Linear Street Extraction Using a Conditional Random Field Model

Padraig Corcoran[a], Peter Mooney[b], Michela Bertolotto[c]

[a]*School of Computer Science and Informatics*
Cardiff University, Cardiff, Wales, UK
[b]*Department of Computer Science*
Maynooth University, Ireland
[c]*School of Computer Science and Informatics*
University College Dublin, Dublin, Ireland

Abstract

A novel method for extracting linear streets from a street network is proposed where a linear street is defined as a sequence of connected street segments having a shape similar to a straight line segment. Specifically a given street network is modeled as a Conditional Random Field (CRF) where the task of extracting linear streets corresponds to performing learning and inference with respect to this model. The energy function of the proposed CRF model is *submodular* and consequently exact inference can be performed in polynomial time. This contrasts with traditional solutions to the problem of extracting linear streets which employ heuristic search procedures and cannot guarantee that the optimal solution will be found. The performance of the proposed method is quantified in terms of identifying those types or classes of streets which generally exhibit the characteristic of being linear. Results achieved on a large evaluation dataset demonstrate that the proposed method greatly outperforms the aforementioned traditional solutions.

Keywords: Conditional Random Field (CRF), Street Network, Pattern

1. Introduction

The automated extraction of geometrical patterns from street networks represents an important component in many geo-spatial applications (Yang et al., 2010). For example, Weiss and Weibel (2014) used a measure of street network centrality to determine the most significant subset of a street network to be represented when reforming map generalization. Porta et al. (2006) extracted connected groups of street segments and used this as a platform for performing network analysis. In order to align two different street network representations of a common area, Koukoletsos et al. (2012) first extracted geometrical patterns from both representations and subsequently aligned these patterns.

A street network can exhibit a wide spectrum of geometrical patterns, a taxonomy of which was proposed by Marshall (2004). In this work we focus on the extraction of the geometrical pattern of linear streets which may be defined as sequences of street segments which have a shape *similar* to that of a straight line segment. Linear streets are also commonly referred to as strokes (Thomson, 2006; Heinzle et al., 2005; Touya, 2010). Some authors define strokes to be sequences of street segments which exhibit good continiuty, that is where the turning angle between adjacent segments is small (Yang et al., 2011), as opposed to sequences of streets segments which have shape *similar* to that of a line segment. For the purposes of this paper we consider linear and stroke patterns to be equivalent and adopt the latter, as opposed to the former, definition. The extraction of linear streets represents a challenging

task for the following reason. Being a linear street is a characteristic of a sequence of street segments. The set of all street segments in a given street network are however interconnected. Therefore in order to extract linear streets, the tasks of segmenting and labelling streets with respect to the characteristic of being linear must be solved simultaneously. This is commonly refereed to as the task of semantic segmentation in the domains of computer vision and robotics (Anand et al., 2012).

Existing methods for extracting linear streets generally employ heuristic search procedures where individual street segments are iteratively expanded or grown to form linear streets. These methods do not guarantee that the optimal solution will be found and no statements regarding the distance from the solution obtained to the optimal solution can be made. In this paper we propose a novel method for extracting linear streets which overcomes these limitations. Specifically we formulate the problem in terms of performing learning and inference with respect to a Conditional Random Field (CRF) which is a type of undirected probabilistic graphical model (Koller and Friedman, 2009). We demonstrate that performing inference with respect to this model corresponds to minimizing a submodular energy function. As a consequence of this fact, the optimal solution can be computed in polynomial time.

The remainder of this paper is structured as follows. In section 2 we review related works on the extraction of geometrical patterns from spatial data. In section 3 the proposed model is described. Finally, in sections 4 and 5 we present results and draw conclusions respectively.

2. Related Works

Existing techniques for extracting geometrical patterns from spatial data generally fall into two broad categories corresponding to those which extract patterns relating to buildings and those which extract patterns relating to street networks. We only considered those techniques in the latter category; the interested reader is directed to the following works in the former category (Lüscher et al., 2009; Zhang et al., 2013). A street network can exhibit a wide spectrum of patterns, a taxonomy of which was proposed by Marshall (2004). Ultimately the actual patterns one attempts to extract depends on the intended application. Some of the most common applications are network analysis (Porta et al., 2006), map generalization (Zhou and Li, 2012) and network matching (Koukoletsos et al., 2012).

Existing approaches to the extraction of linear streets employ heuristic search procedures. We now review these methods. Luan and Yang (2010) and Thomson (2006) describe a simple search procedures which iteratively expands a linear street to contain adjacent street segments. Liu et al. (2010) describes a similar search procedure which is initialized using a single street segment and terminates when the linear street in question cannot be expanded any further. The procedure is initialized using every segment in the street network and a set of criterion is used to evaluate if and how an expansion is performed. Yang et al. (2011) and Luan and Yang (2010) propose search procedures for extracting linear streets which are integrated with methods for extracting dual carriageways and complex junctions. This integrations allows the extraction of linear streets across such features. Zhou and Li (2012) present a comparison of different search procedures which differ in

the criterion used to determine if and how an expansion is performed. The range of criterion consider includes both geometric and semantic attributes of street segments.

A fundamental limitation of the above search procedures is that the result achieved is very dependent on the initialization of the search in question. Given the fact that the initialization is generally done manually or in an automated manner which is heuristic in nature, these methods do not guarantee that the optimal solution will be found. Also no statements regarding the distance from the solution obtained to the optimal solution can be made.

A number of methods have been developed to detect geometrical patterns other than linear streets. The detection of grid patterns has been considered in Heinzle et al. (2005); Yang et al. (2010); Tian et al. (2012). Heinzle et al. (2005, 2006) proposed methods for detecting ring and star patterns. Zhou and Li (2015) proposed a method for detecting interchanges in street networks. Several methods for detecting dual-lane patterns have also been proposed (Yang et al., 2011, 2013; Li et al., 2014).

To date many solutions to the problem of extracting streets from remotely sensed data have been proposed (Barzohar and Coope, 1996; Geman and Jedynak, 1996; Lacoste et al., 2010). This problem is fundamental different from that which is consider in this paper. Despite this fact, similar to the solution proposed in this paper, many of these solutions are also formulated in terms of doing learning and inference respect to an undirected probabilistic graphical model. For example the solution proposed by Tupin et al. (1998) involves performing inference with respect to an undirected probabilistic graphical model using Markov Chain Monte Carlo (MCMC).

3. Methodology

In this section we describe the proposed methodology for extracting linear streets. As discussed in section 1, in order to extract linear streets, the tasks of segmenting and labelling streets with respect to the characteristic of being linear must be solved simultaneously. To overcome this challenge we pose the problem of extracting linear streets in terms of performing learning and inference with respect to a Conditional Random Field (CRF). A CRF is a type of undirected probabilistic graphical model or Markov random field which encodes the conditional dependencies between random variables using a graph (Li, 2009; Koller and Friedman, 2009; Sørbye and Rue, 2014; Law et al., 2014).

The consideration of a CRF model is motivated by the fact that a street network will exhibit conditional dependencies between connected street segments; for example, the probability of two connected street segments both being linear is higher than two unconnected segments. Also, this model has previously been successfully applied in the domain of image analysis to simultaneously solve the problems of image segmentation and labelling where such conditional dependencies are also evident (Li, 2009; Koller and Friedman, 2009). To the author's knowledge, this work represents the first application of a CRF model to the problem of extracting linear streets from a street network. In this section we only describe the specifics of the CRF model implemented. For a broader overview of background material, the interested reader is directed to the following publications (Bishop, 2006; Koller and Friedman, 2009; Wang et al., 2013).

The remainder of this section is structured as follows. Section 3.1 presents

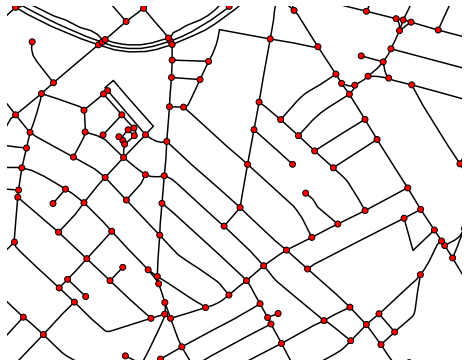


Figure 1: An example of the street network representation $G^s = (V^s, E^s)$ is illustrated. Elements of the sets V^s and E^s are represented by red circles and black lines respectively.

the street network representation used. Section 3.2 describes how the problems of extracting linear streets from this representation is formulated in terms of performing inference with respect to a CRF model. Finally Section 3.3 describes how learning and inference with respect to this model is performed.

3.1. Street Network Representation

We represent a given street network using a graph $G^s = (V^s, E^s)$ where the set of vertices V^s correspond to street intersections and deadends, while the set of edges E^s correspond to street segments connecting these vertices. An example of this representation is illustrated in Figure 1. This is a commonly used street network representation and is known as a *primary representation* (Porta et al., 2006; Corcoran et al., 2013; Corcoran and Mooney, 2013). A UTM coordinate system, where distances measured are in meters, was used to represent the spatial locations of all vertices and edges in G^s.

3.2. Conditional Random Field (CRF) Model

In this section we describe how the task of extracting linear streets is formulated in terms of performing inference with respect to a CRF model.

Let y be a random variable we wish to predict given another random variable x. A CRF is a *discriminative model* which directly models the probability of y given x, denoted $P(y|x)$. This contrasts with a *generative model* which models the joint probability of y and x and uses Bayes rule to infer the probability of y given x (Ng and Jordan, 2002; Koller and Friedman, 2009). Unlike generative models, discriminative models do not assume the set of random variables x are independent and therefore generally offers superior performance with respect to modeling $P(y|x)$ (see section 16.3.2 Koller and Friedman (2009)).

Toward defining the proposed CRF model for extracting linear streets, let x correspond to a set of random variables corresponding to the set of street segments described in the previous section. Let y correspond to a set of binary random variables such that a bijection exists between y and x. Each element of y indicates if the corresponding street segment belongs to a linear street. Let Y be the space of all realizations of y. $P(y|x)$ is subsequently modeled using a CRF as follows. Let $G^c = (V^c, E^c)$ be a graph where a bijection exists between the set of vertices V^c and the set x. An edge is constructed between two vertices in G^c if the corresponding street segments are adjacent (share an end point). A Markov property with respect to G^c is assumed which states that each vertex in G^c is independent of all other vertices given its neighbouring vertices. Given this assumption, by the Hammersley and Clifford theorem (Koller and Friedman, 2009; Li,

2009; Sutton and McCallum, 2011), a conditional probability distribution of y given x is defined by Equation 1 where $E(y|x)$ and Z are defined by Equations 2 and 3 respectively. In Equation 2, y_v denotes the element of y corresponding to v and $\sim$ denotes adjacency between vertices. The term $E(y|x)$ is known as the *energy function*. The variable Z is a normalizing constant which is known as the *partition function*.

$$p(y|x) = Z^{-1} \exp\left(-E(y|x)\right) \tag{1}$$

$$E(y|x) = \sum_{v \in V^c} E_D(y_v, x) + \sum_{v,w \in V^c; w \sim v} E_S(y_v, y_w, x) \tag{2}$$

$$Z = \sum_{y \in Y} E(y|x) \tag{3}$$

The term $E_D(y_v, x)$ in Equation 2 is commonly referred to as the *data term* (Boykov and Kolmogorov, 2004). It is designed to measure the utility of assigning the binary label y_v to the street segment corresponding to the vertex v in G^c. The term $E_S(y_v, y_w, x)$ in Equation 2 is commonly referred to as the *boundary term* (Boykov and Kolmogorov, 2004). It is designed to measure the utility of assigning the labels y_v and y_w to the adjacent street segments corresponding to the vertices v and w in G^c respectively. The *maximum a posteriori* (MAP) solution of Equation 1, that is the most probable binary labelling of the street segments with respect to being linear streets, corresponds to the minimization of the energy defined in Equation 2. In other words, determining the most probable binary labeling of the street segments with respect to belonging to linear streets corresponds to finding a global

minimum of this function.

Minimization of the energy with respect to the data term can be considered as solving the labelling problem. On the other hand, minimization of the energy with respect to the boundary term can be considered as solving the segmentation problem. Therefore minimization of the energy function with respect to both the data and boundary terms can be considered as simultaneously solving the labeling and segmentation problems. Computation of the data and boundary terms is described in the following two subsections.

3.2.1. Data Term

In the context of extracting linear streets, the binary label y_v indicates if the corresponding street segment belongs to a linear street. The term $E_D(y_v, x)$ is designed such that it measures the cost of assigning the label y_v to the corresponding street segment and is computed as follows.

Let S be a sequence of street segments which initially only contains the segment for which the term $E_D(y_v, x)$ is being computed. S is iteratively expanded such that at each iteration that segment adjacent to the first or last segments in S which results in the most linear sequence of segments is added to start or end of S respectively. Using a simple street network, two iterations of this procedure are illustrated in Figure 3. The search is terminated when the length of the sequence of segments in S exceeds a threshold t. Setting the threshold t to a particular value has the effect of encouraging the extraction of linear streets of a length greater than or equal to that value.

The linearity of a sequence of segments S is measured as follows. We first measure the distance d in terms of shape between S and a line segment using the metric of (Arkin et al., 1990). This metric represents each of the

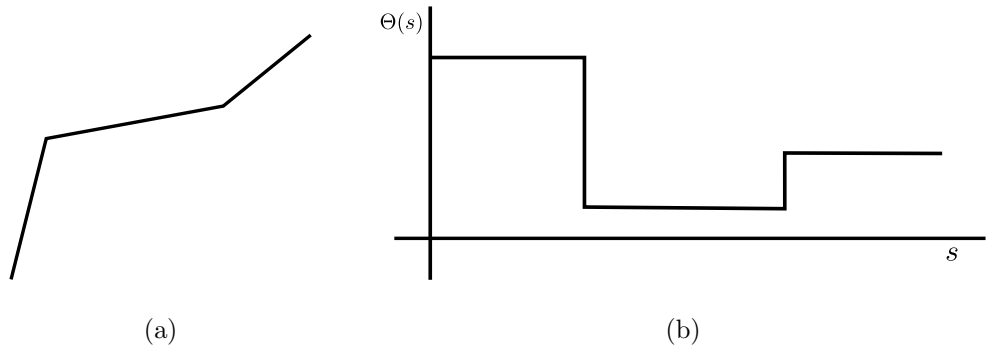


Figure 2: For a given shape represented in (a) the corresponding turning function $\Theta(s)$ is represented in (b).

shapes in question by a turning function Θ which measures tangent angle as a function of s the distance along the shape in question. The concept of a turning function is illustrated in Figure 2. The distance d is subsequently determined to be the minimum area between both turning functions as a function of scalar addition to a single turning function. The linearity of S is then determined by passing d through a non-linear function as defined in Equation 4 where α_l and β_l are model parameters which are subsequently learned. This process of passing the metric distance through a non-linear function is necessary calibration with respect to the CRF model in question (Platt, 1999).

Following termination of the above search, the terms $E_D(1, x)$ and $E_D(0, x)$ are computed using Equations 5 and 6 respectively where l_v is the linearity of the sequence S. The term $E_D(1, x)$ has a range $(0, 1]$ and approaches a value of 0 if the street segment in question belongs to a street which has shape similar to a line segment. The converse of this statement is true for the term $E_D(0, x)$.

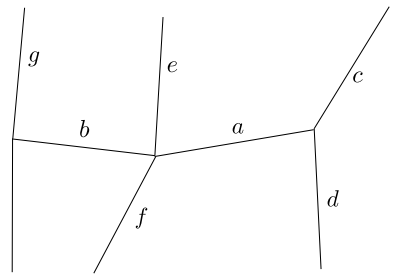


Figure 3: A simple street network is illustrated where each street segment is represented by a line segment and assigned a unique label. In the case of computing the data term for the street segment a, the sequence S initially contains a single element of a; that is $S = \{a\}$. In the first iteration of the search procedure the street segment b is added to the start of S; that is $S = \{b, a\}$. In the second iteration of the search procedure the street segment c is added to the end of S; that is $S = \{b, a, c\}$.

$$l_v = \frac{1}{1 + \exp(\alpha_l d + \beta_l)} \tag{4}$$

$$E_D(1, x) = l_v \tag{5}$$

$$E_D(0, x) = 1 - l_v \tag{6}$$

3.2.2. Boundary Term

The boundary term $E_S(y_v, y_w, x)$ measures the cost of assigning the labels y_v and y_w to the adjacent street segments v and w respectively. It is important that E_s does not over penalize labellings corresponding to boundaries between linear streets and other street segments. This requires that the term E_s be *discontinuity preserving*. In order to achieve this, a *Potts* model

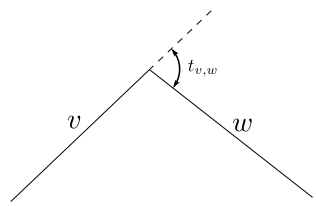


Figure 4: The turning angle $t_{v,w}$ between street segments v and w is illustrated.

for the boundary term was used and is defined in Equation 7 (Boykov and Kolmogorov, 2004).

$$E_S(y_v, y_w, x) = K_{(v,w)} T(y_v \neq y_w) \tag{7}$$

The term $K_{v,w}$ is defined in Equation 8 where $t_{v,w}$ is the turning angle between the line segments formed by joining the first and last points of the street segments v and w. The concept of turning angle is illustrated in Figure 4. The parameters α_t and β_t which are subsequently learned, determine how severely a small turning angle between pairs of segments which do not have the same label are penalized. The function $T(.)$ is 1 if the condition inside the parentheses is true and 0 otherwise.

$$K_{v,w} = \frac{1}{1 + \exp(\alpha_t d + \beta_t)} \tag{8}$$

The use of the above boundary term is motivated by the following fact. The corresponding turning angle between adjacent street segments will be relatively small when both segments belong to linear streets. On the other hand the corresponding turning angle between adjacent street segments will be relatively large when one segment belongs to a linear street and the other does not. It is worth noting that such a smoothness term would not be

appropriate in the context of extracting other patterns such as grid patterns where corresponding adjacent street segments can naturally have a large turning angle between them.

3.3. Model Learning and Inference

Learning corresponds to the task of determining the optimal parameter values for the CRF model from data. Specifically the CRF model has the five parameters of α_l and β_l (see Equation 4), α_t and β_t (see Equation 8) and t (see section 3.2.1) which must be learned. Learning was performed by optimizing the classification accuracy through cross validation using a grid search over the parameter space which may be defined as the set of all possible combinations of different parameter values.

Inference with respect to the CRF model corresponds to determining the MAP solution to Equation 1. Since the terms $E_S(0, 0, x)$ and $E_S(1, 1, x)$ are uniformly zero, the function E_S satisfies the inequality of Equation 9. Any function which satisfies this inequality is sub-modular (Boykov and Kolmogorov, 2004). Sub-modularity in discrete optimization plays a similar role to convexity in continuous optimization (Lovász, 1983; Stobbe and Krause, 2010). That is, sub-modularity allows one can efficiently find provably optimal solutions for large problems. Although other inference techniques such as MCMC also find provably optimal solutions they may not converge quickly and therefore are not considered efficient (Geman and Geman, 1984). In this work we employ the sub-modularity optimization technique of graph-cuts which computes the optimal solution in polynomial time (Boykov and Kolmogorov, 2004; Prince, 2012). We only briefly describe this optimization technique but direct the interested reader to the text-book of Prince (2012)

which contains an in-depth description. This optimization technique first constructs a single-source, single-sink flow network where each s-t cut corresponds to a particular solution to the optimization problem in question. The minimum s-t cut corresponds to the optimal solution to the optimization problem in question. There exists a number of different algorithms for computing the minimum s-t cut; in the work we employed the method of Boykov and Kolmogorov (2004) which computes the solution in polynomial time. It should also be noted that this optimization technique is a direct technique as opposed to an iterative technique. The proposed solution to the problem of extracting linear streets contrasts with existing heuristic search procedures which typically return a local, as opposed to global, optimal solution (Yang et al., 2011; Zhou and Li, 2012; Li et al., 2014).

$$E_S(0,0,x) + E_S(1,1,x) \leq E_S(0,1,x) + E_S(1,0,x) \tag{9}$$

4. Results and Discussion

This section is structured as follows. Section 4.1 describes the data used for evaluation. Section 4.2 describes the evaluation methodology employed. Finally in section 4.3 we present results.

4.1. Data

Two street network representations obtained from OpenStreetMap(OSM) were used in our analysis (Goodchild and Li, 2012). These correspond to non-intersecting sections of the street network representation for the cities of Boston and Cambridge in Massachusetts. One of these street network representations was used for evaluation and is visualized in Figure 5. This

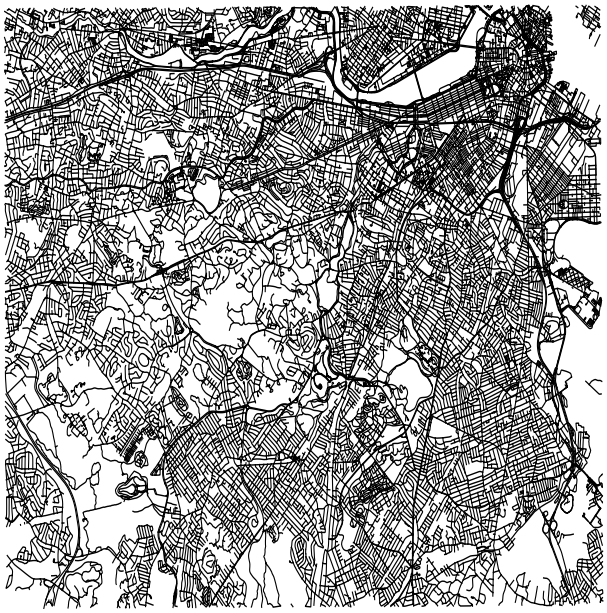


Figure 5: The OpenStreetMap street network representation for the city of Boston is displayed.

network has a corresponding primal representation containing $22,458$ vertices and $32,412$ edges. As such, it represents an extremely large dataset. The other street network representation was used for model learning, and contains $16,368$ vertices and $25,215$ edges.

4.2. Evaluation Methodology

A number of different approaches for evaluating linear street extraction methods have been considered. Jiang et al. (2008) proposed to use the degree of correlation between extracted linear streets and GPS tracks as a measure of correctness. A convincing justification for this approach is not provided. Liu et al. (2010) and Yang et al. (2011) proposed to use a comparison between manually created ground truth data. The creation of large ground

truth dataset is not feasible and has the potential of introducing bias. To overcome these limitations we employed the following approach for evaluation. Through a visual examination of OSM data we observed that the set of streets with types *primary*, *secondary*, *motorway* and *trunk* tend to exhibit the characteristic of being linear streets. A description of the meaning for each of these types can be found on the OSM wiki [1]. All other street types tend to exhibit the characteristic of not being linear street. This observation is illustrated in Figure 6. As consequence we chose to evaluate a linear street extraction method in terms of its ability to discriminate between street segments belonging to the above set of street types and its complement. That is, perform a binary classification of the street segments where positive indicates a linear street while negative indicates a non-linear street. Corresponding classification results are represented using confusion matrices. To allow a direct comparison of results obtained by different methods, the confusion matrices statistics of classification accuracy, error rate, sensitivity and specificity are computed (Han et al., 2006).

As described in section 2, existing methods for extracting linear streets employ heuristic search procedures which perform region growing from arbitrarily chooses seeds. Implementations of these methods are however not available. Therefore in order to perform a comparative evaluation of the proposed method for extracting linear streets, the authors implemented a method which is very similar to the above heuristic search methods. This method first randomly samples k street segments without replacement from

[1] http://wiki.openstreetmap.org/wiki/Key:highway

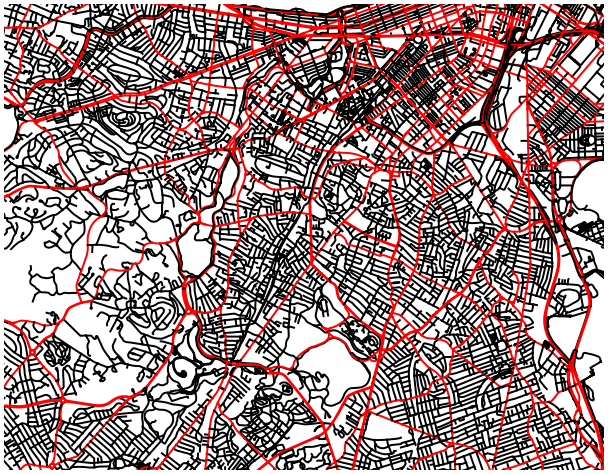


Figure 6: The set of streets with types *primary*, *secondary*, *motorway* and *trunk* are represented by the colour red. It is evident that these street tend to exhibit the characteristic of being linear streets.

the street network. Each of these segments is then iteratively expanded through the addition of those adjacent street segments which result in a sequence of segments most similar in shape to a line segment. Shape similarity is measured using the approach described in section 3.2.1. This iterative expansion terminates when the similarity falls below a threshold h or no adjacent segments exist. The parameters for this method are k and h, and these are learned by optimizing the classification accuracy through cross validation using a grid search over the parameter space.

4.3. Results

Figures 7 and 8 display the linear streets extracted by the heuristic search procedure and CRF model respectively for a subset of the Boston street network. A visual inspection of these results relative to the ground truth data

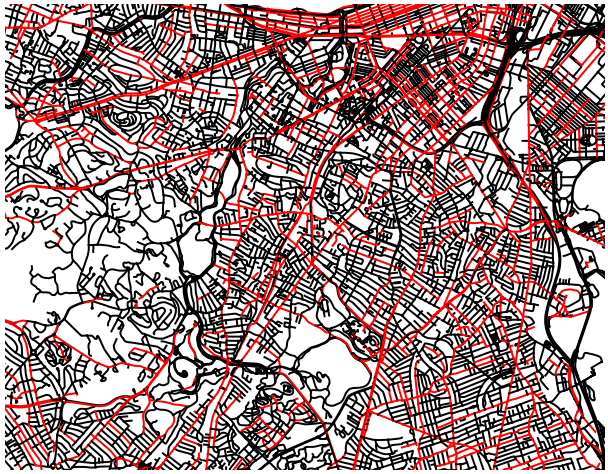


Figure 7: Linear streets extracted by the proposed heuristic search procedure are represented by the colour red.

	Linear	Non-Linear
Linear	3901	3014
Non-Linear	4450	19429

Table 1: Confusion Matrix for the Heuristic search procedure.

of Figure 6 suggests that the proposed CRF model performs considerably better. This is reflected in the corresponding confusion matrices for the entire Boston network which are displayed in Tables 1 and 2 respectively. The classification accuracy for CRF model is 0.84 with an error rate of 0.16. The corresponding sensitivity and specificity are 0.84 and 0.83 respectively. On the other hand, the classification accuracy for heuristic search procedure is 0.76 with an error rate of 0.24. The corresponding sensitivity and specificity are 0.56 and 0.81 respectively.

The relative poor performance of the heuristic search procedure can be

Figure 8: Linear streets extracted by the proposed CRF model are represented by the colour red.

	Linear	Non-Linear
Linear	5816	1099
Non-Linear	4100	20679

Table 2: Confusion Matrix for the CRF model.

attributed to the fact that the k street segments randomly sampled without replacement are not necessarily the most appropriate. To illustrate this point consider Figure 9(a) which displays ground truth for a small subset of the evaluation network where two regions have been highlighed. The corresponding results achieved by the proposed CRF model and heuristic search procedure for this subset are displayed in Figures 9(b) and 9(c) respectively. The heuristic search procedure selects segments to expand which are located in the highlighed region in the center. These segments do not correspond to linear streets and as a consequence a number of false positives occur. On the other hand, it fails to select any segments to expand which correspond to the linear street located in the highlighed region on the right. As a consequence of this fact a number of false negatives occur. The proposed CRF model does not require the specification of segments to expand and therefore does not suffer from these issues.

Although the proposed CRF model performed better overall than the heuristic search procedure, in some subsets of the street network the heuristic search procedure performed the best. Figure 10 illustrates this point for one such subset where it is evident that the proposed CRF model returns a number of additional false positives relative to the heuristic search procedure. We qualify this result with the fact that if the heuristic search procedure was rerun, it could potentially also return similar false positives if seeds were chosen within this subset.

In order to gain a greater insight into the terms in the CRF model we consider the performance of the model when the boundary term is uniformly set to zero. In this case the model does not consider the relationships between

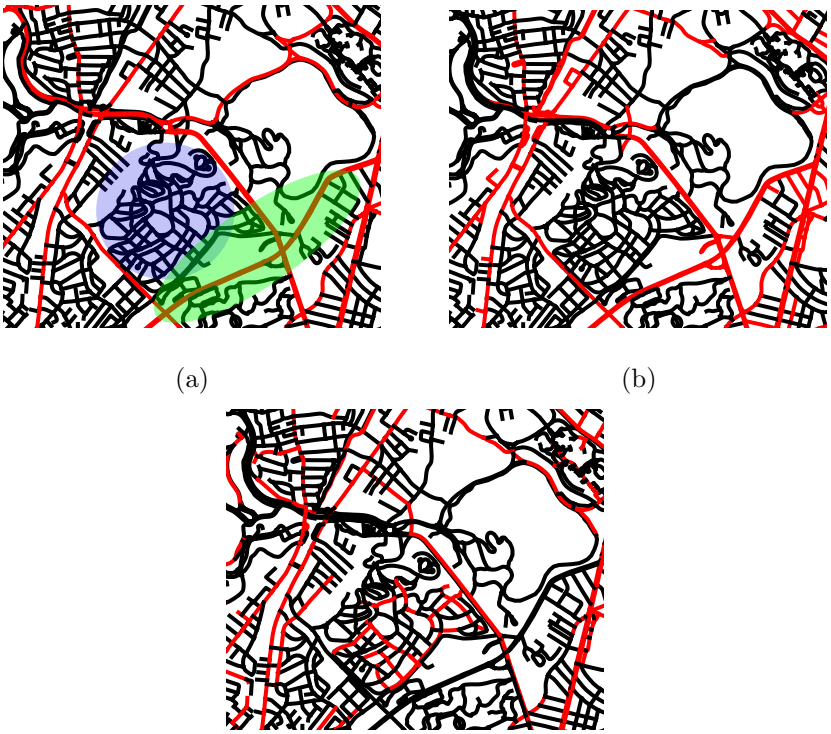
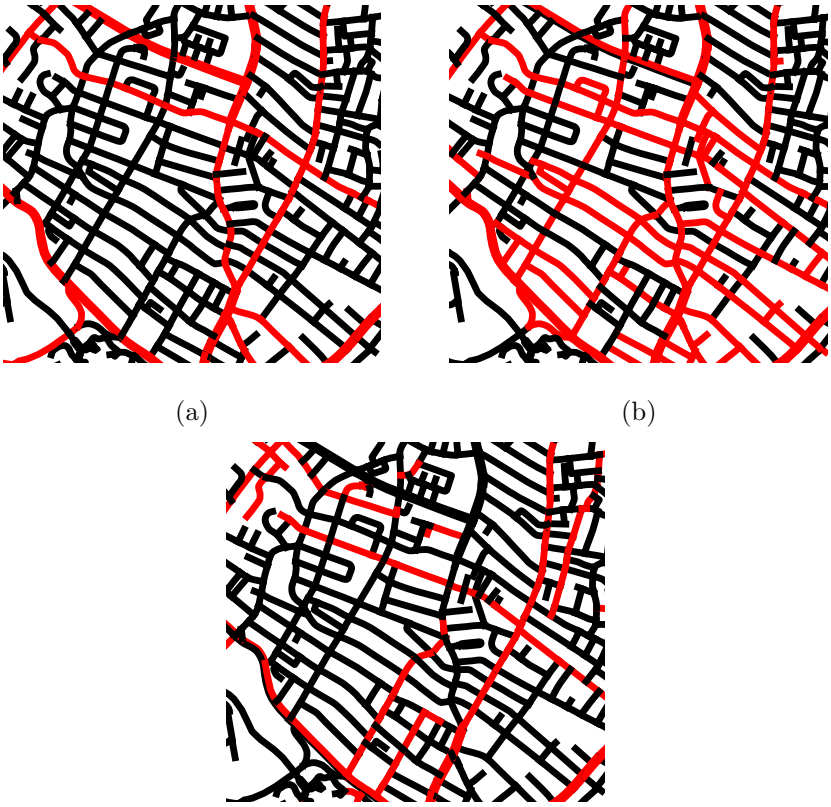


Figure 9: Ground truth for a small subset of the evaluation network along with corresponding results achieved by the proposed CRF model and heuristic search procedure and displayed in (a), (b) and (c) respectively. Two regions of (a), one in the center and one on the right, have been highlighted for discussion.



(a) (b)

(c)

Figure 10: Ground truth for a small subset of the evaluation network along with corresponding results achieved by the proposed CRF model and heuristic search procedure and displayed in (a), (b) and (c) respectively.

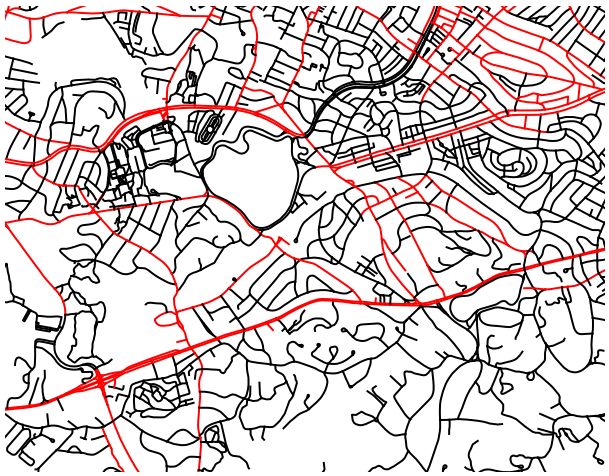


Figure 11: Linear streets extracted by the complete CRF model are represented by the colour red.

those y_v and y_w where the corresponding vertices are adjacent (see Equations 1 and 2). We refer to this model as the reduced CRF model. Figures 11 and 12 display the linear street extracted using the complete CRF model and the reduced CRF model respectively. A visual comparison of these results suggests that the complete CRF model performs considerably better. The confusion matrix corresponding to the reduced CRF model for the entire Boston network is displayed in Table 3. The corresponding classification accuracy is 0.80 with an error rate of 0.20. Comparing these statistics to those of the complete CRF model and the heuristic search procedure we see that the reduced CRF model outperforms the heuristic search procedure but fails to outperform the complete CRF model.

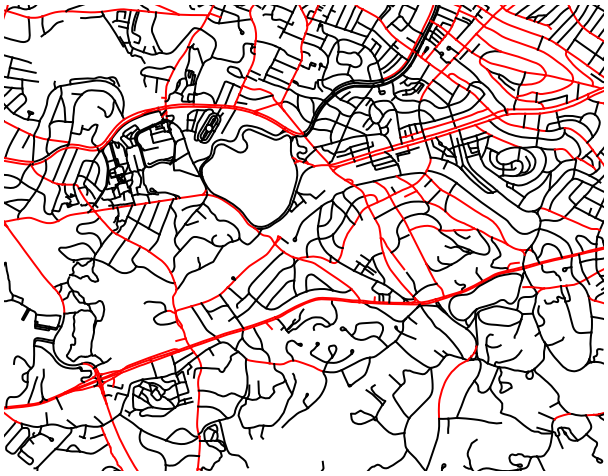


Figure 12: Linear streets extracted by the reduced CRF model are represented by the colour red.

	Linear	Non-Linear
Linear	5615	1300
Non-Linear	5015	19764

Table 3: Confusion Matrix for the reduced CRF model.

5. Conclusions

The extraction of geometrical patterns from street networks represents a first step in many geo-spatial applications such as map generalization and network analysis. In this paper we consider the problem of extracting linear streets which represent one of the most fundamental and commonly considered patterns. A novel methodology is proposed in which a street network is modelled as a Conditional Random Field (CRF) where the task of extracting linear streets corresponds to performing learning and inference with respect to this model. The energy function of the proposed CRF model is *submodular* which allows exact inference to be performed in polynomial time. This contrasts with the traditional solution to the problem of extracting linear streets which employs a heuristic search procedure and cannot make any guarantees regarding the solution found. Results achieved on a large evaluation street network demonstrate the superior performance of the proposed method.

There exists a number of possibilities to further develop the CRF model presented in this paper. In this work we have chosen to model the street network using a CRF where the corresponding energy function is sub-modular. The advantage of using such a function is that it allows exact inference to be performed in polynomial time. The use of a non-sub-modular energy could increase the modelling capacity of the CRF model but would result in an inference problem which could only be solved approximately as opposed to exactly. As such, there exists a trade off between the capacity to model effectively and perform inference efficiently. In future work we hope to investigate this trade off. In this paper we have considered the application of a CRF to the extraction of the single geometrical pattern of linear streets. In future

work we plan to investigate if this model can be adopted to the extraction of other patterns.

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