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Inflation versus price-level targeting and the zero lower bound: Stochastic simulations from the Smets-Wouters US model

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Abstract

Using a version of the Smets-Wouters model of the US economy augmented to include both New Keynesian and New Classical sectors, this paper investigates the performance of inflation targeting and price-level targeting when the zero lower bound on nominal interest rates is *occasionally-binding*. Several notable results emerge. First, the unconditional probability of hitting the lower bound is lower under price-level targeting than inflation targeting, with 'lower bound episodes' being less frequent and lasting for shorter periods of time. Second, the volatilities of key macroeconomic variables are lower under price-level targeting than inflation targeting. Third, the lower frequency and severity of lower bound episodes under price-level targeting appears to have a first-order impact on consumption, investment and output, raising their mean values. Intuitively, price-level targeting performs well because inflation expectations act as automatic stabilisers, reducing the chance of hitting or remaining at the lower bound whilst also providing stability when the economy is away from the lower bound.

Key words: Zero lower bound, occasionally-binding constraint, price-level targeting, inflation targeting.

JEL classification: E52, E58.

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Contents

1 Introduction

With policy rates currently at low levels in major developed economies, both academic economists and central banks have turned their attention to policies that might alleviate welfare costs associated with the 'zero lower bound' (ZLB) on nominal interest rates (see e.g. Chung *et al.* (2011)). This paper investigates the performance of one such policy – namely, price-level targeting (PLT) – using a model of the US economy in which the ZLB on nominal interest rates is an *occasionally-binding constraint*. The main contribution of the paper is to provide a full analysis of this kind using a model that is both micro-founded and has proven empirical performance – the Smets-Wouters model of the US economy. As Smets and Wouters (2007) document, this model, which can be derived from consumers' and firms' first-order conditions, performs comparably to BVAR models in terms of its likelihood and out-of-sample forecast performance.

Motivation for focusing on PLT when the ZLB constrains policy comes from several sources. Firstly, there is the seminal paper by Eggertsson and Woodford (2003), where they derive analytically the optimal commitment policy in the standard New Keynesian model with an occasionally-binding ZLB and show that it takes the form of a credible commitment to PLT. Following up this finding, a number of papers have conducted full quantitative analyses. Adam and Billi (2007) compare the Eggertsson and Woodford's optimal commitment policy (i.e. PLT) with discretionary IT and find that the gains are non-trivial and exceed those in no-ZLB case. Nakov (2008), making the additional point that the optimal commitment policy may be infeasible in practice, provides a similar analysis but focuses instead on simple IT and PLT Taylor-type rules. His analysis shows that the potential gains from targeting the price level are smaller but still non-trivial if policy is represented by simple instrument rules rather than optimal targeting rules. Finally, Cateau and Dorich (2011) point out that with an occasionally-binding ZLB even imperfectly credible PLT will dominate IT so long as credibility is established quite quickly.[1]

A second motivation for focusing on PLT comes from a recent paper by Coibon, Gorodinichenko and Wieland (2010). They focus on a New Keynesian model with an occasionally-binding ZLB, but crucially they allow for a positive rate of steady-state inflation around which the model is log-linearised. Using this model, Coibon *et al.* derive the utility-based loss function and then optimise the steady-state rate of inflation under IT and PLT. They find that the welfare gains of PLT are magnified in this environment because, for any given positive rate of inflation, the likelihood of the ZLB being reached is lower under PLT, such that the welfare gains of lower steady-state inflation can be reaped without a substantial increase in

[1] In an early numerical analysis, Wolman (2005) showed that the favourable performance of PLT when the ZLB is occasionally-binding also holds under the assumption that rigidity in prices arises from Taylor rather than Calvo contracts.

the risk of ZLB episodes. The current paper investigates in detail the likelihood of hitting the ZLB at various horizons under IT and PLT – a task not undertaken formally by Coibon and co-authors.

Third, a number of papers have suggested that PLT would reduce the probability or severity of lower bound episodes, but without modelling the ZLB directly. For example, Gaspar and Smets (2000) set up a simple New Keynesian model and use nominal interest rate variability to assess the likelihood of hitting the lower bound, the idea being that smaller movements in nominal rates will be associated with a lower probability of hitting the ZLB. They find that adding a price level objective into the central bank's loss function reduces nominal rate variability, suggesting that the ZLB would be hit less often in a PLT regime. Amano and Ambler (2008) look instead at frequency with which nominal interest rate are negative. They consider a fully non-linear New Keynesian model but reach an analogous conclusion: negative interest rates are less frequent under PLT than IT, implying that the ZLB would be hit less often. An early paper that did impose the lower bound directly was Coenen and Wieland (2004), but they only examine the performance of PLT conditional on the ZLB having been reached. Their results suggest that, as argued by Svensson (2001), PLT could help the economy to escape from ZLB episodes. In the current paper, by contrast, full stochastic simulations with an occasionally-binding ZLB are used to compare the performance of IT and PLT.

This assessment is made using the Smets and Wouters (2007) model – a medium-scale DSGE model of the US economy that is micro-founded and yet suitable for quantitative policy analysis. In particular, following Le, Minford, Meenagh and Wickens (2011) [forthcoming, *Journal of Economic Dynamics and Control*], the present paper considers a version of the Smets-Wouters model that is augmented to include New Classical wage and price sectors in addition to New Keynesian ones and which is simulated over virtually the whole postwar period – namely from 1948:Q1 to 2004:Q4. As noted by Le *et al.*, a hybrid model of this kind is better able to match the level of nominal variability in the data than a purely New Keynesian specification, leading to an improvement in the overall dynamic fit of the model as evaluated using the method of indirect inference.

Several interesting results are found. First, compared with previous estimates, the probability of hitting the ZLB is relatively high under IT at 10 per cent, reflecting the long sample period used to derive the model's shocks and the fact that flexible price and wage sectors increase nominal variability. PLT reduces the probability of hitting the ZLB to 8.7 per cent, a reduction of more than one-tenth compared to the IT case. Second, ZLB episodes are also less severe under PLT in the sense that they last for shorter periods of time. For instance, conditional on the ZLB being reached in the current quarter, the probability of remaining there for the next two quarters is estimated to be around 18 per cent under IT compared to less than 12 per cent under PLT. Third, the variances of key macro variables are reduced substantially under PLT, with the most notable reductions for labour supply (17 per cent), consumption (8 per cent) and investment (4 per cent).

Most notably, by reducing the probability and severity of ZLB episodes, PLT appears to have a first-order impact on key macro variables. In particular, mean consumption, investment and output are higher in model simulations than under IT, consistent with long-lasting ZLB episodes under IT being sufficiently severe as to cause contractions in investment and consumption that lower their long run averages. On an intuitive level, the result that PLT performs well makes good sense because Svensson (1999) shows that PLT dominates IT with a pure New Classical Phillips curve (and no ZLB), whilst Vestin (2006) shows that this 'free lunch' result also applies in the canonical New Keynesian model. That is to say, what appears to be most important for the performance of PLT is not the extent of nominal rigidity in the economy, but the assumption that economic agents are rational and view PLT as perfectly credible (see Ambler, 2009).

Overall, the results in this paper seem to suggest that adopting PLT could improve macroeconomic stability in the US, but the analysis is incomplete along several dimensions. Most notably, PLT is assumed to be perfectly credible – a strong assumption given that PLT has been adopted only once in history (see Berg and Jonung, 1999). Nevertheless, it is worth noting that the US's neighbour Canada is currently conducting a review of PLT with the aim of establishing whether a switch from IT to PLT would be beneficial from a cost-benefit perspective (see Bank of Canada, 2006). Although the analysis in the current paper focuses specifically on the US economy, the results presented here also contribute to the IT-PLT policy debate that is ongoing in Canada and in other countries.

The remainder of the paper proceeds as follows. First, Section 2 briefly discusses the Smets-Wouters US model, with a focus on the extension of the model to include flexible price and wage sectors à la Le *et al.* (2011). Section 3 turns to model calibration, and Section 4 discusses the methodology used to solve the model with an occasionally-binding ZLB. Finally, results are discussed in Section 5, and Section 6 concludes and discusses implications for policy.

2 Model

A detailed description of the model as a whole is given in Smets and Wouters (2007), whilst the extended hybrid model with flexible price and wage sectors is discussed in Le *et al.* (2011). This section thus provides only a brief overview of the model, though for completeness a full description of the model is given in appendices A and B. Particular attention is paid to the hybrid wage and price-setting equations with which most readers will be less familiar and other (minor) differences relative to the original Smets-Wouters model are noted. Finally, the introduction of a PLT Taylor rule in the model is discussed, along with the zero-truncated specifications of the Taylor rules that are necessary to impose the ZLB constraint.

As in Smets and Wouters (2007), all shocks are assumed to be stationary; there are however some small changes to the assumed shock processes in the Le *et al.* (2011) version of the model. In particular, all shocks are assumed to follow AR(1) processes with white noise innovations, whereas in Smets and Wouters' paper the government spending shock responds to exogenous productivity developments and the wage and price mark-up shocks follow ARMA(1,1) processes. The shocks in the model are derived from postwar US data, but the sample used to derive these shocks is somewhat longer than in Smets and Wouters' analysis, extending from 1948:Q1 to 2004:4 and therefore covering almost the entire postwar period.[2] The main departure of the Le *et al.* model from Smets and Wouters' original paper is the introduction of flexible price and wage sectors which coexist in the economy alongside New Keynesian ones that are characterised by time-dependent nominal rigidity à la Calvo (1983).[3]

The New Classical wage and price sectors are assumed to be perfectly flexible but labour suppliers face a one-period information lag. This situation is modelled formally as follows. Total employment of labour by intermediate firms at time t is given by n_t and consists of labour from imperfectly competitive (i.e. unionised) and competitive markets, so that $n_t = n_{1t} + n_{2t}$, where n_{1t} is employment from the unionised sector and n_{2t} is employment from the competitive sector. The firms are assumed to have production functions that combine these two types of labour in a fixed proportion, hence giving rise to the following demand curve for labour:

$$n_t = v_{w,NK} n_{1t} + (1 - v_{w,NK}) n_{2t} = \left[\int_0^1 n_{1it}{}^{1/(1+\lambda_{w,t})} di \right]^{1+\lambda_{w,t}} + \int_0^1 n_{2it} di \tag{1}$$

where $v_{w,NK}$ is the fraction of total labour demanded from the imperfectly competitive New Keynesian sector; n_{1it} is labour supplied by household i in the imperfectly competitive sector; and n_{2it} is labour supplied by household i to the competitive sector.

Under this specification, n_t can be interpreted as total employment demanded by a 'labour bundler' who uses in fixed proportions a composite of differentiated labour services from the unionised sector and a composite of undifferentiated labour services from the competitive sector. Denoting the nominal wage set by unions W_{NKt} and the competitive wage W_{NCt}, it follows that the aggregate nominal wage in the economy is given by

$$W_t = v_{w,NK} W_{NKt} + (1 - v_{w,NK}) W_{NCt} \tag{2}$$

[2] See Chung *et al.* (2011) on the importance of using a long sample when assessing the implications of the ZLB for the economy.
[3] Also, there is an additional shock in the wage-setting equation that relates to the New Classical part of the Phillips curve for nominal wages (see Appendix A).

The aggregate nominal wage in the economy is thus a weighted average of the competitive wage and the New Keynesian one set by unions. It follows that the economy-wide level of wages is driven partly by a New Keynesian Phillips curve and partly by the competitive wage level. The latter is fully flexible but, due to the assumption that labour suppliers in the competitive market face a one-period information lag, it includes an unanticipated inflation component; see Appendix A. Labour bundlers offer units of labour at this aggregate wage to firms who in turn purchase aggregate labour services n_t for use in production. Consequently, the model can be simulated without distinguishing between unionised and competitive labour, but with the important difference that relative employment in these sectors will affect the economy-wide wage paid to workers, with Smets and Wouters' original model corresponding to the case where $v_{w,NK}$ is assumed to be equal to one.

We can now turn to the retail output sector of the model. By assumption, retail output utilises in a fixed proportion intermediate goods sold in an imperfectly competitive market and intermediate goods sold in a competitive market. Thus, denoting total retail output by y_t, the outputs from the two sectors are related to total output as follows:

$$y_t = y_{1t} + y_{2t} = \left[\int_0^1 y_{1jt}{}^{1/(1+\lambda_{p,t})} dj \right]^{1+\lambda_{p,t}} + \int_0^1 y_{2jt}\, dj \; = \; v_{p,NK} y_t + (1 - v_{p,NK}) y_t \tag{3}$$

where y_{1t} is output produced in the imperfectly competitive sector; y_{2t} is output produced in the competitive sector; and $v_{p,NK}$ is the fraction of retail output produced under conditions of imperfect competition.

The intermediary firm prices composite output from the imperfectly competitive sector at a premium on marginal cost according to the Calvo mark-up equation, whilst composite output from the competitive sector is sold at marginal cost. The retailer is assumed to sell the combined goods in a bundle whose price is given by

$$p_t = v_{p,NK} p_{1t} + (1 - v_{p,NK}) p_{2t} \tag{4}$$

where p_t, the aggregate price level in logs, is equal to a weighted average of the composite price levels in the imperfectly competitive and competitive sectors.[4]

Taking the first difference of Equation (4), inflation is a weighted average of the inflation rates in the two sectors, with the weights reflecting the sectors' relative contributions to retail output:

$$\pi_t = v_{p,NK} \pi_{1t} + (1 - v_{p,NK}) \pi_{2t} \tag{5}$$

[4] Under PLT, it is the aggregate price level on the LHS of this equation on which policy focuses.

Given that retailers themselves operate in a perfectly competitive market, they sell the aggregate output y_t at the price at which it is purchased, such that no distinction need be made between outputs from the two sectors when simulating the model (see Appendix A). It follows that the only change to the model's equations needed to incorporate competitive intermediate goods sectors is that aggregate inflation be given Equation (5), whose dynamics depends crucially on the relative importance of New Keynesian sectors vis-à-vis competitive ones. Of course, Smets and Wouters' purely New Keynesian Phillips curve arises as a special case when the weight on the imperfectly competitive intermediate goods sectors, $v_{p,NK}$, is set equal to one.

Monetary policy is modelled using Taylor rules to which the central bank is fully committed. However, in order to capture the impact of the ZLB on policy, these rules are truncated such that the nominal interest rate cannot fall below a specified value.[5] As in Smets and Wouters (2007), these rules respond to level of output as well as its first-difference, though potential output was set equal to a constant of zero and therefore has no impact on model simulations.[6]

The IT Taylor rule takes the following form:

$$R_t^{nom} = \max\left\{ R^{\min}, \quad \rho_R R_{t-1}^{nom} + (1-\rho_R)(\theta_\pi \pi_t + \theta_y y_t) + \theta_{\Delta y}(y_t - y_{t-1}) + shock_t^{MP} \right\} \tag{6}$$

where R^{nom} is the net nominal interest rate; R^{min} is its minimum permitted value; and shock^{MP} is an AR(1) monetary policy shock to the Taylor rule.

The Taylor rule under PLT takes the same general form but responds to deviations of the price-level from its target. The PLT Taylor rule is thus given by

$$R_t^{nom} = \max\left\{ R^{\min}, \quad \rho_R R_{t-1}^{nom} + (1-\rho_R)(\theta_p (p_t - p_{tar,t}) + \theta_y y_t) + \theta_{\Delta y}(y_t - y_{t-1}) + shock_t^{MP} \right\} \tag{7}$$

where p_t is the log price level at time t; $p_{tar,t}$ is the target price level; and θ_p denotes the response of the nominal interest rate to price-level deviations.

In model simulations, the log price level was calculated recursively using inflation/100 and a log approximation (note that inflation and other nominal variables are expressed in per cent and not fractions in model simulations).[7] The target price level was set equal to the lagged value of the (log) price level in the data over the sample period, because the model is simulated around the

[5] For reasons discussed in the next section, this value was not set at numerical value of zero but a constant deviation below the 'base run' nominal rate around which the model was simulated. This value is referred to as the 'ZLB' in what follows.

[6] Given the simulation methodology employed (see below), any exogenous process for potential output would leave model simulations unaffected.

[7] Namely, the fact that $\ln(1+x) \approx x$ for small x implies that inflation (expressed as a fraction) is approximately equal to the first difference of the log price level.

time path defined by the data – that is, the 'base' or 'base run' simulation. The base price level was calculated recursively over the sample period using actual inflation, with the initial value of the price level index normalised to one.

3 Calibration

The calibration of model parameters is identical to that in Le *et al.* (2011), except that a majority New Keynesian calibration is adopted for the hybrid (or weighted) part of the model. Overall, this calibration is similar to the estimated values reported in Smets and Wouters (2007), with the main difference being that their model is purely New Keynesian and hence does not attribute any weight to New Classical sectors; see Table 1. The only other notable difference between the calibration used here and Smets and Wouters' estimated values is that there is assumed to be a higher Taylor rule response to inflation of 2.30 and lower interest rate persistence of 0.60. These changes reflect the fact that, other things being equal, introducing New Classical wage and price sectors increases the volatility of nominal variables and therefore makes a stronger response of the policy rate to inflation deviations necessary to ensure that inflation variability does not overshoot the level in the data. The lower interest rate persistence coefficient dampens the consequent increase in (long run) nominal interest rate variability whilst enabling policy to respond more rapidly to inflation developments.

As can be seen from the last column in Table 1, these calibrated values are also quite close to those estimated in Le *et al.* (2011), where the rejection rate of the model is minimised over the postwar period using the Wald test statistic from indirect inference. Notably, Le *et al.* report that whilst Smets and Wouters estimated model has a probability (under the null that the model is true) of essentially zero,[8] introducing some New Classical sectors into the model improves its performance because it is better able to match the (higher) level of nominal variability in the data, with the Wald test being passed at the 99th percentile. More specifically, Le *et al.* find Wald-minimising weights on the New Keynesian wage and price-setting sectors respectively of 0.87 and 0.82 (i.e. 87 and 82 per cent).[9] In light of this finding, the current paper considers a version of the model in which both New Keynesian and New Classical sectors play a role. In particular, wage and price-setting sectors are assumed to be 70 per cent New Keynesian and 30 per cent New Classical, fairly close to the optimal weights estimated by Le and co-authors.

One coefficient in Table 1 not estimated by Smets and Wouters or Le *et al.* is the weight on price-level deviations in the PLT Taylor rule. This weight was calibrated, somewhat aribitrarily, at the same level as the weight on inflation deviations under IT, with the aim preventing the

[8] A purely New Classical version of the model is likewise strongly rejected over the postwar period.

[9] Interestingly, however, Le *et al.* find that over the post-1984 Great Moderation period a purely New Keynesian version of the model approximately minimises the Wald statistic: the estimated weights are 0.9966 for wages and 0.9928 for prices (i.e. 99 per cent New Keynesian).

analysis favouring one regime over the other based on calibration rather than macroeconomic performance *per se*. This calibration implies that the price level is only returned to target only at a long horizon. Nevertheless, the dynamic behaviour of the simulated price level varies substantially between IT and PLT regimes (see Section 5.1).

Table 1 – Baseline calibrated values in the model

Parameter	Calibrated value	Smets and Wouters' (2007) mean estimates	Le *et al.* (2011) Wald-min. coefficients
φ	5.74	5.74	5.17
σ_c	1.38	1.38	1.52
λ	0.71	0.71	0.78
ξ_w	0.70	0.70	0.67
σ_l	1.83	1.83	1.65
ξ_p	0.66	0.66	0.60
ι_w	0.58	0.58	0.61
ι_p	0.24	0.24	0.26
ψ	0.54	0.54	0.55
Φ	1.50	1.60	1.35
ρ_R	0.60	0.81	0.54
θ_π	2.3	2.04	2.08
θ_p	2.3	NA	NA
θ_y	0.08	0.08	0.09
$\theta_{\Delta y}$	0.22	0.22	0.24
$100(\beta^{-1}-1)$	0.16	0.16	0.15
L_*	0.53	0.53	0.48
γ	0.43	0.43	0.47
α	0.19	0.19	0.21
$v_{w,NK}$	0.70	NA	0.87
$v_{p,NK}$	0.70	NA	0.82

The calibrated parameters for the AR(1) shock processes in the model are shown in Table 2. These coefficients were obtained by linearly-detrending the data series used by Smets and Wouters (though over the extended sample period used here, 1948:Q1 to 2004:Q4) to make them stationary[10] before estimating first-order autoregressions using ordinary least squares (OLS). The estimated coefficients from OLS were taken as the AR(1) coefficients and the regressions residuals as the innovations to each shock. In order to exploit the information contained in these historical innovations in model simulations, they were randomly reordered in each simulation using bootstrapping. There is no general consensus in the literature on whether

[10] Stationarity was tested using the Dickey-Fuller unit root test.

productivity is stationary or follows a random walk,[11] but the results obtained here suggest that productivity is trend stationary with a first-order autoregressive coefficient of 0.87.

Table 2 – Calibrated AR(1) coefficients for shock processes

Shock	Estimated AR(1) coefficient	Calibrated value
Productivity	0.870005	0.870005
Government spending	0.844399	0.844399
Monetary policy	0.588359	0.588359
Wage mark-up shock	-0.095387	-0.095387
Price mark-up shock	-0.092949	-0.092949
New Classical wage shock	0.929971	0.929971
Risk premium	0.165352	0.165352
Investment price shock	0.842571	0.842571

4 Simulation methodology

The model was solved using an algorithm written in Fortran code. This algorithm implements the Gauss-Siedel method for solving a system of equations (see Judd, 1998) and can easily deal with both the ZLB constraint and non-stationarity. Exploiting the latter feature, the model was simulated on top of a 'base run' given by the actual data over the postwar period, rather than around a deterministic steady-state. The main advantage with this methodology is that one can directly compare simulated paths from the model with the data from the postwar period; some examples of this are presented in the next section. Using this methodology also means that any constants that enter the model equations additively have no impact on the final simulation results. The ZLB constraint was incorporated using a few simple lines of code that tell Fortran to set the nominal rate at its lower bound value whenever it would otherwise be lower than this.

The model is solved under the assumption of certainty equivalence, that is, the assumption that all future shocks are zero; model simulations therefore do not capture higher-order effects from the ZLB constraint, in contrast to the papers by Adam and Billi (2006, 2007) and Nakov (2008) which make use of collocation methods. In effect, the Smets-Wouters model is sufficiently large as to make such methods too computationally intensive, and it is notable that Reifschneider and Williams (2000) also impose certainty equivalence when conducting simulations of the FRB/US model with an occasionally-binding ZLB. It is also worth bearing in mind that standard log-linearisation relies upon the certainty equivalence principle; the difference here of course is that the nominal interest rate is non-linear due to the ZLB constraint and can therefore potentially induce non-linearities in other variables in the model. Certainty equivalence effectively means that whilst such non-linearities can be feature of simulations *ex post*, agents in the model do not take account of such non-linearities when forming expectations about the future.

[11] The model solution algorithm used here can however deal with non-stationary shocks.

As mentioned above, the ZLB was not set at a numerical value of zero in simulations but instead at a constant deviation below the base run for the nominal interest rate (i.e. the nominal interest rate in the data) – namely 1.25 per cent per quarter.[12] This choice was driven by two factors. Firstly, as the model is simulated around the path given by postwar US data, setting a numerical ZLB of zero would automatically increase the chance of hitting the lower bound at times when the nominal interest rate in the data happened to be relatively low, potentially leading to biased results. Secondly, setting the lower bound at 1.25 per cent per quarter below the base run is akin to simulating a model with steady-state nominal interest rate of 5 per cent per annum – consistent with a steady-state real interest rate of 3 per cent per annum,[13] plus an inflation target of 2 per cent. The difference here is simply that the path around which the model is simulated is dynamic rather than constant, such that the base run and not a steady-state must be subtracted to get the pure effect of the shocks. For this reason, the simulation results reported below focus mainly on the deviations of simulated variables from their base run values.

The shocks in model simulations were based upon the historical ones that were derived from data over the sample period, as described in the previous section. Four hundred separate simulations of the model were conducted with these shocks randomly reordered in each simulation using a bootstrapping algorithm, hence giving 92,000 simulated values for each variable from which to compare the performance of IT and PLT. Quantitative results from these simulations are reported in the next section.

5 Results

The results reported in this section focus primarily on the full set of simulation results, though individual simulations are also drawn upon on occasion in order to provide intuition. As mentioned above, the model was simulated around the base run given by the data, so that the pure effect of shocks can be obtained by looking at the deviations of simulated values from base. Moreover, given that all shocks in the model are stationary, meaningful unconditional moments can be estimated for deviations from base – a fact which is exploited in the results that follow.

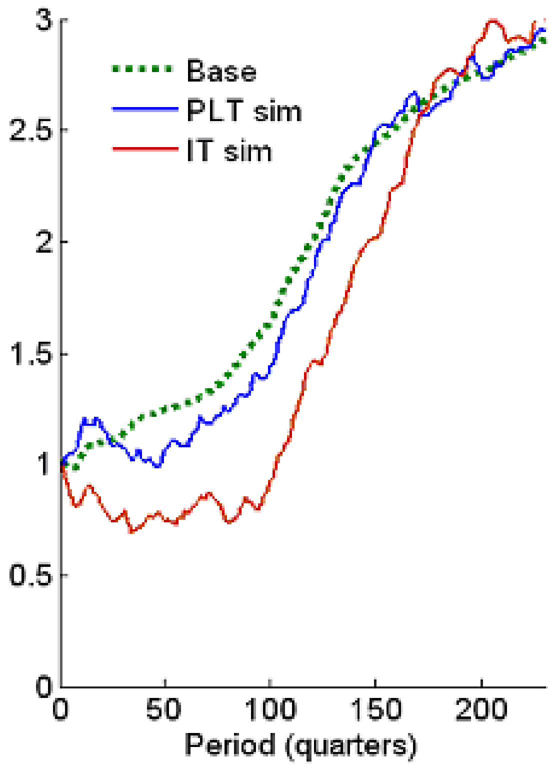
5.1 Price level dynamics under IT and PLT

From a theoretical perspective, the dynamics of the price level should vary somewhat between IT and PLT because the former permits base-level drift in the price level, whilst the latter aims to make the price level trend-stationary. It is therefore instructive to compare the simulated price level under IT and PLT in order to check that these regimes behave as we would expect. Such a

[12] The solution algorithm can deal with a numerical lower bound of zero without any problems.
[13] Reifschneieder and Willliams (2000) set the equilibrium real interest rate in their model constant at 2.5 per cent per annum. The analysis here effectively assumes a long run equilibrium nominal interest rate of 5 per cent per annum, which is close to the average nominal interest rate (in the data) over the sample period.

comparison is made in Figure 1, which shows the simulated (log) price levels under IT and PLT from a single random simulation of the model.[14]

Fig. 1 – Simulated price levels under IT and PLT



Note: The price level was set at 1 in the first period

As anticipated, the price level can deviate substantially from its base value under IT and displays random walk-type behaviour. For example, there is substantial deflation over the first forty quarters, but by the end of the simulation period the price level has 'drifted' up to a level close to the base (i.e. data) value which rises at an average rate of around 0.8 per cent per quarter. Likewise, the simulated price level under PLT behaves as expected: deviations from the base price level (also the target price path under PLT) are much smaller than under IT and are offset by policy, such that the simulated price level is clearly trend-stationary. An interesting feature of the simulated price level under PLT is that substantial deviations from the base price level are permitted and are offset only slowly, suggesting that a stronger response to price level deviations from target in the PLT Taylor rule may be optimal. This proposition, however, was not tested formally in the current paper.

[14] In particular, these results are from the first simulation (of 400) under IT and PLT. The base price level is the same in all simulations, because this is an index derived from actual inflation data over the sample period, with an initial value of one. As noted above, the model is simulated around base values – that is, around the dynamic paths of endogenous variables in the data.

5.2 Probability of ZLB episodes

A number of papers in the literature have argued that the probability of hitting the lower bound would be lower under PLT than IT (e.g. Coibon *et al.*, 2010). In this section, this prediction is tested formally using all 400 stochastic simulations of the model. The results reported focus, first, on the unconditional probability of hitting the ZLB; and second, on the probability of remaining at there in future quarters.

Table 3 reports the unconditional probabilities of hitting the ZLB under IT and PLT based on all 92,000 simulated periods (i.e. quarters). The ZLB is hit 10 per cent of time under IT, compared to 8.7 per cent under PLT. Hence, whilst PLT does reduce the probability of the ZLB being reached, the quantitative impact is not substantial. One possible reason for this relatively small impact is that, as noted in the previous section, deviations of the price level from target are offset only slowly under the baseline calibration of the PLT Taylor rule.

Table 3 – Unconditional probability of hitting the ZLB

IT	PLT
0.100	0.087

Even though the overall impact on the probability of hitting the ZLB is relatively small, PLT could still have a substantial impact on the typical length of ‘ZLB episodes’ as compared to IT. In order to investigate whether this is the case, the probability of remaining at the ZLB in future quarters was calculated conditional on the lower bound having been reached in the current quarter. Table 4 reports these conditional probabilities under IT and PLT for horizons from 1 quarter ahead up to 7 quarters.

Table 4 – Conditional probability of remaining at the ZLB after 1st quarter

No. of quarters ahead	IT	PLT
Next quarter	0.350	0.333
Next 2 quarters	0.180	0.115
Next 3 quarters	0.040	0.034
Next 7 quarters	4.35e-4	0

The probability of remaining at the ZLB after the first quarter is lower under PLT at all horizons. In general, the impact is quite small quantitatively, but it is notable that the probability of 3-quarter ZLB episodes (i.e. 3 consecutive quarters at the ZLB) is almost halved, from 18 per cent under IT to 11.5 per cent under PLT. Hence PLT reduces somewhat the chance of ZLB episodes that last 3 quarters or longer. It is also worth noting that whilst ZLB episodes lasting 2 years or longer have zero probability under PLT, there is a small positive probability attached to such episodes under IT.

The reasoning behind the lower probability of lower bound episodes under PLT runs as follows. The ZLB constraint prevents the central bank from lowering nominal interest rates to boost the economy, so inflation expectations play a pivotal role at times when there is strong downward pressure on output and prices. Under IT, inflation expectations are stabilised at the inflation target rather than stimulated, so the *ex ante* real interest rate is effectively bounded from below when the ZLB is reached. Under PLT, however, inflation expectations are linked to the target price level. Since the price level will typically fall relative to target at times when the ZLB is hit, rational agents anticipate higher future inflation to return the price level to target. This increase in expected inflation then lowers the *ex ante* real interest rate, boosting current consumption, investment and output (and in turn inflation), and moving the economy away from the ZLB, or at least in that direction.[15] Indeed, what the results above indicate is that whilst PLT reduces only slightly the overall probability of hitting the lower bound, it is quite effective at preventing long-lived ZLB episodes.

The intuition given above can be demonstrated formally by examining typical ZLB episodes under IT and PLT from stochastic simulations. A comparison of this kind is made in figures 2 and 3, which express output and inflation as deviations from base for ease of interpretation. In the IT case, depicted in Figure 2, output is initially falling relative to base and inflation is strongly negative. Consequently, the nominal interest rate is set at the lower bound, or close to, for 10 consecutive quarters.[16] Because inflation expectations are not stimulated under IT with the ZLB constraint, inflation initially falls (as does output) and does not become consistently positive until after six consecutive quarters at the ZLB, after which time output was eventually stabilised.

By contrast, a typical ZLB episode under PLT is shown in Figure 3. In this example, output (rather than inflation) is somewhat below base and inflation is initially falling towards its base level, whilst the price level is already below target. Via the PLT Taylor rule, this situation calls for a zero nominal interest rate. However, because the price level is already below target, inflation expectations are automatically stimulated, such that three consecutive quarters at the ZLB are sufficient to make inflation strongly positive and to stabilise output. Hence, what would potentially be a long-lived ZLB episode under IT is a relatively short one under PLT, with the economy soon recovering to a situation of strongly positive nominal interest rates.

[15] That is to say, rational agents who believe that future prices will rise face a lower opportunity cost from not investing or saving, because a given nominal return will purchase less in real terms in the future. It is clear from this discussion why the assumption of perfect credibility of PLT is of great importance at the ZLB; see Cateau and Dorich (2011) for a formal analysis.

[16] 1.25% per quarter was added to the simulated nominal rate so that the ZLB appears at a numerical value of zero in figures 2and 3.

Fig. 2 – A typical ZLB episode under IT

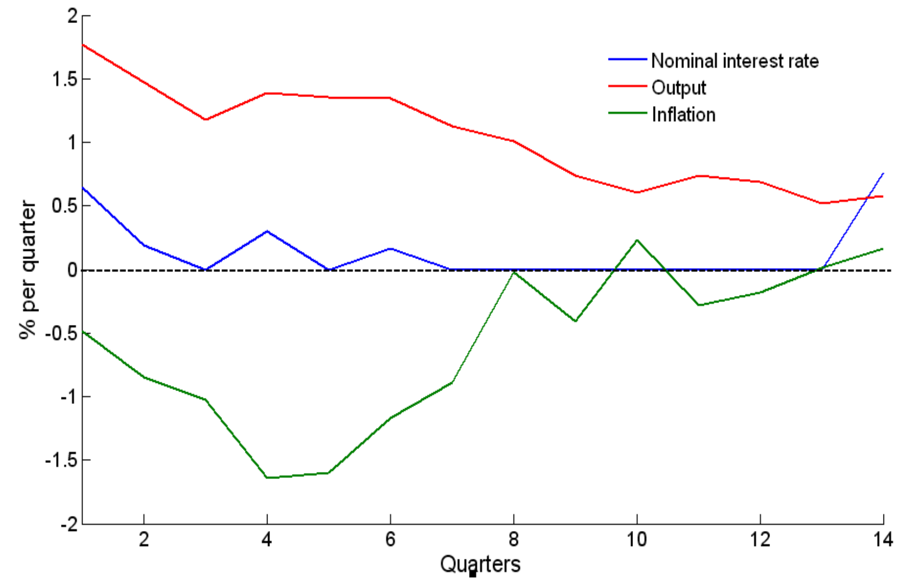
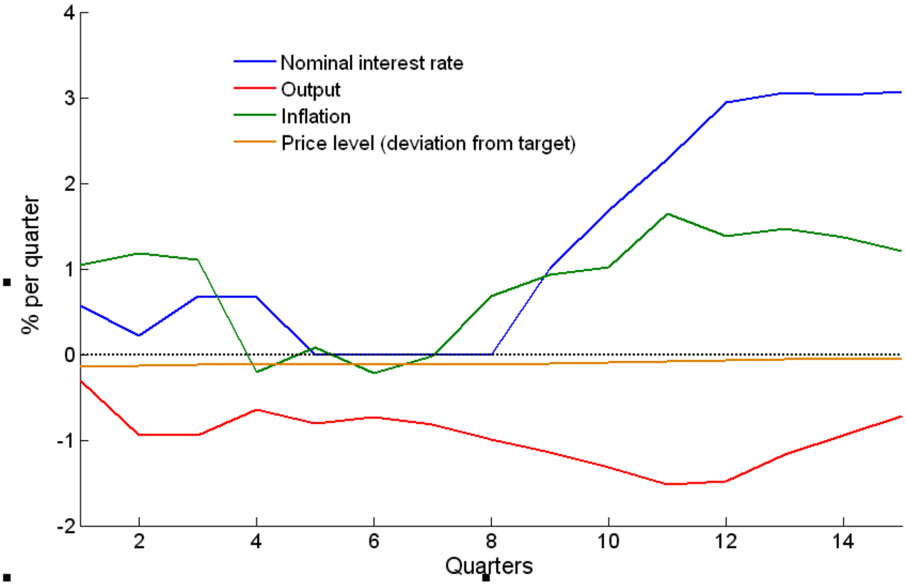


Fig. 3 – A typical ZLB episode under PLT



Note: Inflation is the deviation from the base run; output is the percentage deviation from the base run; the price level is the fractional deviation from target. For the nominal rate, 1.25% is added so that the ZLB appears at 0 and not -1.25%.

5.3 Macroeconomic performance

In order to assess the overall impact of PLT on the economy, this section reports unconditional means and volatilities for key variables based upon all 400 simulations of the model.[17] Table 5 shows that there are some non-trivial differences between IT and PLT in terms of means, with

[17] These results focus on deviations from base for which meaningful unconditional moments will exist given that shocks are stationary.

real variables on average higher under PLT than IT. Nominal variables, i.e. inflation and the nominal interest rate, are also higher under PLT than IT, though the difference is small at 0.01 to 0.03 percentage points per quarter. It thus appears that the lower probability of hitting (or remaining at) the ZLB under PLT has a first-order effect on key variables like investment, consumption, and output, by reducing the frequency and severity of occasions where these variables fall dramatically. Consistent with this intuition, inflation is lower on average under IT.

Table 5 – Simulated means of key variables under IT and PLT

Variable	IT	PLT	Diff (PLT-IT)
Consumption	0.89	1.15	0.26
Investment	-1.52	0.45	1.97
Output	0.81	1.27	0.46
Labour supply	-0.15	-0.10	0.05
Nominal interest rate (%)	0.08	0.11	0.03
Inflation (%)	-0.01	0.00	0.01

Note: variables are in deviations from base

Table 6 also seems to support this intuition since the volatilities of key variables are somewhat higher under IT than PLT, though it is not clear to what extent this is driven by performance away from the ZLB. For example, the variance of consumption is 8 per cent lower under PLT, whilst inflation is almost 12 per cent less volatile. The largest volatility reduction occurs for labour supply at more than 15 per cent.

Table 6 – Standard deviations of key variables under IT and PLT

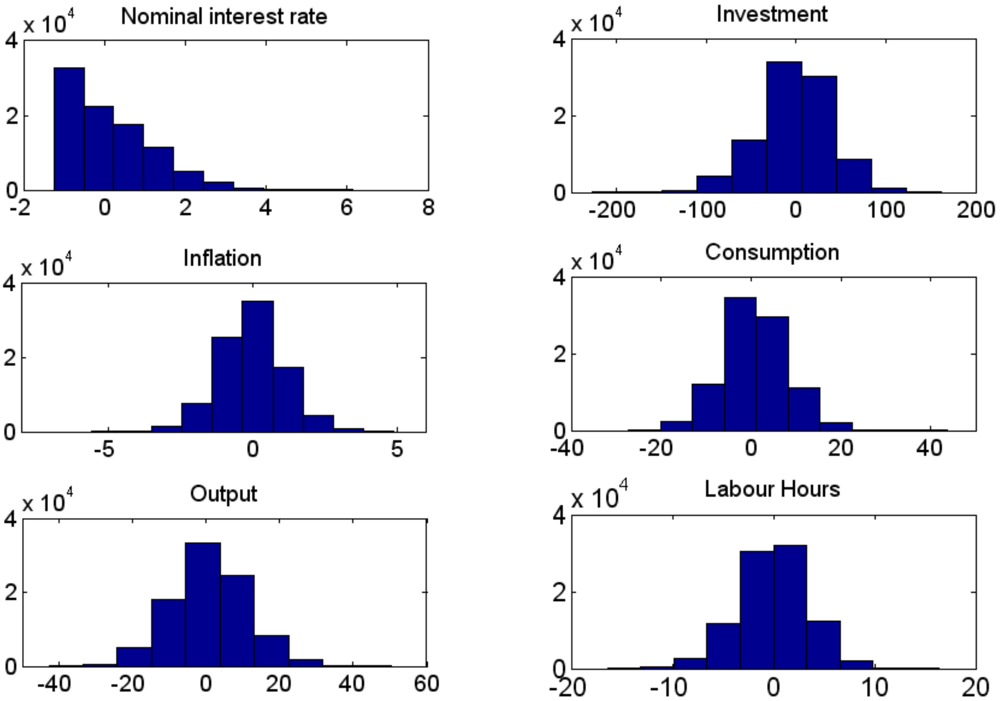
Variable	IT	PLT	Diff (PLT-IT)	% Diff in variance
Consumption	7.14	6.86	-0.28	-7.7
Investment	39.62	38.83	-0.79	-4.0
Output	10.28	10.18	-0.10	-2.0
Labour supply	3.80	3.46	-0.34	-17.1
Nominal interest rate (%)	1.09	1.05	-0.04	-7.3
Inflation (%)	1.10	1.04	-0.06	-11.6

Note: variables are in deviations (in levels) from base

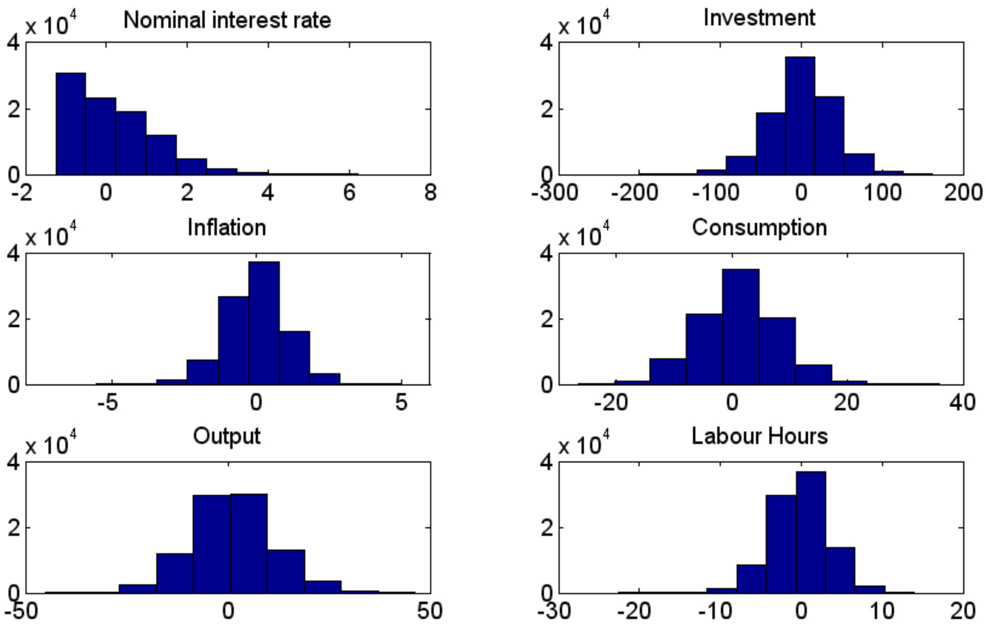
Finally, also consistent with the idea that PLT has a first-order impact, the distributions of key variables are more skewed under IT than PLT. Indeed, as is highlighted by the histograms in panels (a) and (b) of Figure 4, investment, consumption and output are noticeably less skewed under PLT than under IT. Notice also that the nominal interest rate is strongly positively skewed due to the presence of the lower bound constraint which means that there is no probability lying to the left of the ZLB.

Fig. 4 – Histograms of key variables under IT and PLT
(deviations from base)

Panel (a) – IT



Panel (b) – PLT



Note: as variables are in deviations from base, the ZLB occurs at - 1.25% (see Section 4).

The finding that accounting for the ZLB can have a first-order impact on key variables stands in contrast to the findings of several studies where it has essentially no impact on average inflation and output (see Adam and Billi, 2006, 2007; Nakov, 2008). It is therefore worthwhile to consider what might account for this difference. The main difference between these studies and the current one is the size of the model at hand.[18] In Adam and Billi (2006, 2007) and Nakov (2008) the US economy is modelled using the canonical 3-equation New Keynesian model, whilst the Smets-Wouters model includes several additional economic transmission mechanisms, including investment and capacity utilisation; real rigidities such as habit formation and investment adjustment costs; and sources of nominal inertia like price and wage indexation. Taken together, these mechanisms seem to exacerbate the impact of the ZLB on key macro variables.

Consider, for example, the impact of a one-off large negative demand shock. In both models, inflation and output will fall in response, triggering a reduction in the nominal interest rate down to (say) zero via the Taylor rule. In the case of the canonical New Keynesian model, this reduction in the nominal rate should be enough to rapidly return the economy to, or near, its pre-shock state, because there are no other structural sources of inertia in the economy. On the other hand, the same shock is likely to have longer-lasting effects in the Smets-Wouters model. Indeed, a negative demand shock will reduce both consumption and Tobin's Q simultaneously (see Appendix A), with the latter leading to a reduction in the capital stock through the investment Euler equation. Consumption will then remain low due to habit formation, and investment too because of the strong adjustment costs associated with rapidly increasing the capital stock. As a result, a prolonged period of zero nominal rates may be necessary for inflation and output can recover to their pre-shock levels. It is also notable that the presence of New Classical sectors increases nominal volatility compared to a purely New Keynesian version of the model, with the result that the nominal rate must move more to offset inflationary shocks and is therefore more likely to hit the ZLB.[19]

To summarise, the results presented in this section point to PLT having a positive impact on mean consumption, investment, and output, by reducing the probability and severity of ZLB episodes. Moreover, in line with this reasoning, the volatilities of key macro variables are lower under PLT than IT. Though these results are intuitive, the reader should bear in mind that these results are based on only 400 stochastic simulations (rather less than would be ideal) and that the innovations under IT and PLT in each simulation are not constrained to be identical, though they were of course bootstrapped from the same historical innovations. These two factors may

[18] One other difference is that the models analysed by Adam and Billi and Nakov are solved using collocation methods and therefore do not impose the assumption of certainty equivalence. However, it seems unlikely that this could account for the difference of the ZLB on mean values.

[19] In a forthcoming working paper, the author investigates the impact of the extent of nominal rigidity on the probability of hitting the ZLB.

account for at least some of the observed difference in mean values between IT and PLT, and the extent to which they do is an interesting topic for future research

6 Conclusions

The zero lower bound (ZLB) has been an important constraint on monetary policy in recent years. Consequently, economic researchers have begun to focus greater attention on policies that could help alleviate problems posed by the ZLB. This paper provides an assessment of one such alternative – price-level targeting – using full stochastic simulations of the Smets-Wouters US model where the ZLB is an *occasionally-binding constraint*. Given that it is widely believed that the Federal Reserve has an implicit target for inflation, inflation targeting (IT) was used as a benchmark against which the performance of PLT was compared. Moreover, following Le *et al.* (2011), the paper simulates a version of the Smets-Wouters model in which the economy consists of New Classical sectors with perfectly flexible prices, as well as New Keynesian sectors with 'sticky' wages and prices. This 'hybrid' model is better able to match the dynamic behaviour of post-war US data than a purely New Keynesian version, because it raises the extent of nominal volatility in the economy towards the level observed in the data whilst still performing well in terms of matching real variables' dynamics.

Several interesting results were reported. First, the probability of hitting the ZLB is substantial under both IT and PLT, but PLT cuts the risk of hitting the lower bound, consistent with the predictions of past studies that did not impose the ZLB directly (e.g. Gaspar and Smets, 2003; Amano and Ambler, 2008). Second, ZLB episodes – that is, periods of time when the economy has zero interest rates for two or more consecutive quarters – are less frequent under PLT than IT and have shorter durations. Notably, the conditional probability of remaining at the ZLB for two further quarters is reduced from 18 per cent under IT to 12 per cent under PLT, indicating that, in less severe cases, PLT has the ability to move the economy away from the ZLB and prevent an episode of zero nominal rates from developing. Third, the volatility of key macro variables is reduced under PLT, with non-trivial percentage reductions in the variances of labour supply, consumption, investment and output.

Most significantly, the reduction in the length and severity of ZLB episodes under PLT appears to have a first-order impact upon key macro variables like consumption, output and investment – all of which are higher on average under PLT, consistent with there being fewer periods when these variables fall sharply. Although these overall effects are small quantitatively, they are of potential importance from a policy perspective because, as noted by Rudebsuch and Swanson (2008), first-order effects in DSGE models are typically 100 times larger than those arising at second-order – hence raising the question of whether the effects observed here would be sufficient to deliver substantial welfare gains from PLT vis-à-vis IT. A caveat, however, is that the first-order impact ought ideally to be gauged from far more than 400 simulations of each regime, though it seems unlikely that the results reported here are purely spurious. More realistically, they may overstate or understate the average impact of PLT to some extent – thus highlighting the need for future research on this topic.

Overall, the results reported in this paper suggest that the stability of the economy would be improved by adopting PLT, and that by doing so costly ZLB episodes could sometimes be avoided. However, these implications need to be tested along numerous dimensions before any solid policy implications can be drawn. Going forward, future research assessing the performance of PLT will need to confront several issues. First, there is the issue of imperfect credibility of PLT when the ZLB is occasionally-binding. The recent analysis by Cateau and Dorich (2011) demonstrates that allowing for imperfect credibility under PLT can reduce somewhat its attractiveness vis-à-vis IT, but analyses of this kind also need to be conducted in medium or large-scale DSGE models to test robustness. Indeed, previous work at the Bank of Canada investigating imperfect credibility of PLT in the absence of ZLB considerations indicates that the impact depends crucially on the range of economic transmission mechanisms in the model at hand (see Krvtsov *et al.*, 2008 and Cateau *et al.*, 2009).

Second, models with constant parameters and variances are likely to understate the probability of encountering the ZLB (see Chung *et al.* (2011)), and may therefore also understate the potential benefits from PLT vis-à-vis IT. Although allowing for higher-order effects of this kind is a difficult task and computationally-intensive (particularly when also modelling the occasionally-binding ZLB constraint),[20] this is an important and interesting task for future research. Analyses of this kind are likely to be necessary in order to accurately assess the performance of PLT versus IT in the presence of the ZLB. Clearly this is an important consideration for central banks like the Bank of Canada that are considering switching from an IT regime to a PLT regime in the near future.

Finally, the analysis in this paper considers a relatively weak form of PLT in the sense that the Taylor rule response to price level deviations is low and thus permits relatively large deviations of the price level from target for long periods of time. It is an open question whether a stronger response to the price level would reduce the probability of ZLB episodes further – though this certainly seems likely over some range. Likewise, it may be the case that hybrid targeting of inflation and the price level or ‘average inflation targeting’ (see Nessén and Vestin, 2005) would outperform pure PLT in terms of macroeconomic performance when the ZLB is taken into account. The author hopes to pursue these issues in future research using a simulated annealing algorithm with social welfare as the criterion function to be maximised by the policy parameters.

[20] For example, higher-order effects like parameter uncertainty and stochastic volatility can be assessed using perturbation methods (e.g. as in Dynare), but only in the absence of the ZLB constraint (see Ambler, 2009).

Appendix A: Model listing

Consumption Euler equation

$$c_t = \frac{\lambda}{\gamma+\lambda}c_{t-1} + \frac{\gamma}{\gamma+\lambda}E_t c_{t+1} + \frac{(\sigma_c-1)(W_*^h L_* / C_*)}{\sigma_c(1+\lambda/\gamma)}(l_t - E_t l_{t+1}) - \frac{(1-\lambda/\gamma)}{(1+\lambda/\gamma)\sigma_c}(R_t^{nom} - E_t\pi_{t+1}) + e_{1,t} \quad \text{(A1)}$$

Investment Euler Equation

$$inv_t = \frac{1}{1+\beta\gamma^{(1-\sigma_c)}}inv_{t-1} + \frac{\beta\gamma^{(1-\sigma_c)}}{1+\beta\gamma^{(1-\sigma_c)}}E_t inv_{t+1} + \frac{1}{(1+\beta\gamma^{(1-\sigma_c)})\gamma^2\varphi}q_t + e_{2,t} \quad \text{(A2)}$$

Tobin's Q

$$q_t = \frac{1-\delta}{1-\delta+R_*^K}E_t q_{t+1} + \frac{R_*^K}{1-\delta+R_*^K}E_t rk_{t+1} - (R_t^{nom} - E_t\pi_{t+1}) + \frac{(1+\lambda/\gamma)\sigma_c}{1-\lambda/\gamma}e_{1,t} \quad \text{(A3)}$$

Capital accumulation equation

$$k_t = (1-\delta/\gamma)k_{t-1} + (\delta/\gamma)inv_t + (1-\delta/\gamma)(1+\beta\gamma^{(1-\sigma_c)})\varphi\gamma^2 e_{2,t} \quad \text{(A4)}$$

Wage-setting Phillips curve

$$w_t = v_{w,NK}w_{t,NK} + (1-v_{w,NK})w_{t,NC} \quad \text{(A5)}$$

NK real wage

$$\begin{aligned} w_{t,NK} = {} & \frac{\beta\gamma^{(1-\sigma_c)}}{1+\beta\gamma^{(1-\sigma_c)}}(E_t w_{t+1} + E_t\pi_{t+1}) + \frac{1}{1+\beta\gamma^{(1-\sigma_c)}}(w_{t-1} + \iota_w\pi_{t-1}) - \frac{1+\beta\gamma^{(1-\sigma_c)}\iota_w}{1+\beta\gamma^{(1-\sigma_c)}}\pi_t \\ & - \frac{1}{1+\beta\gamma^{(1-\sigma_c)}}\left(\frac{(1-\beta\gamma^{1-\sigma_c}\xi_w)(1-\xi_w)}{(1+\varepsilon_w(\Phi_w-1))\xi_w}\right)\mu_t^w + e_{3,t} \end{aligned} \quad \text{(A6)}$$

NK wage mark-up

$$\mu_t^w = w_t - \sigma_l l_t - (1/(1-\lambda/\gamma))(c_t - (\lambda/\gamma)c_{t-1}) \quad \text{(A7)}$$

NC real wage

$$w_{t,NC} = \sigma_l l_t + (1/(1-\lambda/\gamma))(c_t - (\lambda/\gamma)c_{t-1}) - (\pi_t - E_{t-1}\pi_t) + e_{3,t}^{NC} \quad \text{(A8)}$$

Phillips curve

$$\begin{aligned} \pi_t = {} & v_{p,NK}\left(\frac{\beta\gamma^{(1-\sigma_c)}}{1+\beta\gamma^{(1-\sigma_c)}\iota_p}E_t\pi_{t+1} + \frac{\iota_p}{1+\beta\gamma^{(1-\sigma_c)}\iota_p}\pi_{t-1} + e_{4,t} - \frac{1}{1+\beta\gamma^{(1-\sigma_c)}\iota_p}\left(\frac{(1-\beta\gamma^{1-\sigma_c}\xi_p)(1-\xi_p)}{(1+\varepsilon_p(\phi_p-1))\xi_p}\right)\mu_t^p\right) \\ & + (1-v_{p,NK})\left(\alpha rk_t + (1-\alpha)w_t - e_{5,t}\right) \end{aligned} \quad \text{(A9)}$$

NK price mark-up

$$\mu_t^p = \alpha(k_t^s - l_t) + e_{5,t} - w_t \tag{A10}$$

Capital services

$$k_t^s = k_{t-1} + z_t \tag{A11}$$

Capacity utilisation

$$z_t = \frac{(1-\psi)}{\psi} rk_t \tag{A12}$$

Rental rate on capital

$$rk_t = w_t + (l_t - k_t^s) \tag{A13}$$

Market-clearing in goods market

$$y_t = c_y c_t + i_y inv_t + R_*^K k_y ((1-\psi)/\psi) rk_t + e_{6,t} \tag{A14}$$

Aggregate production function

$$y_t = \phi_p \left(\alpha k_t^s + (1-\alpha) l_t + e_{5,t} \right) \tag{A15}$$

Taylor rule

$$R_t^{nom} = \begin{cases} \max\left\{ R^{\min},\ \rho_R R_{t-1}^{nom} + (1-\rho_R)(\theta_\pi \pi_t + \theta_y y_t) + \theta_{\Delta y}(y_t - y_{t-1}) + e_{7,t} \right\} & \text{under IT} \\ \max\left\{ R^{\min},\ \rho_R R_{t-1}^{nom} + (1-\rho_R)(\theta_p (p_t - p_{tar,t}) + \theta_y y_t) + \theta_{\Delta y}(y_t - y_{t-1}) + e_{7,t} \right\} & \text{under PLT} \end{cases} \tag{A16}$$

Log price level

$$p_t = p_{t-1} + \pi_t / 100 \tag{A17}$$

Target price level

$$p_{tar,t} = p_{base,t-1} \tag{A18}$$

Market-clearing in the labour market

$$n_t = l_t \tag{A19}$$

Shock processes

All follow AR(1) processes with persistence coefficients and innovations $v_{i,t}$ estimated from data

Risk premium

$$e_{1,t} = \rho_1 e_{1,t-1} + v_{1,t} \tag{A20}$$

Investment

$$e_{2,t} = \rho_2 e_{2,t-1} + v_{2,t} \tag{A21}$$

Wage mark-up

$$e_{3,t} = \rho_3 e_{3,t-1} + v_{3,t} \tag{A22}$$

NC wage shock

$$e_{3,t}^{NC} = \rho_{s,NC} e_{3,t}^{NC} + v_{s,t}^{NC} \tag{A23}$$

Price mark-up

$$e_{4,t} = \rho_4 e_{4,t-1} + v_{4,t} \tag{A24}$$

Productivity

$$e_{5,t} = \rho_5 e_{5,t-1} + v_{5,t} \tag{A25}$$

Government spending

$$e_{6,t} = \rho_6 e_{6,t-1} + v_{6,t} \tag{A26}$$

Monetary policy

$$e_{7,t} = \rho_7 e_{7,t-1} + v_{7,t} \tag{A27}$$

Appendix B: Definitions of variables

Variables

c_t = consumption
R_t^{nom} = net nominal interest rate (per cent per quarter)
l_t = labour supply
n_t = labour demand
inv_t = investment
q_t = Tobin's Q
rk_t = real rental rate on capital
w_t = real wage
W_t = nominal wage
k_t = capital
k_t^s = capital services
z_t = capital utilisation rate
μ_t^w = New Keynesian wage mark-up
μ_t^p = New Keynesian price mark-up
y_t = output
π_t = inflation (per cent per quarter)
p_t = log price level
$p_{tar,t}$ = target price level
$p_{base,t-1}$ = (lagged) base price level

Shocks

$e_{1,t}$ = exogenous risk premium shock
$e_{2,t}$ = investment relative price shock
$e_{3,t}$ = wage mark-up shock
$e_{3,t}^{NC}$ = New Classical wage shock
$e_{4,t}$ = price mark-up shock
$e_{5,t}$ = total factor productivity shock
$e_{6,t}$ = exogenous government spending
$e_{7,t}$ = monetary policy shock

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