given set of parameter values. If it is used to obtain parameter estimates that enable the model to fit the data, the procedure is repeated as many times as the economist decides using a simulated annealing algorithm. In the following estimation estimations, the global optimisation algorithm repeats the test up to 100 times. The best estimates of the model parameters are those that generated the lowest TMD. There are several benefits from this procedure. Using the innovations calculated from the model residuals does not require knowledge of the actual distribution of structural shocks, Minford (2015). Le et al. (2015b) evaluated the small sample properties using Monte Carlo analysis and concluded that indirect inference testing is more powerful than an LR test in small samples. Last but not least, indirect inference uses the same specification of the auxiliary model when applied to both simulated and actual data. Therefore, even
if the auxiliary model is not the best representation of the time series, e.g. VAR(1) is used when VARMA(1,1) is most suitable, it would still generate robust results since it is applied uniformly on the two types of data.

It is important to point out that the approach here is defined as directed Wald as opposed to full Wald. In the full Wald the auxiliary model includes all endogenous variables from the DSGE model. This would most certainly lead to a rejection of the model since it is a mere simplified approximation of reality. As a rule of thumb, the more variables and the more lags are included the higher the chance that the model will be rejected. That is why the directed Wald is used. In this case the testing/estimation procedure is focused on a few variables at any given time that are of interest to the researcher. The next section presents the results from testing the model using initial parameter values and stationary data.

4.3 Testing Using Initial Parameter Set

As discussed previously, great care was given to the calculation of some of the parameters in order to provide a better chance for the model to pass the econometric testing. The choice of the auxiliary model was straightforward. The inclusion of output is essential. In order for any model to have some usefulness, it should be able to at least capture the behaviour of output. Since the main contribution of this thesis is to evaluate whether an open economy model augmented by a financial intermediation sector would be able to capture the features of the underlying data,
it was necessary to include at least one variable that is a representative of the financial intermediation sector. A decision was made to use the interest rate spread. There are two main reasons for this. First, interest rate spreads have always been a centre of attention during financial crises and are perceived as a leading indicator. Secondly, using the spread would effectively capture the variation of both: the rate on deposits and the rate on loans. The third variable is another feature that differentiates this model from the others that were tested using the same procedure, namely the external debt elastic interest rate. Although other researchers have included foreign bonds or an equivalent when testing open economy models using indirect inference, most of them have assumed that the interest rate is an exogenous process. Therefore the three variables used in the auxiliary model are: output, interest rate spread and interest on foreign debt.

A description of the full data set used in this thesis is provided in an appendix. For convenience and due to their importance, the description of output, interest rate spread and interest rate on foreign debt are presented here.

The data series for output is the UK GDP. It is a CVM measure and seasonally adjusted. Since output in the model is expressed in real per capita terms there is no need to adjust for inflation but the data was divided by the population to present it in per capita terms.

The data used for the foreign debt interest rate is not directly gathered but
rather it was calculated using data from the ONS. It represents the real interest rate on aggregate foreign debt in per capita terms. Such data series could not be found. However, it can be calculated using the data of net interest income from investment in the UK by the rest of the world and dividing it by the total investment in the UK by foreigners. The ratio is the average return of investment by foreigners in the domestic economy for a given quarter, which is the closest description of the variable in the model.

The choice of the time series that will be used for the deposit and loan rates is a bit more complicated. The complication arises from the fact that there are no data series of total deposits and total loans regardless of the size and maturity dates. The interest rates could be fixed or flexible and vary depending on the maturity. That is why proxies were used. The sterling three-month interbank lending rate was used for the loan rate and the average discount rate on treasury bills – for the deposit rate.

The interbank rate captures the cost at which banks are willing to lend/borrow money from one another and thus could be considered a suitable proxy for the loan rate in the model. The discount rate is considered as one of the closest rates to the risk free rate and could be viewed as the rate at which banks lend to the government, effectively depositing funds. This rate is not the best option to proxy the deposit rate. Other more suitable candidates are: the quarterly average of
sterling certificates of deposit interest rate (three months, mean offer/bid); the 
quartely average of the official bank rate; and the quarterly average of the base 
rates of four UK banks. However, when these data series were compared with the 
LIBOR, there were quarters when the proposed proxies for the deposit rate were 
higher than the lending rate. This would clearly create a problem in the analysis 
since the banking firm would never optimally choose to set the loan rate at a lower 
level than the deposit rate. That is why the only choice left was the discount 
rate, which given how closely it moves with the other three options, should be a 
suitable data choice. Both the deposit and the loan rates were adjusted by the 
CPI to convert them in real terms. The data for the spread is simply the difference 
between the two.

Using the initial parameter values and the data described above and applying 
the procedure explained in the previous section, a test statistic of 2.1912 was 
obtained. Given that this is a one-tailed test with a critical value of 1.645 at the 
95 percentile, the test strictly rejects the model. This result is not a surprise since 
the coefficients used in the initial calibration were not all obtained from the data. 
The next step is to estimate the model using indirect inference.
4.4 Estimation Results

4.4.1 Parameter estimates

Using indirect inference as a method of estimating the model’s parameters, the best estimates were obtained and the value of the test statistic is 1.0532. This statistic generated the values presented in table 4.1. For convenience and to ease comparison the coefficients used in the initial calibration are also presented here. A value for the discount factor is not available since this parameter was not included in the estimation. The value for labour share in output is significantly larger than the initial choice however, a value of 0.6972 is much closer to what other researchers have used/obtained in their work. For example, the value used in Meenagh et al. (2005) is 0.7. The coefficient of relative risk aversion is slightly lower than what was initially thought. However, this value is almost identical to the 1.03, reported by Gandelman and Hernández-Murillo (2014). In contrast the parameter capturing the elasticity of labour supply is larger. The coefficient appears in front of the labour variable in the equation describing the MRS between consumption and leisure. Therefore, $\rho_2$ is the inverse of the elasticity of labour to changes in the real wage rate. Thus an increase in $\rho_2$ would imply a steeper labour supply curve for the given wage rate. The capital depreciation rate is almost twice as much as the annual data indicated.

One very interesting estimate that was not available until now for the UK is
The parameter captures the sensitivity of the interest rates to deviations of the country’s debt level relative to its steady state value. This estimate is even higher than what was calculated using the data and therefore even higher than what is the coefficient for the US. The difference suggests that international capital markets penalise UK residents much more harshly than they do US residents. Another example of this thesis contribution is the estimate for \( \gamma \), the labour share in loan production. The value is smaller than the one for the US (0.11) used in Benk, et al. (2010). The home bias in consumption is significantly lower. A value of 0.52556 would indicate that there is a little bias towards home produced goods. The import demand elasticities are higher, especially the one for the rest of the world. However, they are still within norms.

4.4.2 Impulse response functions using estimates from stationary data.

The estimated values were used to calibrate the model in order to obtain a set of impulse response functions. The objective is to examine if the new parameter values led to any significant changes in the reaction of the endogenous variables to a one period shock. As it can be seen from figures 4.1, 4.2, 4.3, and 4.4, little has changed. The reaction of the endogenous variables compared to the underlying
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Initial Value</th>
<th>Estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
<td>0.985</td>
<td>-</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Labour share in goods production</td>
<td>0.6154</td>
<td>0.6972</td>
</tr>
<tr>
<td>$\rho_1$</td>
<td>Relative risk aversion coefficient</td>
<td>1.2</td>
<td>1.0390</td>
</tr>
<tr>
<td>$\rho_2$</td>
<td>Elasticity of labour supply</td>
<td>1</td>
<td>1.3847</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Capital depreciation rate</td>
<td>0.01299</td>
<td>0.0250</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>Interest elasticity of foreign debt</td>
<td>0.000742</td>
<td>0.0049</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Labour share in loan production</td>
<td>0.066</td>
<td>0.077</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Home bias in consumption</td>
<td>0.7</td>
<td>0.5256</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Import demand elasticity</td>
<td>1</td>
<td>1.0980</td>
</tr>
<tr>
<td>$\sigma^F$</td>
<td>Import demand elasticity (foreign country)</td>
<td>1.2</td>
<td>1.8231</td>
</tr>
</tbody>
</table>

Table 4.1: Initial and Estimated Parameter Values and Stationary Data
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Equation</th>
<th>Initial Value</th>
<th>Estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho^{nc}$</td>
<td>Euler equation</td>
<td>0.7872</td>
<td>0.7830</td>
</tr>
<tr>
<td>$\rho^{nf}$</td>
<td>Imports equation</td>
<td>0.7202</td>
<td>0.7276</td>
</tr>
<tr>
<td>$\rho^{n\eta}$</td>
<td>MRS $n_t$ and $c_t$</td>
<td>0.7846</td>
<td>0.7708</td>
</tr>
<tr>
<td>$\rho^{zg}$</td>
<td>Production function GS</td>
<td>0.6193</td>
<td>0.6761</td>
</tr>
<tr>
<td>$\rho^{zb}$</td>
<td>Production function FS</td>
<td>0.7914</td>
<td>0.7914</td>
</tr>
<tr>
<td>$\rho^{ng}$</td>
<td>MPL equation</td>
<td>0.6125</td>
<td>0.6125</td>
</tr>
<tr>
<td>$\rho^{nk}$</td>
<td>MPK equation</td>
<td>0.5244</td>
<td>0.5146</td>
</tr>
<tr>
<td>$\rho^{nb}$</td>
<td>Spread equation</td>
<td>0.5617</td>
<td>0.6745</td>
</tr>
<tr>
<td>$\rho^F$</td>
<td>Exports equation</td>
<td>0.7023</td>
<td>0.7509</td>
</tr>
<tr>
<td>$\rho^{F}$</td>
<td>Foreign consumption demand</td>
<td>0.8500</td>
<td>0.8500</td>
</tr>
<tr>
<td>$\rho^g$</td>
<td>Government spending</td>
<td>0.3192</td>
<td>0.3192</td>
</tr>
</tbody>
</table>

Table 4.2: Residuals Autocorrelation Coef Based on Initial and Estimated Parameter Values and Stationary Data
theory and economic intuition is sound.

4.5 Test Results from Best Estimates for Parameter Values

The robustness of the estimated values was checked by performing several Wald tests using different endogenous variables but using the estimated coefficients every time. The results are in table 4.3. The table shows that apart from the original model that provided the estimates only two other specifications do not reject the
Figure 4.2: IRF (using estimates from stationary data): Shock to TFP GS (fig.2)
Figure 4.3: IRF (using estimates from stationary data): Shock to TFP FS (fig.1)
Figure 4.4: IRF (using estimates from stationary data): Shock to TFP FS (fig.2)
null hypothesis. These are $y_t, sp_t, df_t$ and $y_t, sp_t, lbt_t$. In both cases the parameters under scrutiny are those that describe the relationship between output and two representatives of the newly introduced features. Unfortunately, in every other case the model is rejected. One statistic that causes concern is the one obtained when capital is one of the variables under investigation. This is also the proxy for the stock of outstanding debt. As discussed in the calibration section of this thesis, there is a possibility that one of the underlying restrictions on this variable is not realistic, given how further away the transformed distance is from the critical value of 1.645. It remains to be seen if this issue will occur in the case when nonstationary data is used to test the model.

4.6 Conclusion

This chapter presented the procedure and application of econometric testing, namely indirect inference, to evaluate the DSGE model featuring financial sector as a form of intermediate good and debt elastic interest rates. The result from the test using the initial calibration, conclusively rejected the null hypothesis. However, once the procedure was used to obtain estimates that minimised the transformed distance between the parameter estimates from the auxiliary model based on the actual and simulated data, the results changed significantly. The
<table>
<thead>
<tr>
<th>Auxiliary model</th>
<th>T statistic</th>
<th>Decision</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_t, sp_t, r_t^f$</td>
<td>1.0532</td>
<td>Best estimate</td>
</tr>
<tr>
<td>$y_t, sp_t, df_t$</td>
<td>1.3213</td>
<td>Do Not Reject</td>
</tr>
<tr>
<td>$y_t, sp_t, Q_t$</td>
<td>2.5539</td>
<td>Reject</td>
</tr>
<tr>
<td>$y_t, sp_t, cx_t$</td>
<td>1.6999</td>
<td>Reject</td>
</tr>
<tr>
<td>$y_t, sp_t, n_t$</td>
<td>2.0634</td>
<td>Reject</td>
</tr>
<tr>
<td>$y_t, sp_t, w_t$</td>
<td>2.0279</td>
<td>Reject</td>
</tr>
<tr>
<td>$y_t, sp_t, lgt$</td>
<td>1.6746</td>
<td>Reject</td>
</tr>
<tr>
<td>$y_t, sp_t, lbt$</td>
<td>1.0974</td>
<td>Do Not Reject</td>
</tr>
<tr>
<td>$y_t, sp_t, im_t$</td>
<td>2.1688</td>
<td>Reject</td>
</tr>
<tr>
<td>$y_t, sp_t, k_t$</td>
<td>47.812</td>
<td>Reject</td>
</tr>
</tbody>
</table>

Table 4.3: Robustness Check Using Best Estimates and Stationary Data
estimated values conform with other coefficients in the literature. For two of the parameter values, estimates on UK data by other researchers have not been found. That is why they were compared to those obtained using US data. Both parameters are within a reasonable distance from those on US data. The significance of the results was checked by performing Wald tests using the estimated coefficients and different combinations of endogenous variables. The results illustrated that when the model contains variables associated with the banking sector and the foreign debt variable, the null hypothesis that the model represents the true data generating process cannot be rejected. However, other test results rejected the model. The results also indicated that there could be a possible issue with one of the assumptions of the model, i.e. that net investment is financed only by using loans from the financial intermediary.
CHAPTER 5

TESTING AND ESTIMATION ON NONSTATIONARY DATA

5.1 Introduction

This chapter replicates the analysis performed in the previous one. The only, but very significant, difference is that in this chapter the model will be tested on nonstationary data. There are several reasons for this. First, many have argued that detrending the data removes valuable information irrespective of the method used. Secondly, despite the widespread use of HP filtering, there has been a growing concern regarding its implications for the data and the cyclical variability. Thirdly the use of a universal multiplier of 1600 in the filter may be to generic to be suitable for every country regardless of the development level. Also, unlike US data which is growing at a relatively identical pace, UK variables are more prone to experiencing fluctuations due to the size of the country and the dependence on the rest of the world. Finally, using nonstationary data would either confirm what was found in the previous chapter and in this way strengthen the arguments made or present results that contradict what was discovered previously and thus highlight possible
investigative avenues.

5.2 Methodology - Adjustments to the Procedure.

It is important to point out that when using nonstationary time series, it is advisable to proceed with caution. A detailed description of time series data properties is available in Hamilton (1994). Given the nature of macroeconomics data, the time series are most likely cointegrated. If the procedure does not take this into account it could lead to misleading results. The first adjustment was made to the way the variables with expectations operator are predicted. In previous discussions it was pointed out that a simple VAR was used; however, this is no longer suitable. That is why the VAR was applied to the differenced variables. The second adjustment was made to the model used to estimate the autocorrelation coefficients of the residuals. The AR(1) model is no longer suitable. To determine if the residuals are trend-stationary or contain unit root, both ADF, Dickey and Fuller (1979), and KPSS, Kwiatkowski et al. (1992), tests were performed. The results are presented in tables 5.1 and 5.2

The tests produce consistent results regarding $\eta_f^c$, $\eta_f^d$, $\eta_i^u$, and $\eta_i^k$ and indicate that the series are trend stationary. They also indicate that the residual for $zg_t$ may contain unit root. However, the results for $zb_t$, $\eta_i^o$, $\eta_i^h$, and $\eta_i^F$ are inconclusive. Employing other tests such as those proposed by Lee and Strazicich (2003) and Ng and Perron (2001) are not suitable within this framework since the model assumes
### ADF Test Results

<table>
<thead>
<tr>
<th>Residual</th>
<th>H₀: A unit root is present in a time series</th>
</tr>
</thead>
<tbody>
<tr>
<td>η₁</td>
<td>Reject H₀</td>
</tr>
<tr>
<td>η₂</td>
<td>Reject H₀</td>
</tr>
<tr>
<td>η₃</td>
<td>Reject H₀</td>
</tr>
<tr>
<td>zg</td>
<td>Failure to reject H₀</td>
</tr>
<tr>
<td>zb</td>
<td>Failure to reject H₀</td>
</tr>
<tr>
<td>η₄</td>
<td>Failure to reject H₀</td>
</tr>
<tr>
<td>η₅</td>
<td>Reject H₀</td>
</tr>
<tr>
<td>η₆</td>
<td>Reject H₀</td>
</tr>
<tr>
<td>η₇</td>
<td>Failure to reject H₀</td>
</tr>
</tbody>
</table>

Table 5.1: Testing for Unit Roots: ADF
<table>
<thead>
<tr>
<th>Residual</th>
<th>$H_0$: The series is trend stationary</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta_t^c$</td>
<td>Failure to reject $H_0$</td>
</tr>
<tr>
<td>$\eta_t^f$</td>
<td>Failure to reject $H_0$</td>
</tr>
<tr>
<td>$\eta_t^n$</td>
<td>Failure to reject $H_0$</td>
</tr>
<tr>
<td>$zg_t$</td>
<td>Reject $H_0$</td>
</tr>
<tr>
<td>$zh_t$</td>
<td>Failure to reject $H_0$</td>
</tr>
<tr>
<td>$\eta_t^g$</td>
<td>Failure to reject $H_0$</td>
</tr>
<tr>
<td>$\eta_t^k$</td>
<td>Failure to reject $H_0$</td>
</tr>
<tr>
<td>$\eta_t^h$</td>
<td>Reject $H_0$</td>
</tr>
<tr>
<td>$\eta_t^f$</td>
<td>Failure to reject $H_0$</td>
</tr>
</tbody>
</table>

Table 5.2: Testing for Unit Roots: KPSS
constant parameters and regime, and therefore there are no structural breaks.

That is why a conjecture, which is not inconsistent with the results from the unit root tests, was made: all residuals are trend-stationary with a drift, apart from one - the Solow residual in the goods sector. The assumption was that if the variables are not trend-stationary removing a trend component would leave the unit root which could be discovered upon investigation of the autoregressive coefficients. Furthermore, any inconsistency would be captured by the Wald test. Following this logic and the outlined steps in the testing procedure proved that the conjecture was correct. Evidence for this are the values for the autoregressive coefficients that were generated using the parameter estimates that produce a test statistic which passes the Wald test. The exact values are in table 5.5. That is why the residuals are modelled as described above.

Last but not least the auxiliary model could no longer be a simple VAR. The remaining possible choices are VARX or VECM. A decision was made to use VECM representation. The estimation was done using spatial econometrics toolbox by James P. LeSage. The estimation function for error correction models (ECM) carries out Johansen’s tests to determine the number of cointegrating relations, which are automatically incorporated in the model. This significantly simplifies the procedure and it was one of the main reasons for choosing this auxiliary model. Given these adjustments the rest of the procedure is exactly the same as the one.
explained in the Chapter 4.

5.3 Testing Using Initial Parameter Set

Although the time series model for the auxiliary model is different, the endogenous variables remain the same. This would ease comparison with previous results and at the same time keeps the focus of this thesis on the proposed model and the additions made. A Wald test was performed using the initial parameter values. The transformed distance is 3.2220. The critical value suggests that the model is rejected at 5% significance level. The value is higher than the one achieved using stationary data. It remains to be seen if the parameter estimates would differ significantly.

5.4 Estimation Results

Using the adjustments mentioned in the beginning of this chapter and the methodology described earlier, an indirect inference approach was used to obtain the best point estimates for the underlying structural parameters. The transformed distance at which these were obtained is 1.5294. The parameter values are presented in table 5.4. Alongside these, the estimates using stationary data, the initial calibration, and some of the relevant estimates obtained by Minford
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Equation</th>
<th>Initial Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho^{rc}$</td>
<td>Euler equation</td>
<td>0.0113</td>
</tr>
<tr>
<td>$\rho^{rf}$</td>
<td>Imports equation</td>
<td>0.8117</td>
</tr>
<tr>
<td>$\rho^{m}$</td>
<td>MRS $n_t$ and $c_t$</td>
<td>0.8826</td>
</tr>
<tr>
<td>$\rho^{zg}$</td>
<td>Production function GS</td>
<td>0.0752</td>
</tr>
<tr>
<td>$\rho^{zb}$</td>
<td>Production function FS</td>
<td>0.9408</td>
</tr>
<tr>
<td>$\rho^{rg}$</td>
<td>MPL equation</td>
<td>0.8722</td>
</tr>
<tr>
<td>$\rho^{rk}$</td>
<td>MPK equation</td>
<td>0.6836</td>
</tr>
<tr>
<td>$\rho^{rb}$</td>
<td>Spread equation</td>
<td>0.8665</td>
</tr>
<tr>
<td>$\rho^{F}$</td>
<td>Exports equation</td>
<td>0.9124</td>
</tr>
<tr>
<td>$\rho^{F}$</td>
<td>Foreign consumption demand</td>
<td>0.9912</td>
</tr>
<tr>
<td>$\rho^{g}$</td>
<td>Government spending</td>
<td>0.9484</td>
</tr>
</tbody>
</table>

Table 5.3: Residuals Autocorrelation Coef Based on Initial Parameter Values and Nonstationary Data
(2015) are presented for comparison purposes. The estimate for the labour share have increased dramatically. Although the value is not unreasonable, it seems very different from the initial calibration. The relative risk aversion has dropped significantly relative to the stationary case and the reported value by Gandelman and Hernández-Murillo (2014). However, this result could be even lower since most macroeconomic studies do not take into account the labour margin and possibility that the representative agent could offset shocks to income by adjusting the working hours as argued by Swanson (2009).

The labour elasticity has increased. This result is consistent with the findings by Minford (2015). The depreciation rate estimate has reverted back to the value calculated using time series data on the consumption of fixed capital. The elasticity of the interest rate with respect to the level of foreign debt has increased further. A pattern begins to emerge. When the results on relative risk aversion and interest rate elasticity coefficients from the initial calibrated values, estimates from stationary data and estimates from nonstationary data are compared it could be seen that as the interest elasticity increases, the relative risk aversion falls. An intuitive explanation of this phenomenon could be the following. A higher level of \( \varphi \) would imply that for any given increase of the home country liabilities, a proportionately higher increase will be observed in the interest rate level on foreign debt. This would reduce the incentive to acquire new debt in order to keep the
cost of borrowing low. Furthermore, if it is assumed that the level of exports and imports remain the same, thus keeping the real effective exchange rate intact, an increase in the cost of borrowing would result in an increase in the interest rate of deposits via the real UIP equation. This would lead to an increase in savings and a substitution of current consumption for future one. The labour share in loan production has dropped by 44% when compared with stationary estimates and by 34% relative to initial value. Given that there is a lack of research in that area when UK data is considered, little can be said regarding which one is more accurate. The result regarding the home bias parameter is consistently estimated around 0.5. The results regarding the import demand elasticities are very different but they all are within the bounds of what other researchers have presented.
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
<td>-</td>
<td>0.985</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Labour share in goods production</td>
<td>0.7846</td>
<td>0.6154</td>
<td>0.6972</td>
<td>-</td>
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<tr>
<td>$\rho_1$</td>
<td>Relative risk aversion coefficient</td>
<td>0.7936</td>
<td>1.2</td>
<td>1.0390</td>
<td>0.9712</td>
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<tr>
<td>$\rho_2$</td>
<td>Elasticity of labour supply</td>
<td>1.6380</td>
<td>1</td>
<td>1.3847</td>
<td>1.5198</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Capital depreciation rate</td>
<td>0.0166</td>
<td>0.01299</td>
<td>0.0250</td>
<td>-</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>Interest elasticity of foreign debt</td>
<td>0.0097</td>
<td>0.000742</td>
<td>0.0049</td>
<td>-</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Labour share in loan production</td>
<td>0.0437</td>
<td>0.066</td>
<td>0.077</td>
<td>-</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Home bias in consumption</td>
<td>0.5095</td>
<td>0.7</td>
<td>0.5256</td>
<td>0.5264</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Import demand elasticity</td>
<td>1.0410</td>
<td>1</td>
<td>1.0980</td>
<td>0.7676</td>
</tr>
<tr>
<td>$\sigma^F$</td>
<td>Import demand elasticity (foreign country)</td>
<td>1.1744</td>
<td>1.2</td>
<td>1.8231</td>
<td>0.8819</td>
</tr>
</tbody>
</table>

Table 5.4: Initial Parameter Values and Estimates Using Stationary and Nonstationary Data
5.5 Test Results from Best Estimates for Parameter Values

This section, in a similar fashion to the corresponding one in the previous chapter, presents the results from testing the model using the Wald statistic on a different set of endogenous variables but using the estimates provided by the estimation procedure on nonstationary data. Several things stand out when the results from the robustness test on stationary data is taken into account. Similarly, like the results in the previous chapter, the combinations of variables that contain the labour share in loan production and the foreign debt are not rejected by the procedure. Another interesting result is the inability of the test to reject the model when consumption is included. Given that the inclusion of an additional variable increases the possibility of rejection as is evident from the other examples of VECMs with 4 variables, this result is surprising. The test statistic on the auxiliary model which has capital as one of the variables is still very high in relation to the others. This confirms the suspicion that the underlying assumption is too restrictive and forcing the variable to capture elements from both financial intermediation and goods producing sectors leads to underperformance.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Equation</th>
<th>Initial Value</th>
<th>Estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_{nc}$</td>
<td>Euler equation</td>
<td>0.0113</td>
<td>0.0074</td>
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<tr>
<td>$\rho_{nf}$</td>
<td>Imports equation</td>
<td>0.8117</td>
<td>0.8162</td>
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<tr>
<td>$\rho_{nm}$</td>
<td>MRS $n_t$ and $c_t$</td>
<td>0.8826</td>
<td>0.8342</td>
</tr>
<tr>
<td>$\rho_{zg}$</td>
<td>Production function GS</td>
<td>0.7520</td>
<td>0.0384</td>
</tr>
<tr>
<td>$\rho_{zb}$</td>
<td>Production function FS</td>
<td>0.9408</td>
<td>0.9408</td>
</tr>
<tr>
<td>$\rho_{ng}$</td>
<td>MPL equation</td>
<td>0.8722</td>
<td>0.8722</td>
</tr>
<tr>
<td>$\rho_{nk}$</td>
<td>MPK equation</td>
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<td>0.6808</td>
</tr>
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<td>$\rho_{nh}$</td>
<td>Spread equation</td>
<td>0.8665</td>
<td>0.8578</td>
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<tr>
<td>$\rho_{F}$</td>
<td>Exports equation</td>
<td>0.9124</td>
<td>0.9120</td>
</tr>
<tr>
<td>$\rho_{FE}$</td>
<td>Foreign consumption demand</td>
<td>0.9912</td>
<td>0.9912</td>
</tr>
<tr>
<td>$\rho_{g}$</td>
<td>Government spending</td>
<td>0.9484</td>
<td>0.9484</td>
</tr>
</tbody>
</table>

Table 5.5: Residuals Autocorrelation Coef Based on Initial Parameter Values and Estimates Using Nonstationary data
<table>
<thead>
<tr>
<th>Auxiliary model</th>
<th>T statistic</th>
<th>Decision</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_t, sp_t, r_t^f$</td>
<td>1.2558</td>
<td>Best Estimate</td>
</tr>
<tr>
<td>$y_t, sp_t, df_t$</td>
<td>1.5294</td>
<td>Do Not Reject</td>
</tr>
<tr>
<td>$y_t, sp_t, cx_t$</td>
<td>2.6267</td>
<td>Reject</td>
</tr>
<tr>
<td>$y_t, sp_t, im_t$</td>
<td>2.4273</td>
<td>Reject</td>
</tr>
<tr>
<td>$y_t, sp_t, w_t$</td>
<td>1.6466</td>
<td>Reject</td>
</tr>
<tr>
<td>$y_t, sp_t, lb_l$</td>
<td>1.3602</td>
<td>Do Not Reject</td>
</tr>
<tr>
<td>$y_t, sp_t, k_t$</td>
<td>44.7346</td>
<td>Reject</td>
</tr>
<tr>
<td>$y_t, sp_t, df_t, lb_l$</td>
<td>2.0715</td>
<td>Reject</td>
</tr>
<tr>
<td>$y_t, sp_t, df_t, Qt$</td>
<td>1.8046</td>
<td>Reject</td>
</tr>
<tr>
<td>$y_t, sp_t, r_t^f, n_t$</td>
<td>2.1739</td>
<td>Reject</td>
</tr>
<tr>
<td>$y_t, sp_t, r_t^f, g_t$</td>
<td>6.1039</td>
<td>Reject</td>
</tr>
<tr>
<td>$y_t, sp_t, r_t^f, c_t$</td>
<td>1.4257</td>
<td>Do Not Reject</td>
</tr>
</tbody>
</table>

Table 5.6: Robustness Check Using Best Estimates and Nonstationary Data
5.6 Conclusion

This chapter concluded the empirical analysis of the proposed model. The utilisation of nonstationary data lead to slightly different results with respect to parameter estimates but the overall ability of the model to match the data remained consistent with the results from the previous chapter. One main similarity is that in both cases the test statistic strongly rejects the model when capital is one of the variables used in the auxiliary model. This confirmed the suspicion that firms should be allowed to use not only debt financing but also reinvestment of profits and equity financing to invest in capital goods.
The objective of this thesis was to present the reader with an alternative economic framework that incorporates features from three well established branches of macroeconomics and using this unique combination, gain an insight into the workings of the UK economy. The constructed model can be defined as a two sector small open economy dynamic stochastic general equilibrium real business cycle model with a financial intermediation sector that plays a major role in the final goods production. The main economic questions posed and answered are as follows. “Is it possible to use a real business cycle model without any nominal and price rigidity to account for the joint behaviour of output, interest rate spread and interest on foreign debt?” The answer is “Yes”. “What are the parameter values in the loan production function based on UK data?” The analysis suggests that it depends on what data is used. Using stationary (nonstationary) data, the best estimate for the labour share to loan production is 0.077 (0.0437). “What is the parameter estimate of the foreign debt interest rate elasticity?” According to the results it is equal to 0.0049 (0.0097) when filtered (raw) data is used.

Although the analysis accomplished the objectives, it raised new questions as
well. First, the consistent rejection of a test statistic based on capital combined with the small residual of the TFP shock suggest that one of the assumptions is too restrictive. Chapter 3 showed that firms were assumed to use debt financing in the investment in new capital. This assumption implies that, if present since the beginning of time (period zero), it leads to the solution that all capital is bought on credit. Not allowing for these variables to move independently of each other imposes a strong and unrealistic assumption, since in reality most firms use a combination of debt and equity financing as well as reinvestment of profits. As it was suggested in earlier chapters, relaxing this assumption could solve the problem indicated by the test results.

One possible solution would be to assume that only a fraction of gross investment is bought on credit and that the fraction is time varying. This adjustment should break the one-to-one relationship between capital and loans and possibly provide better results. This adjustment would also require an explicit modelling of other sources of raising capital. Another possible solution could be to replace the representative firm with a N number of firms that operate in a perfectly competitive market and produce the same final good. Capital accumulation would follow a law of motion where new capital is a result of investment in risky projects. An assumption would be made that the risk-return profile is not public knowledge. Thus lenders would have to build expectations regarding the return on their assets.
If it is assumed that the firms have access to both domestic and foreign financial intermediary, for every investment project they would have to choose from which lender to borrow. To minimise costs, the firm would pick the lender that offers the lower rate, ceteris paribus. If this is valid for every firm, on aggregate the goods producing sector would have a fraction of firms financed by the home country financial intermediary and another fraction that borrows funds from international markets. In this way, aggregate capital and loans would not be the same. An alternative solution is to introduce firm specific idiosyncratic shocks and adjustment costs of investment decisions to new information in the manner of Kwark (2002).

Another very strong assumption that could be the reason for the bad test results on capital data is the simplistic balance sheet of the financial intermediary. According to the balance sheet constraint deposits must equal loans. However as explained above, loans are equal to capital. Therefore, three variables are tied together. This suggests that either the balance sheet should be augmented to become more realistic or it should not be considered in the optimisation problem. Prior to any further investigation into the properties of this model and possible extensions, an adjustment should be made to these two assumptions.

The next stage would be to introduce realism in the model by incorporating monetary and fiscal authorities. This would also permit a welfare analysis of different government policies. The simplest way to introduce monetary policy is
via cash-in-advance constraint in the manner of Benk et al. (2005, 2010) and Gillman and Kejak (2004, 2008). This would allow for an investigation of the impact of monetary shocks on the cyclical properties of the model.

The impact of prudential regulation could also be analysed by introducing, for example, reserve requirements or debt-to-income ratios. Including a constant reserve requirement would simply create a wedge between the deposit and loan rates which would increase the interest rate spread by a constant fraction without altering the dynamic properties. However, if the reserve requirement is endogenous and a function of the TFP shocks in the two sectors it would have amplification effects. For example, a negative shock in either of the industries would result in tighter reserve requirements. This would increase the spread and the loan rate, lowering the demand for loans, thus reducing investment and output. This would be an additional decrease in the level of GDP which would have fallen as a direct effect of the negative TFP. Given these propositions, the inclusion of monetary and prudential control would be a beneficial research avenue.

The model proposed in this thesis assumes that government spending is exogenous and that the effects of changes in capital and income tax are captured by error terms. This specification prevents the analysis of the impact of government interventions and fiscal shocks. One way to introduce government is via a budget constraint in the manner of Meenagh et al. (2005) which states that any deficit
is covered by the issuance of new debt. Furthermore, allowing the government to issue debt would result in a portfolio choice for both the representative agent and the financial intermediary.

A straightforward extension of the existing model is to allow the world interest rate to vary over time and be subjected to exogenous shocks. Given the importance of international financial markets and the effect they have on relatively small countries, especially emerging economies, shocks in the world interest rate would affect the cost of borrowing via the debt elastic interest rate. In a similar fashion, a shock to the risk premium could be included that would capture the effects of factors other than the level of debt, such as investors’ perceptions and rating agencies. Other extensions could be the inclusion of a sector for nontraded goods, habit formation, capital adjustment costs, credit constraints, default risk, exchange credit, and news to name a few.
APPENDICES

A. Log-Linearisation

The system of equations is log-linearisation around the steady state using Taylor expansion series. To achieve this the following steps are performed. First, take natural logs. Second, apply first order Taylor expansion about the steady state. Finally, express the variables as a percentage deviation from the steady state. For any variable $x_t$, let $\bar{x}$ be the steady state value and $\tilde{x}_t$ be the percentage deviation from the steady state of that variable.

$$\tilde{x}_t = \frac{(x_t - \bar{x})}{\bar{x}}$$

For any exogenous variable $\eta^i_t$, that is defined as an AR(1) process of the form $\eta^i_t = (\eta^i_{t-1})^{\rho^i} + \varepsilon^i_t$, where $\varepsilon^i_t \sim iid (0, 1)$ and $\rho^i \neq 0$, the steady state value, denoted as $\bar{\eta}^i$, is equal to 1.
Proof.

\[ \eta^i_t = (\eta^i_{t-1})^\sigma + \varepsilon^i_t \]
\[ \bar{\eta}^i = (\bar{\eta}^i)^\sigma + 0 \]
\[ \bar{\eta}^i = (\bar{\eta}^i)^\sigma \]
\[ \bar{\eta}^i = 1 \]

(3.17):

\[ p^h_t = \left( \chi^{-\sigma} - \left( \frac{1-\chi}{\chi} \right)^\sigma (\eta^i_t)^\sigma (Q_t)^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \]

\[ \ln(p^h_t) = \ln \left( \chi^{-\sigma} - \left( \frac{1-\chi}{\chi} \right)^\sigma (\eta^i_t)^\sigma (Q_t)^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \]

\[ \ln(p^h_t) = \frac{1}{1-\sigma} \ln \left( \chi^{-\sigma} - \left( \frac{1-\chi}{\chi} \right)^\sigma (\eta^i_t)^\sigma (Q_t)^{1-\sigma} \right) \]

\[ \ln(p^h_t) + \frac{1}{\bar{p}^h}(p^h_t - \bar{p}^h) = \frac{1}{1-\sigma} \ln \left( \chi^{-\sigma} - \left( \frac{1-\chi}{\chi} \right)^\sigma (\bar{\eta}^i)^\sigma (\bar{Q})^{1-\sigma} \right) + \]
\[ \frac{1}{1-\sigma} \left( \chi^{-\sigma} - \left( \frac{1-\chi}{\chi} \right)^\sigma (\bar{\eta}^i)^\sigma (\bar{Q})^{1-\sigma} \right) \left( - \left( \frac{1-\chi}{\chi} \right)^\sigma (\bar{\eta}^i)^{\sigma-1} (\bar{Q})^{1-\sigma} \right) (\eta^i_t - \bar{\eta}^i) + \]
\[ \frac{1}{1-\sigma} \left( \chi^{-\sigma} - \left( \frac{1-\chi}{\chi} \right)^\sigma (\bar{\eta}^i)^\sigma (\bar{Q})^{1-\sigma} \right) \left( - \left( \frac{1-\chi}{\chi} \right)^\sigma (\bar{\eta}^i)^\sigma (1-\sigma) (\bar{Q})^{-\sigma} \right) (Q_t - \bar{Q}) \]

In Steady State:

\[ \bar{p}^h = \left( \chi^{-\sigma} - \left( \frac{1-\chi}{\chi} \right)^\sigma (\bar{\eta}^i)^\sigma (\bar{Q})^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \]
\[ (\bar{p}^h)^{1-\sigma} = \left( \chi^{-\sigma} - \left( \frac{1-\chi}{\chi} \right)^\sigma (\bar{\eta}^i)^\sigma (\bar{Q})^{1-\sigma} \right) \]
\[ \ln(\bar{p}^h) = \frac{1}{1-\sigma} \ln \left( \chi^{-\sigma} - \left( \frac{1-\chi}{\chi} \right)^\sigma (\bar{\eta}^i)^\sigma (\bar{Q})^{1-\sigma} \right) \]

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\[
\tilde{p}_t^h = \frac{1}{1 - \sigma} \left( \frac{1}{\tilde{p}_t^h} \right)^{1-\sigma} \left( -\left( \frac{1 - \chi}{\chi} \right)^\sigma \left( \tilde{Q} \right)^{1-\sigma} \right) \tilde{n}_t^f + \\
\frac{1}{1 - \sigma} \left( \frac{1}{\tilde{p}_t^h} \right)^{1-\sigma} \left( -\left( \frac{1 - \chi}{\chi} \right)^\sigma (1 - \sigma) \left( \tilde{Q} \right)^{1-\sigma} \right) \tilde{Q}_t
\]

\[
\tilde{p}_t^h = \frac{1}{1 - \sigma} \left( \frac{1}{\tilde{p}_t^h} \right)^{1-\sigma} \left( -\left( \frac{1 - \chi}{\chi} \right)^\sigma \left( \tilde{Q} \right)^{1-\sigma} \right) \tilde{n}_t^f + \\
\frac{1}{1 - \sigma} \left( \frac{1}{\tilde{p}_t^h} \right)^{1-\sigma} \left( -\left( \frac{1 - \chi}{\chi} \right)^\sigma (1 - \sigma) \left( \tilde{Q} \right)^{1-\sigma} \right) \tilde{Q}_t
\]

\[
= -\frac{1}{1 - \sigma} \left( \frac{1}{\tilde{p}_t^h} \right)^{1-\sigma} \left( \frac{1 - \chi}{\chi} \right)^\sigma \left( \tilde{Q} \right)^{1-\sigma} \left( \tilde{n}_t^f + (1 - \sigma) \tilde{Q}_t \right)
\]

\[
= -\frac{1}{1 - \sigma} \left( \frac{1}{\tilde{p}_t^h} \right)^{1-\sigma} \left( \frac{1 - \chi}{\chi} \right)^\sigma \left( \tilde{Q} \right)^{1-\sigma} (1 - \sigma) \left( \frac{\sigma}{(1 - \sigma)} \tilde{n}_t^f + \tilde{Q}_t \right)
\]

\[
\approx -\left( \frac{1 - \chi}{\chi} \right)^\sigma \left( \tilde{n}_t^f + \tilde{Q}_t \right)
\]

(3.22):

\[
n_t = l_{gt} + l_{bt}
\]

\[
\ln n_t = \ln (l_{gt} + l_{bt})
\]

\[
\ln \tilde{n} + \frac{1}{\bar{n}} (n_t - \bar{n}) = \ln \left( \tilde{l}_g + \tilde{l}_b \right) + \frac{1}{l_g + l_b} \left( l_{gt} - \tilde{l}_g \right) + \frac{1}{l_g + l_b} \left( l_{bt} - \tilde{l}_b \right)
\]

\[
\tilde{n}_t = \frac{l_g}{l_g + l_b} \tilde{l}_{gt} + \frac{l_b}{l_g + l_b} \tilde{l}_{bt}
\]

\[
\tilde{n}_t = \frac{l_g}{\bar{n}} \tilde{l}_{gt} + \frac{l_b}{\bar{n}} \tilde{l}_{bt}
\]

(3.32):

\[
\beta (1 + r_{t+1}^d) = \frac{\tilde{n}_t^{C_t - \rho_1}}{E_t \left[ \tilde{n}_{t+1}^{C_{t+1} - \rho_1} \right]}
\]

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\ln \left( \beta (1 + r_{t+1}^d) \right) = \ln \left( \frac{c_t^{\rho_1}}{E_t (c_{t+1}^{\rho_1})} \right)

\ln \beta + \ln (1 + r_{t+1}^d) = \ln \eta_t^c - \rho_1 \ln c_t - \ln E_t \left[ \eta_{t+1}^c \right] + \rho_1 \ln E_t [c_{t+1}]

\ln \beta + \ln \bar{r}^d + \frac{\bar{r}^d}{1 + \bar{r}^d} (r_{t+1}^d - \bar{r}^d) \approx \ln \left( \frac{\bar{c}_t^{\rho_1}}{\bar{c}_{t+1}^{\rho_1}} \right) + \frac{1}{\bar{c}_t^c} (\eta_t^c - \bar{\eta}) - \frac{1}{\bar{\eta}} (E_t \eta_{t+1}^c - \bar{\eta}) - 

- \rho_1 \frac{1}{\bar{c}_t^c} (c_t - \bar{c}) + \rho_1 \frac{1}{\bar{c}_t^c} (E_t c_{t+1} - \bar{c})

\frac{\bar{r}^d}{1 + \bar{r}^d} \bar{r}^d_{t+1} \approx \eta_t^c - E_t \eta_{t+1}^c - \rho_1 (c_t - E_t c_{t+1})

\frac{\bar{r}^d}{1 + \bar{r}^d} \bar{r}^d_{t+1} \approx - \rho_1 (c_t - E_t c_{t+1}) + \tilde{\eta}_t^c

(3.31):

(1 - \omega) \eta_t^n (1 - n_t)^{-\rho_2} = \omega \eta_t^c c_t^{\rho_1} p_t^h w_t

\ln \left( (1 - \omega) \eta_t^n (1 - n_t)^{-\rho_2} \right) = \ln \left( \omega \eta_t^c c_t^{\rho_1} p_t^h w_t \right)

\ln (1 - \omega) + \ln \eta_t^n - \rho_2 \ln (1 - n_t) = \ln \omega + \ln \eta_t^c - \rho_1 \ln c_t + \ln p_t^h + \ln w_t

\ln \left( (1 - \omega) \tilde{\eta}_t^n (1 - \tilde{n}_t)^{-\rho_2} \right) + \frac{1}{\tilde{\eta}_t^n} (\eta_t^n - \tilde{n}_t) - \rho_2 \frac{1}{1 - \tilde{n}} (-1) (n_t - \tilde{n}) = \ln \left( \omega \eta_t^c \bar{c}_{t+1}^{\rho_1} \bar{p}_t^h \bar{w} \right) + 

+ \frac{1}{\eta_t^c} (\eta_t^c - \tilde{\eta}_t^c) - \rho_1 \frac{1}{\bar{c}_t} (c_t - \bar{c}) + \frac{1}{\bar{p}_t^h} (p_t^h - \bar{p}_t^h) + \frac{1}{\bar{w}} (w_t - \bar{w})

\tilde{\eta}_t^n + \rho_2 \frac{\bar{n}}{1 - \bar{n}} \tilde{n}_t = \eta_t^c - \rho_1 \bar{c}_t + \bar{p}_t^h + \bar{w}_t

\rho_2 \frac{\bar{n}}{1 - \bar{n}} \tilde{n}_t \approx - \eta_t^n - \rho_1 \tilde{c}_t + \tilde{p}_t^h + \tilde{w}_t

(3.33)

\ln \left( 1 + r_{t+1}^d \right) = \ln \left( 1 + r_{t+1}^f \right) \frac{E_t Q_{t+1}}{Q_t}

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\[ \ln(1 + r_{t+1}^d) = \ln \left( (1 + r_{t+1}^f) \frac{E_t Q_{t+1}}{Q_t} \right) \]
\[ \ln(1 + r_{t+1}^d) = \ln(1 + r_{t+1}^f) + \ln \frac{E_t Q_{t+1}}{Q_t} \]
\[ \ln \bar{r}^d + \frac{1}{1 + \bar{r}^d} (r_{t+1}^d - \bar{r}^d) \approx \ln \bar{r}^f + \frac{1}{1 + \bar{r}^f} (r_{t+1}^f - \bar{r}^f) + \ln \bar{Q} + \frac{1}{\bar{Q}} (E_t Q_{t+1} - \bar{Q}) \]
\[ - \ln Q - \frac{1}{Q} (Q_t - \bar{Q}) \]
\[ \frac{\bar{r}^d}{1 + \bar{r}^d} \bar{r}^d_{t+1} \approx \frac{\bar{r}^f}{1 + \bar{r}^f} \bar{r}^f_{t+1} + E_t \bar{Q}_{t+1} - \bar{Q}_t \]

(3.34)

\[ y_t = A_g z_g t^\alpha k_t^{1-\alpha} \]

\[ \ln y_t = \ln (A_g z_g t^\alpha k_t^{1-\alpha}) \]

\[ \ln \bar{y} + \frac{1}{\bar{y}} (y_t - \bar{y}) = \ln (A_g z_g t^\alpha k_t^{1-\alpha}) + \frac{1}{z_g} (z_{gt} - \bar{z}_g) + \alpha \frac{1}{t_g} (l_{gt} - \bar{l}_g) + (1 - \alpha) \frac{1}{k} (k_t - \bar{k}) \]

\[ \bar{y}_t = \tilde{z}_{gt} + \alpha \bar{l}_{gt} + (1 - \alpha) \tilde{k}_t \]

(3.35)

\[ k_{t+1} = i_{kt} + (1 - \delta) k_t \]

\[ \ln k_{t+1} = \ln (i_{kt} + (1 - \delta) k_t) \]

\[ \ln \bar{k} + \frac{1}{\bar{k}} (k_{t+1} - \bar{k}) = \ln (\bar{i}_k + (1 - \delta) \bar{k}) + \frac{1}{\bar{k}} (i_{kt} - \bar{i}_k) + \frac{1}{\bar{k}} (1 - \delta) (k_t - \bar{k}) \]

\[ \tilde{k}_{t+1} = \frac{\bar{v}_k}{\bar{k}} i_{kt} + (1 - \delta) \tilde{k}_t \]

(3.37)

\[ k_t = q_t^d \]

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\[ \ln \tilde{k} + \frac{1}{k} (k_t - \tilde{k}) = \ln \tilde{q}^d + \frac{1}{q^d} (q_t^d - \tilde{q}^d) \]

\[ \tilde{k}_t = \tilde{q}_t^d \]

(3.40):

\[ \alpha A_g z_{gt} \left( \frac{l_{gt}}{k_t} \right)^{\alpha - 1} = \eta^q_{it} w_t \]

\[ \ln \left( \alpha A_g z_{gt} \left( \frac{l_{gt}}{k_t} \right)^{\alpha - 1} \right) = \ln \eta^q_{it} + \ln w_t \]

\[ \frac{1}{\tilde{\eta}^\varphi} (\eta^q_{it} - \bar{\eta}^\varphi) + \frac{1}{w} (w_t - \bar{w}) = \ln \left( \alpha A_g \tilde{z}_g \left( \frac{l_{gt}}{k_t} \right)^{\alpha - 1} \right) + \]

\[ \frac{1}{\tilde{z}_g} (z_{gt} - \bar{z}_g) + (\alpha - 1) \left( \frac{1}{l_g} (l_{gt} - \bar{l}_g) - \frac{1}{k} (k_t - \tilde{k}) \right) \]

\[ \tilde{z}_{gt} + (\alpha - 1) \left( \tilde{l}_{gt} - \tilde{k}_t \right) = \tilde{\eta}^q_{it} + \tilde{w}_t \]

\[ \tilde{y}_t - \tilde{l}_{gt} = \tilde{\eta}^q_{it} + \tilde{w}_t \]

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(3.41):

\[(1 - \alpha) A_g z_{gt} \left( \frac{l_{gt}}{k_t} \right)^\alpha = \left( r_t^q + \delta + \eta_t^k \right)\]
\[(1 - \alpha) A_g z_{gt} r_o \left( k_t^{-\alpha} \right) = \left( r_t^q + \delta + \eta_t^k \right)\]
\[(1 - \alpha) A_g z_{gt} r_o \left( k_t^{-\alpha} \frac{k_t}{k_t} \right) = \left( r_t^q + \delta + \eta_t^k \right)\]
\[(1 - \alpha) A_g z_{gt} r_o \left( 1 - \alpha \right) \frac{1}{k_t} = \left( r_t^q + \delta + \eta_t^k \right)\]
\[(1 - \alpha) \frac{y_t}{k_t} = \left( r_t^q + \delta + \eta_t^k \right)\]
\[\ln \left((1 - \alpha) \frac{y_t}{k_t}\right) = \ln \left( r_t^q + \delta + \eta_t^k \right)\]
\[\ln \left((1 - \alpha) \frac{y_t}{k_t}\right) + \frac{1}{\delta} \left( y_t - \bar{y} \right) - \frac{1}{k_t} (k_t - \bar{k}) = \]
\[= \ln \left( r_t^q + \delta + \eta_t^k \right) + \frac{1}{\delta} \left( r_t^q - \bar{r}^q \right) + \frac{1}{\delta + \bar{r}^q} \left( \eta_t^k - \bar{\eta}^k \right)\]
\[\hat{y}_t - \hat{k}_t = \frac{1}{\delta + \bar{r}^q} \left( \hat{r}_t^q + \hat{\eta}_t^k \right)\]

(3.48)

\[A_b z_{bt} l_{bt}^{\gamma} q_{l_{t+1}}^{1-\gamma} = q_{t+1}\]
\[A_b z_{bt} l_{bt}^{\gamma} q_{l_{t+1}}^{1-\gamma} = 1\]
\[\ln A_b z_{bt} l_{bt}^{\gamma} q_{l_{t+1}}^{1-\gamma} = \ln 1\]
\[\ln A_b z_{bt} l_{bt}^{\gamma} q_{l_{t+1}}^{1-\gamma} + \frac{1}{z_b} (z_{bt} - \bar{z}_b) + \gamma \frac{1}{l_b} (l_{bt} - \bar{l}_b) - \gamma \frac{1}{q} (q_{t+1} - \bar{q}) = 0\]
\[\hat{z}_{bt} - \gamma \hat{q}_{t+1} + \gamma \hat{l}_{bt} = 0\]

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$$q_{t+1} = d_{t+1}$$

$$\ln q_{t+1} = \ln d_{t+1}$$

$$\ln \tilde{q} + \frac{1}{\tilde{q}} (q_{t+1} - \tilde{q}) = \ln \tilde{d} + \frac{1}{\tilde{d}} (d_{t+1} - \tilde{d})$$

$$\tilde{q}_{t+1} = \tilde{d}_{t+1}$$

$$l_{bt} = \left( \frac{1}{A_b z_{bt}} \right)^{\frac{1}{\gamma}}$$

$$\ln \frac{l_{bt}}{q_{t+1}} = \ln \left( \frac{1}{A_b z_{bt}} \right)^{\frac{1}{\gamma}}$$

$$\ln \frac{l_{bt}}{q_{t+1}} + \frac{1}{l_{bt}} (l_{bt} - \tilde{l}_{bt}) - \frac{1}{\tilde{q}} (q_{t+1} - \tilde{q}) = \ln \left( \frac{1}{A_b \tilde{z}_b} \right)^{\frac{1}{\gamma}} - \frac{1}{\gamma} z_{bt} z_{bt}$$

$$\tilde{l}_{bt} - \tilde{q}_{t+1} = -\frac{1}{\gamma} \tilde{z}_{bt}$$

$$\left( r_t^q - r_t^d \right) = p_t^h \eta_t^b w_t \frac{l_{bt}}{q_t}$$

$$\ln \left( r_t^q - r_t^d \right) = \ln \left( p_t^h \eta_t^b w_t \frac{l_{bt}}{q_t} \right)$$

$$\ln \left( r_t^q - r_t^d \right) + \frac{1}{r_t^q - r_t^d} (r_t^q - r_t^d) - \frac{1}{r_t^d - r_t^d} (r_t^d - r_t^d) = \ln \left( \tilde{p}_t^h \tilde{\eta}_t^b \tilde{w}_t \frac{\tilde{l}_{bt}}{\tilde{q}_t} \right) + \frac{1}{\tilde{p}_t^h} (\eta_t^b - \tilde{\eta}_t^b) + \frac{1}{\tilde{w}_t} (w_t - \tilde{w}_t) + \frac{1}{\tilde{l}_{bt}} (l_{bt} - \tilde{l}_{bt}) - \frac{1}{\tilde{q}_t} (q_t - \tilde{q}_t)$$

$$\frac{1}{r_t^q - r_t^d} \left( r_t^q r_t^q - r_t^d r_t^d \right) = \tilde{p}_t^h + \tilde{\eta}_t^b + \tilde{w}_t + \tilde{l}_{bt} - \tilde{\eta}_t$$

$$\frac{1}{r_t^q - r_t^d} \left( r_t^q r_t^q - r_t^d r_t^d \right) = \tilde{p}_t^h + \tilde{\eta}_t^b + \tilde{w}_t + \frac{1}{\gamma} \tilde{z}_{bt}$$
\[
sp_t = r_t^q - r_t^d
\]

\[
\ln sp_t = \ln (r_t^q - r_t^d)
\]

\[
\ln \bar{sp} + \frac{1}{\bar{sp}} (sp_t - \bar{sp}) = \ln \left( \bar{r}^q - \bar{r}^d \right) + \frac{1}{\bar{r}^q - \bar{r}^d} \left( r_t^q - \bar{r}^q \right) - \frac{1}{\bar{r}^q - \bar{r}^d} \left( r_t^d - \bar{r}^d \right)
\]

\[
\hat{sp}_t = \frac{1}{\bar{r}^q - \bar{r}^d} (\bar{r}^q r_t^q - \bar{r}^d r_t^d)
\]

\[
d_{t+1}^f = \left( 1 + r_t^f \right) d_t^f - \left( \frac{ex_t}{Q_t} - im_t \right)
\]

\[
\ln d_{t+1}^f = \ln \left( \left( 1 + r_t^f \right) d_t^f - \left( \frac{ex_t}{Q_t} - im_t \right) \right)
\]

\[
\ln \bar{d}^f + \frac{1}{\bar{d}^f} \left( d_{t+1}^f - \bar{d}^f \right) = \ln \left( \left( 1 + \bar{r}^f \right) \bar{d}^f - \left( \frac{ex}{\bar{Q}} - i\bar{m} \right) \right) + \frac{1}{\bar{d}^f} \bar{d}^f \left( r_t^f - \bar{r}^f \right) + \frac{1}{\bar{d}^f} (1 + \bar{r}^f) \left( d_t^f - \bar{d}^f \right) + \frac{1}{\bar{d}^f} \left( -\frac{1}{\bar{Q}} \right) (ex_t - \bar{ex}) + \frac{1}{\bar{d}^f} (\bar{ex} - ex) (Q_t - \bar{Q}) + \frac{1}{\bar{d}^f} (im_t - \bar{im})
\]

\[
\bar{d}_{t+1}^f = \bar{r}^f \bar{r}_t^f + (1 + \bar{r}^f) \bar{d}_t^f + \frac{1}{\bar{d}^f} \left( -\frac{ex}{\bar{Q}} \bar{ex}_t - \frac{ex}{\bar{Q}} \bar{Q}_t + i\bar{m} \bar{im}_t \right)
\]

\[
r_t^f = \bar{r}^f + \varphi \left( e^{\left( d_t^f - \bar{d}^f \right)} - 1 \right)
\]

\[
\ln r_t^f = \ln \left( \bar{r}^f + \varphi \left( e^{\left( d_t^f - \bar{d}^f \right)} - 1 \right) \right)
\]

\[
\ln \bar{r}^f + \frac{1}{\bar{r}^f} \left( r_t^f - \bar{r}^f \right) = \ln \left( \bar{r}^f + \varphi \left( e^{\left( d_t^f - \bar{d}^f \right)} - 1 \right) \right) + \frac{1}{\bar{r}^f} (\varphi) e^{\left( \bar{d}^f - \bar{a}^f \right)} \left( d_t^f - \bar{d}^f \right)
\]

\[
\bar{r}_t^f = \frac{\varphi \bar{d}^f}{\bar{r}^f} d_t^f
\]

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\[ y_t = c_t + i_{kt} + g_t + e x_t - im_t \]

\[ \ln y_t = \ln (c_t + i_{kt} + g_t + e x_t - im_t) \]

\[ \ln \bar{y} + \frac{1}{\bar{y}} (y_t - \bar{y}) = \ln \left( \bar{c} + \bar{i}_k + \bar{g} + \bar{e} \bar{x} - \bar{im} \right) + \frac{1}{\bar{y}} (c_t - \bar{c}) + \frac{1}{\bar{y}} (i_{kt} - \bar{i}_k) + \frac{1}{\bar{y}} (g_t - \bar{g}) + \frac{1}{\bar{y}} (ex_t - \bar{e} \bar{x}) - \frac{1}{\bar{y}} (im_t - \bar{im}) \]

\[ \bar{y} \bar{y}_t = \bar{c} \bar{c}_t + \bar{i}_k \bar{i}_{kt} + \bar{g} \bar{g}_t + \bar{e} \bar{e} \bar{x}_t - \bar{im} \bar{im}_t \]

\[ \bar{y} \bar{y}_t = \bar{c} \bar{c}_t + \bar{k} \left( \bar{k}_{t+1} - (1 - \delta) \bar{k}_t \right) + \bar{g} \bar{g}_t + \bar{e} \bar{e} \bar{x}_t - \bar{im} \bar{im}_t \]

\[ q_{t+1}^d = \frac{q_{t+1}}{p_t^h} \]

\[ \ln q_{t+1}^d = \ln \frac{q_{t+1}}{p_t^h} \]

\[ \ln \bar{q} + \frac{1}{\bar{q}} (q_{t+1}^d - \bar{q}^d) = \ln \frac{\bar{q}}{\bar{p}^h} + \frac{1}{\bar{q}} (q_{t+1} - \bar{q}) - \frac{1}{\bar{p}^h} (p_{t+1}^h - \bar{p}^h) \]

\[ \tilde{q}_t^d = \tilde{q}_t - \tilde{p}_t^h \]

\[ im_t = (1 - \chi)^\sigma \left( \eta_t^f \right)^\sigma (Q_t)^{-\sigma} c_t \]

\[ \ln (im_t) = \ln \left( (1 - \chi)^\sigma \left( \eta_t^f \right)^\sigma (Q_t)^{-\sigma} c_t \right) \]

\[ \ln (im_t) = \ln (1 - \chi)^\sigma + \sigma \ln \left( \eta_t^f \right) - \sigma \ln (Q_t) + \ln c_t \]

\[ \ln(im) + \frac{1}{im} (im_t - im) = \ln (1 - \chi)^\sigma + \ln \left( \eta_t^f \right)^\sigma + \ln (Q)^{-\sigma} + \ln \bar{e} + \sigma \frac{1}{\eta_t^f} \left( \eta_t^f - \bar{\eta}^f \right) - \sigma \frac{1}{Q} (Q_t - Q) + \frac{1}{\bar{e}} (c_t - \bar{c}) \]

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\[ \tilde{m}_t = \sigma \tilde{n}_t^F - \sigma \tilde{Q}_t + \tilde{c}_t \]

(3.62)

\[ ex_t = (1 - \chi^F)^{\sigma^F} (\eta_t^F)^{\sigma^F} (Q_t)^{\sigma^F} c_t^F \]

\[
\ln (ex_t) = \ln \left((1 - \chi^F)^{\sigma^F} (\eta_t^F)^{\sigma^F} (Q_t)^{-\sigma^F} c_t^F\right) \\
\ln (ex_t) = \ln \left((1 - \chi^F)^{\sigma^F} + \sigma^F \ln (\eta_t^F) - \sigma^F \ln (Q_t) + \ln c_t \right) \\
\ln(ex) + \frac{1}{ex} (ex_t - ex) = \ln \left((1 - \chi^F)^{\sigma^F} + \ln (\eta_t^F)^{\sigma^F} + \ln (Q)^{-\sigma^F} + \ln \bar{c} + \right) \\
\sigma^F \frac{1}{\eta_t^F} (\eta_t^F - \bar{\eta}^{F}) - \sigma^F \frac{1}{Q} (Q_t - \bar{Q}) + \frac{1}{\bar{c}} (c_t^F - \bar{c}^F) \\
\tilde{e}x_t = \sigma^F \tilde{n}_t^F - \sigma^F \tilde{Q}_t + \tilde{c}_t^F \]

For any equation of the form \( x_t^i = (x_{t-1}^i)^{\alpha^i} + \varepsilon_t^i \), the steps to log-linearise it are as follows:

\[
x_t^i = (x_{t-1}^i)^{\alpha^i} + \varepsilon_t^i \\
\ln x_t^i = \ln \left((x_{t-1}^i)^{\alpha^i} + \varepsilon_t^i\right) \\
\ln \bar{x} + \frac{1}{\bar{x}} (x_t - \bar{x}) = \ln \left((\bar{x})^{\alpha^i} + 0\right) + \rho^i \frac{1}{\bar{x} + 0} (x_{t-1}^i - \bar{x}) + \frac{1}{\bar{x} + 0} (\varepsilon_t^i - 0) \\
\tilde{x}_t^i = \rho^i \tilde{x}_{t-1}^i + \tilde{\varepsilon}_t^i \\
\]

Therefore,

\[ c_t^F = (c_{t-1}^F)^{\rho^F} + \varepsilon_t^{cF} \]
\[ \tilde{c}_t^F = \rho^{cF} \tilde{c}_{t-1}^F + \tilde{\varepsilon}_t^{cF} \]
\[ g_t = (g_{t-1})^{\rho_G} + \varepsilon_t^G \]
\[ \tilde{g}_t = \rho^G \tilde{g}_{t-1} + \tilde{\varepsilon}_t^G \]
\[ z_{gt} = (z_{gt-1})^{\rho^{zg}} + \varepsilon_t^{zg} \]
\[ \tilde{z}_{gt} = \rho^{zg} \tilde{z}_{gt-1} + \tilde{\varepsilon}_t^{zg} \]
\[ z_{bt} = (z_{bt-1})^{\rho^{zb}} + \varepsilon_t^{zb} \]
\[ \tilde{z}_{bt} = \rho^{zb} \tilde{z}_{bt-1} + \tilde{\varepsilon}_t^{zb} \]
\[ \eta^f_t = \left( \eta^f_{t-1} \right)^{\rho^f} + \varepsilon_t^f \]
\[ \tilde{\eta}^f_t = \rho^f \tilde{\eta}^f_{t-1} + \tilde{\varepsilon}_t^f \]
\[ \eta^c_t = \left( \eta^c_{t-1} \right)^{\rho^c} + \varepsilon_t^c \]
\[ \tilde{\eta}^c_t = \rho^c \tilde{\eta}^c_{t-1} + \tilde{\varepsilon}_t^c \]
\[ \eta^n_t = \left( \eta^n_{t-1} \right)^{\rho^n} + \varepsilon_t^n \]
\[ \tilde{\eta}^n_t = \rho^n \tilde{\eta}^n_{t-1} + \tilde{\varepsilon}_t^n \]
\[ \eta^q_t = \left( \eta^q_{t-1} \right)^{\rho^q} + \varepsilon_t^q \]
\[ \tilde{\eta}^q_t = \rho^q \tilde{\eta}^q_{t-1} + \tilde{\varepsilon}_t^q \]
\[ \eta^k_t = \left( \eta^k_{t-1} \right)^{\rho^k} + \varepsilon_t^k \]
\[ \tilde{\eta}^k_t = \rho^k \tilde{\eta}^k_{t-1} + \tilde{\varepsilon}_t^k \]
\[ \eta^b_t = (\eta^b_{t-1})^{\rho^b} + \varepsilon^b_t \]

\[ \tilde{\eta}^b_t = \rho^b \eta^b_{t-1} + \varepsilon^b_t \]

\[ \eta^F_t = (\eta^F_{t-1})^{\rho^F} + \varepsilon^F_t \]

\[ \tilde{\eta}^F_t = \rho^F \eta^F_{t-1} + \varepsilon^F_t \]
B. Data

The data set used for the analysis in this thesis is UK time series quarterly data for the period 1978 Q3 : 2013 Q3. The list of the original series can be found in table 6.1. The data providers, series codes, units of measurement and base periods, where appropriate, are also given. These series are then transformed in a manner to conform with the variables that are used in the model specification. The proposed model in this thesis is a macroeconomic model with microfoundations and explores only the real side of a small open economy. That is why the actual data had to be transformed in real per capita terms. What follows is a description of the data gathering process and all transformation made to the actual time series prior to any testing and estimation.

Macroeconomic data is published at aggregate level. To be compatible with the model’s structure and assumptions all variables (excluding relative prices, interest rates and exchange rates) have to be transformed in per capita terms. ONS provides various measures for population, labour force and employment covering various age ranges. As the model is subjected to econometric analysis it requires a large data set. This causes a limitation to the number of series that can be used in the transformation. Several series were considered but the most appropriate one is MGSL. This series was used to transform the data set in per capita terms.

Most of the variables are in real terms so no adjustment for the price level
was necessary. However, the series for the interest rates were in nominal terms. The CPI Index from OECD was used to calculate the inflation rate. The reason why this one was used instead of the measure provided by ONS was to ensure consistency with between how the consumer price index and the real effective exchange rate are calculated. Both measures use a consumption basket with specific weights that were not available to the general public. But since both measures were obtained from the OECD, the measure should be consistent. The series used for the real effective exchange rate is relative consumer price index. The inverse of that measure was used since OECD calculates it as the home price level relative to a basket of goods from the rest of the world adjusted by the nominal exchange rate.
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<tr>
<td>JWS7</td>
<td>ONS</td>
<td>UK workforce Jobs: Financial and Insurance</td>
<td>000s</td>
<td>SA</td>
<td>-</td>
<td>-</td>
<td>Q</td>
</tr>
<tr>
<td>DYDC</td>
<td>ONS</td>
<td>UK workforce jobs: Total</td>
<td>000s</td>
<td>SA</td>
<td>-</td>
<td>-</td>
<td>Q</td>
</tr>
<tr>
<td>HBQB</td>
<td>ONS</td>
<td>UK International Inv. Position - Liabs</td>
<td>£m</td>
<td>SA</td>
<td>CUR</td>
<td>-</td>
<td>Q</td>
</tr>
<tr>
<td>YBGB</td>
<td>ONS</td>
<td>GDP deflator</td>
<td>£m</td>
<td>SA</td>
<td>DEFL</td>
<td>2010</td>
<td>Q</td>
</tr>
<tr>
<td>IUQAAJNB</td>
<td>BOE</td>
<td>Avg discount rate Treasury Bills</td>
<td>%</td>
<td>-</td>
<td>CUR</td>
<td>-</td>
<td>Q</td>
</tr>
<tr>
<td>IUQAVCD</td>
<td>BOE</td>
<td>Sterling 3m interbank lending rate</td>
<td>%</td>
<td>-</td>
<td>CUR</td>
<td>Q</td>
<td></td>
</tr>
</tbody>
</table>

Table 6.1: Data Series Description I
<table>
<thead>
<tr>
<th>Data Series</th>
<th>Source</th>
<th>Description</th>
<th>Unit</th>
<th>Seasonal Adj.</th>
<th>Price</th>
<th>Base</th>
<th>Period</th>
</tr>
</thead>
<tbody>
<tr>
<td>IUQAVCD</td>
<td>BOE</td>
<td>Sterling 3m interbank lending rate</td>
<td>%</td>
<td>-</td>
<td>CUR</td>
<td>Q</td>
<td></td>
</tr>
<tr>
<td>HMBO</td>
<td>ONS</td>
<td>Total investment income from inv. in UK</td>
<td>£m</td>
<td>SA</td>
<td>CON</td>
<td>2010</td>
<td>Q</td>
</tr>
<tr>
<td>-</td>
<td>OECD</td>
<td>World exports in G&amp;s</td>
<td>£b</td>
<td>SA</td>
<td>CON</td>
<td>2005</td>
<td>Q</td>
</tr>
<tr>
<td>SP.POP.TOTL</td>
<td>WB</td>
<td>World population</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>A</td>
</tr>
<tr>
<td></td>
<td>OECD</td>
<td>CPI All items index</td>
<td>INX</td>
<td>-</td>
<td>-</td>
<td>2010</td>
<td>Q</td>
</tr>
<tr>
<td></td>
<td>OECD</td>
<td>Relative CPI</td>
<td>INX</td>
<td>2010</td>
<td>Q</td>
<td></td>
<td></td>
</tr>
<tr>
<td>P36K</td>
<td>ONS</td>
<td>Workplace based compensation: Fin and ins.</td>
<td>£m</td>
<td>-</td>
<td>CUR</td>
<td>A</td>
<td></td>
</tr>
<tr>
<td>HAEEA</td>
<td>ONS</td>
<td>Workplace based compensation: Fin and ins.</td>
<td>£m</td>
<td>-</td>
<td>CUR</td>
<td>A</td>
<td></td>
</tr>
<tr>
<td>TMPV</td>
<td>ONS</td>
<td>Total GVA</td>
<td>£m</td>
<td>-</td>
<td>CUR</td>
<td>A</td>
<td></td>
</tr>
<tr>
<td>LPMB6VG</td>
<td>BOE</td>
<td>Monthly amounts outstanding</td>
<td>£m</td>
<td>-</td>
<td>CUR</td>
<td>M</td>
<td></td>
</tr>
<tr>
<td>MS2L</td>
<td>ONS</td>
<td>Gross capital stock to output ratio CVM</td>
<td>£m</td>
<td>NSA</td>
<td>CON</td>
<td>A</td>
<td></td>
</tr>
</tbody>
</table>

Table 6.2: Data Series Description II
The series that were used as proxies for the loan rate and the deposit rate were obtained from the Bank of England. The collected values were adjusted by the inflation rate calculated from the CPI data. The data for the interest rate on foreign debt was calculated using the data of net interest income from investment in the UK by the rest of the world divided by the total investment in the UK by foreigners.

The consumption per capita was calculated by dividing total household consumption by the total population. Since it is a CVM measure no adjustment was made by the price level. The variable for world consumption was extracted as follows. Exactly the same approach was done for investment output, imports exports and debt. The investment series is calculated by adding the values of changes in inventories including alignment adjustment and total gross fixed capital formation. These values are then adjusted by the population.

The capital data was calculated using investment and output data. From the capital/output ratio the average annual ratio was calculated. This was equal to 4.43. Since capital is a stock measure and output is a flow measure the value was multiplied by four to obtain the average quarterly capital output ratio. This quarterly ratio was used to calculate the first period of capital by multiplying 17.2 by the data for output. The rest of the series was constructed by iterating one period at a time using the investment and output data and the capital accumulation rate.
The employment rate was calculated by dividing the population in employment by the total population. This series was then split using a variable ratio that determines what fraction of the labour force is employed in the financial industry. By dividing the workforce jobs in financial and insurance activities by the total number of workforce jobs, a percentage is obtained. If this is multiplied by the employment rate, it would generate the fraction of people that are employed in the financial industry. Using these values and subtracting them from the total employment rate would generate the fraction of employment outside the financial industry.

The data for exports in goods and services was used as a proxy for world demand. The data was rebased to reflect the 2010 base period. Then the variable was divided by the extrapolated world population data in order to represent demand per capita. World population was available as annual data only. First, the difference in population between two consecutive years was found. Second, the value was divided by 4, assuming that there was equal growth in world population every quarter for that period. Then the average value per quarter was subtracted by the value of the next year to obtain the value for the last quarter of this year. Then this value was used to calculate the value for Q3, and so on. Once the approximated series was obtained, the value for world demand per capita could be calculated.
Figure 6.1: Residual From Export Equation

C. Residuals from Stationary Data
Figure 6.2: Residual From Import Equation
Figure 6.3: Residual from MPK Equation
Figure 6.4: Residual from Spread Equation
Figure 6.5: Residual from MPL equation
Figure 6.6: Residual from Euler Equation
Figure 6.7: Residual from MRS Equation
Figure 6.8: IRF (initial calibration): Consumption Preference Shock (fig.1)

D. Impulse Response Functions - Initial Calibration
Figure 6.9: IRF (initial calibration): Consumption Preference Shock (fig.2)
Figure 6.10: IRF (initial calibration): Ommited Labour Tax in FS (fig.1)
Figure 6.11: IRF(initial calibration): Ommited Labour Tax FS (fig.2)
Figure 6.12: IRF (initial calibration): Leisure Preference Shock (fig.1)
Figure 6.13: IRF (initial calibration): Leisure Preference Shock (fig.2)
Figure 6.14: IRF (initial calibration): Ommited Labour Tax Shock GS (fig.1)
Figure 6.15: IRF (initial calibration): Ommited Labour Tax in GS (fig2)
Figure 6.16: IRF (initial calibration): Ommited Capital Tax in GS (fig.1)
Figure 6.17: IRF(initial calibration): Ommited Capital Tax GS (fig.2)
E. Programs

MATLAB Version: 8.1.0.604 (R2013a)

DYNARE Version: 4.3.2

Spatial Econometrics Toolbox
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