AUTOMATED SPACE LAYOUT PLANNING FOR ENVIRONMENTAL SUSTAINABILITY

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Abstract

There is a growing global interest in low/zero carbon buildings in response to the increased CO_2 in the atmosphere, nearly half of which comes from building energy consumption. Buildings are built for a considerably longer lifespan and enhancing energy efficiency in buildings can play a significant role in reducing CO_2 emissions. Energy efficiency features need to be incorporated at the earliest, as alterations to the design at latter stages may prove to be difficult and sometimes expensive. Building design is concerned with satisfying various objectives (e.g. cost, efficiency of a space layout, energy consumption), which are sometimes in conflict with each other. Performance of various indicators, therefore, needs to be assessed as a whole rather than in isolation.

Space layout planning is considered as the starting point of building design. Most performance indicators; i.e. cost, energy efficiency, etc. are closely linked with the layout. Researchers have attempted at automating space layout planning since the 1960s with a view to effectively search the solution space. Diverse approaches are adopted in space layout planning that ranges from the analysis of spatial proximity to the application of 'space syntax' theory. Developments in whole building energy simulation and integration of simulation in the design process imply that the search for optimum space layout could be better guided by incorporating detailed-based simulation as response generators as opposed to the ones with a simplified representation of the problem domain.

This paper describes a framework for sustainable space layout planning that uses evolutionary computation methods to search the solution space. Whole building simulation programs are used as response generators to guide the search for energy efficient layouts. The integrated approach enables the consideration of energy consumption, in addition to the geometry and topology, for decision making during space layout planning.

1. Introduction

Rapid increases of global average surface temperature and most recently, the increase in extreme weather events have been attributed to the increased concentration of atmospheric CO_2, believed to be a result of the humanity's consumption of fossil fuel for energy (IPCC 2007). Worldwide, the focus has been on the reduction of CO_2 emissions to mitigate the impacts of climate change. In the UK, the government aims to reduce CO_2 emissions by at least 26% by 2020 and at least 60% by 2050, compared to 1990 levels (OPSI 2007). The 60% target is, however, set to increase to 80% (BBC 2008). Buildings use about 46% of UK's energy and initial findings suggest further increases in the future (Levermore et al. 2004). Therefore, the reduction of CO_2 emissions from buildings is vital in meeting the national and/or global emissions reduction target. Early stages of a building lifecycle are found to be the most influential in determining its energy/carbon footprint. Moreover, the errors made at these stages can hardly be rectified later (Miles et al. 2001) without substantial financial involvement.

Building design concerns with different aspects such as cost, aesthetics, use of space, lighting, energy use, etc. Design by considering one aspect may cause to sacrifice building quality in other aspects; e.g. trying to

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improve natural lighting by using large windows causes large heat gains/losses, resulting in low energy performance of the building (Caldas and Norford 2002). Therefore, it is essential to identify low energy measures using building performance simulations for various aspects of performance at the earliest in the design process (Bouchlaghem et al. 2005). However, using simulations alone is a tedious approach towards a satisfactory design (Radford and Gero 1980) and only a few scenarios can be evaluated from a range of possible choices. Conversely, coupling a simulation programme with an optimisation tool enables the designer to evaluate a large number of candidate designs (Caldas and Norford 2002).

Space layout planning (SLP) can be considered as the starting point of building design (Miller 1971). Hence, the low energy measures taken at this stage would be the most influential to the building's performance over its lifecycle. Space layouts are found to be affecting all three aspects of sustainability: social, economic and environmental. Social logic of space and the associating concept of "space syntax", presented by Hillier and Hanson (1983), have shed light on social aspects of SLP. Economic aspects of SLP can be found in literature on facilities planning (Armour and Buffa 1963), while, the lifecycle environmental dimensions of SLP are discussed in Neilsen and Svendsen (2002).

This paper presents an approach for integrating the concepts of sustainability (e.g. energy conservation in buildings) into automated space layout planning. A genetic algorithm is implemented to optimise the size, orientation and relative locations of spatial units. The objectives are to minimise energy use and maximise the satisfaction of topological and geometrical requirements of a given design brief. The next section describes the nature of space layout planning and a review of previous works on SLP. Section 2 describes the approach taken in this study, including the integration of space layout planning with a detailed-based building energy simulation program: EnergyPlus (Crawley et al. 2003). Section 3 briefly describes the genetic algorithm as the optimisation tool used, and the method for evaluating layouts for their geometrical and topological attributes. The paper concludes with a discussion on the presented approach and future directions of research.

2. Automated Space Layout Planning

Space layout planning is concerned with finding a set of locations for a set of activities such that the resulting layout satisfies both the relationship requirements between the activities and the tangible design goals, which are usually determined a priori. In addition, the emotional feel of the building, flow of space, transition between spaces, philosophical ideals and theories of design (e.g. rhythm, harmony, variety, emphasis, pattern, texture etc.) are taken into consideration by designers. Hence, it is evident that a computer program alone may not be sufficient to incorporate many non-tangible design aspects for a satisfactory layout. Approaches to SLP, before the advent of computer aided methods, can be broadly thought of as a threefold process of: information gathering, trial and error design and solution presentation (Miller 1971). Computer aided SLP, on the other hand, is considered an interactive process between the human designer and the computer program that aids in the effective search of the solution space.

Space planning problems belong to the category non-polynomial time hard (NP-hard) problems (Jagielski and Gero1997); i.e. the best solution for the problem can be found if the solution space is investigated exhaustively. Due to the combinatorial nature (the size of the solution space grows exponentially with the size of the problem considered) of SLP problem (Jo and Gero 1996), the solution space becomes large and complicated for a human designer to find solutions (Liggett 1981). Also, a human designer may only explore a limited area of this large solution space to find a layout and may not find the optimum one. In order to investigate the solution space, longer time and expensive computation techniques such as an automated SLP is required. Approaches to the automated SLP use strategies to control the combinatorial behaviour; e.g. setting constraints to the problem, enumerating only a fixed number of solutions at a time and reducing the number of design variables in the problem considered (Michalek and Choudhary 2002). Further, combinatorial behaviour of the problem limits the number of spatial units that can be handled by a particular search strategy. Therefore, the quality of solutions found depends on three aspects that are strongly interrelated: method of representation, problem formulation and solution strategy (Jo and Gero 1998).

Objectives of SLP extend beyond mere satisfaction of geometrical and topological relationships to the objectives to group co-working units, minimising material handling costs, minimising travelling distances, improving productivity, comfort of occupants (Liggett and Mitchell 1981) and maximising the use of daylight (Caldas 2008). SLP is, therefore, a multi objective design optimisation problem that aims to achieve these objectives through finding the most suitable combination of design variables without sacrificing the user requirements (Liggett and Mitchell 1981).

Attempts to automate space layout planning started nearly 40 years ago (Liggett and Mitchell 2000). Many researchers such as Eastman (1973), Flemming (1978), Grason (1978), Baykan and Fox (1989), Charman (1993) and Medjdoub and Yannou (2001) have used heuristic based approaches while, observed current trend is to use methods of artificial intelligence such as genetic algorithm, Genetic programming and Artificial neural networks (Kazakov and Gero 1997; Michalek et al. 2006 and Caldas 2008). Also, physically based methods such as the 'spring damper kinematic model' used by Scott and House (2002) have been used.

Software Environment to support Early phases of building Design (SEED) by Flemming et al. (1995) is an interactive program intended for frequently recurring building types (i.e. schools, hospitals, fire stations etc). In addition to rapid generation of design representations (i.e. layout), SEED provided support for analysis and evaluation at early stages of design. Two earlier generative design methods: (a) LOOS/ABLOOS for synthesis of layouts of rectangles and (b) GENESIS for the generation of assemblies of 3-dimensional solids have been used in SEED for design generation.

In summary, the evolution of automated space layout planning techniques over the past decades focussed predominantly on locating spatial units within a set building boundary, which is not necessarily the focus of the majority of activities at early design stages. In most design endeavours, space layout design is considered as the primordial activity, which results in the shape and form of the building and impacts on most other indicators, including, but not limited to, sustainability. Secondly, the integration of building performance assessment as part of decision making has been dealt with in a few SLP research, but was limited to the use of simplified models of building energy and lacked a comprehensive understanding of the interdependent aspects such as of layout, energy, spatial quality, daylighting availability, etc.

The authors propose that (a) the decision making for sustainable buildings are evidence based, which can be obtained through an integrated assessment of sustainability indicators, (b) assessments for sustainability need to be incorporated at the earliest in the design stage, in particular during conceptual stages; e.g. space layout planning/design and (c) the application of automated SLP and mathematical optimisation methods can help in achieving (a) and (b). The following section describes the proposed framework.

3. The sustainable space layout planning framework

For flexibility and ease of extension, the sustainable space layout planning (SSLP) framework has been designed in 4 modular components, namely (a) design and decision dashboard, (b) optimiser, (c) performance evaluator and (d) space layout planner, as illustrated in

. The scope of the framework is the broader sustainability, hence it is called the SSLP. The implementation is, however, limited to environmental sustainability at present.

3.1 Design and decision dashboard

SSLP activity starts with the decoding of the design brief in the design and decision dashboard. The brief describes: (a) spatial units (SU) area and aspect ratio, (b) the container unit (CU) area and shape, (c) a matrix of adjacencies between SUs, (d) a matrix of proximity between SUs and (e) a matrix of relationship between SUs. Based on the specifications: (a)-(e), the required parameters are passed on to the optimiser for further processing. Results from optimiser are transferred to the dashboard for visualisation and decision making. The decision dashboard is also designed to allow effective interactions between the designer and the SSLP system.

3.2 Optimiser

The optimiser is designed as an open-end container for implementation of various optimisation algorithms. The algorithm-agnostic implementations allow for the selection of a suitable algorithm for a particular type of layout planning problem; e.g. whether the layout problem is assignment or assimilation in nature. The optimiser initialises design variables and passes on to the space layout planner. The resulting layout is then evaluated for various performance indicators through the evaluator module.

3.3 Performance evaluator

The performance evaluator module allows for connection between the optimiser and the simulation engine/program of interest. Architectural Design Optimisation Tool (ArDOT), an interoperability based design optimisation tool (Mourshed et al. 2003) has been used to connect standards based simulation programs without the loss of semantics. EnergyPlus (Crawley et al. 2001) has been closely integrated in the SSLP for detailed based evaluations of building energy and resulting emissions.

3.4 Space layout planner

The modularisation of space layout planner enables integration of various SLP methods. Interactive, heuristics and sequential based are the three space layout planning algorithms implemented in the SSLP.
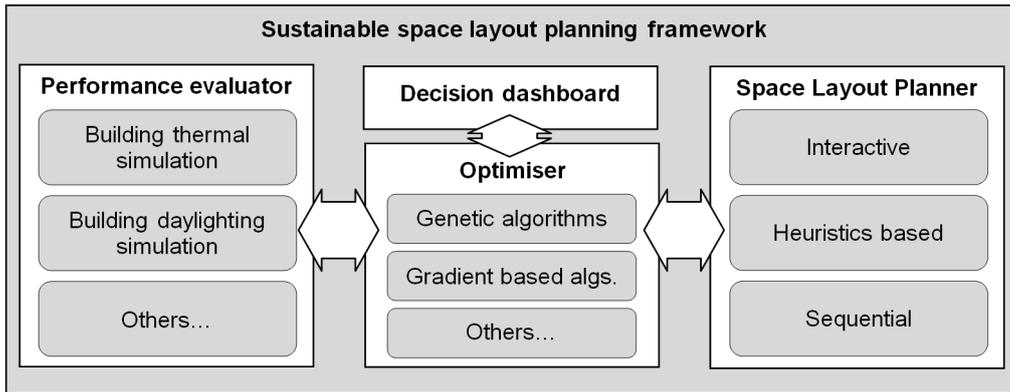
Figure 1   Components of the sustainable space layout planning framework

4. Implementation

The framework has been implemented using genetic algorithm (GA) as the optimisation method. The workflow is given in Figure 2. The candidate layouts, sequentially generated by the space layout planner, are evaluated for four aspects: topology, geometry, compactness of the layout and energy consumption. Evaluation for topology, geometry and compactness is carried out using an evaluator, coded primarily for this research. Total annual energy consumption, which accounts for heating, cooling, lighting and equipments' energy consumption is evaluated using EnergyPlus. In order to allow for the most energy efficient building design, the building footprint is allowed to be determined by the spatial arrangements, instead of a fixed building boundary to start with. The problem of SLP is set up considering suitable representations for spatial units, interrelationships between the spatial units, constraints that are not to be violated and the objectives that are optimised. Mathematical relationships were built to evaluate layouts with regards to the degree of infeasibility (if the constraints were violated) and the value of the objective function. Next, GA is coded to optimise using constrained optimisation and stochastic ranking.
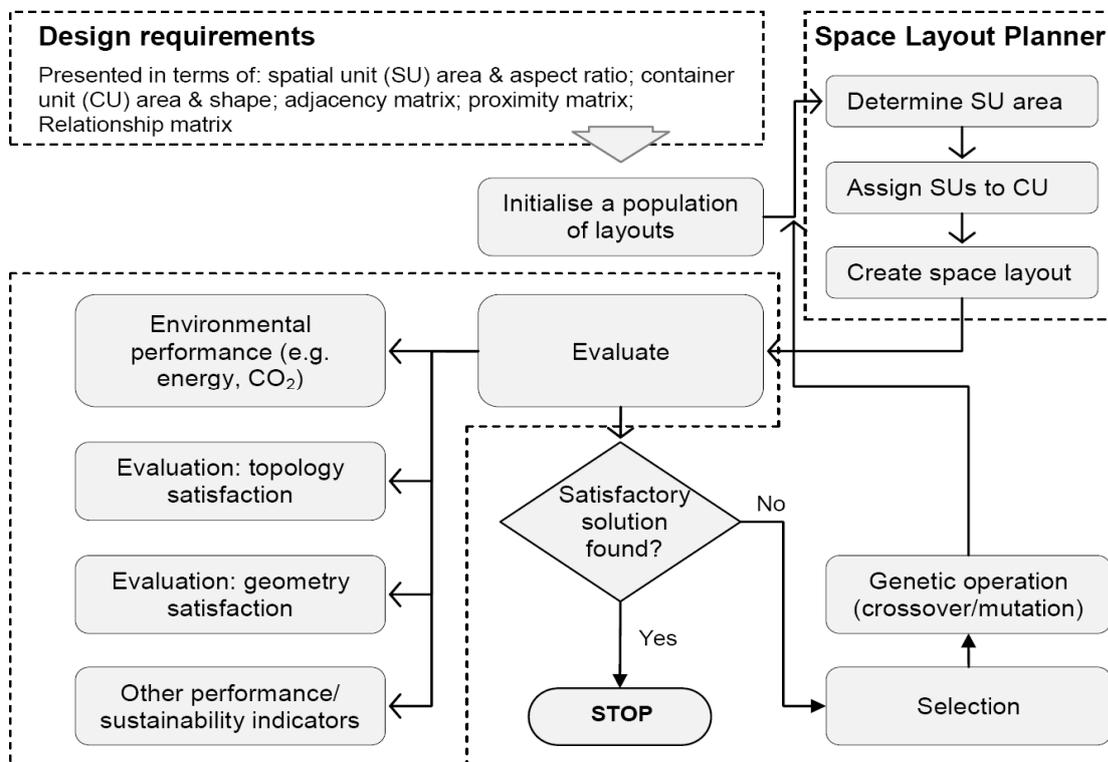

Figure 2   Workflow diagram of the SSLP framework using GA as the optimisation algorithm.

The following four sections describe the formulation of an SLP optimisation problem implemented in this research.

4.1 Representation

Spaces for each activity is represented by a rectangle with a reference point and its dimensions aligned with North-South coordinates (see Figure 3a). The container space for the spatial units (or activities) is the building site, which can be of any arbitrary shape defined by its boundary point coordinates. Every SU is associated with a corridor block on any arbitrary side of it with a constant width and length equal to that of the side of SU it is attached to.
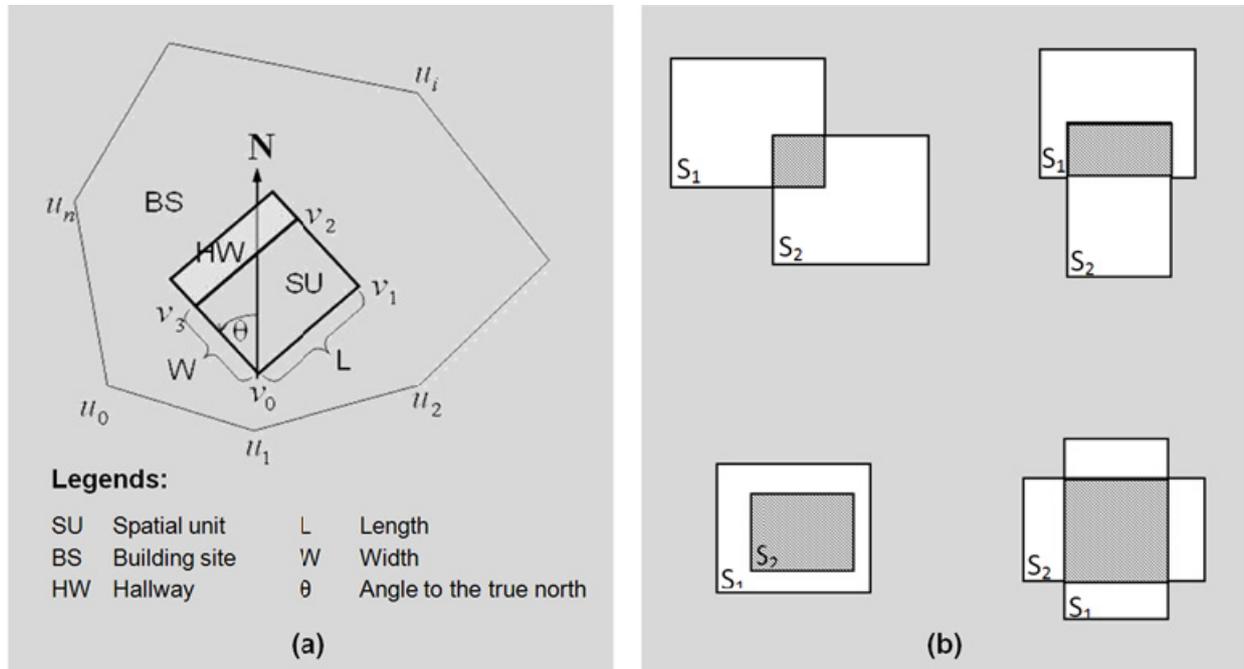

Figure 3   (a) Representation of spatial units in building site.

(b) Possible scenarios of overlapping spatial units.

4.2 Variables

Independent variables of an SU were its location (x and y coordinates) and position of the attached corridor block. Location of an SU varies between the maximum and minimum coordinates of the building site with an increment of 0.1m. Dependent variables are coordinates of other vertices of each SU and geometrical interrelations between SUs; i.e. overlap and adjacency.

4.3 Constraints

The implicit constraints are, not allowing any overlap of SUs (corridors can overlap with rooms or other corridors) and all SUs are placed within the building site. Requirements on adjacency and accessibility are classified as objectives, rather than constraints.

4.3.1 Degree of infeasibility in overlapping

Each pair of SU is tested and overlapped area is calculated considering the possible scenarios of overlap as illustrated in Figure 3b. The worst case being all the SUs having overlapped on to each other, the degree of infeasibility of the available layout in overlapping is determined as a percentage of the worst scenario.

4.3.2 Degree of infeasibility in falling off building site

Each SU is tested for falling out of the building site. The area that is lying out of the building site is calculated. Worst case being all SUs falling off building site, degree of infeasibility of SUs is calculated as a percentage of the worst scenario. The explicit constraints are quality of access path, approximate travelling distance and the adjacency requirements.

4.3.3 Accessibility

Two SUs are considered as accessible from each other if a path exists from one SU to the other. This path may fall through hallways and or SUs. Also, the two SUs may be directly accessible; i.e. either the hallway of one SU is overlapping/adjacent with the other SU or hallway, or the two SUs are adjacent or overlapping. Although overlapping is considered as infeasible, it can be accepted as an accessibility condition. Based on

the direct accessibilities, an 'available accessibility tree' is constructed. Dijkstra's shortest path algorithm (Kleinberg and Tardos 2005) is used for searching the shortest path between pairs of SUs for which the accessibility requirement is specified.

Accessibility is considered as two separate constraints one for quality of paths and the other for travelling distance of each path. Paths that fall through hallways are given preference over those that fall through SUs. A point is added each time a path falls through a SU. Total of these points for all paths is the total value of path quality constraints. Value of travelling distance constraint is obtained by subtracting maximum allowed distance from available approximate travelling distance. Only the positive values are considered. Squared sum of accessibility of each accessibility constraint is considered as the total value.

4.3.4 Adjacency

Two SUs are considered to be adjacent if a side of one SU is touching a side of the other SU. For each adjacency requirement a minimum desired length of adjacency is also specified. Two constraints are considered for adjacency: existence of an adjacency and the length of adjacency. The total of existing adjacencies, added to the squared sum of constraint function values of adjacent length (determined in a similar way to the accessibility), is considered as the total adjacency constraint function value.

Addition of all constraint functions is considered as the total constraint function.

4.4 Objectives

The objectives considered for the optimisation are: annual energy consumption and compactness of the layout. The aim of the optimisation is to minimise the operational energy consumption (the sum of heating, lighting and cooling energy requirements) and the spread of the layout.

Compactness of the layout is measured using the spread area of the layout and total area of SUs. Spread area is the area of the largest rectangle that encloses all SUs. Compactness objective is determined as the percentage of the sum of areas of SUs to the spread area.

5. Optimisation algorithm

Considering the combinatorial nature of the space layout planning problem, as described earlier, the use of manual computation for this problem is prohibitive. Therefore, an algorithm needs to be used that produces satisfactory results within a reasonable timeframe. Genetic algorithm is a powerful search mechanism to explore large solution spaces and have been used successfully in many building design problems. Therefore, GA has been the choice of algorithm in this research.

5.1 Genetic algorithms

Genetic algorithm is inspired from natural genetics. In which, a population of individuals, comprising of genes that determine each individual's fitness to survive in a particular environment, evolve through a number of generations until majority of the population become fit for that environment. This environment is represented by a fitness function in the optimisation problem. When having a minimisation objective, lower the value of objective function higher the fitness of an individual. From a given initial a set of individuals are selected to participate in reproduction based on their relative fitness. An individual (chromosome) is a potential solution; i. e. a layout. It comprises of n number of SUs resulting in $n \times 3$ number of genes, because each SU has three variables: length, width and position of the hallway.

5.1.1 Fitness evaluation

Fitness evaluation criteria used in this study is constrained stochastic ranking (Runarsson and Yao 2000). The individuals are ranked based on their degree of infeasibility and the objective function values. Ranking is performed in a bubble sort manner, where slightly greater probability (55%) is given to the infeasibility component where, value of objective function is given 45% probability. i.e. the selection of an individual relies more on its degree of infeasibility. The rank of each individual is taken as its fitness.

5.1.2 Selection

Tournament selection with a score is used for selection of individuals. The population of individuals are grouped and fitness of each individual in a group is compared with that of others in the same group. Each time the fitness of the individual considered is higher than another one it gets a 'win'. The individuals with most number of 'win's are included in the mating pool.

5.1.3 Reproduction

Reproduction operation undergoes two processes: recombination and mutation. A pair of parents are selected randomly from the above selected mating pool for recombination. Genes of each parent is subject

to swapping with the corresponding gene of the other parent based on the probability of crossover. Also, in order to introduce further variations, the genes are subjected to slight modifications if the two genes are not holding the same values. After recombination, offspring is subjected to mutation depending on the probability of mutation.

The generated new population undergoes evaluation, selection and reproduction process in a cyclic manner until a satisfactory set of solutions are reached or a certain number of generations are completed, illustrated in Figure 2.

6. Conclusion

An extensible framework for enhancing building sustainability has been presented in this paper. The framework considers space layout planning as the primordial activity in the building lifecycle. The aim was to generate evidence based sustainable building solutions through an informed decision making process by taking into account the complex interrelationship between various aspects of building design, construction and operation. The evidence is provided through the application of detailed-based evaluations of building performance indicators. The resulting solution space, which is combinatorial in nature, is searched using mathematical optimisation methods. An implementation of the framework, using GA as the optimisation method and EnergyPlus as the engine for evaluation of building energy performance, has been described.

Future work on the framework can focus on its application in various sustainability scenarios such as lifecycle modelling and in the development of space layout planning algorithms that is less expensive, computationally.

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