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Unemployment, Crime and Social Insurance∗

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Abstract
We study an individual’s incentive to search for a job in the presence of random criminal opportunities. These opportunities extenuate moral hazard, as the individual sometimes commits crime rather than searching. Even when he searches, he applies less effort. We then revisit the design of optimal unemployment insurance in this environment. If the individual is more likely to remain unemployed and unpunished when he commits crime than when he searches for a job (as suggested by empirical studies), declining unemployment benefits reduce the payoff from crime relative to that from searching. Compared to the canonical models of optimal unemployment insurance, this provides a further incentive to reduce benefits over time.

Keywords: Unemployment insurance; Moral hazard; Crime; Recursive contracts.

JEL Classification: C61; D82; H55; J65; K42.

1 Introduction

It has long been accepted that, when deciding whether to commit crime, individuals weigh up the gains from the act against, amongst other things, the income they forego if they are punished. Those with higher income thus commit less crime. However, this argument is incomplete. We present a possible counterexample grounded in a dynamic model of unemployment, crime and social insurance with moral hazard.

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An unemployed agent faces a decline in benefits. Each period, he can apply effort searching for a job or can commit an opportunistic crime\(^1\). As the agent’s benefit declines, the marginal utility from accepting any given criminal opportunity increases. This static effect increases the agent’s criminality. However, declining benefits cause the continuation payoff from remaining unemployed to also fall over time. This generates a second, dynamic, effect. If he searches, he weights the future in which he remains unemployed (and hence receives said continuation payoff) by the probability of finding a job. If he commits crime instead, he weights it by the probability of being punished. Over time, these different weights mean that crime could become less attractive relative to searching for a job. As the agent’s legitimate income from benefits declines, he commits less crime. Static and dynamic effects exist under a range of assumptions. We discuss both simultaneous job-search and criminal activities, as well as crime committed by employed agents. Neither substantive affect the results of the model, although they do make it more complicated.

The existence of crime also increases the moral hazard problem identified by Shavell and Weiss \(1979\). The government is unable to observe the effort that the agent applies in searching for a job. Criminal opportunities reduce this effort (and hence increase the expected duration of unemployment) through two channels. Firstly, when the agent commits crime, he is unable to search at all. He will definitely not find a job in the current period. Secondly, as he will only accept criminal opportunities that are sufficiently profitable to increase his utility, the welfare he expects to enjoy whilst unemployed is higher. He is less concerned about finding a job, and hence applies lower effort in the periods when he does choose to search.

We consider the optimal benefit schedule in such a setting. As with previous studies, the government commits to providing the agent with a certain level of expected discounted utility. It minimises the cost of this provision. By decreasing benefits over time, it induces the agent to greater search effort. The introduction of opportunistic crime has two effects. Firstly, since the agent receives additional utility by committing crime, the government is able to offer lower benefits in every period without breaking its promise. Secondly, as the continuation payoff from unemployment declines, the agent commits less crime due to the dynamic effect. Since crime generates an additional cost to society,\(^1\)

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\(^1\)We refer to crime that has a direct financial motivation, undertaken by low-income individuals to increase their means of subsistence. This definition mainly includes property crimes such as shoplifting, burglary, robbery, theft and handling of stolen goods. This approach is adopted in the search literature on crime and unemployment (c.f. Burdett et al. 2003, 2004; Imai and Krishina 2004). This literature focuses on the opportunistic nature of a lot of property crime, rather than viewing it as a professional activity.
this provides a further incentive to front-load benefits. The continuation payoff falls more rapidly. This result holds irrespective of the instantaneous utility function.

Studying the relationship between crime and the legitimate labour market has a long history (starting with Ehrlich [1973]). Since then, an extensive literature - both theoretical and empirical - has developed. Recent work has focused on the decision to commit crime in a search and matching setting (Burdett et al. 2003, 2004, Huang et al. 2004, Engelhardt et al. 2008). Like our work, these contributions consider criminal opportunities that arrive stochastically to individuals. There are, however, differences. Unemployment benefits are constant, but wage offers vary. We endogenise the benefit schedule, but all employed individuals receive the same wage. Additive separability of legitimate and illegal income is assumed. As we shall see, this proves less innocuous than hitherto imagined. Finally, since agents do not apply search effort, these papers assume that they can simultaneously commit crime and look for a job. In this sense, our contribution harks back to Ehrlich [1973] or Block and Heineke [1975]. Accepted criminal opportunities are time consuming, and so the agent must choose between searching and crime.

Empirical studies into crime and unemployment abound. These invariably show that there is a negative relationship between earnings per worker and crime (Grogger [1998], Raphael and Winter-Ebmer 2001, Gould et al. 2002). Our model supports this assertion. Increases in the wage workers receive do indeed reduce criminality. However, changes in unemployment benefits are a different matter. Surprisingly, little work has been devoted to this. One exception is a recent study that attempts to estimate the effect of not receiving benefits on different categories of crime (Fougère et al. 2009). Their results are inconclusive. Whilst some crimes do increase, for others the effect is insignificant or may even lead to a decline. These latter crimes tend to be more opportunistic in nature, for example shoplifting or receiving stolen goods. Our results may go some way towards explaining their findings.

Another example reviews the effect of a policy that toughened the eligibility criteria for unemployment benefits in the United Kingdom (Machin and Marie 2006). Comparing criminal activity before and after the policy was introduced, the authors did indeed find that crime increased. This is indicative of the static effect. As the policy change had been announced some time earlier, unemployed individuals would have already taken it into account when considering future payoffs. Expecting their benefits to decline in the future, the dynamic effect suggests that they would have begun to reduce their criminal activity ex ante. As their current benefits were unaffected, the static effect was zero. When the policy was finally introduced, the dynamic effect would have been zero, as
the decline was already expected. The static effect, on the other hand, would be large, explaining the rise in crime that the authors report.

For falling continuation payoffs from unemployment to reduce criminality, we require that a searching agent is more likely to find a job than a criminal agent is to be punished. Empirical estimates support this claim (Imrohoroglu et al. 2004, Engelhardt et al. 2008). They find that the probability of finding a job is significantly higher than the probability of being punished.\(^2\) Moreover, there are intrinsic biases in the estimates. The probability of finding a job incorporates all those who are unemployed, including criminals. The estimate conditional on searching should be higher. Similarly, estimates of the probability of punishment are based upon police clearance rates. However, a large proportion of crimes are not reported to the police. We would therefore expect the true likelihood of being caught to be lower than the empirical estimates. One exception to this criticism estimates a structural model to derive these values (Engelhardt 2010). The estimates are more stark, and suggest that the probability of finding a job may be an order of magnitude higher over one month than the probability of a criminal being punished.

Whilst unemployment benefits provide an important safety net to those who lose a job, it has often been suggested that it reduces the incentive to search for a job (for a discussion, see Ehrenberg and Oaxaca 1976). The optimal unemployment insurance literature considers the most cost-effective way to provide a safety net, whilst mitigating the problems of reduced search effort (Shavell and Weiss 1979). It prescribes that benefits are gradually reduced over time. By doing so, an unemployed agent faces a declining continuation payoff to remaining unemployed. This provides them with a greater (and increasing) incentive to apply effort to search for a job. The approach has been shown to be desirable, even incorporating its funding (Hopenhayn and Nicolini 1997) or its consequences for voters (Pollak 2007).

We adopt a model similar to that of Shavell and Weiss, but expand it to include outside criminal opportunities as in the search literature on crime. To the best of our knowledge, we are the first to attempt to do this. In many advanced economies, committing crime more than participating in informal sector activities is the main way for low-income individuals facing liquidity constraints to finance their expenditures. Foley 2011 finds that committing crime to smooth consumption patterns between welfare payments is a recurrent phenomenon in the United States among welfare beneficiaries. The Ministry of Justice and Department for Work and Pensions (2011) in the United King-

\(^2\)One exception is Burdett et al. 2004 who use a far lower probability of finding a job than comparable papers.
dom finds that about one-third of unemployment benefit claims are made by offenders and that about 70 per-cent of offenders committing theft and handling of stolen goods offences were claiming benefits at some point in the month before their sentence date.

Some have, however, considered the problem of optimal unemployment insurance when individuals access informal labour markets to compensate for their income loss during unemployment. This seems particularly relevant for developing countries, with large shares of the informal sector relative to the official economy. Pavoni 2007 suggests that concerns about unemployed individuals participating in the informal labor market justify the provision of a constant nonzero minimum level of benefits to those individuals in long-term unemployment. Álvarez Parra and Sánchez 2009 derive the nonzero lower bound as an outcome of the optimal contract when the return from activities in the informal labor market is accounted for by the planner. The nonzero lower bound has the effect of smoothing consumption for long-term unemployed as the income from labor in the informal sector compensates for the loss of unemployment insurance when benefits reach zero under the optimal contract. We find a similar result in the solution to the optimal unemployment contract with crime. A nonzero lower bound on the consumption of long-term unemployed is implicit in our model as they always have access (in expectations) to income from criminal opportunities.

Previous work has also investigated optimal sanctioning of benefits following monitoring of search effort (Boone et al. 2007). The optimality in their contribution comes not from varying benefits over time, but from the choice of the monitoring technology and sanction for unacceptable search effort. We adopt the opposite approach, taking the monitoring technology and sanction as given, and instead focusing on using benefits to influence both search effort and criminality simultaneously. Monitoring is costly, whereas adjusting the benefit schedule received by the agent is relatively cheap to do.

The remainder of the paper is organised as follows. Section 2 presents a model of unemployment insurance with opportunistic crime. Section 3 discusses how the agent responds to the benefits on offer. Section 4 reports results for constant benefits. This provides a simpler environment in which to derive analytical results, and provides a point of comparison with the previous literature. Section 5 considers optimal unemployment insurance. The paper then turns to numerical methods. Sections 6 provides details of the calibration and computational techniques. Section 7 then illustrates numerically the main theoretical results of the paper. Section 8 concludes. Major proofs and details of the computational method are given in the appendices.
2 The Economic Environment

We present an infinite-horizon model of unemployment insurance, similar to that of Shavell and Weiss [1979]. The government is committed to providing an unemployed agent with expected lifetime utility $V$. It provides a stream of unemployment benefits $\{b_t\}_{t=0}^\infty$ in order to meet this commitment.

We introduce criminal activity into this model, as in the search literature on crime [Burdett et al. 2003, 2004; Imai and Krishna 2004]. Each period, the agent receives a private criminal opportunity denoted by $c_t \geq 0$. Each $c_t$ is i.i.d., and drawn from some distribution $F(c)$ with density $f(c)$. The agent then chooses whether to search for work ($s_t = 1$) or accept the opportunity ($s_t = 0$). His instantaneous utility is $u(y_t, (1 - s_t)c_t)$ where $y_t$ is the agent’s legitimate income and $(1 - s_t)c_t$ is the income from an accepted criminal opportunity. We assume that utility is increasing and concave in both arguments, and that the Inada conditions hold.

When searching for a job, the agent exerts effort $a_t \geq 0$. He receives instantaneous utility $u(b_t, 0) - a_t$ and finds a job with probability $p(a_t)$. $p(\cdot)$ is increasing and concave. No effort implies no job ($p(0) = 0$), but neither is a job ever guaranteed ($p(\infty) < 1$). Should his search be successful, the agent receives a wage, $y_t = w$, and hence utility, $u(w, 0)$, in all future periods. The model is one of imperfect information, as the government cannot observe either $c_t$, $s_t$ or $a_t$.

If instead the agent accepts the criminal opportunity, he receives instantaneous utility $u(b_t, c_t)$. However, he is unable to search for a job. He may also be punished, which begins in the following period with probability $\phi$. A criminal who is punished is no longer entitled to welfare under the current scheme.

The agent seeks to maximise his discounted utility:

$$\max_{\{a_t, s_t\}_{t=0}^\infty} \mathbb{E}_0 \left[ \sum_{t=0}^\infty \beta^t \{u(y_t, (1 - s_t)c_t) - s_t a_t\} \right].$$

(1)

where $\beta$ is the discount factor and $\mathbb{E}_0[\cdot]$ denotes the expectation conditional on the agent’s information in $t = 0$. Each period, an unemployed agent chooses whether to search. If he searches ($s_t = 1$), he then chooses optimal search effort and receives $u(b_t, 0) - a_t$. Otherwise (if $s_t = 0$) he receives $u(b_t, c_t)$. Uncertainty exists over whether the agent

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3This approach captures the opportunistic nature of crime. As we will show, the agent commits crime in some periods, but not in others. An alternative timing protocol could consist of the agent choosing whether to search for a job before observing his criminal opportunity. This would imply that his decision would be based upon the expected value from criminal income. In equilibrium, the agent either always searches or always commits crime.
receives a job offer when searching, gets punished when committing crime, as well as the value of future criminal opportunities.

The government aims to minimise the expected discounted cost of providing unemployment insurance, incorporating the potential criminal activities of the agent. Crime not only provides an alternative source of income, it also inflicts an external cost upon society (c.f. Becker 1968, Shavell 1987). The size of this damage is given by $d(c_t, \delta)$. More intensive crime inflicts more damage ($d(\cdot) > 0$). $\delta$ reflects how destructive crime is ($d(\cdot) > 0$) and will be used in later sections to derive comparative statics results. The government’s problem is:

$$\min_{\{b_t\}_{t=0}^{\infty}} \mathbb{E}_0 \left[ \sum_{t=0}^{\infty} \beta^t \{b_t + (1 - s_t)d(c_t, \delta)\} \right],$$

subject to the expected lifetime utility of the agent given by (1) being greater than $V$.

During each period of unemployment, the government provides $b_t$. If the agent commits crime ($s_t = 0$), the government also suffers the cost of this activity. The government is unable to observe the agent’s search effort or his criminality (there is moral hazard), and hence forms expectations over these actions.

![政府设定福利计划：

政府设定福利计划：

1. 政府设定福利计划：

2. 期望失业期的福利

3. 被雇用：

4. 领取福利

图1：模型的时间流程。

图1：模型的时间流程。政府承诺福利计划，$\{b_t\}_{t=0}^{\infty}$. 在每个失业时期，代理人会得到一个犯罪机会，$c_t \geq 0$. 当代理人犯罪（$s_t = 0$），政府也承担这一活动的成本。政府无法观察到代理人的搜索努力或其犯罪行为（存在道德风险），因此需要对这些行为形成期望。
which he can accept ($s_t = 0$) or reject ($s_t = 1$). If he accepts, he commits the crime, receives $u(b_t, c_t)$ and either remains unemployed or gets punished (an absorbing state). If he rejects, he exerts effort searching for a job and receives $u(b_t, 0) - a_t$. In this case, he is either successful (another absorbing state), or he remains unemployed. We proceed to solve the model by backwards induction, starting with the agent’s response to the announced benefits schedule.

Note that both employment and punishment are absorbing states in this environment. Allowing for movement back into unemployment will have no qualitative effect the agent’s decisions. Upon becoming unemployed again, his expected discounted utility will simply be $V$. With exogenous transition probabilities, this is just a constant. Employment is, nevertheless, often modelled as an absorbing state in this literature (c.f. Shavell and Weiss 1979; Hopenhayn and Nicolini 1997; Pavoni 2007). This simplifies the government’s decision substantially, as it does not need to worry about future promises of insurance when minimising the cost of its provision.

3 Agent Decisions

The agent’s utility maximisation problem (1) can be represented recursively by the following system of Bellman equations:

\begin{align}
V^e &= \frac{u(w, 0)}{1 - \beta}, \\
V^u_t &= \max_{a_t \geq 0} \left\{ u(b_t, 0) - a_t + \beta p(a_t) V^e + \beta (1 - p(a_t)) V_{t+1} \right\}, \\
V^c_t &= u(b_t, c_t) + \beta \phi V^p + \beta (1 - \phi) V_{t+1}, \\
V_t &= \max_{s_t \in \{0, 1\}} \left\{ s_t V^u_t + (1 - s_t) V^c_t \right\}, \\
V_{t+1} &= E_t \left[ V_{t+1} \right].
\end{align}

Equation (4) addresses search effort, conditional on searching for a job. The agent receives his benefits, but suffers the cost of effort. With probability $p(a_t)$ he finds a job, and receives value of employment, $V^e$, given by (3). Otherwise he remains unemployed, and receives a continuation payoff $V_{t+1}$. The agent maximises this payoff over search effort.

Equation (5) gives the payoff from committing a crime in the current period. The agent receives his benefits from the government, as well as income from accepting the criminal opportunity. In the following period, with probability $\phi$ he gets punished, and
receives continuation payoff $V^p$. With probability $1 - \phi$ he remains unpunished, and receives the same continuation payoff as an unemployed agent.

Next, (6) addresses the decision whether to search for a job or accept a criminal opportunity. The agent will search if and only if $V^u_t \geq V^c_t$.

Finally, (7) states the continuation payoff from remaining unemployed. The agent knows that he will choose $s_{t+1}$ to maximise his payoff next period, given the benefit schedule provided by the government. He will thus receive $V_{t+1}$. However, he does not know the value of the criminal opportunity, and so forms an expectation.

The agent has two decisions to make. Firstly, conditional on searching for a job, he chooses search effort. His optimal effort, $a^*_t$, is derived from the first order condition from (4):

$$\beta p'(a^*_t) (V^e - V_{t+1}) \leq 1.$$  

The form of this first order condition is similar to Shavell and Weiss 1979. The left hand side represents the marginal benefit of effort. This is proportional to the utility gain the agent enjoys when moving from unemployment into work, $V^e - V_{t+1}$, starting from the following period. The right hand side gives the (constant) marginal cost. Since $p(a_t)$ is concave, the agent will apply more effort if the payoff from getting a job increases relative to that from remaining unemployed.

Note that, since the marginal cost of effort is positive, there exists a $V^{max} < V^e$ such that $a^*_t = 0$ for all $V_{t+1} \geq V^{max}$:

$$V^{max} = V^e - \frac{1}{p'(0)}.$$  

When $V_{t+1} = V^{max}$, there still exists a positive marginal benefit to the agent of applying effort to searching for a job: the utility gain is positive. Unfortunately, this marginal benefit is insufficient to compensate for the marginal cost, and the agent optimally applies no effort to searching. We will refer to $V^{max} - V_{t+1}$ as the adjusted utility gain from employment throughout the paper. Whilst it tracks the actual utility gain one-for-one, it has the added benefit that the agent will apply effort if and only if the adjusted utility gain from employment is positive.

Consideration of (8) leads us to our first result:

**Proposition 1 (Crime exacerbates Moral Hazard)** The presence of criminal opportunities extends the expected duration of the agent’s unemployment relative to the canonical model.

Relative to the canonical model, the determinants of the continuation value from
remaining unemployed, \( V_{t+1} \), are different. If the agent receives an unprofitable criminal opportunity in the following period, he will search for a job and receive \( V_{t+1}^u \) (by (6)), as per the model without crime. However, if the opportunity is sufficiently profitable to induce him to accept it, he must receive a higher payoff, \( V_{t+1}^c \geq V_{t+1}^u \). The expected payoff from remaining unemployed is therefore higher. Even if the agent does choose to search, he applies lower effort in equilibrium. Moral hazard is exacerbated. In this sense, crime can lead to unemployment, as the agent is less likely to find a job.

Secondly, whether the agent searches at all depends upon the criminal opportunity he receives. By committing crime, the agent gives up future utility. He will not get a job, and may get arrested. In order to compensate for this, the criminal opportunity must be sufficiently large:

**Proposition 2 (Threshold Criminal Opportunities)** For any benefits schedule, there exists a sequence of threshold criminal opportunities, \( \{c^*_t\}_{t=0}^{\infty} \), each defined by:

\[
u(b_t, c^*_t) \equiv u(b_t, 0) - a^*_t + \beta p(a^*_t) V^c + \beta[\phi - p(a^*_t)] V_{t+1} - \beta \phi V^p, \tag{10}
\]

such that the agent will search for a job in period \( t \) if and only if \( c_t \leq c^*_t \).

**Proof.** See Appendix A.

This period’s opportunity has no impact upon the continuation payoff conditional on not finding a job or being punished, \( V_{t+1} \). It therefore does not affect \( V_{t}^c \) (by (4)). Conversely, by (5) higher criminal opportunities increase \( V_{t}^c \). The agent enjoys the payoff from his crimes, \( u(b_t, c_t) \). From (6), the agent will only accept an opportunity if \( V_{t}^c \geq V_{t}^u \). \( c^*_t \) is the opportunity that makes the agent indifferent. Any \( c_t > c^*_t \) raises \( V_{t}^c \) above \( V_{t}^u \), and induces the agent to accept. Likewise any \( c_t < c^*_t \) lowers \( V_{t}^c \) below \( V_{t}^u \) and causes the agent to search.

Careful consideration of (10) provides a surprising result alluded to in the introduction. Casual intuition would suggest that, as unemployment benefits decline over time, the agent would commit more crime. With lower legitimate income in each period, the marginal utility of illegal income increases. We call this the static effect. This need not be the case in a dynamic setting, as future considerations may offset this incentive (a dynamic effect).

**Proposition 3 (Criminality and the Continuation Payoff)** There exists \( \hat{V} \) such that the agent’s criminality decreases as the continuation payoff from unemployment declines if and only if:

\[ V_{t+1} \leq \hat{V}. \]
Proof. See Appendix B.

The probability that the agent commits a crime in any period of unemployment is $1 - F(c^*_t)$, the probability of receiving a worthwhile criminal opportunity. So for his criminality to decrease as the continuation payoff from unemployment shrinks, we need that $c^*_t$ increases. We can write:

$$- \frac{\partial c^*_t}{\partial V_{t+1}} = - \frac{\beta(1 - p(a^*_t))}{u_c(b_t, c^*_t)} + \frac{\beta(1 - \phi)}{u_c(b_t, c^*_t)}.$$ (11)

Reducing the continuation payoff has an impact upon the threshold criminal opportunity through two channels. The expected discounted future utility from searching for a job is $p(a^*_t)V^e + (1 - p(a^*_t))V_{t+1}$. A lower continuation payoff reduces this, and the risk of remaining jobless looms larger. For any given effort level, this static effect makes crime more appealing, and tends to lower the threshold opportunity (the first term in (11)). However, reducing $V_{t+1}$ also reduces $V^c$ for the same reason. The expected discounted future utility from crime is $\phi V^p + (1 - \phi)V_{t+1}$. This dynamic effect tends to increase the threshold opportunity (the second term in (11)). If the agent is more likely to get a job when searching than get punished after committing crime (i.e. if $p(a^*_t) > \phi$), his expected discounted future utility from crime will fall faster than that from searching. The agent’s criminality will decline.

In order for the agent to discount the likelihood of remaining unemployed whilst searching heavily, he must be induced to a reasonably high search effort. From (8), this requires that the continuation payoff from unemployment must be sufficiently smaller than that of employment. This assumption fits with the observed transition probabilities, as noted in the introduction.

The results of this section are robust to a range of alternative assumptions. The agent bases his decision on a stationary value of employment. This could incorporate criminal opportunities taken whilst working, without affecting the main results. Since wages are constant, the employed agent’s threshold criminal opportunity would also be constant. Allowing on-the-job crime would simply raise $V^e$, without affecting any other aspect of the model.

Empirical estimates of the different job-finding rates between criminals and non-criminals also suggest that the results would hold even if the agent could commit crime and search simultaneously. Suppose that if the agent does not commit crime he applies effort $a^N_t$, whereas if he commits crime he applies $a^C_t$. Since the agent may be punished as a result of criminal activity (superseding any job offer) the marginal benefit to search
effort is lower: $a_t^C < a_t^N$. In this case, Proposition 3 would require that:

$$[1 - p(a_t^N)] - (1 - \phi)[1 - p(a_t^C)] < 0$$

$$\iff p(a_t^N) > \phi + p(a_t^C)(1 - \phi).$$

Engelhardt 2010 reports that the monthly $p(a_t^N) \geq 0.378$ at the 95% confidence level, whereas $p(a_t^C) \leq 0.179$ and $\phi \leq 0.014$. For these values, $\phi + p(a_t^C)(1 - \phi) = 0.190$, well below the job finding rate for an agent who does not commit crime.

4 Constant Unemployment Insurance

The literature on unemployment and crime is dominated by the assumption of constant benefits (c.f. Burdett et al. 2003, 2004, Engelhardt et al. 2008, Engelhardt 2010). This provides a useful benchmark to compare some of the analytical results deriving from our framework. The agent’s problem is still described by (1) - (6), but with the additional restriction that $b_t = b$. In this case, however, the problem is stationary. The agent’s optimal choices $a^*_cb$ and $c^*_cb$ (cb denotes constant benefits), along with the optimal continuation payoff $V_{cb}$ are constant, and given by the solution to the following system of simultaneous equations:

$$\beta p'(a^*_cb) (V^e - V_{cb}) = 1, \quad (12)$$

$$u(b, c^*_cb) = u(b, 0) - a^*_cb + \beta p(a^*_cb)V^e + \beta(\phi - p(a^*_cb))V_{cb} - \beta \phi V^p, \quad (13)$$

$$V_{cb} = \frac{\int_{c=c^*_cb}^\infty u(b, c)f(c)dc + F(c^*_cb)u(b, c^*_cb) + \beta \phi V^p}{1 - \beta(1 - \phi)}, \quad (14)$$

The first two equations are equivalent to (8) and (10). The third equation defines the continuation payoff from remaining unemployed. With probability $1 - F(c^*_cb)$ a sufficiently good opportunity comes along to induce the agent to commit crime. He expects to receive $\int_{c=c^*_cb}^\infty u(b, c)\frac{f(c)}{1-F(c^*_cb)}dc$ in the current period and $(1 - \phi)V_{cb} + \phi V^p$ in the future. Otherwise (with probability $F(c^*_cb)$) the agent receives $V^u$. Substituting for $V^u$ using (4) and (13), and then solving for $V_{cb}$ yields (14).

Suppose that the government increases the unemployment benefit it offers. Higher benefits increase the continuation payoff from remaining unemployed for the obvious reason. As a result, if the agent does not receive an acceptable criminal opportunity, he lowers his search effort. The effect on his criminality is interesting:

**Lemma 1 (Criminality and Benefits)** If $\frac{\partial^2 u}{\partial b \partial c} \geq 0$, the agent’s criminality declines
as benefits declines:

\[ -\frac{\partial c^{*}_{cb}}{\partial b} > 0. \]

The proof follows a simple application of the Implicit Function Theorem to (12) - (14). This yields that \(-\frac{\partial c^{*}_{cb}}{\partial b} > 0\) if and only if:

\[
\left\{ u_{b}(b, c^{*}_{cb}) - u_{b}(b, 0) \right\} - \beta [\phi - p(a^{*}_{cb})] \frac{\int_{c^{*}_{cb}}^{\infty} u_{b}(b, c) f(c) dc + F(c^{*}_{cb}) u_{b}(b, c^{*}_{cb})}{1 - \beta (1 - \phi)} > 0. \tag{15}
\]

The first term in parentheses in (15) relates to the static effect. It compares the marginal utility of benefits with and without accepting the threshold criminal opportunity. If \(\frac{\partial^{2}u}{\partial b \partial c} < 0\) then the term is negative. A fall in benefits increases the static incentive to commit crime. Conversely, if \(\frac{\partial^{2}u}{\partial b \partial c} \geq 0\) as, for example, would be the case with additive separable utility, then falling benefits weakly reduce the static incentive to commit crime.

The second term in (15) describes the dynamic effect. The fall in benefits reduces the continuation payoff from remaining unemployed. Following (11), this increases the threshold criminal opportunity. The expected continuation payoff from crime, \(\phi V^{p} + (1 - \phi)V_{cb}\), decreases faster than the expected continuation payoff from searching, \(p(a^{*}_{cb})V^{e} + (1 - p(a^{*}_{cb}))V_{cb}\), as \(1 - \phi > 1 - p(a^{*}_{cb})\). This makes crime less appealing, and, at the margin, tends to increase \(c^{*}_{cb}\).

Summarising, if \(\frac{\partial^{2}u}{\partial b \partial c} \geq 0\), both static and dynamic effects lead to a reduction in criminality. Crime is (weakly) less appealing in terms of instantaneous utility, and (strictly) less appealing in terms of future discounted utility. If \(\frac{\partial^{2}u}{\partial b \partial c} < 0\), the two effects counteract one another. Whilst crime is more appealing immediately, it is still less appealing in terms of future utility. Criminality may therefore increase or decrease depending upon the magnitude of each effect.

Whilst Lemma 1 seems to conflict with a lot of the empirical work, it is worth noting that this literature tends to focus on workers’ earnings. Our model is entirely consistent with their findings:

Lemma 2 (Criminality and Wages) Criminality declines as wages from employment increase:

\[ \frac{\partial c^{*}_{cb}}{\partial w} > 0. \]

As wages increase, the payoff from getting a job rises relative to remaining unemployed. This directly increases the expected payoff from searching. Moreover, the marginal benefit of search effort increases, raising effort and causing the agent to place a higher probability on a successful search. This further increases the payoff from searching, reducing the criminality of the agent.
We can also assess the impact of anti-crime policies in this setting. Consider an increase in the probability of conviction, $\phi$. If $V_{cb} > V^p$, this lowers the continuation payoff to crime. The agent is less willing to accept criminal opportunities. $c_{cb}^*$ increases in (13). Although the increase in $c_{cb}^*$ raises $V_{cb}$ in (14), it is discounted more heavily due to the high likelihood of being punished. Overall, $V_{cb}$ declines. In (12), the optimal search effort thus increases.

More severe punishment (a fall in $V^p$) will have a similar effect. The agent will be less willing to commit crime, as the continuation payoff from being caught has fallen in (13). It also reduces the expected continuation payoff from crime in (14). Once again, since unemployment is less comfortable than previously, optimal effort increases.

5 Optimal Unemployment Insurance

We now consider optimal unemployment insurance. The government announces the benefit schedule prior to any decisions by the agent. Although it does not observe these decisions, it knows how the agent will react to its policy. It is thus in a position to not only choose benefits in each period, but also (implicitly) the agent’s effort and threshold criminal opportunity.

The agent’s reaction to the benefits schedule allow us to be more explicit about three elements of the model that are important for the government’s decision. The probability of remaining unemployed:

$$P(\text{Unemployed in } t + 1 | \text{Unemployed in } t) = (1 - F(c_t^*)) (1 - \phi) + F(c_t^*) (1 - p(a_t^*))$$

$$= 1 - \phi + F(c_t^*) (\phi - p(a_t^*)) .$$

The probability that the agent receives a worthwhile criminal opportunity in period $t$ is $1 - F(c_t^*)$. In this case, he remains unemployed with probability $1 - \phi$. Otherwise, he remains unemployed with probability $1 - p(a_t^*)$.

The government does not observe the criminal opportunity that the agent receives. However, it knows that the agent only accepts opportunities above $c_t^*$. The expected payoff from an accepted opportunity is:

$$E[u(b_t, c) | c \geq c_t^*] = \int_{c = c_t^*}^{\infty} u(b_t, c) \frac{f(c)}{1 - F(c_t^*)} dc.$$
Similarly the expected damage caused by an accepted criminal opportunity is:

\[
\mathbb{E}[d(c, \delta)|c \geq c^*_t] = \int_{c=c^*_t}^{\infty} d(c, \delta) \frac{f(c)}{1-F(c^*_t)} dc.
\]

The government never observes the criminal opportunity received by the agent, nor whether he chose to accept it. So, in period \(t\), it has the same information about the agent’s welfare as the agent himself had in period \(t-1\). The government’s expectation of the agent’s discounted lifetime utility from being unemployed in period \(t\) is simply \(V_t\). Following the recursive contracts literature (Spear and Srivastava 1987; Phelan and Townsend 1991), the problem of minimising the expected discounted cost to the government of providing these benefits given by (2) can be represented recursively as follows:

\[
C(V_t) = \min_{b_t, a^*_t, c^*_t, V_{t+1}} \left\{ b_t + \int_{c=c^*_t}^{\infty} d(c, \delta) f(c) dc + \beta[1 - \phi + F(c^*_t)(\phi - p(a^*_t))]C(V_{t+1}) \right\}
\]

subject to:

\[
V_t \leq F(c^*_t) \left\{ u(b_t, 0) - a^*_t + \beta p(a^*_t)V^e + \beta(1-p(a^*_t))V_{t+1} \right\} + (1 - F(c^*_t)) \left\{ \int_{c=c^*_t}^{\infty} u(b_t, c) \frac{f(c)}{1-F(c^*_t)} dc + \beta(1-\phi)V_{t+1} + \beta\phi V^p \right\}
\]

(16) and (17)

where \(V_1 = V^u\), the amount promised by the government upon becoming unemployed.

Equation (16) describes the cost of providing unemployment insurance. In the current period, the government provides benefit \(b_t\). It only suffers damage from crime if the agent chooses to accept his opportunity rather than searching for a job (if \(c_t > c^*_t\)). In the following period, assuming that the agent neither finds a job or gets punished, the government is still committed to providing \(V_{t+1}\) expected lifetime utility. Following the work of Hopenhayn and Nicolini 1997, we assume that \(C(\cdot)\) is increasing and convex.

Equation (17) is the promise-keeping constraint. It requires that the unemployed agent’s expected discounted utility is at least \(V_t\). With probability \(F(c^*_t)\), the criminal opportunity is insufficient to induce the agent to commit crime. He receives \(V^u_t\), and apply effort \(a^*_t\) to finding a job. Otherwise, he commits crime and receives (in expectation) \(\int_{c=c^*_t}^{\infty} V^c f(c) dc\). Constraints (8) and (10) incorporate the agent’s responses into the policy decision.
The solution to \((16)\) is as follows:

**Proposition 4 (Optimal Unemployment Insurance)**  For any instantaneous utility function, \(u(y_t, (1 - s_t)c_t)\), if \(V_t \leq \tilde{V}\) then the optimal unemployment insurance schedule involves reducing the continuation payoff from unemployment over time.

**Proof.** See Appendix C. □

The intuition behind this result is very similar to that of the canonical model. By paying the majority of the promised utility in the early stages of unemployment, the government creates a declining expected continuation payoff from remaining unemployed. Since the agent knows that his situation will only get worse in the future, he applies more effort to search for a job. When crime is introduced, a second channel opens up through the dynamic effect. Reducing the continuation payoff from unemployment causes the payoff from crime to decline faster than the payoff from searching. The agent foregoes criminal opportunities that would previously have been attractive, in order to try to get a job. Reducing the continuation payoff thus reduces crime as well.

The impact of the dynamic effect on the optimal unemployment insurance schedule with opportunistic crime can be further drawn out by considering how the schedule changes when the cost of crime (as measured by \(\delta\)) increases:

**Lemma 3 (Crime Increases Front-Loading of Welfare)**  For any instantaneous utility function, \(u(y_t, (1 - s_t)c_t)\), when the damage associated with opportunistic crime increases, the continuation payoff from unemployment declines.

When the damage from crime increases, its significance in the cost of supporting the unemployed agent increases. The government becomes more inclined to use the benefit schedule as an anti-crime policy tool. Taking advantage of the dynamic effect, it reduces the continuation payoff more rapidly. This tends to lower the agent’s utility in every period, as well as reduce his utility from crime. The agent becomes worse off. The government has breached its promise to provide the agent with a given amount of welfare. To adjust for this, whilst still reducing crime, the government further front-loads welfare. In the early stages of unemployment, it is more generous than it had previously been. This enables it to lower the agent’s welfare more rapidly, eventually reducing the continuation payoff below that of the original scheme.

The static effect does create added complications for optimal benefit payments, as we saw in Lemma 1. On the one hand, falling benefits lower the continuation payoff over time, as already mentioned. However, it also potentially increases the utility gain from a given criminal opportunity. Legitimate income is lower, so the marginal utility of crime
may be higher. This muddies the water regarding whether benefits should optimally decline. However, if we make an additional assumption about the form of the utility function, we can be more explicit:

**Proposition 5 (Optimal Unemployment Insurance with Additive Separability)**

If $V_t \leq \tilde{V}$ and utility from legitimate income and crime is additively separable, then the optimal unemployment insurance schedule involves reducing benefit payments over time.

**Proof.** See Appendix D. □

If utility from legitimate and illegal income is additively separable, declining benefits will have no impact upon the marginal utility of crime. There is no static effect. Only the dynamic effect remains. The optimal policy will still lead to the continuation payoff from unemployment falling, increasing the agent’s incentive to search for a job and unambiguously lowering his criminality.

### 6 Calibration and computation

Table I summarizes the calibration of the parameters for the numerical analysis. Unless otherwise stated, parameters will be maintained at these baseline values. Instantaneous utility is given by $u(y_t, (1 - s_t)c_t) = y_t^{1-\sigma} + \frac{[(1-s_t)c_t]^{1-\sigma}}{1-\sigma}$. Following Hopenhayn and Nicolini 1997, we set $\sigma = 0.5$. The probability of finding a job is $p(a) = 1 - \exp(-ra)$. The parameter $r > 0$ is calibrated so that $p(a^*_t) = 0.1$ in the stationary equilibrium without unemployment insurance. The discount rate is $\beta = 0.999$, and the weekly wage is $w = 100$.

Criminal opportunities are specified over a finite and discrete grid of $n = 4001$ equally spaced points; $c_i, i = 1, .., n$. It varies from zero ($c_1 = 0$) to twice the weekly wage ($c_n = 200$). The probability of receiving a criminal opportunity is given by a discretised exponential probability density function:

$$q_i = \frac{f(c_i|\mu)}{\sum_{j=1}^{n} f(c_j|\mu)},$$

(18)

with $f(c_i|\mu) = \frac{1}{\mu} e^{-\frac{c_i}{\mu}},$ and $i = 1, ... , n$. An increase in $\mu$ leads to an increase in the probability of earning high income from crime. As $\mu$ becomes infinitely large the PDF of each criminal opportunity tends towards the uniform distribution. In the numerical analysis we examine the sensitivity of the results to different values of $\mu$.

---

5 This assumption is ubiquitous in the search and matching literature, and is employed by Burdett et al. 2003, 2004, Engelhardt et al. 2008 and Engelhardt 2010.
When using the job-finding probability function of Hopenhayn and Nicolini [1997], \( V_{\text{max}} = V^e - \frac{1}{r\beta} \) from (9). The adjusted utility gain from employment is therefore:

\[
V_{\text{max}} - V_{t+1} = V^e - V_{t+1} - \frac{1}{r\beta}
\]

For given \( r \) and \( \beta \), the adjusted utility gain from employment and actual utility gain from employment move one-to-one. However, when \( V_{\text{max}} - V_{t+1} = 0 \), then \( a^* = 0 \).

The benchmark conviction rate is set to \( \phi = 1.56E - 4 \). This is derived starting from the annual unemployment incarceration rate of 18.5 per cent and the average prison duration (APD) of 12 months for property crime reported by Imrohoroglu et al. [2004]. The weekly probability of being convicted is then calculated as 0.0039. Since we treat punishment as an absorbing state, we then rescale \( \phi \) so that the expected punishment from 12 months in prison with a conviction probability of 0.0039 is the same as the that for a life sentence (an APD of 25 years). The benchmark conviction rate \( \phi \) is thus measured as 4 per cent \( (= \frac{1}{25}) \) of this value. We also examine the sensitivity of the results when the APD is 6, 9, 15 and 18 months by rescaling appropriately.

The damage from crime is proportional to the utility from a criminal opportunity:

\[
d(c_i) = \delta^{\frac{\nu}{1-\eta}}, \quad i = 1, 2, ..., n.
\]

In the benchmark computation we assume \( \delta = 1 \). To derive this, we first collect an estimate of the average social cost of property crime reported by Cohen and Piquero [2009]. To link this to the criminal’s payoff, we then compute the average value of stolen property per crime from the FBI’s Uniform Crime Report [Federal Bureau of Investigation 2010]. Comparing these two estimates suggests that the damage from property crime is of the same order as the criminal’s payoff. Again, different values for the factor \( \delta \) are used to examine the sensitivity of the results to changes in the cost of crime.

The solution to the model under constant unemployment insurance is derived by iterating the agent’s Bellman equations (4) - (6), given (3) and (7). Starting from an initial arbitrary value of \( V \), \( r \) is determined from the FOC (8) using the stationary equilibrium value of the search probability when \( b = 0 \). This is then replaced in \( p (a^*_t) \) to determine the stationary equilibrium search intensity \( a^*_t \), which in turn can be used to compute \( V^u \) and \( V^c \) in (4) and update \( V \) in (7). Once convergence is achieved, the stationary equilibrium threshold income from crime is computed from (10). This allows

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6The probability of being convicted at some point during the year (0.185) is equal to the probability of being convicted in the first week (\( \phi \)) plus the probability of escaping in the first week, but being convicted in the second ((1 - \( \phi \))\( \phi \)), plus the probability of escaping in the first two weeks, but being convicted in the third ((1 - \( \phi \))^2\( \phi \)) etc. Summing the geometric progression over all 52 weeks yields 

\[
\frac{\phi(1-\phi)^{52}}{1-(1-\phi)^2} = (1-\phi)^{53} = 0.185.
\]

Solving yields \( \phi = 0.0039 \).
Table 1: Benchmark calibration of the model parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.999</td>
<td>Discount rate</td>
</tr>
<tr>
<td>$w$</td>
<td>100</td>
<td>Wage</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.5</td>
<td>Relative risk aversion unemployed</td>
</tr>
<tr>
<td>$\phi$</td>
<td>$1.56E - 04$</td>
<td>Incarceration prob., based on 12 months APD</td>
</tr>
<tr>
<td>$\delta$</td>
<td>1</td>
<td>Damage from crime in % of utility from crime</td>
</tr>
<tr>
<td>$c_1$</td>
<td>0</td>
<td>Minimum payoff from crime</td>
</tr>
<tr>
<td>$c_n$</td>
<td>200</td>
<td>Maximum payoff from crime</td>
</tr>
<tr>
<td>$n$</td>
<td>4001</td>
<td>Number of possible criminal opportunities</td>
</tr>
<tr>
<td>$V^p$</td>
<td>0</td>
<td>Value of punishment</td>
</tr>
<tr>
<td>$\mu$</td>
<td>5000</td>
<td>Parameter of PDF income from crime</td>
</tr>
</tbody>
</table>

the computation of the expected value of a criminal opportunity and the cumulative probability of the utility from not committing crime - the first and second terms respectively on the numerator of (14) - consistent with the stationary equilibrium solution for the continuation value $V_{cb}$. This algorithm can be implemented for any constant value of $b$.

The numerical solution under optimal insurance is derived using an algorithm based upon Chebyshev polynomials. We perform the computation in three stages. Firstly, the original government problem of minimizing the cost function (16) with respect to four variables, is reduced to minimization over a single variable. This is done using the constraints in (8), (10) and (17) to eliminate $b_t$, $a^*_t$ and $c^*_t$ as choice variables from the Bellman equation (16), thereby reducing the minimization over $V_{t+1}$ alone. Chebyshev polynomial approximation applies to functions on bounded intervals. For this reason, in the second stage, we derive upper and lower bounds for the continuation payoff from remaining unemployed $V_{t+1}$. The agent can never be worse off than in the absence of unemployment insurance. From (12) - (14), substituting $b = 0$ identifies the lower bound $V_{t+1}^{\text{min}}$. Similarly the government would never wish to give sufficient insurance as to provide no incentive to search for a job. This identifies an upper bound for the continuation value $V_{t+1}^{\text{max}}$ from (8) with $a^*_t = 0$. We are now in a position in which we can solve the government’s problem.

In the third stage, we implement the algorithm for the numerical derivation of the solution. The approximator is based on a fifth-order Chebyshev polynomial with 15 interpolation nodes. The approximate solution is derived by adapting the Chebyshev regression algorithm proposed by Judd 1998 (pp. 223) to the minimisation problem defined in the first stage. See Appendix E for a detailed discussion of the computational method.
7 Results

7.1 Constant Unemployment Insurance

Figure 2 depicts the stationary equilibrium solution with constant \( b = 0 \) for three alternative distributions of criminal opportunities. The panels on the left side plot the associated PDFs. Those on the right side display the corresponding values of unemployment \( V^u_t \), crime \( V^c_t \) and the expected continuation payoff from remaining unemployed \( V^c_b \) (given by (4), (5) and (14) respectively) for different realisations of the current period criminal opportunity. The equilibrium \( V^c_t \) is a concave function of the criminal opportunity drawn in the current period, reflecting the concavity of the instantaneous utility function. Conversely, \( V^u_t \) and \( V^c_b \) are independent of the current realisation. The threshold criminal opportunity is given by the intersection of the \( V^u \) and \( V^c \) curves. Any opportunity with higher value induces the agent to commit crime: \( V^c_t > V^u_t \). Likewise, any opportunity with a lower value is rejected in favour of searching for a job.

A second feature can be discerned from Figure 2. As the probability of receiving high value criminal opportunities increases (as we move from the top panels to the bottom),

![Graphical representation of the stationary equilibrium solution with constant unemployment insurance for different distributions of criminal opportunities.](image_url)
the expected continuation payoff from remaining unemployed, $V_{cb}$ grows. From (4) and (5), this raises both $V_u$ and $V_c$. However, since $\beta(1 - \phi)V_{cb} > \beta(1 - p(a_{cb}^*))V_{cb}$, the expected lifetime payoff from crime, $V_c$, increases more. Accordingly, the threshold criminal opportunity declines, as predicted by Proposition 3.

Figure 3: The effect of increase in the expected criminal opportunity on the utility gain from employment, search effort and the threshold criminal opportunity in stationary equilibrium with constant unemployment insurance ($b = 0$).

Figure 3 illustrates how increases in the likelihood of receiving high value criminal opportunities (measured by raising the parameter $\mu$) exacerbate moral hazard deriving from unobserved search effort. The top panel displays how the utility gain from employment (the difference between the continuation payoffs from employment and unemployment) varies. Changes in $\mu$ have no effect upon the value of employment. $V_e$ is constant. However, $V_{cb}$ is increasing in $\mu$, following the argument outlined for Figure 2. $V_e - V_{cb} - 1/r\beta$ declines. The higher continuation payoff makes getting a job less urgent, encouraging the agent to engage in crime. Accordingly, the threshold criminal opportunity declines. As benefits are constant, only the dynamic effect is in action. The agent’s criminality increases, contributing to a longer expected duration of unemployment.

When $V_{cb}$ increases, it gets closer to $V_{\text{max}}$. The utility gain from employment declines. Following (8), the marginal benefit of search effort is reduced. Since the marginal cost is
constant (and therefore unaffected), the prospect of greater criminal opportunities also lowers $a_{cb}^\ast$. Since an unemployed agent expects to receive higher lifetime utility, hunting for a job becomes less urgent. Moral hazard is indeed made worse (Proposition 1). Note that, when $\mu = 0$, we are in an crime-free environment (almost surely). In this case, the stationary equilibrium search effort, $a_{cb}^\ast$, is equal to that of Hopenhayn and Nicolini [1997].

Table 2: Stationary solution under $b_t = 0$ for increasing probabilities of receiving high criminal opportunities and increasing punishment.

<table>
<thead>
<tr>
<th>APD</th>
<th>$a_{cb}^\ast$</th>
<th>$r$</th>
<th>$c_{cb}^\ast$</th>
<th>$V^u$</th>
<th>$V_{cb}$</th>
<th>$\mathbb{E}[V_c]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Absence of Criminal Opportunities</td>
<td>0</td>
<td>307</td>
<td>3.43E-04</td>
<td>0</td>
<td>16759</td>
<td>16759</td>
</tr>
<tr>
<td>Criminal Opportunities ($\mu = 1000$)</td>
<td>6</td>
<td>295</td>
<td>3.57E-04</td>
<td>76</td>
<td>16884</td>
<td>16885</td>
</tr>
<tr>
<td>9</td>
<td>297</td>
<td>3.54E-04</td>
<td>83</td>
<td>16860</td>
<td>16861</td>
<td>16861</td>
</tr>
<tr>
<td>12</td>
<td>299</td>
<td>3.52E-04</td>
<td>90</td>
<td>16840</td>
<td>16841</td>
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<td>15</td>
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<td>3.50E-04</td>
<td>97</td>
<td>16824</td>
<td>16824</td>
<td>16823</td>
</tr>
<tr>
<td>18</td>
<td>302</td>
<td>3.49E-04</td>
<td>104</td>
<td>16810</td>
<td>16810</td>
<td>16808</td>
</tr>
<tr>
<td>Criminal Opportunities ($\mu = 5000$)</td>
<td>6</td>
<td>232</td>
<td>4.54E-04</td>
<td>49</td>
<td>17544</td>
<td>17549</td>
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<tr>
<td>9</td>
<td>247</td>
<td>4.27E-04</td>
<td>60</td>
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<tr>
<td>12</td>
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<tr>
<td>15</td>
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<td>3.83E-04</td>
<td>91</td>
<td>17091</td>
<td>17093</td>
<td>17090</td>
</tr>
<tr>
<td>Criminal Opportunities ($\mu = \infty$)</td>
<td>6</td>
<td>183</td>
<td>5.75E-04</td>
<td>33</td>
<td>18058</td>
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<tr>
<td>12</td>
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<td>4.61E-04</td>
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<td>18</td>
<td>257</td>
<td>4.09E-04</td>
<td>82</td>
<td>17280</td>
<td>17283</td>
<td>17280</td>
</tr>
</tbody>
</table>

Note: $V^c = 20000$.

Table 2 provides numerical values of the stationary equilibrium solution under constant unemployment insurance reported in Figure 2. It also shows how this is affected by changes in punishment, represented by the monthly APD. For reference we also report the solution in the absence of criminal opportunities. $\mathbb{E}[V_c]$ is the expected utility from committing crime, before any opportunity has been realised. It is computed by calculating $V^c$ for each possible $c_t$, and then weighting this by the associated probability $q_t$ from (18). As higher criminal opportunities become more likely, or as punishment becomes less severe, $\mathbb{E}[V_c]$ increases.

---

7This is calculated assuming that the probability that $c_t = 0$ is unity in all periods.
Table 2 show that greater search effort is exerted in the absence of crime. When criminal opportunities are available, an increase in the severity of punishment reduces the expected continuation payoff from remaining unemployed. In turn, the threshold criminal opportunity increases (from (10)) or, equivalently, the agent’s criminality declines. It also increases search effort from (8), as the utility gain from finding a job is higher.

![Graph showing (Adjusted) Utility Gain From Finding Job, V_e - V_{cb} - 1/rβ](image)

![Graph showing Effort, a*](image)

![Graph showing Threshold Criminal Opportunity, c*](image)

Figure 4: Increasing unemployment benefits under constant unemployment insurance.

Figure 4 illustrates how the stationary solution changes in response to increases in the unemployment benefits (Lemma 1). The figure reports adjusted utility gain from employment, $V_{max} - V_{cb}$, search effort, $a^*_{cb}$, and the threshold criminal opportunity, $c^*_{cb}$. When $b = 0$ the equilibrium solution corresponds to that in the second panel of Figure 2 with numerical values reported in Table 2. The probability of getting a job is $0.1$ when $b = 0$, but falls as $b$ increases. The figure clearly shows that an increase in the level of constant benefits paid by the government increases the value of remaining unemployed. The incentive to search declines (as per (8)), as the utility gain from getting a job ($V_e - V_{cb}$) is smaller. $p(a^*_{cb})$ declines, exacerbating moral hazard and increasing the expected duration

---

8 The solution is derived following the same algorithm described in the previous section under constant unemployment insurance with the only difference that $r$ is fixed at 4.08E-04 throughout.
of unemployment. At the same time the threshold criminal opportunity declines, due to the dynamic effect. $V^c_t$ increases faster than $V^u_t$. The agent’s criminality increases. This further raises the expected duration of unemployment, as the agent is less likely to search in any given period (as per Proposition 1).

When benefits increase to $b = 30$, the continuation payoff from remaining unemployed reaches $V^{\max}$. The adjusted utility gain from employment is zero. The marginal cost of effort always exceeds the marginal benefit, and the agent chooses $a^*_{cb} = 0$. At this point, increasing benefits has no additional effect.

![Graph of (Adjusted) Utility Gain From Finding Job, $V^e - V^u_{cb} - 1/\beta$](image1)

![Graph of Effort, $a^*$](image2)

![Graph of Threshold Criminal Opportunity, $c^*$](image3)

Figure 5: Increase in wages under constant unemployment insurance.

Figure 5 shows how the solution changes in response to an increase in wages (Lemma 2). Higher wages increase the payoff of getting a job relative to remaining unemployed (top panel). From (8) the agent’s optimal search effort increases (bottom-left panel).

Although higher wages feed back into a higher continuation value from unemployment (which, by the dynamic effect, reduces $c^*_{cb}$), they also have a direct effect on the threshold opportunity (in (10)). With an improvement in the formal labour market, the opportunity cost of any criminal activity is higher. This more than offsets the dynamic effect, and the threshold criminal opportunity increases (bottom-right panel). Higher wages result in the both the agent’s expected duration of unemployment and his criminality declining.
7.2 Optimal Unemployment Insurance

Figure 6: Optimal unemployment insurance with low criminal opportunities ($\mu = 200$, solid line) and in the absence of crime (dotted line).

We now focus on optimal unemployment insurance. Figure 6 displays the situation with low expected criminal opportunities (the solid line). Over time, the government reduces the benefit it provides to the agent (top-left panel). By doing so, it creates a declining continuation payoff schedule from remaining unemployed (top-right panel). As $V_{t+1}$ falls, the marginal benefit to search effort increases from (8), as the welfare gain from finding a job gets larger. The agent applies more effort whenever he searches (bottom-left panel).

Declining benefits and continuation payoff also affect the incentive to commit crime through the static and dynamic effects. Clearly, the dynamic effect dominates. As the continuation payoff from remaining unemployed declines, the agent begins to reject criminal opportunities that he would previously have accepted. The need to find a job is increasingly pressing. The threshold criminal opportunity increases (as per Proposition 4, see the bottom-right panel).

The dotted line in Figure 6 shows the situation without crime, and allows for a comparison. Since the expected criminal opportunity is very small, $V_{t+1}$ is very close to
$V_{t+1}^u$ (the continuation payoff from unemployment in the absence of crime). The expected utility that the agent receives from any given benefit schedule is thus very similar in either case. In spite of this, the two schedules are very different. When crime is present there is greater front-loading of benefits, as the government takes advantage of the dynamic effect (Lemma 3). By generating an increasing threshold criminal opportunity, the government is able to reduce the expected damage from the agent’s criminal activities. This additional incentive causes the government to reduce benefits faster. However, the total amount of welfare it must provide is the same with or without crime. It therefore offers much higher benefits during the early stages of unemployment, and lower benefits in the later stages. More extensive front-loading of benefits results.

Figure 7 illustrates a second feature of the optimal benefits schedule. When more valuable criminal opportunities become more likely ($\mu = 600$), the government expects the agent’s instantaneous utility to be higher in each period. The agent will supplement his formal income with crime. Whilst the government suffers from this additional crime, it is able to reduce the benefit payments (top-left panel) made to the agent in every period whilst still maintaining the promise-keeping constraint (equation (17)). Note that, once
again, we have more extensive front-loading of benefits. Even with lower benefits in every period, the government still reduces them by a much greater proportion in the presence of crime during the early stages of unemployment. In turn, this causes the continuation payoff $V_{t+1}$ to diverge from that in the absence of crime (top-right panel).

Comparing optimal search effort and threshold criminal opportunity in Figures 6 and 7 yields some interesting insights. When more profitable criminal opportunities become more likely, the expected damage caused by the agent’s criminal activities increases. This creates an incentive for the government to pay an even greater proportion of the promised welfare in the early stages of unemployment. Although the payments are relatively costly, the rapidly declining continuation payoff induces much higher search effort and higher threshold criminal opportunities (the bottom panels). It becomes more likely that the agent will both search for a job, and be successful in that search. Consequently, in every period, the agent is less likely to accept a criminal opportunity.

![Graphs showing optimal unemployment insurance with low criminal opportunities (μ = 200) as crime becomes more damaging: δ = 1 (solid line); δ = 4 (dashed line); and δ = 8 (dotted line).]

Figure 8 illustrates the effect of an increase in the social cost of crime on the optimal unemployment benefit schedule and its implications for crime and search effort (Lemma 3). As the damage from crime increases, the agent’s criminal activity features more
prominently in the decision of the government. It thus seeks to reduce the amount of crime committed by reducing the continuation payoff from remaining unemployed (top-right panel) even more rapidly. This affects the incentives of unemployed individuals to search for a job: firstly he applies more effort as the continuation payoff from employment is now significantly higher than that from unemployment (bottom left panel). Secondly, he foregoes criminal opportunities he would previously accepted in order to devote more time to searching for a job (bottom-right panel). In order to reduce the continuation payoff without changing the promise-keeping constraint the government front-loads benefits to a greater and greater extent (top-left panel).

![Graphs showing the effect of unemployment insurance on various parameters](image_url)

Figure 9: Optimal unemployment insurance with low criminal opportunities ($\mu = 600$) as crime becomes more damaging: $\delta = 1$ (solid line); $\delta = 4$ (dashed line); and $\delta = 8$ (dotted line).

Figure 9 shows the effect of increasing the damage from crime when the expected criminal opportunity is relatively high. The intuition is identical to that of Figure 8. As crime becomes more damaging, the government has an increasing incentive to front-load benefits. However, in a high criminal opportunity environment, the expected damage caused by crime becomes even more acute. The government front-loads to such an extent that benefits rapidly decline to zero. At this point, the government has reached the end of its influence. We enter a situation equivalent to constant benefits (with $b_t = 0$). The
agent maintains a constant search effort and threshold for criminal opportunities. His continuation payoff derives solely from expected future criminality and the possibility of receiving a job. In a similar manner to Alvarez Parra and Sánchez (2009), this continuation payoff is endogenously bounded above zero by the presence of crime.

8 Conclusions

Engaging in crime whilst unemployed is a double-edged sword. On the one hand, crime supplements legitimate income from unemployment benefits. On the other, it makes it more likely that an agent will remain unemployed. Knowing that he can supplement future income with criminal activity reduces the incentive to apply search effort, lowering the likelihood of successfully acquiring a job. Time taken to commit crime further reduces searching. In this sense, crime exacerbates moral hazard deriving from the fact that the government cannot observe the agent’s actions.

When benefits decline, the marginal benefit of search effort increases. Conditional on looking for a job, the agent applies more effort. The effect on criminality is unclear. The instantaneous marginal utility from criminal opportunities increases (a static effect) providing a greater incentive to commit crime. Each opportunity is worth more to the agent. However, the continuation payoff from remaining unemployed also declines. This provides a counterbalancing incentive (a dynamic effect), potentially reducing criminality. With a higher probability of remaining unemployed, a criminal suffers more from this decline. For a reasonable calibration, this dynamic effect dominates.

The existence of opportunistic crime has interesting implications for optimal unemployment insurance. Firstly, the government can offer lower benefits whilst still providing individuals with sufficient welfare. Secondly, the dynamic effect provides an additional reason to reduce benefits over time. By doing so, the continuation payoff from remaining unemployed falls. This not only provides greater incentives to apply search effort, but may also lower the criminality of the unemployed agent.

Our findings are in tune with insights from dynamic social contracting theory with private information. In the absence of crime, the government wants to provide insurance to smooth the allocation of consumption among employed and unemployed. At the same time, it recognises that the provision of this insurance induces moral hazard behaviour. By front-loading benefits, the government increases the effectiveness of the insurance in the interest of the society as a whole. Front-loading is driven by an efficiency, rather than a punitive, motive. Recent work shows that front-loading distortions are ultimately an optimality principle and illustrate this for a broad class of environments (Albanesi and
We have illustrated this for a special type of economic environment: one with random criminal opportunities. As crime induces even greater moral hazard, society has a further reason to provide greater incentive for the unemployed to search for jobs. The increase in the front-loading of unemployment benefits is a natural implication of social contracting theory when applied to an economic environment with crime.

Appendices

A Proof of Proposition 2

Proof. Given \( \{b_t\}_{t=0}^{\infty} \), the agent will commit a crime in period \( t \) if and only if:

\[
V_t^c \geq V_t^u
\]

\[
\iff u(b_t, c_t) \geq u(b_t, 0) - a_t^* + \beta p(a_t^*) V_{t+1} + \beta (\phi - p(a_t^*)) V_{t+1} - \beta \phi V^p,
\]

where \( a_t^* \) is the agent’s optimal choice of effort given \( \{b_t\}_{t=0}^{\infty} \). The left-hand side is strictly increasing in \( c_t \), whereas the right-hand side is independent of \( c_t \). Therefore, there exists a unique \( c_t^* \) such that (10) holds. The agent will commit a crime if and only if \( c_t \geq c_t^* \).

This completes the proof.

B Proof of Proposition 3

Proof. The probability that the agent commits a crime in any given period during which he is unemployed is \( 1 - F(c_t^*) \). His criminality is increasing in the continuation payoff from unemployment if and only if \( c_t^* \) declines as \( V_{t+1} \) gets larger.

From equation (10), we have that:

\[
\frac{\partial c_t^*}{\partial V_{t+1}} = \frac{\beta [\phi - p(a_t^*)]}{u_c(b_t, c_t^*)}.
\]

\( c_t^* \) declines with the continuation payoff from unemployment if and only if \( \phi - p(a_t^*) < 0 \). Rearranging this condition to solve for \( a_t^* \), we require that \( a_t^* > p^{-1}(\phi) \). For this to hold, there must still be a gain for the agent from applying additional search effort when \( a_t = p^{-1}(\phi) \). From (8), we need that the marginal benefit of search effort when
\(a_t = p^{-1}(\phi)\) exceeds the marginal cost:
\[
\beta p'(p^{-1}(\phi)) (V^e - V_{t+1}) > 1
\]
\[
\iff V_{t+1} < V^e - \frac{1}{\beta p'(p^{-1}(\phi))}.
\]

Define \(\tilde{V} \equiv V^e - \frac{1}{\beta p'(p^{-1}(\phi))}\). So long as \(V_{t+1} < \tilde{V}\), \(c_t^*\) is declining in the continuation payoff from unemployment. This completes the proof. ■

**C Proof of Proposition 4**

**Proof.** The Lagrangian for the government’s problem:
\[
\mathcal{L}(b_t^*, a_t^*, c_t^*, V_{t+1}; \theta, \eta, \lambda) = b_t^* + \int_{c=c_t^*}^{\infty} d(c, \delta) f(c) dc + \beta [1 - \phi + F(c_t^*)[\phi - p(a_t^*)]] C(V_{t+1})
\]
\[
- \theta \left\{ \int_{c=c_t^*}^{\infty} u(b_t^*, c) f(c) dc + F(c_t^*) u(b_t, 0) - F(c_t^*) a_t^* \right.
\]
\[
+ \beta F(c_t^*) p(a_t^*) V^e + \beta [1 - F(c_t^*)] \phi V^p
\]
\[
+ \beta [1 - \phi + F(c_t^*)[\phi - p(a_t^*)]] V_{t+1} - V_t \right\}
\]
\[
- \eta \{ \beta p'(a_t^*) (V^e - V_{t+1}) - 1 \} - \lambda \{ u(b_t^*, c_t^*) - u(b_t, 0) + a_t^* - \beta p(a_t^*) V^e
\]
\[
- \beta [\phi - p(a_t^*)] V_{t+1} + \beta \phi V^p \}.
\]

Taking appropriate first-order conditions:
\[
\frac{\partial \mathcal{L}}{\partial b_t^*} = 1 - \theta \left\{ \int_{c=c_t^*}^{\infty} u(b_t^*, c) f(c) dc + F(c_t^*) u_b(b_t^*, 0) \right\}
\]
\[
- \lambda \{ u_b(b_t^*, c_t^*) - u_b(b_t^*, 0) \} \equiv 0,
\]
\[
\frac{\partial \mathcal{L}}{\partial a_t^*} = - \beta F(c_t^*) p'(a_t^*) C(V_{t+1}) - \eta \beta p''(a_t^*) (V^e - V_{t+1}) \equiv 0,
\]
\[
\frac{\partial \mathcal{L}}{\partial c_t^*} = f(c_t^*) \{ \beta [\phi - p(a_t^*)] C(V_{t+1}) - d(c_t^*, \delta) \} + \lambda u_c(b_t^*, c_t^*) \equiv 0,
\]
\[
\frac{\partial \mathcal{L}}{\partial V_{t+1}} = \beta [1 - \phi + F(c_t^*)[\phi - p(a_t^*)]] [C'(V_{t+1}) - \theta]
\]
\[
+ \eta \beta p'(a_t^*) + \lambda [\phi - p(a_t^*)] \equiv 0.
\]
The envelope condition yields $C'(V_t) = \theta$. Moreover, since $V_t < \tilde{V}$, $\phi - p(a^*_t) < 0$.

From (20) and (21), we have that:

$$
\eta = \frac{p'(a^*_t) F(c^*_t) C(V_{t+1})}{p''(a^*_t) (V^e - V_{t+1})} > 0,
$$

$$
\lambda = \frac{f(c^*_t) \{ \beta [\phi - p(a^*_t)] C(V_{t+1}) - d(c^*_t, \delta) \}}{u_t(b^*_t, c^*_t)} < 0.
$$

Rearranging (22) yields:

$$
C'(V_{t+1}) = \theta - \eta p'(a^*_t) + \lambda [\phi - p(a^*_t)]
$$

Since $\phi - p(a^*_t) < 0$, $\lambda [\phi - p(a^*_t)] > 0$. $C'(V_{t+1}) < \theta = C'(V_t)$ by the envelope condition.

Since, by assumption, $C$ is convex, $V_t > V_{t+1}$. The expected continuation payoff from unemployment falls from period $t$ to period $t + 1$.

A declining continuation payoff implies that $V_{t+1} < \tilde{V}$. Thus $\phi - p(a^*_{t+1}) < 0$. By the same analysis, $V_{t+2} < V_{t+1}$. An inductive argument completes the proof.

### D Proof of Proposition 5

**Proof.** Suppose that we can write $u(b, c) = u(b) + v(c)$. The threshold criminal opportunity from (10) is defined as follows:

$$
v(c^*_t) = -a^*_t + \beta p(a^*_t) V^e + \beta [\phi - p(a^*_t)] V_{t+1} - \beta \phi V^p.
$$

The Lagrangian is:

$$
\mathcal{L} (b^*_t, a^*_t, c^*_t, V_{t+1}; \theta, \eta, \lambda) = b^*_t + \int_{c = c^*_t}^{\infty} d(c, \delta) f(c) dc + \beta \{1 - \phi + F(c^*_t) [\phi - p(a^*_t)] \} C(V_{t+1})
$$

$$
- \theta \left\{ u(b^*_t) + \int_{c = c^*_t}^{\infty} v(c) f(c) dc - F(c^*_t) a^*_t + \beta F(c^*_t) p(a^*_t) V^e
$$

$$
+ \beta [1 - F(c^*_t)] \phi V^p + \beta [1 - \phi + F(c^*_t) [\phi - p(a^*_t)] V_{t+1} - V_t
$$

$$
- \eta \{ \beta p'(a^*_t) (V^e - V_{t+1}) - 1 \}
$$

$$
+ \lambda \{ v(c^*_t) + a^*_t - \beta p(a^*) V^e - \beta [\phi - p(a^*_t)] V_{t+1} \}.
$$

This is a special case of Proposition 4 and so $V_t$ is declining. The first-order condition
for effort is:
\[
\frac{\partial L}{\partial b^*_t} \equiv 0 \equiv 1 - \theta u'(b^*_t),
\]
(23)
and the envelope condition yields:
\[
C'(V_t) \equiv \theta.
\]
Rearranging (23) and substituting for \( \theta \) yields:
\[
C'(V_t) = \frac{1}{u'(b^*_t)}.
\]
As \( V_t \) is declining and \( C \) is convex, diminishing marginal utility implies that \( b^*_t \) must also decline. This completes the proof. ■

E Computational Method

E.1 Reduction of the government minimisation problem
The reduction of the dynamic optimisation of (16) to the minimisation over a single choice variable \( V_{t+1} \) is done as follows. Solving (8) for \( a^*_t \) yields:
\[
a^*_t = \max \left\{ 0, \frac{\ln [\beta r (V^e - V_{t+1})]}{r} \right\},
\]
(24)
which expresses \( a^* \) as a function of \( V \). The above can be replaced into (10) with additive separability to obtain:
\[
c^*_t = v^{-1} \left( \frac{-\ln [\beta r (V^e - V_{t+1})]}{r} + \beta V^e - \beta (1 - \phi) V_{t+1} - \frac{1}{r} \right),
\]
(25)
which expresses \( c^*_t \) as a function of \( V_{t+1} \). Then the promise-keeping constraint in equation (17) can be solved as
\[
b^*_t = u^{-1} \left( \int_{c=c^*_t}^{\infty} v(c) f(c) dc + F(c^*_t) a^*_t - \beta F(c^*_t) p(a^*_t) V^e - \beta \{1 - \phi + F(c^*_t) [\phi - p(a^*_t)]\} V_{t+1} \right),
\]
(26)
which formulates \( b^*_t \) in terms of \( V_{t+1} \), both directly and through \( a^*_t \) and \( c^*_t \) derived from (24) and (25) respectively. Using these results, the Bellman equation (16) can be written
as

\[
C(V_t) = \min_{V_{t+1}} \left\{ b_t^* + \int_{c=c_t^*}^{\infty} d(c, \delta) f(c) \, dc + \beta \{ 1 - \phi + F(c_t^*) \, [\phi - p(a_t)] \} \, C(V_{t+1}) \right\},
\]

where \(a_t^*, c_t^*\) and \(b_t^*\) are derived from (24), (25) and (26).

\[\text{E.2 Upper and lower bounds}\]

The continuation value \(V_{t+1}\) is included within a bounded interval \([V^{\min}, V^{\max}]\). The upper bound is the continuation value paid by the government that would induce zero search effort. When imposing \(a_t^* = 0\) into equation (8), the upper bound is therefore calculated as:

\[V^{\max} = V^e - (r\beta)^{-1} \]

The lower bound is the continuation value paid by the government that would induce maximum search effort. This is consistent with the stationary equilibrium solution for \(V_{t+1}\) when the government provides no insurance. In this case, the agent’s problem is still described by equations (3)-(7), for \(b_t = 0\). This problem is stationary, as the continuation payoffs, conditional on being employed or unemployed are independent of the current period action. The agent’s optimal choices \(a_t^*\) and \(c_t^*\), along with the optimal continuation payoff \(V^{\min}\) are given by the solution to the following system of simultaneous equations:

\[
v(c_t^*) = -a_t^* + \beta p(a_t^*) V^e + \beta [\phi - p(a_t^*)] V^{\min} - \beta \phi V^p,
1 \geq \beta p'(a_t^*) (V^e - V^{\min})
V^{\min} = \frac{\int_{c=c_t^*}^{\infty} v(c) \, f(c) \, dc + F(c_t^*) \, v(c_t^*) + \beta \phi V^p}{1 - \beta (1 - \phi)}.
\]

The first equation defines the threshold criminal opportunity, \(c_t^*\), that makes the agent indifferent between searching or committing crime. The second defines the optimal search effort, \(a_t^*\), in the absence of unemployment insurance. The third equation gives the resulting continuation payoff.

\[\text{E.3 Chebyshev regression algorithm}\]

We recall that the \(n\)-degree Chebyshev polynomial is \(p_n(x) = \sum_{i=0}^{n} \alpha_i T_i(x)\), with \(T_0(x) = 1, T_1(x) = x\) and \(T_{i+1}(x) = 2xT_i(x) - T_{i-1}(x)\) for \(i = 1, ..., n\). The Chebyshev
regression algorithm works as follows.

1. Compute the $m > n + 1$ Chebyshev interpolation nodes on the interval $[-1, 1]$; 
   
   $$ z_k = -\cos \left( \frac{2k-1}{2m} \pi \right), \quad k = 1, \ldots, m. $$

2. Adjust the nodes to the interval $[V_{\text{min}}, V_{\text{max}}]$; 
   
   $$ V_k = \left( z_k + 1 \right) \left( \frac{V_{\text{max}} - V_{\text{min}}}{2} \right) + V_{\text{min}}, \quad k = 1, \ldots, m. $$

3. Set initial arbitrary values for the Chebyshev coefficients, $\alpha_i$, $i = 1, \ldots, n$. Given a 
   
   $V_k \in [V_{\text{min}}, V_{\text{max}}]$ form the approximation of $C(V)$ on the right side of equation 
   
   (27): 
   
   $$ \hat{C}_j (V) = \sum_{i=0}^{n} \alpha_i T_i \left( 2 \frac{V_k - V_{\text{min}}}{V_{\text{max}} - V_{\text{min}}} - 1 \right). $$

   Use this in combination with $a^*, c^*$ and $b$ from equations (24), (25) and (26) to compute the right side of equation (27).

   This gives the value $\hat{C}_j (V)$ of the approximated cost function at iteration $j$ given 
   
   $V = V_k$ and $V = V_j$ both included in $[V_{\text{min}}, V_{\text{max}}]$. Use a numerical optimization 
   
   routine to search within the interval $[V_{\text{min}}, V_{\text{max}}]$ the value of $V_j$ that gives the 
   
   minimum $\hat{C}_j (V)$ when $V = V_k$.

   Denote this as $\hat{C}(V_k)$.

4. Repeat steps 3 for all $V_k$, $k = 1, \ldots, m$, derived in step 2. This gives the value of 
   
   the cost function approximated at the $m$ Chebyshev nodes; $\hat{C}(V_k)$, $k = 1, \ldots, m$.

5. Update the Chebyshev coefficients using 
   
   $$ \alpha_i = \frac{\sum_{k=1}^{m} \hat{C}(V_k) T_i (z_k)}{\sum_{k=1}^{m} T_i (z_k)^2}. $$

6. Replace the Chebyshev coefficients and repeat steps 3 - 5 until convergence.

The MATLAB programme used for the numerical computation is available upon request from the authors. This was constructed starting from a programme that replicates the solution to Hopenhayn and Nicolini 1997 available at http://dge.repec.org/codes/sargent/hugo/.

References


Specifically, we use the MATLAB function fminbnd.


