Journal of Applied Physics

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Citation: J. Appl. Phys. 93, 8480 (2003); doi: 10.1063/1.1540059
View online: http://dx.doi.org/10.1063/1.1540059
View Table of Contents: http://jap.aip.org/resource/1/JAPIAU/v93/i10
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Modeling of the magnetomechanical effect: Application of the Rayleigh law to the stress domain

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(Presented on 15 November 2002)

Stress is one of the principal external factors affecting the magnetization of materials. The magnetomechanical effect, that is, the change of magnetization of a magnetic material resulting from the application of stress, has attracted attention because of its scientific complexity. An improved model equation for interpreting the magnetomechanical effect has been developed based on extension of the previous equation to include the Rayleigh law. According to the previous theory of the magnetomechanical effect, which is based on the ''law of approach,'' application of stress induces changes in magnetization toward anhysteretic magnetization which itself is stress dependent, and the rate of change of magnetization with the input elastic energy is dependent on the displacement of the prevailing magnetization from the anhysteretic magnetization. The theory has been refined by including a linear term in the model equation in addition to the well-known quadratic term. It was found that the modified theory provides a much better description of the magnetization changes under stress, particularly at small applied stress amplitudes and when the stress changes sign. © *2003 American Institute of Physics.* [DOI: 10.1063/1.1540059]

PREVIOUS MODEL THEORIES OF MAGNETOMECHANICAL EFFECTS

Recent research has shown that the magnetization curves of materials can be modeled in a variety of configurations.[1] In addition, stress, whether uniaxial[2] or torsional,[3] strongly affects the measured magnetic properties. The effect of stress on magnetization can be described as a perturbation of the magnetic field, since the stress affects the orientation of magnetic moments through magnetoelastic coupling.[4] But this ''effective field theory'' and the ''law of approach'' have their limitations.

Experimental results of Craik and Wood[5] are shown in Fig. 1 (which is taken from Fig. 5 of Ref. 5). At zero stress, the slope of the magnetic induction versus stress curve is positive, on both the positive and negative stress sides.

As shown in previous research,[4] the effect of stress on the magnetization can be considered as an effective field which can be derived from thermodynamics as the derivative of the appropriate free energy with respect to the magnetization,

$$H_\sigma = \frac{3}{2}\frac{\sigma}{\mu_0}\left(\frac{d\lambda}{dM}\right) \tag{1}$$

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where σ is the stress, which is negative for compression and positive for tension, λ is the magnetostriction, M is the magnetization of the material, and μ_0 is the permeability of free space. This equation can be used under suitable conditions for description of the effects of uniaxial, multiaxial, and torsional stresses on anhysteretic magnetization. The total effective field H_{eff}, including the stress contribution, can be represented as

$$H_{\mathrm{eff}} = H + \alpha M + H_\sigma \tag{2}$$

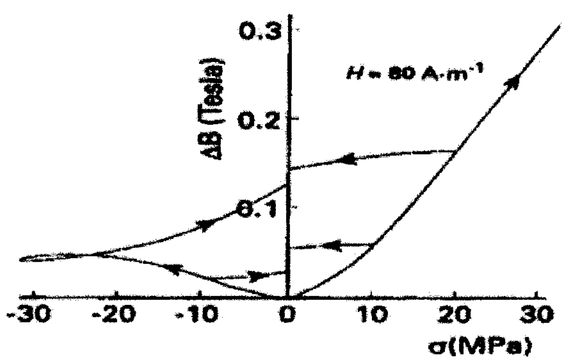

FIG. 1. The variation in magnetic induction B with stress for a specimen of mild steel, after Craik and Wood (Ref. 5). The slope at zero stress is positive.

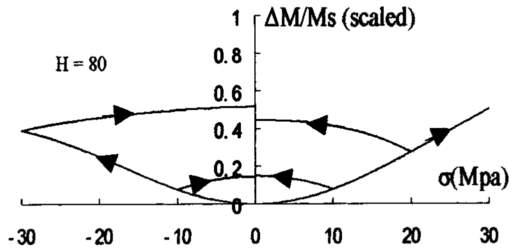

FIG. 2. Calculated result using effective field theory under conditions similar to those employed in Ref. 5.

where α is a dimensionless mean field parameter representing interdomain coupling and H is the applied field.

Then the total effective field H_{eff} including the stress contribution is

$$H_{\text{eff}}=H+\alpha M+\frac{3}{2}\frac{\sigma_0}{\mu_0}\left(\frac{d\lambda}{dM}\right)_{\sigma}(\cos^2\theta-\nu\sin^2\theta) \tag{3}$$

where θ is the angle between the axis of the applied stress σ_0 and the axis of the magnetic field H, and ν is Poisson's ratio.

Based on symmetry, an empirical model for magnetostriction can be given as

$$\lambda=\sum_{i=0}^{\infty}\gamma_i M^{2i}. \tag{4}$$

If we use an approximation to the magnetostriction by including the terms up to $i=1$, this gives

$$H_{\text{eff}}=H+\alpha M+\frac{3\gamma_1\sigma_0}{\mu_0}(\cos^2\theta-\nu\sin^2\theta)M. \tag{5}$$

Figure 2 is the calculated result based on Eq. (5) which uses the effective field theory. From this figure, we can see that the slope of the magnetization versus stress curve at zero stress is zero, which is not in total agreement with experimental results.

In many cases the stress can be included in the form of a perturbation to the magnetic field. The key to this description is to provide a means by which both magnetic field and stress can be treated similarly in the equations. However, not all magnetomechanical behavior can be explained by the effective field theory. For example, at larger stresses this approximation is no longer valid since magnetic field and stress have different effects on the magnetization.

A model theory of the changes in magnetization that a ferromagnetic material undergoes when subjected to an applied uniaxial stress has been described previously.[6] The change in magnetization on application of stress can be described by Eq. (6), in which the rate of change of magnetization with elastic energy is proportional to the displacement of the magnetization from the anhysteretic magnetization:

$$\frac{dM}{d\sigma}=\frac{1}{\varepsilon^2}\sigma(1-c)(M_{\text{an}}-M_{\text{irr}})+c\frac{dM_{\text{an}}}{d\sigma}. \tag{6}$$

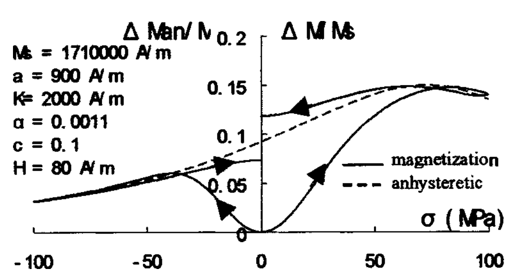

FIG. 3. The calculated variation of magnetization with stress under conditions similar to those employed in Ref. 5. The slope at zero stress is zero.

Figure 3 is a calculated result based on Eq. (6). Without the linear term, the slope of the magnetization versus stress curve must be zero at zero stress. This does not occur in practice. In other words, this model equation needs to be modified in order to give predictions that are in agreement with observations.

DEVELOPMENT OF MODEL THEORY OF MAGNETOMECHANICAL EFFECTS

The Rayleigh law, which describes hysteretic behavior in magnetization at low field strengths, can be expressed as

$$M=\chi_a H\pm\eta H^2 \tag{7}$$

where χ_a is the initial susceptibility and η is called the Rayleigh constant; the + is for positive field use, and the − for negative field for the initial magnetization curve.

Rayleigh also showed that the hysteresis loop was composed of two parabolas

$$M=(\chi_a-\eta H_-)H+\frac{\eta}{2}(H^2-H_-^2), \tag{8}$$

$$M=(x_a+\eta H_+)H-\frac{\eta}{2}(H^2-H_+^2), \tag{9}$$

where Eqs. (8) and (9) present ascending and descending portions of the loop, respectively. H_+ and H_- are the maximum fields applied.

From Eq. (7), we will have $M_+=\chi_a H_+ +\eta H_+^2$ and $M_-=\chi_a H_- -\eta H_-^2$, where M_+ and M_- are the magnetization at maximum magnetic field H_+ and H_-. Substituting these into Eqs. (8) and (9), we obtain

$$M-M_+=\chi_a(H-H_+)-\frac{\eta}{2}(H-H_+)^2, \tag{10}$$

$$M-M_-=\chi_a(H-H_-)+\frac{\eta}{2}(H-H_-)^2. \tag{11}$$

According to Brown,[7] the effect of stress on magnetization can be expressed using an equation that is very similar to the Rayleigh law. In this derivation the fractional change in volume is

$$\Delta V=\frac{V-V_0}{V_{\text{tot}}}=\alpha|\sigma|+\beta\sigma^2 \tag{12}$$

where V_0 is the original volume before any domain wall movement and V_{tot} is the total volume of the sample; α and β are constants depending on domain wall type. From this, in the simplest case of spin-up and spin-down domains, it is easily shown[8] that

$$\Delta M=2M_s\Delta V; \tag{13}$$

therefore if there is a change in volume of the domains ΔV a corresponding change of magnetization ΔM occurs. Beginning from these definitions Eqs. (7) and (8), we can see that the equivalent expression for changes in magnetization is

$$\Delta M = 2M_s(\alpha|\sigma| + \beta\sigma^2), \tag{14}$$

and this equation is true whether stress is increasing in the positive direction (tension) or the negative direction (compression). That is the important difference between this stress dependent equation and the normal field dependent Rayleigh region equation.

From this we can develop stress dependent equations for the Rayleigh region. When the stress is reduced from σ_+ along the descending branch, the equation governing this is

$$\Delta M - \Delta M_+ = 2M_s\left[\alpha(|\sigma| - |\sigma_+|) - \frac{\beta}{2}(\sigma - \sigma_+)^2\right]; \tag{15}$$

when the stress is reduced from σ_- along the ascending branch, the equation governing this is

$$\Delta M - \Delta M_- = 2M_s\left[\alpha(|\sigma| - |\sigma_-|) - \frac{\beta}{2}(\sigma - \sigma_-)^2\right]. \tag{16}$$

In other words, these curves are actually symmetric, unlike the analogous curve of magnetization versus field.

RESULTS OF CURRENT INVESTIGATION

Based on Eqs. (15) and (16), a model equation with an additional linear term has been developed:

$$\frac{dM}{d\sigma} = \frac{1}{\varepsilon^2}(1-c)(M_{\mathrm{an}} - M_{\mathrm{irr}})(\sigma \pm \eta E) + c\left(\frac{\sigma}{E} \pm \eta\right)\frac{dM_{\mathrm{an}}}{d\sigma} \tag{17}$$

where M_{an} is the anhysteretic magnetization, σ is the stress, M_{irr} represents the irreversible component of magnetization, E is the relevant elastic modulus, c describes the flexibility of the magnetic domain walls, ε has been defined previously,[6] and η is a coefficient that represents the irreversible change in the magnetization with the action of a stress.[7]

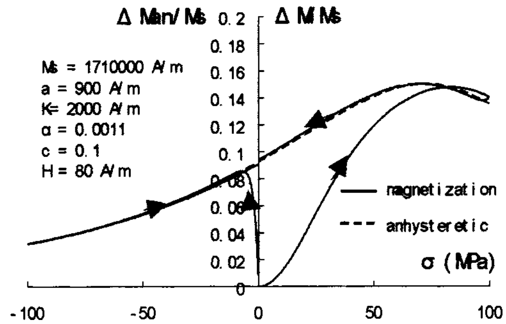

FIG. 4. The calculated variation of magnetization with stress under conditions similar to those employed in Ref. 5. The slope at zero stress is positive.

The calculated result using this model equation is shown in Fig. 4. At zero stress, the slope of the induction versus stress curve is positive, on both the positive and negative stress sides. This result agrees well with experimental data.

CONCLUSIONS

Both effective field theory and the law of approach have their limitations. Rayleigh's law gives us a basis for the solution of problems that arose due to disagreement between theory and observation. The law of approach can be modified by adding a linear component into the equation in addition to the usual quadratic term. This model equation gives results that are in better agreement with experimental observations.

ACKNOWLEDGMENT

This work was supported by the NSF Industry/University Cooperative Research Program at the Center for Nondestructive Evaluation.

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