

**Cardiff Economics  
Working Papers**

Michael Hatcher

*Price-level targeting versus inflation targeting  
over the long-term*

E2011/5

Cardiff Business School  
Cardiff University  
Colum Drive  
Cardiff CF10 3EU  
United Kingdom  
t: +44 (0)29 2087 4000  
f: +44 (0)29 2087 4419  
[www.cardiff.ac.uk/carbs](http://www.cardiff.ac.uk/carbs)

ISSN 1749-6101  
March 2011

This working paper is produced for discussion purpose only. These working papers are expected to be published in due course, in revised form, and should not be quoted or cited without the author's written permission. Cardiff Economics Working Papers are available online from: <http://www.cardiff.ac.uk/carbs/econ/workingpapers>  
Enquiries: [EconWP@cardiff.ac.uk](mailto:EconWP@cardiff.ac.uk)

# Price-level targeting versus inflation targeting over the long-term<sup>1</sup>

Michael Hatcher  
PhD Student, Cardiff Business School, Cardiff University  
Aberconway Drive, Cardiff CF10 3EU, UK.  
Email: [hatcher@cf.ac.uk](mailto:hatcher@cf.ac.uk)  
Phone: +44 029 2087 6449

March 2011

Preliminary Draft

---

## Abstract

This paper investigates the long-term impact of price-level targeting on social welfare in an overlapping generations model in which the young save for old age by investing in productive capital and indexed and nominal government bonds. A key feature of the model is that the extent of bond indexation is determined endogenously in response to monetary policy as part of an optimal commitment Ramsey policy. Due to the absence of base-level drift under price-level targeting, long-term inflation risk is reduced by an order of magnitude compared to inflation targeting. Consequently, real bond returns are stabilised somewhat, and consumption volatility for old generations is reduced by around 15 per cent. The baseline welfare gain from price-level targeting is equivalent to a permanent increase in aggregate consumption of only 0.01 per cent, but this estimate is strongly sensitive on the upside.

**Keywords:** inflation targeting, price-level targeting, optimal indexation, government bonds.

**JEL Classification:** E52, E58.

---

---

<sup>1</sup> The author thanks his supervisors, Professor Patrick Minford and Dr. Panayiotis Pourpourides, for their help and guidance, and would also like to thank participants at the 3<sup>rd</sup> PhD Conference in Economics (May 2010, Athens) and the 27<sup>th</sup> Symposium on Money, Banking and Finance (June 2010, Bordeaux) for useful comments on an earlier version of this paper. This research was funded by the ESRC.

## 1. Introduction

This paper investigates the long-term impact of price-level targeting on social welfare. The motivation for studying this issue can be traced back to a result that is well-known to monetary economists, namely, that inflation targeting (IT) implies ‘base drift’ in the price level, whereas price-level targeting (PLT) introduces trend-stationarity. Hence, whilst inflation risk increases with the forecast horizon under IT, it is bounded under PLT. A number of papers have investigated the consequences of this result for inflation uncertainty at a long horizon, concluding that uncertainty would be reduced by an order of magnitude under PLT (Dittmar, Gavin and Kydland, 1999; Bordo, Dittmar and Gavin, 2007; Gavin, Keen and Pakko, 2009). However, no papers have yet investigated, within a dynamic stochastic general equilibrium (DSGE) model, the social welfare impact of PLT through the long-term inflation risk channel.<sup>2</sup> Crucially, as Gavin, Keen and Pakko (2009) point out, standard New Keynesian models are not suitable for this purpose, because only short-term inflation fluctuations lead to welfare losses for the representative agent (see Woodford, 2001).

This paper presents a DSGE model in which long-term inflation risk matters for social welfare, and uses it to estimate the long-term welfare impact of PLT. Making such a calculation is of particular importance in light of the fact that the Bank of Canada is currently conducting a review of PLT in anticipation of its next policy agreement with the Government in 2011.<sup>3</sup> Indeed, as emphasised by Ambler (2009), evidence on the long-term impact of PLT on social welfare is necessary so that a full cost-benefit analysis of PLT can be undertaken. For this reason, the Bank of Canada has identified this topic as a key area in which further research is needed (see Bank of Canada, 2009). Moreover, other central banks and policy institutions have begun to investigate PLT for themselves – a partial list includes the Bank of Finland (Mayes, 2008), the Bundesbank (Bundesbank, 2010), the ECB (Gaspar, Smets and Vestin, 2007), the OECD (Cournède and Moccero, 2009) and US Federal Reserve banks (e.g. Kahn, 2009) –, making the long-term impact of PLT of potential policy importance more widely.<sup>4</sup>

The model put forth in this paper is an overlapping generations (OLG) model of life-cycle saving in which the young save for old age using productive capital and indexed and nominal government bonds whose payoffs are vulnerable to inflation risk. The motivation for studying the welfare impact of PLT in a model with long-term bonds that offer imperfect insurance against inflation is set out clearly by Carlstrom and Fuerst (2002):

---

<sup>2</sup> That is, to the author’s knowledge.

<sup>3</sup> The review was announced in Bank of Canada (2006).

<sup>4</sup> There is a large literature on the impact of PLT at a short horizon; see the excellent surveys by Ambler (2009) and Cournède and Moccero (2009).

“[T]he base-drift problem with IT leads to a great deal of uncertainty about what the price level 5, 10, or 30 years in the future will be. The central bank may miss its inflation target by a very small percentage in each year, but if these misses are not offset, they will accumulate and become quite large after 30 years. Therefore, a price-level target will reduce the uncertainty associated with buying and selling long-term fixed bonds.”

A key feature of the model is that indexation of government bonds is determined endogenously in response to monetary policy as part of an optimal commitment Ramsey policy. This feature of the model is important because social welfare comparisons across monetary policy regimes that are vulnerable to the Lucas critique can give seriously misleading results (Ambler, 2009). This point has been clearly demonstrated for IT versus PLT in the context of wage indexation. Indeed, optimal wage indexation is substantially lower under PLT than IT (Minford, Nowell and Webb, 2003; Amano, Ambler and Ireland, 2007), and failure to account for this difference can give highly misleading welfare results that are of little practical use to policymakers (Minford and Peel, 2003). A second critical feature of the model is that monetary policy is implemented via money supply rules that capture the long-term impact of base-level drift under IT and its absence under PLT. This is done by building up the long-term money supply rules in the model from a yearly horizon.

Each generation in the model lives for two periods – youth and old age – that last 30 years each; hence the young can be thought of as saving over a 30-year horizon from ‘average youth’ until ‘average old age’ before consuming the lifetime savings they have accumulated. Young generations can save for old age by holding money balances, productive capital and indexed and nominal government bonds, and there is aggregate uncertainty due to real and nominal disturbances. The government, the monopoly supplier of bonds in the economy, sets the supplies of indexed and nominal bonds so as to maximise social welfare, subject to the monetary policy regime in place and consumers’ first-order conditions for optimal saving. Thus the government solves an optimal commitment Ramsey problem whose solution determines the economy’s equilibrium and, ultimately, the level of social welfare under IT and PLT.

The main findings from the model are as follows. Firstly, due to the presence of base-level drift under IT and its absence under PLT, inflation volatility is reduced by an order of magnitude under PLT. Secondly, as a result of the reduction in inflation volatility under PLT, real returns on indexed and nominal bonds are substantially less volatile, so that consumption volatility across old generations is reduced by around 15 per cent. Thirdly, because consumption volatility is reduced under PLT, social welfare is increased relative to IT. The estimated welfare gain under the baseline calibration is small at 0.01 per cent of aggregate consumption, but this estimate is highly sensitive on the upside. Countries that have little (or no) indexation, high nominal volatility,

relatively high risk aversion, and in which the public sector plays a dominant role in providing retirement income, could gain more substantially from price-level targeting. The main policy implication arising from these results is that the long-term welfare gain from PLT through the inflation risk channel is likely to be relatively small, but not sufficiently so that it can safely be ignored in a cost-benefit analysis of PLT.

The paper proceeds as follows. Section 2 sets out the model, including the monetary policy rules under IT and PLT; Section 3 discusses model calibration; Section 4 describes how the model was solved; Section 5 presents simulation results; and Section 6 tests robustness. Finally, Section 7 concludes and discusses policy implications.

## 2. The Model

The model is an OLG model of life-cycle saving in which consumers hold money, capital, and indexed and nominal government bonds. Consumers have homogenous preferences and live for two periods of 30 years: in the first they are ‘young’ and receive an exogenous endowment income; and in the second they are ‘old’ and consume the proceeds from their savings in youth, including output produced using capital. Population growth is set equal to zero for simplicity<sup>5</sup> and, without loss of generality, each generation is assumed to have a constant size of one.<sup>6</sup>

Although ‘fiat money’ is a popular way of justifying money holdings in OLG models (e.g. McCandless and Wallace, 1991), this approach is not theoretically convincing because money must offer the same return as non-monetary assets to have value, implying deflation if other assets offer real returns. Money is instead introduced by a cash-in-advance constraint, an approach taken by a number of recent contributions that investigate optimal monetary policy in OLG economies (e.g. Michel and Wigniolle, 2005; Gahvari, 2007). Monetary policy takes the form of IT and PLT money supply rules that are implemented by the government.

The government also carries out fiscal policy by taxing young consumers in order to meet a long run government spending target<sup>7</sup> and issuing government bonds. The total bond supply is set to ensure optimal consumption smoothing (in expected terms) for each generation, along the lines of the standard OLG model where government bonds are ‘net wealth’ (see Barro, 1974; Minford and Peel, 2002). The mix between indexed

---

<sup>5</sup> Constant population growth would introduce an additional parameter (the population growth rate) but would not change model dynamics or, therefore, the social welfare results.

<sup>6</sup> There is no loss of generality because the focus throughout is on per-capita values. All model equations would be left unchanged if generations had a constant size greater than one and were populated by homogenous consumers. The only difference is that per-capita values would need to be multiplied by the constant generation size in order to get economy-wide aggregates.

<sup>7</sup> This spending is used up in projects that have no direct effect on consumption or utility.

and nominal bonds, or the ‘indexation share’, is chosen to maximise social welfare, subject to consumers’ first-order conditions, monetary policy, and the long run government spending target. Since indexed and nominal bonds are priced to rule out arbitrage by consumers’ first-order conditions, all indexation shares in the range  $[0,1]$  are feasible equilibria. In effect, the government’s indexation problem is to select the equilibrium from this feasible set that maximises social welfare under IT and PLT.

The model is solved using a second-order approximation in Dynare++ (Julliard, 2001). This point is crucial since a linear approximation would ignore asset risk-premia. More generally, it is well-known that linear approximation can lead to an inaccurate social welfare ranking of alternative policies because it ignores the impact of uncertainty on the stochastic means of endogenous variables in the model (Kim and Kim, 2003; Schmitt-Grohé and Uribe, 2004).

## 2.1 The consumer problem

Consumers live for two periods of 30 years and have constant relative risk aversion (CRRA) preferences:

$$u_t = u_{t,Y}(c_{t,Y}) + E_t u_{t+1,O}(c_{t+1,O}) \quad (1)$$

where  $u_{t,Y}(c_{t,Y}) \equiv c_{t,Y}^{1-\delta} / (1-\delta)$  is utility in youth and  $u_{t+1,O}(c_{t+1,O}) \equiv c_{t+1,O}^{1-\delta} / (1-\delta)$  is utility in old age. Consumption in period  $t$  when young is denoted by  $c_{t,Y}$ , and  $c_{t+1,O}$  is consumption in period  $t+1$  when old.<sup>8</sup>

The budget constraint faced by the young can be expressed in real terms as follows:

$$c_{t,Y} + b_t^{i,d} + b_t^{n,d} + m_t^d + k_t = \varpi(1 - \tau^j) \quad (2)$$

where  $\varpi$  is a young consumer’s constant real endowment income;  $b_t^{i,d} \equiv B_t^{i,d} / P_t$  is real demand for indexed bonds;  $b_t^{n,d} \equiv B_t^{n,d} / P_t$  is real demand for nominal bonds; and  $m_t^d \equiv M_t^d / P_t$  is demand for real money balances. Note that uppercase values are nominal and  $P_t$  is the aggregate price level. Capital in real terms is given by  $k_t$ , and  $\tau^j$  for  $j \in (IT, PLT)$  is the constant rate of income tax.

---

<sup>8</sup> Consumers do not discount consumption in old age, as is often assumed in models with overlapping generations. Examples from the literature that use this assumption include Champ and Freeman (1990) and Brazier, Harrison, King and Yates (2006).

Following Artus (1995), consumers' demand for money arises from a cash-in-advance (CIA) constraint which states that real monetary savings are a fraction  $0 < \theta < 1$  of consumption when young:<sup>9</sup>

$$m_t^d \geq \theta \times c_{t,Y} \quad (3)$$

As shown in Appendix A, the CIA constraint will bind with strict equality if the monetary return on nominal bonds exceeds one. Intuitively, since money is a perfect store of nominal value, an optimising consumer will not hold monetary savings in excess of the proportion  $\theta$  required by the CIA constraint if nominal bonds pay a higher return. The CIA constraint is taken to be strictly binding, i.e.  $m_t^d = \theta c_{t,Y}$ .<sup>10</sup>

As in Lungu and Minford (2006), capital is used to produce output (which is consumed in old age) via a production function with diminishing returns. The depreciation rate on capital is 100 per cent, so capital lasts for only one period.<sup>11</sup> Given that the amount of output produced using capital depends on the stochastic level of productivity, capital is a claim to an uncertain amount of real output in old age.

Output in old age is given by the following production function:

$$y_{t+1,O} = A_{t+1} k_t^\alpha \quad 0 < \alpha < 1 \quad (4)$$

where  $\alpha$  is the elasticity of output with respect to capital.

Productivity  $A_t$  follows an AR(1) process in logs:

$$\ln A_t = (1 - \rho) \ln A_{mean} + \rho \ln A_{t-1} + e_t \quad 0 < \rho < 1 \quad (5)$$

where the productivity innovation  $e_t$  is an IID-Normal random variable with mean zero and variance  $\sigma_e^2$ .

Consumption in old age consists of output produced using capital and savings income from holding money and bonds. Real consumption by the old is therefore given by

$$\begin{aligned} c_{t+1,O} &= A_{t+1} k_t^\alpha + r_{t+1}^i b_t^{i,d} + r_{t+1}^n b_t^{n,d} + r_{t+1}^m m_t^d \\ &= A_{t+1} k_t^\alpha + (a r_{t+1}^i + (1-a) r_{t+1}^n) b_t^d + r_{t+1}^m m_t^d \end{aligned} \quad (6)$$

<sup>9</sup> Cited by Crettez, Michel and Wigniolle (1999). This constraint is interpreted as a 'cash-in-advance' specification in the OLG literature.

<sup>10</sup> Note that the nominal (or money) return on nominal bonds was greater than one in all simulations.

<sup>11</sup> Given that each period lasts 30 years, the assumption of full depreciation is empirically plausible. See Nadiri and Prucha (1996) and studies cited therein.

where  $a \equiv b_t^{i,d} / b_t^d$  is the share of indexed bonds in consumers' bond portfolios;  $b_t^d \equiv b_t^{i,d} + b_t^{n,d}$  is total demand for government bonds in real terms;  $r_{t+1}^m \equiv 1 / (1 + \pi_{t+1})$  is the gross real return on money balances held from youth to old age; and  $\pi_t \equiv (P_t / P_{t-1} - 1)$  is the rate of inflation in period  $t$ . The real returns on indexed bonds and nominal bonds,  $r_{t+1}^i$  and  $r_{t+1}^n$  respectively, are explained in detail below.

Indexed bonds pay an *ex ante* riskless gross real return  $r_t$  that is endogenously determined.<sup>12</sup> However, due to the presence of 'indexation bias' and lagged indexation, the *ex post* real return on an indexed bond will differ from this riskless return. In particular, the *ex post* real return on an indexed bond held from period  $t$  to period  $t + 1$  is given by

$$r_{t+1}^i = r_t \times \left[ \frac{(1 + \pi_{t+1}^{ind})}{(1 + \pi_{t+1})} + v_{t+1} \right] \quad (7)$$

where  $\pi_t^{ind}$  is the biased rate of inflation to which indexed bonds are linked,  $\pi_t$  is the true rate of inflation, and  $v_t$  is a Gaussian 'white noise' innovation whose variance  $\sigma_v^2$  is based on the indexation lag length.

The first term in square brackets reflects indexation bias: its value will deviate from one if 'true' and 'biased' inflation are not equal. Indexation is biased because the price index used for indexation differs from the true one that defines consumers' standard of living. In the UK case, for example, index-linked gilts are indexed to the Retail Prices Index (RPI), whereas the Retail Prices Index excluding mortgage interest payments (RPIX) may better reflect the inflation rate faced by the majority of pensioners (i.e. 'old generations') who do not make mortgage repayments (Leceister, O'Dea and Oldfield, 2008). The extent of indexation bias depends on the correlation between true and biased inflation, and on the similarity of the inflation variances.

The second term in square brackets captures the impact of lagged indexation on the *ex post* real return received on indexed bonds. Indexation is lagged in practice due to data publication and collection lags, and the indexation lag on government bonds varies across countries. For example, the large majority of outstanding index-linked gilts in the UK are indexed to the RPI with an 8-month lag (DMO, 2010), whereas indexed bonds in the US and Canada are linked to the Consumer Prices Index (CPI) with a 3-month lag. The indexation lag is modelled by a white noise innovation since this is a simple way to capture volatility arising from lagged indexation when the lag

---

<sup>12</sup> The return  $r_t$  ensures that the market for indexed bonds clears.

length is small relative to the holding period.<sup>13</sup> The innovation  $v_t$  is exogenous and invariant to monetary policy, reflecting the assumption that the length of indexation lag is not affected by a shift in monetary policy regime.

Nominal bonds pay a riskless nominal return  $R_t$ . The *ex post* real return on nominal bonds is certain but for inflation risk and is given by

$$r_{t+1}^n = R_t / (1 + \pi_{t+1}) = R_t \times r_{t+1}^m \quad (8)$$

where the nominal return  $R_t$  is endogenously determined.<sup>14</sup>

Finally, the initial old are endowed with  $m_0$  units of real money balances, an initial stock of government debt  $b_0 = b_0^i + b_0^n$ , and capital  $k_0$ ; their corresponding level of consumption is  $c_{1,O}$ . In model simulations, these initial values are set equal to the deterministic steady-state values. Trivially, the utility of the initial old is given by

$$u_{1,O} = \frac{c_{1,O}^{1-\delta}}{1-\delta} \quad (9)$$

## 2.2 Consumers' first-order conditions

Consider the following Lagrangian:

$$L_t \equiv E_t \left[ \frac{1}{1-\delta} (c_{t,Y}^{1-\delta} + c_{t+1,O}^{1-\delta}) + \lambda_{t,Y} (\varpi(1-\tau^j) - m_t^d - b_t^{i,d} - b_t^{n,d} - k_t - c_{t,Y}) \right. \\ \left. + \mu_t (m_t^d - \theta c_{t,Y}) + \lambda_{t+1,O} (A_{t+1} k_t^\alpha + r_{t+1}^i b_t^{i,d} + r_{t+1}^n b_t^{n,d} + r_{t+1}^m m_t^d - c_{t+1,O}) \right] \quad (10)$$

where  $\lambda_{t,Y}$  ( $\lambda_{t+1,O}$ ) is the Lagrange multiplier on young (old) consumers' budget constraints, and  $\mu_t$  is the Lagrange multiplier on the CIA constraint.

First-order conditions are as follows:

$$c_{t,Y} : c_{t,Y}^{-\delta} = \lambda_{t,Y} + \theta \mu_t \quad (11)$$

$$c_{t+1,O} : \lambda_{t+1,O} = c_{t+1,O}^{-\delta} \quad (12)$$

$$b_t^{i,d} : \lambda_{t,Y} = E_t (\lambda_{t+1,O} r_{t+1}^i) \quad (13)$$

$$b_t^{n,d} : \lambda_{t,Y} = E_t (\lambda_{t+1,O} r_{t+1}^n) \quad (14)$$

<sup>13</sup> Modelling the lag explicitly by indexing to past inflation is not an option given that each period lasts 30 years.

<sup>14</sup> In particular,  $R_t$  ensures that the market for nominal bonds clears.

$$m_t^d : \lambda_{t,Y} = E_t(\lambda_{t+1,O} r_{t+1}^m) + \mu_t \quad (15)$$

$$k_t : \lambda_{t,Y} = E_t(\lambda_{t+1,O} \alpha A_{t+1} k_t^{\alpha-1}) \quad (16)$$

Substituting for the Lagrange multipliers on the budget constraints when young and old gives the following consumption Euler equations for indexed bonds, nominal bonds, and capital respectively:

$$c_{t,Y}^{-\delta} = E_t(c_{t+1,O}^{-\delta} r_{t+1}^i) + \theta \mu_t \quad (17)$$

$$c_{t,Y}^{-\delta} = E_t(c_{t+1,O}^{-\delta} r_{t+1}^n) + \theta \mu_t \quad (18)$$

$$c_{t,Y}^{-\delta} = E_t(c_{t+1,O}^{-\delta} r_{t+1}^k) + \theta \mu_t \quad (19)$$

where  $r_{t+1}^k \equiv \alpha A_{t+1} k_t^{\alpha-1}$  is the real return on capital.

The Lagrange multiplier on the cash-in-advance constraint is given by

$$\mu_t = E_t(c_{t+1,O}^{-\delta} (r_{t+1}^i - r_{t+1}^m)) = E_t(c_{t+1,O}^{-\delta} (r_{t+1}^n - r_{t+1}^m)) = E_t(c_{t+1,O}^{-\delta} (r_{t+1}^k - r_{t+1}^m)) \quad (20)$$

where the multiple equalities follow from the absence of arbitrage opportunities across assets due to bond returns being endogenously determined. Intuitively, Equation (20) states that, absent uncertainty, a sufficient condition for the CIA constraint to be strictly binding (i.e.  $\mu_t > 0 \quad \forall t$ )<sup>15</sup> is that money be rate of return dominated by other assets.

### 2.3 Government

The government finances spending by taxing young consumers, printing money and issuing indexed and nominal government bonds. The government budget constraint in real terms is thus given by

$$g_t = \tau^j \varpi + m_t^s - r_t^m m_{t-1}^s + b_t^{i,s} - r_t^i b_{t-1}^{i,s} + b_t^{n,s} - r_t^n b_{t-1}^{n,s} \quad (21)$$

where  $\tau^j \varpi$  is revenue from taxing young consumers,  $b_t^{i,s}$  ( $b_t^{n,s}$ ) is the real supply of indexed (nominal) bonds issued by the government in period  $t$ , and  $m_t^s$  is the real money supply in circulation in period  $t$ .

The government sets the income tax rate on young consumers' endowment incomes  $\tau^j$  (where  $j \in (IT, PLT)$ ) in order to achieve a long run target level of real

<sup>15</sup> See Hodrick, Kocherlakota and Lucas (1991).

government spending of  $E(g_t) = g^*$ <sup>16</sup> and controls the money supply in the economy via money supply rules. The total bond supply  $b_t^s \equiv b_t^{i,s} + b_t^{n,s}$  is set to ensure that the marginal utility of consumption in youth is equated with the expected marginal utility of consumption in old age, or  $c_{t,Y}^{-\delta} = E_t(c_{t+1,O}^{-\delta})$ . This policy ensures perfect consumption-smoothing in expected terms for each generation, thereby increasing lifetime utility as in the standard OLG model in which government bonds are ‘net wealth’ (Barro, 1974).<sup>17,18</sup> The supplies of indexed and nominal bonds are constrained to be non-negative, so  $b_t^{i,s} \geq 0$  and  $b_t^{n,s} \geq 0$  for all  $t$ .

The division of the total bond supply between indexed and nominal bonds, as defined by the indexation share  $a \in [0, 1]$ , is chosen by the government to maximise social welfare, taking into account consumers’ first-order conditions, the money supply rule, and the necessity of meeting the long run government spending target. The indexation problem and the corresponding results are presented in detail in Hatcher (2011). The current paper therefore discusses optimal indexation only briefly in Sections 4 and 6.

## 2.4 Long-term inflation risk, lifetime utility and social welfare

It was noted in the introduction that the workhorse New Keynesian model does not allow researchers to explicitly evaluate the impact of long-term inflation risk on social welfare. One important advantage of the OLG model presented above is that, by contrast, it posits a direct link between long-term (i.e. 30-year) inflation volatility and lifetime utility – and hence also to social welfare. As a result, the impact of long-term inflation risk on social welfare can be modelled explicitly, laying the foundations for an evaluation of the welfare impact of PLT through the long-term inflation risk channel.

In order to demonstrate the link between long-term inflation risk and social welfare in the OLG model, consider Equation (1), the lifetime utility of a given generation  $t$ . Taking a second-order Taylor expansion of lifetime utility around expected values yields the following expression:

$$\tilde{u}_t \approx -\frac{1}{2} \left[ \frac{\delta}{(E_t c_{t+1,O})^{1+\delta}} \right] \text{var}_t(c_{t+1,O}) \quad (22)$$

<sup>16</sup> That is, the government has risk-neutral preferences over government spending.

<sup>17</sup> The ‘net wealth’ result was first demonstrated formally by Barro, but he argues against government bonds being a source of net wealth because introducing a bequest motive into the OLG model resurrects the Ricardian equivalence proposition.

<sup>18</sup> In a model without uncertainty, the government can set the total bond supply so that all generations enjoy perfect consumption smoothing *ex post*, thereby maximising lifetime utility for all generations. See Minford and Peel (2002) for a simple example.

where  $\tilde{u}_t \equiv u_t - u_t(c_{t,Y}, E_t c_{t+1,O})$  is the difference between lifetime utility  $u_t$  and the level of lifetime utility when consumption levels are at their time- $t$  expected values.

The impact of long-term inflation risk is captured by the conditional variance of consumption in old age (which reduces lifetime utility due to risk aversion). Appendix B shows this point explicitly by log-linearising the model around the deterministic steady-state to obtain neat expressions for  $E_t c_{t+1,O}$  and  $\text{var}_t(c_{t+1,O})$ .

The resulting expression for  $\tilde{u}_t$  is given by

$$\tilde{u}_t = -\frac{1}{2} \chi_t \left[ \begin{array}{c} \text{var}_t(\pi_{t+1}) + \varphi \text{var}_t(\pi_{t+1}^{ind}) - 2\kappa \text{cov}_t(\pi_{t+1}, \pi_{t+1}^{ind}) \\ + \eta \sigma_v^2 + \vartheta \sigma_e^2 \end{array} \right] \quad (23)$$

where  $\chi_t, \varphi, \kappa, \eta$  and  $\vartheta$  are positive coefficients; see Appendix B for the details.

The loss in lifetime utility increases with long-term inflation risk,  $\text{var}_t(\pi_{t+1})$ , since this increases the extent of return risk faced by bondholders.<sup>19</sup> The loss in utility also depends on the extent of indexation bias, volatility arising due to the indexation lag, and long-term productivity volatility (since this causes fluctuations in output produced for consumption in old age). The covariance between actual and biased inflation enters with a positive sign because it denotes the extent to which indexed bonds can protect consumers against inflation fluctuations. However, given that indexed bonds are imperfect, they can provide only partial insurance.<sup>20</sup> As social welfare is measured by average lifetime utility across all generations (see Section 2.6), Equation (23) implies that long-term inflation risk will reduce social welfare.<sup>21</sup>

## 2.5 Monetary Policy

The major difference between IT and PLT is that the former implies base-level drift in the price level, whilst the latter prevents base-level drift. To allow for this difference, the 30-year (i.e. one period) money supply rules under IT and PLT are built up from a *yearly horizon*. With this approach, equilibrium inflation in the model reflects the

<sup>19</sup> An increase in long-term inflation risk also increases (real) return risk from holding money.

<sup>20</sup> If indexed bonds are excluded from the model, the loss in lifetime utility will depend only on inflation risk and productivity (since  $\varphi = \kappa = \eta = 0$ ), thus highlighting the negative impact of inflation risk more clearly.

<sup>21</sup> There are several alternative ways to reach the same conclusion. For instance, the whole model can be solved by log-linearisation (in which case risk-premia are ignored) and the resulting consumption levels can be substituted into a second-order expansion of the social welfare function, or the conditional variance of consumption in Equation (22) can be calculated without log-linearising the budget constraint of old agents. These derivations are available from the author on request.

presence of base-level drift under IT, and its absence under PLT. Since the derivation of 30-year money supply rules from yearly ones is long-winded, the details are presented in Appendix C.

Given the long-term horizon embedded in the model, monetary policy does not respond directly to output or productivity, and since the government can commit to money supply rules, no time-inconsistency or credibility issues arise in relation to monetary policy. The money supply rules given below are stated in terms of the nominal money supply (which is non-stationary), but the money supply is converted back into real terms in order to solve the model in Dynare++.

### The IT money supply rule

The nominal money supply rule under IT takes the following form:

$$\ln(M_t^{s,IT} / M_{t-1}^{s,IT}) = 30 \times \pi + \sum_{i=1}^{30} \varepsilon_{i,t} + \ln(c_{t,Y} / c_{t-1,Y}) \quad (24)$$

where  $\pi$  is the annual inflation target and the  $\varepsilon_{i,t}$ s are Gaussian white noise money supply innovations in year  $i$  of period  $t$ , each with variance  $\sigma^2$ .

Notice that under IT the aggregate money supply innovation is simply the sum of the 30 yearly money supply innovations that accumulate in each period due to base-level drift. In the absence of money supply innovations, Equation (24) implies perfect stabilisation of inflation at the inflation target.

Money market equilibrium (i.e.  $M_t^d = M_t^s$ , where  $M_t^d = P_t m_t^d$ ) implies that inflation under IT is given by<sup>22</sup>

$$\pi_t^{IT} = 30 \times \pi + \sum_{i=1}^{30} \varepsilon_{i,t} \quad (25)$$

Therefore, expected inflation is equal to the 30-year inflation target, and the inflation variance is thirty times the yearly money supply innovation variance:

$$E_t \pi_{t+1}^{IT} = 30 \times \pi \quad (26)$$

$$\text{var}(\pi_t^{IT}) = 30 \sigma^2 \quad (27)$$

---

<sup>22</sup> To arrive at this expression for inflation, take the first difference of the natural log of (nominal) money demand and use the approximation  $\pi_t \approx \ln P_t - \ln P_{t-1}$ . Then set money demand equal to money supply and solve for inflation.

Intuitively, expected inflation is equal to the inflation target because the government makes a fully credible commitment to an IT money supply rule. The inflation variance is thirty times the yearly innovation variance because of base-level drift: money supply innovations cause inflation to deviate from target in each year, and over thirty years these innovations accumulate, with each one adding to long-term inflation uncertainty.

### The PLT money supply rule

The nominal money supply rule under PLT is given by

$$\ln(M_t^{s,PLT} / M_{t-1}^{s,PLT}) = \ln(P_t^* / P_{t-1}^*) + \varepsilon_{30,t} - \varepsilon_{30,t-1} + \ln(c_{t,Y} / c_{t-1,Y}) \quad (28)$$

where  $P_t^*$  is the target price level and  $\varepsilon_{30,t}$  is the money supply innovation in year 30 of period  $t$ .

In the absence of money supply innovations, this money supply rule implies perfect stabilisation of the price level at target; excepting this, the price level will deviate from its target value in each period. The presence of a lagged money supply innovation in Equation (28) reflects the response of the PLT money supply rule to the price-level deviation in the previous period – a response that is necessary to return the price level to its target path.

It assumed that the target log price level under PLT increases at the target rate of inflation under IT:<sup>23</sup>

$$\ln P_t^* = p_0 + (30 \times \pi)t \quad (29)$$

where  $p_0$  is the initial target price level.

The money supply rule in Equation (29) can therefore be written as follows:

$$\ln(M_t^{s,PLT} / M_{t-1}^{s,PLT}) = 30 \times \pi + \varepsilon_{30,t} - \varepsilon_{30,t-1} + \ln(c_{t,Y} / c_{t-1,Y}) \quad (30)$$

In contrast to the IT case, the 30-year money supply rule contains only two yearly money supply innovations which relate to year 30 in adjacent periods. The reasoning is as follows: innovations that occur in years 1-29 are offset in the following year in order to bring the price level back to its target path. For instance, a shock in year 29

---

<sup>23</sup> The rate of inflation implied by the target price path is assumed to be equal to the inflation target to ensure direct comparability of IT and PLT. With this assumption, IT and PLT are identical in the absence of money supply innovations and PLT can be interpreted as ‘average inflation targeting’.

will be offset in year 30, the last year of the current period. However, the innovation in year 30 of each period cannot be offset until year 1 of the next period. Hence the innovations  $\varepsilon_{30,t}$  and  $\varepsilon_{30,t-1}$  enter the money supply rule. The first is the innovation in year 30 of the current period, and the second is the innovation from year 30 of the previous period (which is then offset in year 1 of the current period).

Money market equilibrium implies that inflation under PLT is given by

$$\pi_t^{PLT} = 30 \times \pi + \varepsilon_{30,t} - \varepsilon_{30,t-1} \quad (31)$$

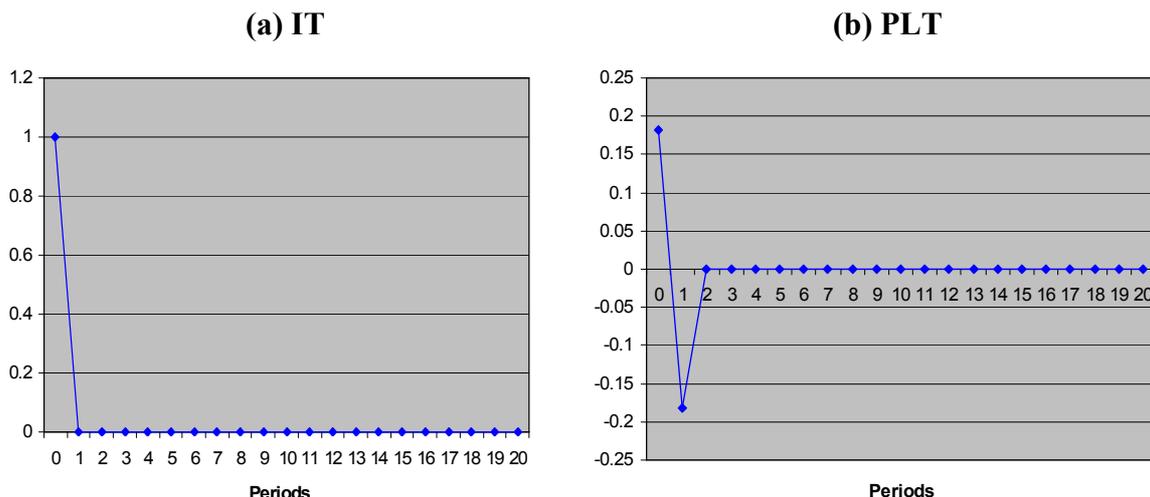
Hence expected inflation is state-contingent, and the 30-year inflation variance is two times the yearly innovation variance:

$$E_t \pi_{t+1}^{PLT} = 30 \times \pi - \varepsilon_{30,t-1} \quad (32)$$

$$\text{var}(\pi_t^{PLT}) = 2\sigma^2 \quad (33)$$

Both of these results have been discussed in the PLT literature (e.g. Svensson, 1999; Minford, 2004). First, expected inflation varies because past deviations from the target price path are subsequently offset, and rational agents take this into account when forming their inflation expectations. Second, the 30-year inflation variance is 15 times lower under PLT since inflation depends on only 2 yearly money supply innovations, compared to 30 under IT. The reasoning is simply that yearly deviations from the inflation target do not accumulate to increase long-term inflation uncertainty, because PLT precludes base-level drift.

In order to make the differences between IT and PLT concrete, Panels (a) and (b) of Figure 1 report impulse responses of inflation to a period- $t$  money supply innovation. As the yearly money supply innovation variance has not yet been calibrated, the innovation was normalised to one in the IT case and scaled accordingly in the PLT case. The differences between IT and PLT are clear: the initial impact is somewhat smaller under PLT because of the lower (30-year) money supply innovation variance; and the inflationary shock is reversed in the following period under PLT but is treated as a bygone under IT.



**Fig. 1 – Inflation impulse responses to a money supply innovation**

Finally, the biased inflation rate to which indexed bonds are linked is given by an exogenous process that has the same functional form as true inflation. In particular, the mean is given by the 30-year inflation target, and inflation responds only to current innovations under IT but to current and past innovations under PLT. As a result, the 30-year variance is also 15 times lower under PLT than IT.

The biased inflation rate used for indexation is given by

$$\left. \begin{aligned} \pi_t^{ind,IT} &= 30 \times \pi + \sum_{i=1}^{30} \varepsilon_{i,t}^{ind} \\ \pi_t^{ind,PLT} &= 30 \times \pi + \varepsilon_{30,t}^{ind} - \varepsilon_{30,t-1}^{ind} \end{aligned} \right\} \begin{array}{l} \text{under IT} \\ \text{under PLT} \end{array} \quad (34)$$

where  $\varepsilon_{i,t}^{ind} \sim N(0, \sigma_{ind}^2)$ , and  $\sigma_{ind}^2$  is the yearly innovation variance to biased inflation.

The  $\varepsilon_{i,t}^{ind}$  s are serially-uncorrelated but are contemporaneously cross-correlated with innovations to true inflation, with the cross-correlation reflecting the extent of indexation bias. Both the cross-correlation between innovations and the innovation variances (for true and biased inflation) are estimated using UK data.

## 2.6 Social welfare

The government maximises the unconditional expectation of social welfare, that is, the average across all possible histories of shocks (see Damjanovic, Damjanovic and Nolan, 2008). The unconditional welfare criterion was first proposed by Taylor (1979), and has been used in numerous papers in the monetary policy literature,

including Rotemberg and Woodford (1998), Clarida, Gali and Gertler (1999), and Schmitt-Grohé and Uribe (2007).<sup>24</sup>

Given consumers' lifetime utility function and the utility of the initial old, average lifetime utility across  $T$  generations is given by

$$\begin{aligned}
U^T &= \frac{1}{T} \left[ u_{1,O}(c_{1,O}) + \sum_{t=1}^T u_t \right] \\
&= \frac{1}{T} \left[ u_{1,O}(c_{1,O}) + \sum_{t=1}^{T-1} (u_{t,Y}(c_{t,Y}) + E_t u_{t+1,O}(c_{t+1,O})) + u_{T,Y}(c_{T,Y}) \right] \quad (35) \\
&= \frac{1}{T} \left[ \sum_{t=1}^T u_{t,Y}(c_{t,Y}) + u_{1,O}(c_{1,O}) + \sum_{t=1}^{T-1} E_t u_{t+1,O}(c_{t+1,O}) \right]
\end{aligned}$$

Social welfare is defined as the unconditional expectation of this expression, or

$$U^{society} \equiv E[U^T] = \frac{1}{T} E \left[ \sum_{t=1}^T (u_{t,Y}(c_{t,Y}) + u_{t,O}(c_{t,O})) \right] \quad (36)$$

## 2.7 Steady state and market-clearing conditions

The model's deterministic steady state and market-clearing conditions are presented in Appendix D, and Appendix E gives a full model listing.

## 3. Calibration

Model calibration is described in detail in Hatcher (2011). Therefore, the current paper provides only a brief discussion.

### 3.1 Money supply rules and biased inflation

The money supply rules and the stochastic process for biased inflation were calibrated using UK inflation since 1997. The Retail Prices Index excluding mortgage interest payments (RPIX) was chosen as the measure of 'true' inflation and the Retail Prices Index (RPI) as the 'biased' measure, with the sample period running from 1997Q3 to 2010Q2.<sup>25</sup> The RPIX was chosen as the measure of true inflation because it excludes mortgage interest payments, which are not faced by the majority of pensioners in the

<sup>24</sup> Examples of OLG models in which monetary policy is evaluated using an unconditional social welfare criterion include Brazier, Harrison, King and Yates (2006) and Kryvtsov, Shukayev and Ueberfeldt (2007).

<sup>25</sup> The Bank of England was assigned an inflation target soon after 'Black Wednesday' in 1992, but was not given full operational independence until May 1997.

UK (Leicester, O’Dea and Oldfield, 2008). It also includes council tax and housing costs, both of which are relatively more important costs for pensioners that are excluded from the Consumer Prices Index (CPI). Given that indexed bonds in the UK are linked to the Retail Prices Index (RPI), the stochastic process for biased inflation was calibrated using the RPI.

The annual inflation target was set equal to 2.5 per cent, which is close to mean RPIX inflation over the sample period, and the innovation variances, which are yearly, were calibrated by applying the unit root hypothesis to the quarterly variances of RPIX and RPI inflation.<sup>26</sup> Finally, the covariance between innovations to true inflation and biased inflation was calculated using the sample correlation of 0.89. The results are summarised in Table 1.

**Table 1 – Money supply and biased inflation calibration**

<b>Model parameter</b>	<b>Role in the model</b>	<b>Calibrated value</b>
$30 \times \pi$	Inflation target over 30 years	0.75
$\text{var}(\varepsilon_{i,t})$	Yearly money supply innovation variance	$1.44 \times 10^{-4}$
$\text{var}(\varepsilon_{i,t}^{ind})$	Yearly biased inflation innovation variance	$2.13 \times 10^{-4}$
$\text{cov}(\varepsilon_{i,t}, \varepsilon_{i,t}^{ind})$	Yearly covariance between innovations	$1.56 \times 10^{-4}$ $(= 0.89 \times sd(\varepsilon_{i,t}) \times sd(\varepsilon_{i,t}^{ind}))$

### 3.2 Calibrating stochastic productivity

To calibrate the productivity process, a typical quarterly calibration from the real business cycle (RBC) literature was extended over a 30-year horizon.

Consider an AR(1) process for log productivity at a quarterly horizon  $q$ :

$$\ln A_q = (1 - \rho_q) \ln A_{q,mean} + \rho_q \ln A_{q-1} + e_q \quad 0 < \rho_q < 1 \quad (37)$$

where  $e_q$  is an IID-Normal productivity innovation with mean zero and variance  $\sigma_q^2$ .

By substituting repeatedly for lagged productivity terms, productivity over a 30-year (i.e. 120 quarters) horizon can be obtained as follows:

$$\ln A_q = (1 - \rho_q^{120}) \ln A_{q,mean} + \rho_q^{120} \ln A_{q-120} + \sum_{j=0}^{119} \rho_q^j e_{q-j} \quad (38)$$

<sup>26</sup> Dickey-Fuller tests on the quarterly RPIX and RPI could not reject the null hypothesis of a unit root in the price level.

Therefore, productivity in any given period  $t$  is given by

$$\ln A_t = (1 - \rho) \ln A_{mean} + \rho \ln A_{t-1} + e_t \quad (39)$$

where  $\ln A_{mean} \equiv (1 - \rho_q^{120}) \ln A_{q,mean} / (1 - \rho)$ ,  $\rho \equiv \rho_q^{120}$  and  $e_t \equiv \sum_{j=0}^{119} \rho_q^j e_{q-j}$ .

Equation (39) is used as basis for calibrating the stochastic productivity process in the model. Following Gavin, Keen and Pakko (2009), the quarterly productivity innovation standard deviation is set at 0.005, and the quarterly autocorrelation coefficient at  $\rho_q = 0.996$ , consistent with the bulk of the RBC literature. Finally, steady-state productivity was set equal to 3/4. The calibration of the productivity process is summarised in Table 2.

**Table 2 – Calibration of stochastic productivity**

<b>Model parameter</b>	<b>Role in the model</b>	<b>Calibrated value</b>
$\rho$	Persistence in productivity at a 30-year horizon	0.618
$\sigma_e$	Productivity innovation standard deviation (at a 30-year horizon)	0.04398
$A_{mean}$	Steady-state level of productivity	0.75

### 3.3 Calibrating the indexation lag

The random innovation  $v_t$  is used to proxy for the impact of an indexation lag on the *ex post* real return on indexed bonds. In order to calibrate its variance, a number of points should be considered. First, given the specification of the real return on indexed bonds, it should have the same units as the term  $(1 + \pi_t^{ind}) / (1 + \pi_t)$  which it appears in brackets alongside. Hence  $v_t$  is interpreted as the impact of the indexation lag, in percentage points, on the inflation-indexed component of an indexed bond. Second, the variance of  $v_t$  should reflect the volatility of the inflation rate to which indexed bonds are linked, measured over a horizon defined by the length of the indexation lag.

Given that the indexation lag on the majority of outstanding index-linked gilts in the UK is 8 months, this variance was calibrated using the standard deviation of RPI

inflation measured over a three-quarter horizon.<sup>27</sup> Based on this standard deviation of 0.0121 (1.2 per cent), the variance of  $v_t$  was set equal to  $\sigma_v^2 = 0.0121^2 = 0.000146$ .

### 3.4 Model parameter calibration

The calibration of other parameters in the model is given in Table 3. The model was calibrated so as to match approximately, at steady-state, the ratio of key variables to GDP in UK data. A key ratio in the model is that of capital to bond holdings. This ratio is calibrated so as to match (i) the ratio of investment to government bonds in UK data, and (ii) the ratio of private sector to public sector pension spending, with capital interpreted as private pensions and government bonds as public sector pensions.<sup>28</sup> The endowment income of young consumers was chosen so that steady-state GDP was equal to 2.

**Table 3 –Calibration of model parameters**

Model parameter	Role in the model	Calibrated value
$\theta$	Proportion of consumption when young held as money balances	0.10
$\delta$	Coefficient of relative risk aversion	3
$\varpi$	Endowment income of young consumers	1.641
$g^*$	Long run government spending target	0.40
$\alpha$	Elasticity of output produced for old age with respect to capital	0.375

The deterministic steady-state of the model under this calibration is shown in Table 4. Aggregate consumption accounts for 73 per cent of GDP and is split equally between consumption by young and old generations.<sup>29</sup> Money holdings are approximately 3.7 per cent of GDP (i.e.  $0.073/2$ ), which is similar to the UK share of notes and coins in GDP over the past decade (ONS Financial Statistics, 2010), and the government spending target of 20 per cent of GDP is similar to the government spending share in UK data (ONS Blue Book, 2010). The remaining 7 per cent of GDP is accounted for by investment, which is given by the level of capital holdings as there is full depreciation. The ratio of investment to GDP is lower than in UK data (around 15 per cent), but the ratio of capital holdings to bond holdings (41 per cent) is close to the average ratio of investment to government bonds in the UK over the past decade

<sup>27</sup> Using 3 quarters (9 months) meant that the same quarterly RPI data could be used in estimation throughout the paper.

<sup>28</sup> There are a number of similarities between UK public sector pensions and long-term government bonds, including the holding period, the length of the indexation lag (8 months), and the price index used for indexation (RPI in both cases). Moreover, as UK public sector pensions are predominantly ‘defined benefit’, they are in effect long-term nominal contracts; see Whitehouse (2009) for a discussion.

<sup>29</sup> The reason for the equal split is that the government sets the total bond supply so that consumption is smoothed between youth and old age.

(ONS Blue Book, 2010; ONS Financial Statistics, 2010), and is equal to the ratio of private pension spending to public pension spending in the UK in 2006 (OECD, 2009). Finally, steady-state inflation is equal to the 30-year inflation target of 0.75 – i.e. a 75 per cent increase in prices over a 30-year horizon.

**Table 4 – Key variables at steady-state**

<b>Model variable</b>	<b>Steady-state value</b>
$c_{t,Y}$	0.730
$c_{t,O}$	0.730
$b_t^d (= b_t^s)$	0.343
$m_t^d (= m_t^s)$	0.073
$k_t$	0.140
$\pi_t$	0.75

Note: Steady-state GDP is equal to 2

#### 4. Model solution

The model was solved using second-order approximation in Dynare++ (Julliard, 2001) for two reasons. First, linearizing the model would remove covariance risk, thus eliminating risk-premia in asset returns. Second, when comparing social welfare across alternative monetary policy regimes, linear approximation can easily lead to an inaccurate ranking of policies because it neglects the impact of second-order terms on the stochastic means of endogenous variables in the model. For instance, Kim and Kim (2003) present a simple two-agent economy in which linearization leads to the spurious conclusion that autarky delivers higher social welfare than full risk sharing. In the model at hand, spurious conclusions regarding optimal indexation and social welfare could be drawn if linear approximation methods were employed.

The model solution was carried out in two stages. In the first stage, the optimal indexation shares under IT and PLT were identified from the feasible range [0,1] using a method akin to grid search. Then, in the second stage, the model was solved with the optimal indexation shares imposed so as to obtain full simulation results under optimal commitment. Each solution of the model was based upon 1000 simulations of 5000 periods each, with the simulation seed chosen randomly in each simulation.

## 5. Results

### 5.1 Optimal indexation

The optimal indexation results are summarised in Table 5. Only a brief discussion is provided here as the results are explained in detail in Hatcher (2011). As noted therein, the optimal indexation shares approximately minimise consumption volatility across old generations. The reason is that changes in indexation affect consumption risk (primarily for old generations) but not the average level of aggregate consumption.

**Table 5 – Optimal indexation under IT and PLT**

<b>Monetary policy</b>	<b>Optimal indexation share</b>	<b>Share at which <math>\text{var}(c_{t,o})</math> minimised</b>
IT	76%	77%
PLT	26%	27%

Optimal indexation is substantially lower under PLT. The intuition is that targeting the price level reduces long-term inflation risk by an order of magnitude, hence making nominal bonds a better store of value compared to indexed bonds, and enabling old generations' consumption volatility to be reduced by substitution from indexed bonds to nominal bonds. Nine-tenths of the reduction in indexation under PLT arises because real return volatility falls more sharply on nominal bonds than indexed bonds: nominal bonds pay a return that is certain except for inflation risk, whilst the return paid on indexed bonds depends, additionally, on risk arising from the indexation lag – risk which is not reduced under PLT. The remainder of the reduction in indexation is due to the real payoffs from holding money and nominal bonds being less closely correlated under PLT than IT (i.e. a reduction in covariance risk) due to expected inflation varying over time.<sup>30</sup>

### 5.2 Social welfare and consumption volatility

In order to measure the impact of PLT on social welfare in a way that is relevant for policy, the consumption equivalent welfare gain was calculated. Formally, the consumption equivalent welfare gain  $\lambda$  is defined as the change in consumption by young and old generations, as a fraction of consumption under IT, that is necessary to equate social welfare under the two policies, i.e.

$$(1 + \lambda)^{1-\delta} U_{society}^{IT} = U_{society}^{PLT} \quad (40)$$

---

<sup>30</sup> Recall that expected inflation is constant under the IT regime.

where  $U_{society}^{IT}$  is social welfare under IT and  $U_{society}^{PLT}$  is social welfare under PLT.

If  $\lambda$  is positive, this indicates that social welfare is higher under PLT than IT, and that the gain in welfare is equivalent to an increase in consumption for all young and old generations of  $\lambda \times 100$  per cent. On the other hand, if social welfare is higher under IT, then  $\lambda$  will be negative. Note that since aggregate consumption is 73 per cent of steady-state GDP, the consumption equivalent welfare gain can be expressed in terms of a gain or loss in per cent of GDP by multiplication of  $\lambda$  by 0.73.

Table 6 reports social welfare under IT and PLT, plus consumption means and variances across young and old generations. The latter provide intuition for the social welfare results, as is explained in detail below.

**Table 6 – Social welfare and consumption**

<b>Simulated value</b>	<b>IT</b>	<b>PLT</b>
$Ec_{t,Y}$	0.730	0.730
$Ec_{t,O}$	0.731	0.731
$\text{var}(c_{t,Y}) \times 1000$	0.0385	0.0387
$\text{var}(c_{t,O}) \times 1000$	0.3401	0.2946
$\lambda$ (in % terms)	0.010%	

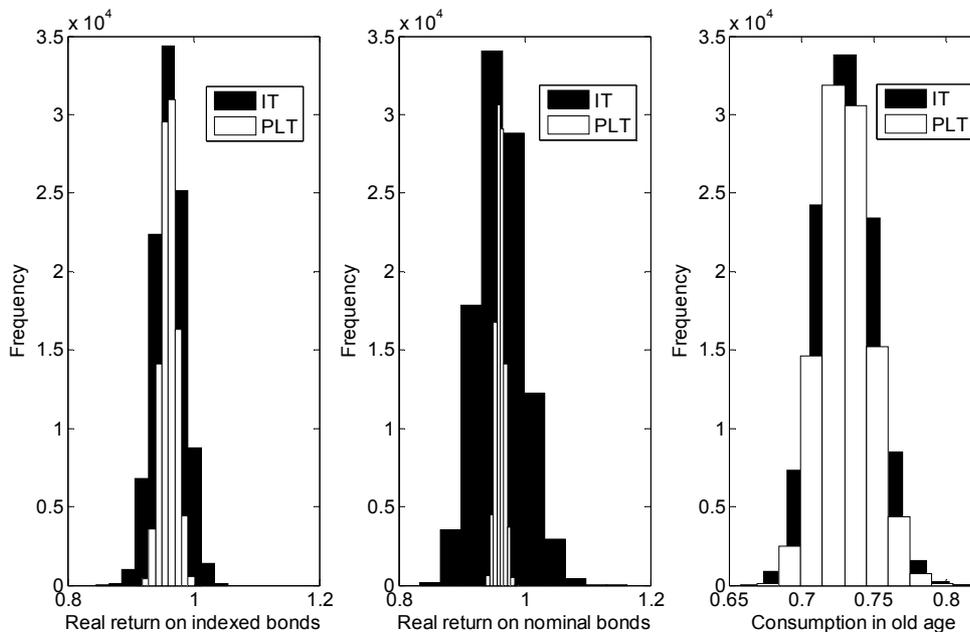
Notes: Mean consumption levels are rounded to 3 d.p.

Social welfare is increased under PLT. However, the welfare gain is rather small at an increase in consumption for young and old generations of 0.010 per cent, or an increase in GDP of 0.007 per cent. Based on UK data, this increase in consumption is equal to only £10.80 per pensioner, or £3.13 per member of the working population. It should be noted, however, that in principle these gains would apply to all current and future generations (i.e. the gain would be permanent). Moreover, the level of consumption (or GDP) on which these gains are calculated would grow over time.

Mean consumption levels for young and old generations are the same under IT and PLT, but there are non-trivial differences in terms of consumption risk. In particular, consumption volatility across young generations rises slightly under PLT (an increase of 0.5 per cent), but this is more than offset by a substantive reduction in consumption volatility across old generations of 13.1 per cent. Consumption volatility across young generations increases under PLT because expected inflation is time-varying, and this causes successive generations of young consumers to substitute between money and non-monetary assets (capital, indexed bonds and nominal bonds) since the expected real return on money rises when expected inflation falls, and falls when inflation

expectations rise. The increase in consumption volatility is minimal reflecting the fact that these portfolio substitution effects are small.<sup>31</sup>

On the other hand, consumption volatility across old generations falls somewhat under PLT due to the reduction in inflation risk by an order of magnitude. The reason is that the reduction in inflation risk under PLT dramatically reduces real return volatility on indexed and nominal bonds, as Figure 2 clearly illustrates.<sup>32</sup> It is striking that the proportional reduction in consumption risk under PLT is much smaller than the reductions in risk on bond returns (compare the third panel of Figure 2 to the first two). This result is driven by the fact that risk arising from holding capital is the main source of consumption volatility for old generations – risk which is unaffected by a shift from IT to PLT because it is driven by the exogenous level of productivity. Consequently, PLT can reduce only a relatively small proportion of overall consumption risk via its stabilising impact on bond returns.



**Fig. 2 – Real bonds returns and consumption in old age**

It is also notable that the welfare gain from PLT is far smaller, in percentage terms, than the reduction in consumption risk it engenders. As demonstrated formally in the next section, this result arises because changes in consumption volatility have only a second-order impact on social welfare, whilst mean consumption levels have a first-order impact.

<sup>31</sup> Note that the variance of expected inflation under PLT is equal to  $\sigma^2$ .

<sup>32</sup> The histograms in Figure 2 are based on the first 100,000 periods of the simulation in Table 6.

### 5.3 Intuition for the welfare results

As discussed above, the gain in welfare under PLT arises due to a reduction in consumption risk. This section provides intuition for this result based upon the social welfare function. Social welfare is given by Equation (36), but that expression is cumbersome to work with analytically. Hence consider the following equation:

$$U^{society} = E(u(c_{t,Y}) + u(c_{t,O})) \quad (41)$$

This expression arises exactly if the utility of the initial old is excluded from social welfare (or equivalently if the limit of Equation (36) is taken as the number of generations  $T$  tends to infinity). The reason is that all generations, except the initial old, are *ex ante* homogenous and hence have the same long run average level of utility.

Given that the model is solved using a second-order perturbation method, we can work with a second-order Taylor expansion of the above equation around unconditional mean consumption levels. Making use of the CRRA specification of utility, this expansion results in the following social welfare criterion:

$$U^{society} \approx \left( \frac{(Ec_{t,Y})^{1-\delta} + (Ec_{t,O})^{1-\delta}}{1-\delta} \right) - \frac{1}{2} \left( |U_{c_Y c_Y}^{society}| \text{var}(c_{t,Y}) + |U_{c_O c_O}^{society}| \text{var}(c_{t,O}) \right) \quad (42)$$

where  $U_{c_Y c_Y}^{society} = -\delta(Ec_{t,Y})^{-(1+\delta)}$  ( $U_{c_O c_O}^{society} = -\delta(Ec_{t,O})^{-(1+\delta)}$ ) is the second derivative of  $U^{society}$  with respect to  $c_{t,Y}$  ( $c_{t,O}$ ).

Therefore, social welfare can be expressed in a mean-variance form in which welfare is positively related to mean consumption levels by young and old generations, but negatively related to consumption risk. The above expression also makes clear that mean consumption levels have a first-order impact on social welfare, whilst consumption risk has only a second-order impact. In order to gain intuition for the equivalence of mean aggregate consumption levels under IT and PLT (see Table 6), consider a first-order Taylor expansion of the first term on the right hand side of Equation (42) around the deterministic steady-state.<sup>33</sup>

---

<sup>33</sup> This approximation is employed only to provide intuition. When the model was simulated, the full expression for social welfare was evaluated up to second-order.

Using this approximation results in the following social welfare criterion:

$$U^{society} \approx c_o^{1-\delta} \left( Ec_{t,Y} + Ec_{t,O} + \frac{2\delta \times c_o}{1-\delta} \right) - \frac{1}{2} \left( \left| U_{c_Y c_Y}^{society} \right| \text{var}(c_{t,Y}) + \left| U_{c_o c_o}^{society} \right| \text{var}(c_{t,O}) \right) \quad (43)$$

where  $Ec_{t,Y} + Ec_{t,O}$  is the average level of aggregate consumption.

The goods market-clearing condition  $c_{t,Y} + c_{t,O} + k_t + g_t = \varpi + A_t k_{t-1}^\alpha$  can be used to show that  $Ec_{t,Y} + Ec_{t,O}$  is approximately invariant to a switch in monetary policy from IT to PLT (or *vice versa*). In particular, taking the unconditional expectations operator through the market-clearing condition gives  $Ec_{t,Y} + Ec_{t,O} = \varpi - g^* + E(A_t k_{t-1}^\alpha - k_t)$ , which is approximately independent of the money supply rule, since capital is pure real asset whose real return is uncorrelated with inflation. The key to the invariance result is that the government must meet its long run government spending target of  $Eg_t = g^*$ , regardless of whether it targets inflation or the price level.

Thus, using notation employed by Woodford (2001), the expression for social welfare in Equation (43) can be written as follows:

$$U^{society} \approx -\frac{1}{2} \left( \left| U_{c_Y c_Y}^{society} \right| \text{var}(c_{t,Y}) + \left| U_{c_o c_o}^{society} \right| \text{var}(c_{t,O}) \right) + t.i.p. \quad (44)$$

where *t.i.p.* stands for ‘terms independent of policy’, which in this context should be taken to mean ‘terms that are approximately independent of a change in monetary policy regime from IT to PLT’ (or *vice versa*).

That the gain in welfare under PLT arises directly from a reduction in consumption risk can be seen clearly from Equation (44). Indeed, PLT leads to an increase in welfare because there is a substantial reduction in consumption volatility across old generations, but only a small increase in volatility across young generations.

## 6. Sensitivity analysis

This section analyses the sensitivity of the social welfare gain from PLT to model specification and key calibrated parameters and variances.

### 6.1 Indexation and social welfare

An issue that can be investigated using the model is the impact of indexation being optimised. This issue is of interest for two reasons. Firstly, an important finding from the PLT literature is that optimal indexation of *wages* is substantially lower under PLT than IT and that this has important implications for social welfare comparisons. For example, Amano, Ambler and Ireland (2007) find that assuming wage indexation is exogenous understates substantially the potential welfare gain of PLT vis-à-vis IT. An even stronger result is found by Minford and Peel (2003) – holding indexation fixed under PLT gives the incorrect conclusion that social welfare is higher under IT. An interesting question for the current model is whether the change in optimal bond indexation under PLT (from 76 per cent to 26 per cent) is important for the welfare gain from PLT. This issue is investigated by calculating the welfare gain from PLT under the assumption that indexation remains fixed at the optimal level under IT.

Secondly, indexation of government bond portfolios is rather low in developed economies, and well below the optimal of 76 per cent implied by the model under IT. In fact, amongst developed countries, indexed government bonds are most prevalent in the UK at roughly one-quarter of the total government bond portfolio, compared to around a 10 per cent share in the US (Campbell, Shiller and Viceira, 2009) and 2 per cent in Japan (Kitamura, 2009). It is therefore instructive to compute the welfare gain from PLT when the extent of bond indexation in the model is fixed at a low level that is comparable to the experiences of developed economies. For this purpose, the indexation share was set at 21 per cent, the share of indexed bonds in the UK portfolio as of March 2010 (see DMO, 2010).<sup>34</sup>

The results from these two sensitivity exercises are shown in Table 7. For comparison purposes, the consumption equivalent welfare gain from the baseline calibration is shown in bold in the middle column.

**Table 7 – Indexation and the welfare gain from PLT,  $\lambda$**

<b>Indexation fixed at IT optimal (76 per cent)</b>	<b>Optimal indexation (Baseline)</b>	<b>Indexation fixed at UK level (21 per cent)</b>
0.008%	<b>0.010%</b>	0.020%

<sup>34</sup> Note also that public sector pensions in most developed economies are indexed (Whitehouse, 2009), but only once in payment and not during the ‘holding period’.

The welfare gain from PLT is underestimated by one-fifth if indexation under PLT is held fixed at the optimal level under IT. The reasoning is simply that there is a rise in consumption risk across old generations. On the other hand, if indexation is fixed at the current UK level under both IT and PLT, the welfare gain from PLT is doubled relative to the baseline case. Intuitively, low indexation works in favour of PLT and against IT since optimal indexation is substantially lower under the former. Furthermore, as bond indexation is below 20 per cent in other developed economies, this estimate can be interpreted as a lower bound estimate of the welfare gain from PLT if issuance of indexed government debt remains limited.

## 6.2 Model parameters

The welfare results are robust to variations in model parameters, with the exception of the extent of risk aversion and the correlation between true inflation and biased inflation. Therefore, this section focuses on sensitivity with respect to the risk aversion coefficient  $\delta$  and the indexation bias correlation.<sup>35</sup>

### *Risk aversion*

The baseline calibration of 3 is close to the mid-range of values considered plausible in the literature - in particular, risk aversion coefficients in a range from 1 to 5 seem most relevant from an empirical perspective. On this basis, sensitivity is tested when the risk aversion coefficient is equal to alternative values of 3/2 and 5.<sup>36</sup> Given that indexation is endogenously determined, the optimal indexation share varies as risk aversion is changed. Table 8 therefore reports both welfare gains and the corresponding optimal indexation shares as risk aversion is varied. The baseline results are highlighted in bold in the middle column.

**Table 8 – Sensitivity to risk aversion**

Simulated value	Coefficient of relative risk aversion, $\delta$		
	$\delta = 3/2$	$\delta = 3$	$\delta = 5$
IT optimal indexation share	77%	<b>76%</b>	78%
PLT optimal indexation share	24%	<b>26%</b>	27%
$\lambda$	0.006%	<b>0.010%</b>	0.014%

The extent of risk aversion has relatively little impact on optimal indexation but has a substantial impact on the estimated welfare gain from PLT. When risk aversion is

<sup>35</sup> The sensitivity results for the other parameters are available from the author on request.

<sup>36</sup> Using US stock return data from 1926-2002, Tödter (2008) estimates 95 per cent confidence interval of (1.4, 7.1) for the coefficient of relative risk aversion.

relatively low, the welfare gain from PLT falls to 0.006 per cent of aggregate consumption, whilst with ‘high’ risk aversion it rises to 0.014 per cent – deviations of four-tenths from the baseline welfare gain. The reasoning behind the impact of risk aversion on the welfare gain is that an increase (decrease) in risk aversion increases (reduces) the relative importance of consumption risk for social welfare. Hence, for instance, a reduction in old generations’ consumption volatility is valued more heavily in social welfare when risk aversion is increased, thus increasing the potential welfare gain from PLT.

### ***Indexation bias***

In order to investigate the impact of changes in the extent of indexation bias on the welfare gain from PLT, the model was simulated for alternative correlations between innovations to the money supply (i.e. innovations to true inflation) and innovations to biased inflation. In the baseline calibration, this correlation was set at 0.89; as a result, true and biased inflation were strongly positively correlated. In this section, sensitivity is investigated to with respect to alternative correlations of 0.80 and 0.98, thus providing results applicable to countries with more substantial indexation bias than in the baseline calibration, and also to those in which indexation bias is more or less absent. The results from these sensitivity tests are presented in Table 9.

**Table 9 – Sensitivity to indexation bias**

<b>Simulated value</b>	<b>Inflation correlation, <math>corr(\pi, \pi^{ind})</math></b>		
	<b>0.80</b>	<b>0.89</b>	<b>0.98</b>
IT optimal share	71%	<b>76%</b>	85%
PLT optimal share	22%	<b>26%</b>	30%
$\lambda$	0.013%	<b>0.010%</b>	0.005%

Intuitively, optimal indexation increases as the correlation between true and biased inflation is increased (a reduction in indexation bias), and falls as it is reduced. Optimal indexation is reasonably robust to this correlation, but the welfare gain from PLT is rather sensitive. When the inflation correlation is low (i.e. 0.80), the welfare gain from PLT increases by roughly one-third. Intuitively, with indexation bias increased, indexed bonds are less able to provide protection against the relatively high level of inflation risk under IT. By the same token, an increase in the indexation correlation to 0.98 reduces the welfare gain from PLT, because indexed bonds will provide much better insurance against inflation risk, reducing to a large extent the cost of having high inflation risk under IT. In the low indexation bias case, the welfare gain from PLT is halved to 0.005 per cent.

### 6.3 Model variances

As the welfare gain from PLT is robust to changes in the productivity innovation variance,<sup>37</sup> this section focuses on sensitivity with respect to nominal volatility and the indexation lag length (as captured by  $\sigma_v^2$ ).

#### *Nominal volatility*

In order to investigate sensitivity to nominal volatility, alternative money supply innovation standard deviations of 0.051 and 0.081 are considered – approximately 25 per cent deviations from the baseline calibration of 0.066. Moreover, in order to ensure that indexation bias and the indexation lag are held constant as the money supply innovation variance is changed,<sup>38</sup> the innovation variance to biased inflation and the indexation lag innovation variance are changed in the same ratio. The innovation variances under the high and low nominal volatility calibrations are given in Table 10.

**Table 10 – Nominal volatility sensitivity calibrations**

<b>Money supply innovation variance</b> $\text{var}(\varepsilon_{i,t})$		
Low $0.87 \times 10^{-4}$	<b>Baseline</b> $1.44 \times 10^{-4}$	High $2.20 \times 10^{-4}$
<b>Indexed inflation innovation variance</b> $\text{var}(\varepsilon_{i,t}^{ind})$		
Low $1.30 \times 10^{-4}$	<b>Baseline</b> $2.13 \times 10^{-4}$	High $3.20 \times 10^{-4}$
<b>Indexation lag innovation variance</b> $\sigma_v^2$		
Low $8.7 \times 10^{-5}$	<b>Baseline</b> 0.00015	High 0.000220

The corresponding sensitivity results are reported in Table 11. Increasing nominal volatility increases optimal indexation slightly and increases welfare gain from PLT by one-fifth to 0.012 per cent. Intuitively, the welfare gain from PLT rises because nominal volatility is increased whilst real risk from holding capital is held constant, so that inflation volatility becomes a relatively more important factor in consumption risk for old generations. Conversely, a reduction in nominal volatility reduces optimal indexation compared to the baseline case, and the welfare gain from PLT falls to 0.007 per cent.

<sup>37</sup> Again, results are available from the author on request.

<sup>38</sup> Indexation bias results from the less than perfect correlation between true inflation and the biased inflation rate used for indexation, and also from the difference in their variances.

**Table 11 – Sensitivity to nominal volatility**

Simulated value	Nominal volatility		
	Low	Baseline	High
IT optimal indexation share	75%	<b>76%</b>	79%
PLT optimal indexation share	23%	<b>26%</b>	28%
$\lambda$	0.007%	<b>0.010%</b>	0.012%

***Indexation lag length***

In order to investigate sensitivity along this dimension, alternative calibrations are considered for the white noise innovation  $v_t$  that enters the return on indexed bonds, and whose variance was calibrated based on the length of the indexation lag. The innovation variance in the baseline calibration was set at  $\sigma_v^2 = 0.00015$  based on estimation results for RPI inflation over a three quarter horizon – a horizon which is approximately equal to the indexation lag on index-linked gilts in the UK of 8 months. However, since many developed countries issue indexed bonds with an indexation lag of three months, it is instructive to consider the impact of reducing the indexation lag length.<sup>39</sup> The sensitivity analysis below considers the impact of calibrating the model for an indexation lag of three months (i.e.  $\sigma_v^2$  one-third of the baseline value), and also the impact of a longer indexation lag of 15 months (an increase in  $\sigma_v^2$  of two-thirds on the baseline value). The calibrations and results are reported in Table 12.

**Table 12 – Sensitivity to the indexation lag**

Simulated value	Indexation lag length		
	High (5 quarters) $\sigma_v^2 = 0.00027$	Baseline (3 quarters) $\sigma_v^2 = 0.00015$	Low (1 quarter) $\sigma_v^2 = 5.3 \times 10^{-5}$
IT optimal indexation share	72%	<b>76%</b>	83%
PLT optimal indexation share	17%	<b>26%</b>	44%
$\lambda$	0.010%	<b>0.010%</b>	0.008%

The indexation lag length has a non-trivial impact on optimal indexation. For instance, when calibration is based on a one-quarter indexation lag, optimal indexation increases under both IT and PLT, and the IT-PLT indexation differential is

<sup>39</sup> The UK also issues indexed bonds with a lag of three months, but the majority of bonds outstanding have an eight-month indexation lag (DMO, 2010).

reduced to from 50 per cent to 39 per cent. Optimal indexation increases because the indexation lag becomes a less important source of volatility, thus making indexed bonds less risky relative to nominal bonds. Indexation rises more sharply under PLT because a reduction in the indexation lag length is a more important factor in indexed return volatility than under IT, since the extent of inflation risk to which indexed bond returns are vulnerable is markedly lower under PLT. By the same reasoning, an increase in the length of the indexation lag to 5 quarters reduces optimal indexation under both IT and PLT and increases the optimal indexation differential. Interestingly, the increase in the indexation lag length has no impact on the welfare gain from PLT (to 3 decimal places), though reducing the indexation lag length to one quarter (i.e. 3 months) reduces the welfare gain by one-fifth. Intuitively, a reduction in the indexation lag length lessens the potential gains from PLT because consumption risk for old generations becomes less dependent on the payoff from nominal bonds, and is therefore less strongly linked to the level of long-term inflation risk.

#### 6.4 Model specification

This section considers sensitivity of the welfare results to the inclusion of indexed bonds and capital in the model. The motivation for studying robustness along these dimensions is that there is substantial heterogeneity across developed countries in terms of the extent of indexation and the importance of the public sector in retirement income provision (OECD, 2009). For example, in the US and UK, the shares of indexed bonds in the government bond portfolio are non-trivial and both the public and private sector pay an important role in provision of retirement income via private and public sector pensions. At the other end of the spectrum, Germany does not issue indexed government bonds, and public sector pensions, which are linked to wages and not prices (once in payment), account for more than 80 per cent of retirement income (Börsch-Supan, 2000).

Table 13 reports results for three different versions of the model. Firstly, Model 1 is the full baseline model. Secondly, Model 2 is the full model but with indexed bonds excluded – i.e. an indexation share of zero, so that  $b_t^{i,s} = b_t^{i,d} = 0$  for all  $t$ . Finally, Model 3 excludes both indexed bonds and capital; hence  $b_t^{i,s} = b_t^{i,d} = k_t = 0$  for all  $t$ .<sup>40</sup> Given that the welfare results are rather sensitive to risk aversion and the extent of nominal volatility, the welfare gain is also reported for the high and low values of risk aversion and nominal volatility discussed, respectively, in sections 6.2 and 6.3.

---

<sup>40</sup> Note that since capital is excluded from the model, Model 3 is a pure endowment economy. The endowment income of the young is normalised to one in this model, since this means that asset holdings can be interpreted as proportions of GDP. Accordingly, the government spending target was halved to 0.20, so that it remained at one-fifth of GDP.

**Table 13 – Sensitivity to model specification**

<b>Model</b>	<b>(1) Full model</b>	<b>(2) Nominal bonds + capital</b>	<b>(3) Nominal bonds</b>
Consumption equivalent welfare gain $\lambda$	0.010%	0.026%	0.103%
Risk aversion sensitivity (low, high)	0.006%,0.014%	0.012%, 0.044%	0.052%, 0.173%
Nominal volatility sensitivity (low, high)	0.007%,0.012%	0.015%, 0.039%	0.062%, 0.154%
Baseline reduction in $\text{var}(c_{t,0})$	13.1%	39.4%	96.4%

The welfare gain from PLT is positive in all three models, but its magnitude varies substantially across the different specifications. For instance, the welfare gain is equivalent to 0.026 per cent of aggregate consumption in Model 2 (when indexed bonds are removed) – more than double the original estimate. The reasoning is that indexed bonds provide excellent insurance against the high level of inflation risk under IT, so that their absence is somewhat costly under an IT regime. When capital is also absent in Model 3, the welfare gain from PLT increases even further to 0.103 per cent of aggregate consumption, and close to one-tenth of one per cent of GDP. This marked increase arises because removing capital increases the share of inflation-vulnerable assets in consumers’ savings portfolios, and hence eliminates a substantial source of consumption risk (*viz.* productivity) that is unaffected by monetary policy.

Both of these effects work to increase the proportional impact of PLT on consumption risk through the inflation risk channel, thus increasing the welfare gain from PLT. This link is made strikingly clear by the last row of Table 13: consumption volatility across old generations is reduced by less than 15 per cent in the full model (i.e. Model 1), compared to more than a 95 per cent reduction in Model 3! Given that Model 3 posits the extreme case in which consumers hold only nominal assets, the results in this case can be considered an upper bound estimate of the long-term welfare gain from PLT.

The Model 2 and Model 3 results are highly sensitive to both risk aversion and the extent of nominal volatility. In Model 2, reducing the risk aversion coefficient to 3/2 lowers the welfare gain from PLT to 0.012 per cent, whilst an increase up to 5 increases the welfare gain to 0.044 per cent – an increase of almost seven-tenths on the baseline estimate. Sensitivity is also strong with respect to nominal volatility, with a slightly narrower range from 0.015 per cent (low volatility) to 0.039 per cent (high

volatility). The variability of results is increased even further in Model 3. For example, when risk aversion is low (a coefficient of  $3/2$ ), the welfare gain is roughly halved to 0.052 per cent, whilst when risk aversion is high (a coefficient of 5) the welfare gain is equal to 0.173 per cent – more than 15 times the baseline estimate in Model 1 and equivalent to a permanent increase in GDP of more than one-eighth. As in Model 2, sensitivity with respect to nominal volatility is almost as strong. Overall, the sensitivity results in Table 13 suggest that the welfare gain from PLT will lie between 0.005 and 0.173 per cent of aggregate consumption.

Several important policy implications follow from the model specification sensitivity analysis reported in Table 13. Firstly, the welfare gain from PLT is likely to vary substantially across countries, since the ‘best model’ for any country will depend upon which assets are the most important in the provision of retirement income. For example, in the case of the UK and US, the full model with capital and indexed and nominal bonds (i.e. Model 1) is likely to be most applicable, suggesting a relatively small potential welfare gain from switching to PLT. On the other hand, public sector pensions account for a substantial fraction of retirement income in Germany and indexed government bonds are not issued. Therefore, Model 3 may be most relevant in this case, suggesting a somewhat higher potential welfare gain from PLT. Secondly, the welfare gain from PLT varies substantially with nominal volatility and risk aversion, and is hence likely to vary across countries, being higher in countries in which IT is less successful and society is more risk-averse. Finally, although the long-term welfare gain from PLT is estimated to be small in the baseline model, the presence of substantial sensitivity on the upside implies that this gain cannot, in general, be safely ignored by a cost-benefit analysis of PLT.

## **7. Conclusions and policy implications**

An important area neglected by recent research comparing inflation targeting and price-level targeting is the impact of price-level targeting on social welfare via the long-term inflation risk channel. In this paper, a dynamic stochastic general equilibrium (DSGE) model was presented in which long-term inflation risk has a direct impact on social welfare, and the model was subsequently used to evaluate the welfare impact of price-level targeting. The literature on price-level targeting predicts that its long-term welfare impact will be positive (e.g. Duguay, 1994; Minford, 2004; Bank of Canada, 2009), but has stopped short of modelling this impact explicitly in a DSGE framework. The main goal of the current paper was to quantify – in a way that is meaningful for policy – the long-term welfare impact of price-level targeting.

In order to focus on the long-term, the model presented consists of overlapping generations of consumers that live for two periods of 30 years each. In effect, the model is one of optimal pension provision: young consumers save for old age by

holding money and capital, plus indexed and nominal government bonds. Since both indexed and nominal bonds are imperfect stores of purchasing power, the model captures a potentially important channel from long-term inflation risk to social welfare. The two key features of the model are that (i) the extent of indexation of government bonds depends on monetary policy regime and is computed as the solution to an optimal commitment Ramsey problem, and (ii) monetary policy is modelled via money supply rules that capture the long-term outcomes of inflation targeting and price-level targeting – *viz.* ‘base-level drift’ in the price level versus trend-stationarity.

The main result from the model is that price-level targeting leads to small gain in welfare compared to inflation targeting – a permanent increase in aggregate consumption of 0.01 per cent in the baseline case. The reasoning behind this welfare gain runs as follows. Long-term inflation uncertainty is substantial under inflation targeting because of base-level drift: even if the central bank misses its inflation target by only a small percentage in each year, these misses can accumulate and become large after 30 years. Consequently, real bonds returns are relatively volatile in an inflation targeting regime, increasing consumption volatility somewhat for old generations. Under price-level targeting, by contrast, past deviations from the inflation target do not accumulate so that long-term inflation volatility is reduced by an order of magnitude. Bonds returns are therefore stabilised, and consumption risk is reduced for old generations by around 15 per cent. Though this reduction in consumption risk is responsible for the increase in welfare under price-level targeting, the welfare gain itself is much smaller, in percentage terms, than the reduction in risk, because volatility has only a second-order impact on social welfare. Although optimal indexation is substantially lower under price-level targeting (26 per cent versus 76 per cent), the endogeneity of indexation is not necessary for welfare to increase under price-level targeting. Indeed, holding indexation fixed at the optimal level under inflation targeting produces a welfare gain, albeit a lower one by one-fifth.

The welfare results obtained are highly sensitive along a number of important dimensions. Most notably, both indexed bonds and capital play a crucial role in the model. If consumers do not have access to indexed government bonds (or if the share of such bonds is close to zero), the welfare gain from price-level targeting is more than doubled, because consumers are unable to insure themselves against the high level of inflation risk under inflation targeting by holding indexed bonds. If, in addition, capital is removed from the model, the welfare gain from price-level targeting is ten times the baseline estimate. The reasoning is that with consumers’ savings entirely in nominal assets, consumption risk remains substantial under inflation targeting but is largely eliminated by price-level targeting.

There is also considerable sensitivity to key calibrated quantities in the model. In particular, both risk aversion and the extent of nominal volatility play an important role. For instance, increasing risk aversion raises the welfare gain from price-level targeting markedly because it makes a reduction in consumption volatility (a second-order impact) relatively more important for social welfare. Increasing the level of nominal volatility also raises the welfare gain somewhat, though there is less sensitivity than with respect to risk aversion. The intuition for this result is simply that the benefit from targeting the price level is proportional to the extent of nominal volatility; if such volatility is low then, intuitively, there is little price-level targeting can do to improve upon the situation. On the other hand, if nominal volatility is substantial, then there is scope for price-level targeting to reduce a substantial proportion of consumption risk faced by old generations. Overall, sensitivity analysis suggests that the welfare gain from price-level targeting will range from 0.005 to 0.173 per cent of aggregate consumption. Hence, the potential welfare gain from price-level targeting through the inflation risk channel is relatively small, but not sufficiently so that it can safely be ignored in a cost-benefit analysis.

A number of notable policy implications arise from these results. Firstly, although the baseline estimated welfare gain from price-level targeting is small, this gain is driven by a reduction in consumption volatility for old generations of around 15 per cent. Hence old generations (i.e. pensioners) stand to gain from a non-trivial reduction in consumption risk under a price-level targeting regime. Secondly, the potential welfare gain from price-level targeting is rather sensitive, in particular on the upside. The potential gains from targeting the price level are higher for countries with more nominal volatility and greater risk-aversion, and also for countries in which indexation is not widespread (or is non-existent) and the government is the primary provider of retirement income. Based on these results, countries like Germany – where the large majority of retirement income comes from state pensions that are not indexed to prices, and indexed bonds are not issued – would have more to gain from targeting the price level. Finally, this paper has demonstrated that an OLG model of life-cycle saving can be fruitfully used to study optimal monetary policy over a long-term horizon. Indeed, a key advantage of the model over the workhorse New Keynesian model is that the impact of long-term inflation risk on social welfare can be modelled explicitly using consumer utility.

Future research should look to test the robustness of the conclusions reached in this paper in more comprehensive models of the kind used by central banks for quantitative policy analysis. For example, two recent contributions that utilise models that are calibrated in detail for individual economies are the papers by Dopeke and Schneider (2006) and Meh, Rios-Rull and Terajima (2010), which study the redistributive effects of inflation through revaluation effects on nominal claims. The paper by Meh, Rios-Rull and Terajima (2010) is particularly notable, because it

compares redistribution under inflation and price-level targeting within a model of the Canadian economy, and computes changes in welfare arising from these redistribution effects. Lastly, it should be noted that a key assumption in the current paper is that the price level is returned to its target path within one year under a price-level targeting regime. However, returning the price level to trend this quickly could lead to excessive output volatility, so that a horizon somewhat longer than one year may be optimal (see Smets, 2003). Importantly, a longer horizon for returning the price level to target would eliminate some of the reduction in long-term inflation risk under price-level targeting, and would hence reduce its positive impact on social welfare. The importance of the target horizon is an interesting issue that is left for future research.

## References

Amano, R., Ambler, S. and P. Ireland (2007), Price-level targeting, indexation and welfare, working paper.

Ambler, S. (2009), Price-level targeting and stabilisation policy: A survey, *Journal of Economic Surveys*, 23, 5, 974-997.

Artus, P. (1995), Macro Economie (collection *Economica Poche*), Economica, Paris.

Bank of Canada (2006), Renewal of the inflation-control target: Background information, Bank of Canada: Ottawa.

Bank of Canada (2009), Bank of Canada *Review*, Spring 2009, Ottawa: Bank of Canada.

Barro, R.J. (1974), Are government bonds net wealth? *Journal of Political Economy* 82, 1095-1117.

Bordo, M.D., Dittmar, R.D. and Gavin, W.T. (2007), Gold, Fiat Money and Price Stability, B.E. *Journal of Macroeconomics*, Berkeley Electronic Press, 7(1).

Börsch-Supan, A. (2000), Pension reform in Germany, *World Economics* 1, 81-101.

Brazier, A., Harrison, R., King, M. and Yates, T. (2006), The danger of inflating expectations of macroeconomic stability: heuristic switching in an overlapping generations monetary model, Bank of England Working Paper No. 303.

Bundesbank (2010), Price-level targeting as a monetary policy strategy, Deutsche Bundesbank Monthly Report, January 2010.

Campbell, J.Y., Shiller, R.J. and Viceira, L.M. (2009), Understanding inflation-indexed bonds markets, *Brookings Papers on Economic Activity* 1:2009, 79-120.

Carlstrom, C.T. and Fuerst, T.S. (2002), Monetary policy rules and stability: Inflation targeting versus price-level targeting, Federal Reserve Bank of Cleveland *Economic Commentary*, Feb 15.

Champ, B. and Freeman, S. (1990), Money, output and the nominal national debt, *American Economic Review* 80, 390-97.

Clarida, R., Gali, J. and Gertler, M. (1999), The science of monetary policy: A New Keynesian perspective, *Journal of Economic Literature*, 37, 1661-1707.

Cournède, B. and Moccero, D.N. (2009), Is there a case for price-level targeting?, OECD Working Paper No. 721.

Crettez, B., Michel, P. and Wigniolle, B. (1999), Cash-in-advance constraints in the Diamond overlapping generations model: Neutrality and optimality of monetary policies, *Oxford Economic Papers* 51, 431-52.

- Damjanovic, T. and Damjanovic, V. and Nolan, C. (2008), Unconditionally optimal monetary policy, *Journal of Monetary Economics* 55, 491-500.
- Dittmar, R., Gavin, W.T. and Kydland, F.E. (1999), Price-level uncertainty and inflation targeting, Federal Reserve Bank of St. Louis *Review*, July/August 1999, 23-33.
- Doepke, M. and Schnieder, M. (2006), Inflation and the redistribution of nominal wealth, *Journal of Political Economy*, 114, 1069-1097.
- DMO (2010), Annual Review 2009-10, Debt Management Office, UK.
- Duguay, P. (1994), Some thoughts on price stability versus zero inflation, Working Paper, Bank of Canada – presented at the conference on Central Bank Independence and Accountability, Universita Bocconi, Milan, March 1994.
- Gahvari, F. (2007), The Friedman rule: Old and new, *Journal of Monetary Economics* 54, 581-589.
- Gaspar, V., Smets, F. and Vestin, D. (2007), Is the time ripe for price level path stability? European Central Bank Working Paper No. 818.
- Gavin, W.T., Keen, B.D. and Pakko, M.R. (2009), Inflation risk and optimal monetary policy, *Macroeconomic Dynamics* 13, 58-75.
- Hatcher, M. (2011), Optimal indexation of government bonds and monetary policy, Cardiff University Economics Working Paper E2011/3.
- Hodrick, R.J., Kocherlakota, N. and Lucas, D. (1991), The variability of velocity in cash-in-advance models, *Journal of Political Economy* 99, 358-384.
- Julliard, M. (2001), Dynare: A program for the simulation of rational expectations models, *Computing in Economics and Finance* 213, Society for Computational Economics.
- Kahn, G. (2009), Beyond inflation targeting: Should central banks target the price level? Federal Reserve Bank of Kansas City *Economic Review*, Third quarter 2009, 35-64.
- Kim, H.K. and Kim, J. (2003), Spurious welfare reversals in international business cycle models, *Journal of International Economics* 60, 471-500.
- Kitamura, Y. (2009), Experiences in Japan: Inflation indexed bond markets, Working Paper, Hitotsubashi University.
- Kryvtsov, O., Shukayev, M. and Ueberfeldt (2007), Optimal monetary policy and price stability over the long run, Bank of Canada Working Paper No. 2007-26.
- Leceister, A., O’Dea, C. and Oldfield, Z. (2008), The inflation experience of older households, Commentary No. 106, Institute for Fiscal Studies, London.

- Ljungqvist, L. and Sargent, T.J. (2000), *Recursive macroeconomic theory*, The MIT Press: Cambridge.
- Lungu, L. and Minford, P. (2006), Explaining the equity risk premium, *Manchester School* 74, 670-700.
- Mayes, D.G. (2008), A new look at price-level targeting, Bank of Finland Online, No. 11.
- McCandless, G.T. and Wallace, N. (1991), *Introduction to dynamic macroeconomic theory: An overlapping generations approach*, Harvard University Press: Cambridge.
- Meh, C.A., Rios-Rull, V.J. and Terajima, Y. (2010), Aggregate and welfare effects of redistribution of wealth under inflation and price-level targeting, *Journal of Monetary Economics* 57, 637-52.
- Michel, P. and Wigniolle, B. (2005), Cash-in advance constraints, bubbles and monetary policy, *Macroeconomic Dynamics* 9, 28-56.
- Minford, P. (2004), Monetary Policy – Should it Move to a Price Level Target? The A.W. Phillips memorial lecture to the New Zealand Association of Economists.
- Minford, P. and Peel, D. (2003), Optimal Monetary Policy: Is price-level targeting the next step? *Scottish Journal of Political Economy* 50, 650-667.
- Minford, P., Nowell, E, and Webb, B. (2003), Nominal contracting and monetary targets – drifting into indexation, *Economic Journal*, 113, 65-100.
- Minford, P. and Peel, D. (2002), *Advanced macroeconomics: A primer*, Blackwell: London.
- Nadiri, M.I. and Prucha, I.R. (1996), Estimation of the depreciation rate of physical and R&D capital in the US total manufacturing sector, *Economic Inquiry* 34, 43-56.
- OECD (2009), *Private pensions outlook 2008*, Paris.
- ONS Blue Book (2010), *The UK National Accounts*, Palgrave Macmillan: Office for National Statistics.
- ONS Financial Statistics (2010), Oct 2010, Palgrave Macmillan: Office for National Statistics.
- Rotemberg, J.J. and Woodford, M. (1998), An optimization-based econometric framework for the evaluation of monetary policy, NBER Technical Working Paper No. 233.
- Schmitt-Grohé, S. and Uribe, M. (2007), Optimal simple and implementable monetary and fiscal rules, *Journal of Monetary Economics* 54, 1702-1725.

Schmitt-Grohé, S., Uribe, M. (2004), Solving dynamic general equilibrium models using a second-order approximation to the policy function, *Journal of Economic Dynamics and Control* 28, 755-775.

Smets, F. (2003), Maintaining price stability: How long is the medium term? *Journal of Monetary Economics*, 50, 1293-1309.

Svensson, L.E.O. (1999), Price-level targeting versus inflation targeting: A free lunch? *Journal of Money, Credit and Banking* 31, 277-295.

Tödter, K.-H. (2008), Estimating the uncertainty of relative risk aversion, *Applied Financial Economics Letters* 4, 25-27.

Whitehouse, E.R. (2009), Pensions, purchasing-power risk, inflation and indexation, Social, Employment and Migration Working Paper No. 77, OECD, Paris.

Woodford, M. (2001), Inflation stabilization and welfare, NBER Working Paper No. 8071.

**Appendix A – Proof that the CIA constraint binds with strict equality when  $R_t > 1$**

In this appendix it is shown that the CIA constraint is strictly binding if the gross money return on a nominal bond exceeds the gross return on money of one. The Lagrangian from the main text can be used to derive this result, with allowance made for the possibility that the CIA constraint may not hold with strict equality. Consequently, the Lagrangian will additionally give rise to Kuhn-Tucker conditions relating to the Lagrange multiplier on the CIA constraint.

**Proposition: The CIA constraint binds with strict equality when  $R_t > 1$**

**Proof.**

From the main text the first-order conditions for indexed bonds, nominal bonds and money holdings are as follows:

$$c_{t,Y}^{-\delta} = E_t(c_{t+1,O}^{-\delta} r_{t+1}^i) + \theta \mu_t \quad (\text{A1})$$

$$c_{t,Y}^{-\delta} = E_t(c_{t+1,O}^{-\delta} r_{t+1}^n) + \theta \mu_t \quad (\text{A2})$$

$$c_{t,Y}^{-\delta} = E_t(c_{t+1,O}^{-\delta} r_{t+1}^m) + (1 + \theta) \mu_t \quad (\text{A3})$$

where  $\mu_t$  is the Lagrange multiplier on the CIA constraint.

The Kuhn-Tucker conditions associated with  $\mu_t$  are summarised in the following equation:

$$\{\mu_t \geq 0 \quad \text{and} \quad \mu_t (m_t - \theta c_{t,Y}) = 0\} \quad (\text{A4})$$

where the second equation, the complementary slackness condition, implies that the CIA constraint will be strictly binding if  $\mu_t > 0$  for all  $t$ .

Using Equations (A2) and (A3), the Lagrange multiplier  $\mu_t$  will strictly positive iff

$$E_t(c_{t+1,O}^{-\delta} r_{t+1}^n) > E_t(c_{t+1,O}^{-\delta} r_{t+1}^m) \quad \forall t \quad (\text{A5})$$

Substitution of the real return on nominal bonds into Equation (A5) gives

$$R_t \times E_t(c_{t+1,O}^{-\delta} r_{t+1}^m) > E_t(c_{t+1,O}^{-\delta} r_{t+1}^m) \quad \forall t \quad (\text{A6})$$

Dividing Inequality (A6) by  $E_t(c_{t+1,O}^{-\delta} r_{t+1}^m)$  yields the following necessary condition for the nominal interest rate:

$$R_t > 1, \quad \forall t \quad (\text{A7})$$

Finally, notice that  $E_t(c_{t+1,O}^{-\delta} r_{t+1}^n) = E_t(c_{t+1,O}^{-\delta} r_{t+1}^i)$ , such that Inequality (A7) ensures that holding indexed bonds is also strictly preferred to holding money, i.e.

$$E_t(c_{t+1,O}^{-\delta} r_{t+1}^i) > E_t(c_{t+1,O}^{-\delta} r_{t+1}^m), \quad \forall t \quad \text{iff} \quad R_t > 1 \quad \text{Q.E.D.} \quad (\text{A8})$$

## Appendix B: The link between lifetime utility and long-term inflation risk

A second-order expansion of the lifetime utility of generation  $t$  can be written in the following form:

$$\tilde{u}_t \approx -\frac{1}{2} \left[ \frac{\delta}{(E_t c_{t+1,O})^{1+\delta}} \right] \text{var}_t(c_{t+1,O}) \quad (\text{B1})$$

where  $\tilde{u}_t \equiv u_t - u_t(c_{t,Y}, E_t c_{t+1,O})$  is the deviation of lifetime utility from its value when consumption in youth and old age are at their time- $t$  expected values.

In order to evaluate the term  $\text{var}_t(c_{t+1,O})$ , note that log-linearising the budget constraint faced in old age around the deterministic steady-state gives the following expression:

$$c_O \hat{c}_{t+1,O} = Ak^\alpha [\hat{A}_{t+1} + \alpha \hat{k}_t] + r^i b^i [\hat{r}_{t+1}^i + \hat{b}_t^{i,d}] + r^n b^n [\hat{r}_{t+1}^n + \hat{b}_t^{n,d}] + r^m m [\hat{r}_{t+1}^m + \hat{m}_t^d] \quad (\text{B2})$$

where ‘hats’ indicate percentage deviations from the deterministic steady-state; steady-state values are defined by the absence of time subscripts; and  $\hat{A}_{t+1} = \rho \hat{A}_t + e_t$ .

The log-linearised real returns on money balances, indexed bonds and nominal bonds are as follows:

$$\hat{r}_{t+1}^m = -\frac{1}{1 + \pi^{ss}} (\pi_{t+1} - \pi^{ss}) \quad (\text{B3})$$

$$\hat{r}_{t+1}^i = \hat{r}_t + \frac{1}{1 + \pi^{ss}} (\pi_{t+1}^{ind} - \pi^{ss}) - \frac{1}{1 + \pi^{ss}} (\pi_{t+1} - \pi^{ss}) + v_{t+1} \quad (\text{B4})$$

$$\hat{r}_{t+1}^n = \hat{R}_t + \hat{r}_{t+1}^m = \hat{R}_t - \frac{1}{1 + \pi^{ss}} (\pi_{t+1} - \pi^{ss}) \quad (\text{B5})$$

where  $\pi^{ss}$  is the steady-state rate of inflation.

The right hand side of Equation (B2) can be written in terms of actual inflation and biased inflation using equations (B3) to (B5). Carrying out these steps and noting that  $c_{t+1,O} = c_O (1 + \hat{c}_{t+1,O})$ , consumption in old age can be expressed as follows:

$$c_{t+1,O} = \left( c_O + Ak^\alpha [\hat{A}_{t+1} + \alpha \hat{k}_t] + r^i b^i \hat{b}_t^{i,d} + r^n b^n \hat{b}_t^{n,d} + r^m m \hat{m}_t^d + r^i b^i \hat{r}_t + r^n b^n \hat{R}_t \right) - \left( \frac{r^i b^i + r^n b^n + r^m m}{1 + \pi^{ss}} (\pi_{t+1} - \pi^{ss}) + \frac{r^i b^i}{1 + \pi^{ss}} (\pi_{t+1}^{ind} - \pi^{ss}) + r^i b^i v_{t+1} \right) \quad (\text{B6})$$

Expected consumption in old age is given by

$$E_t c_{t+1,O} = \left( c_O + Ak^\alpha [\rho \hat{A}_t + \alpha \hat{k}_t] + r^i b^i \hat{b}_t^{i,d} + r^n b^n \hat{b}_t^{n,d} + r^m m \hat{m}_t^d + r^i b^i \hat{r}_t + r^n b^n \hat{R}_t \right) - \left( \frac{r^i b^i + r^n b^n + r^m m}{1 + \pi^{ss}} (E_t \pi_{t+1} - \pi^{ss}) + \frac{r^i b^i}{1 + \pi^{ss}} (E_t \pi_{t+1}^{ind} - \pi^{ss}) \right) \quad (\text{B7})$$

Thus, the conditional variance of consumption in old age is given by

$$\text{var}_t(c_{t+1,O}) = \left( \begin{array}{l} \frac{(r^i b^i + r^n b^n + r^m m)^2}{(1 + \pi^{ss})^2} \text{var}_t(\pi_{t+1}) + \frac{(r^i b^i)^2}{(1 + \pi^{ss})^2} \text{var}_t(\pi_{t+1}^{ind}) + (r^i b^i)^2 \sigma_v^2 \\ - \frac{2r^i b^i (r^i b^i + r^n b^n + r^m m)}{(1 + \pi^{ss})^2} \text{cov}_t(\pi_{t+1}, \pi_{t+1}^{ind}) + k^{2\alpha} \sigma_e^2 \end{array} \right) \quad (\text{B8})$$

where  $\text{var}_t(\pi_{t+1})$  is the conditional variance of actual inflation,  $\text{var}_t(\pi_{t+1}^{ind})$  is the conditional variance of biased inflation, and the fact that  $\text{var}_t(\hat{A}_{t+1}) = A^{-2} \text{var}_t(A_{t+1}) = A^{-2} \sigma_e^2$  has been used.

Therefore, by Equation (B1), the utility loss of generation  $t$  can be written as follows:

$$\tilde{u}_t \approx -\frac{1}{2} \chi_t \left[ \begin{array}{l} \text{var}_t(\pi_{t+1}) + \varphi \text{var}_t(\pi_{t+1}^{ind}) - 2\kappa \text{cov}_t(\pi_{t+1}, \pi_{t+1}^{ind}) \\ + \eta \sigma_v^2 + \vartheta \sigma_e^2 \end{array} \right] \quad (\text{B9})$$

where  $\chi_t \equiv \delta(r^i b^i + r^n b^n + r^m m)^2 (1 + \pi^{ss})^{-2} (E_t c_{t+1,O})^{-(1+\delta)} > 0$ , and  $E_t c_{t+1,O}$  is given by Equation (B7).

Coefficients are defined as follows:

$$\varphi \equiv (r^i b^i)^2 / (r^i b^i + r^n b^n + r^m m)^2 \quad (\text{B10})$$

$$\kappa \equiv \varphi (r^i b^i + r^n b^n + r^m m) / r^i b^i \quad (\text{B11})$$

$$\eta \equiv \varphi (1 + \pi^{ss})^2 \quad (\text{B12})$$

$$\vartheta \equiv (r^i b^i + r^n b^n + r^m m)^{-2} (1 + \pi^{ss})^2 k^{2\alpha} \quad (\text{B13})$$

Note that in the special case when indexed bonds and capital are excluded from the model (i.e.  $b^i = k = 0$ ), the loss in lifetime utility is given by  $\tilde{u}_t \approx -\frac{1}{2} \chi_t \text{var}_t(\pi_{t+1})$  since  $\varphi = \kappa = \eta = \vartheta = 0$ .

## Appendix C – Derivations of the IT and PLT money supply rules from a yearly horizon

### Inflation targeting

Consider the following *yearly IT money supply rule* that aims at a constant inflation target and is subject to exogenous monetary innovations in each year  $i$ :

$$\ln M_i^{s,IT} = \ln M_{i-1}^{s,IT} + \pi + \varepsilon_i + \ln c_{i,Y} - \ln c_{i-1,Y} \quad (C1)$$

where  $\pi$  is the yearly inflation target and  $\varepsilon_i$  is an IID-normal money supply innovation with mean zero and variance  $\sigma^2$ .

To derive a 30-year money supply rule from this yearly specification, substitute repeatedly for the lagged money supply term on the right-hand side of Equation (C1) until the following 30-year money supply rule is reached:

$$\ln M_i^{s,IT} = \ln M_{i-30}^{s,IT} + 30 \times \pi + \sum_{j=0}^{29} \varepsilon_{i-j} + \ln c_{i,Y} - \ln c_{i-30,Y} \quad (C2)$$

This equation states that the 30-year growth rate of the nominal money supply has three components: a 30-year inflation target  $30 \times \pi$ ; the sum-total of 30 separate yearly money supply innovations; and the 30-year rate of growth of consumption by the young.

Given that each period  $t$  lasts 30 years, Equation (C2) implies that the money supply rule in any period  $t$  can be represented in the following form:

$$\ln M_t^{s,IT} - \ln M_{t-1}^{s,IT} = 30 \times \pi + \sum_{i=1}^{30} \varepsilon_{i,t} + \ln c_{t,Y} - \ln c_{t-1,Y} \quad (C3)$$

where, for ease of exposition, the Gaussian white noise money supply innovations have been re-indexed from years 1 to 30, and the time subscript indicates that all 30 innovations belong to period  $t$ .

### Price-level targeting

Consider the following *yearly PLT money supply rule* that aims at a target yearly (log) price level of  $\ln P_i^* = p_0 + \pi \times i$  in each year  $i$ :

$$\ln M_i^{s,PLT} = \ln M_{i-1}^{s,PLT} + \pi + \varepsilon_i - \varepsilon_{i-1} + \ln c_{i,Y} - \ln c_{i-1,Y} \quad (C4)$$

where  $\pi$  is the constant yearly inflation target that is consistent with the target price path, and  $\varepsilon_i$  is an IID-Normal innovation with mean zero and variance  $\sigma^2$  (exactly as in the IT case).

To derive the implied money supply rule over a 30-year horizon, substitute for the lagged money supply term on the right hand side of Equation (C4) until the following expression is reached:

$$\ln M_i^{s,PLT} = \ln M_{i-30}^{s,PLT} + 30 \times \pi + \varepsilon_i - \varepsilon_{i-30} + \ln c_{i,Y} - \ln c_{i-30,Y} \quad (C5)$$

Given that each period  $t$  lasts 30 years, Equation (C5) implies a period  $t$  money supply rule of the form

$$\ln M_t^{s,IT} - M_{t-1}^{s,IT} = 30 \times \pi + \varepsilon_{30,t} - \varepsilon_{30,t-1} + \ln c_{t,Y} - \ln c_{t-1,Y} \quad (C6)$$

where again the money supply innovations have been indexed to reflect the year in which they occur, and the  $t$  subscript indicates the period to which innovations belong.

## Appendix D: Deterministic steady-state and market-clearing conditions

### Deterministic steady state<sup>41</sup>

$$c_Y + b^{i,d} + b^{n,d} + m^d + k = \varpi(1 - \tau^j), \quad \text{for } j \in (IT, PLT) \quad (D1)$$

$$c_O = Ak^\alpha + r^i b^{i,d} + r^n b^{n,d} + r^m m \quad (D2)$$

$$R = (1 + \pi^{ss})r^n \quad (D3)$$

$$r^i = \frac{(1 + \pi^{ind})}{(1 + \pi^{ss})} r \quad (D4)$$

$$r^m = 1/(1 + \pi^{ss}) \quad (D5)$$

$$g = \tau^j \varpi + (1 - r^i)b^{i,s} + (1 - r^n)b^{n,s} + m^d \pi^{ss} / (1 + \pi^{ss}) \quad (D6)$$

$$\pi^{ss} = 30 \times \pi \quad (D7)$$

$$\pi^{ind} = \pi^{ss} \quad (D8)$$

$$m^s = m^d \quad (D9)$$

$$m^d = \theta c_Y \quad (D10)$$

$$b^{i,d} = b^{i,s} = a \times b^s \quad (D11)$$

$$b^{n,d} = b^{n,s} = (1 - a) \times b^s \quad (D12)$$

$$b^d = b^{i,d} + b^{n,d} = b^s = \frac{\varpi(1 - \tau^j) - (1 + r^m)m^d - (1 + Ak^\alpha)k}{1 + r} \quad (D13)$$

(from bond supply rule,  $c_Y^{-\delta} = c_O^{-\delta}$  in SS)

$$c_Y^{-\delta} = c_O^{-\delta} \left( (1 + \theta)r^i - \frac{\theta}{1 + \pi^{ss}} \right) \quad (D14)$$

$$c_Y^{-\delta} = c_O^{-\delta} \left( (1 + \theta)r^n - \frac{\theta}{1 + \pi^{ss}} \right) \quad (D15)$$

$$r^i = r^n = r = \frac{1 + \theta + \pi^{ss}}{(1 + \pi^{ss})(1 + \theta)} \quad (\text{implied by the previous two equations}) \quad (D16)$$

$$\alpha Ak^{\alpha-1} = r^n = r^i \quad (\text{from the Euler equations for capital and bonds}) \quad (D17)$$

$$A = A_{mean} \quad (D18)$$

---

<sup>41</sup>  $\pi^{ss}$  denotes the steady-state rate of inflation.

### Market-clearing conditions

A monetary equilibrium in the OLG economy is a set of allocations  $\{c_{t,Y}, c_{t,O}, b_t^{i,d}, b_t^{i,s}, b_t^{n,d}, b_t^{n,s}, k_t, m_t^d, m_t^s, g_t, \pi_t, r_t^n, R_t, r_t^m, \tau^j\}_{t=1}^T$  with the following properties for all  $t$ :

(1) Allocations  $\{c_{t,Y}, c_{t+1,O}, b_t^{i,d}, b_t^{n,d}, k_t, m_t^d\}_{t=1}^T$  solve the maximisation problem of a young consumer born at time  $t$ ;

(2) The goods, money and bond markets clear:

$$\varpi + A_t k_{t-1}^\alpha = c_{t,Y} + c_{t,O} + g_t + k_t \quad (\text{D19})$$

$$m_t^d = m_t^s \quad (\text{D20})$$

$$b_t^{i,d} = b_t^{i,s} \quad (\text{D21})$$

$$b_t^{n,d} = b_t^{n,s} \quad (\text{D22})$$

(3) The government's budget constraint and long run government spending target are satisfied:

$$g_t = \tau^j \varpi + m_t^s - r_t^m m_{t-1}^s + b_t^{i,s} - r_t^i b_{t-1}^{i,s} + b_t^{n,s} - r_t^n b_{t-1}^{n,s} \quad (\text{D23})$$

$$E(g_t) = g^* \quad (\text{D24})$$

## Appendix E: Model listing

$$u_t = \frac{c_{t,Y}^{1-\delta}}{1-\delta} + E_t \frac{c_{t+1,O}^{1-\delta}}{1-\delta} \quad \text{Lifetime utility of generation } t \quad (\text{E1})$$

$$c_{t,Y} + b_t^{n,d} + b_t^{i,d} + m_t^d + k_t = \varpi(1-\tau^j) \quad \text{Budget constraint when young} \quad (\text{E2})$$

$$c_{t+1,O} = A_{t+1}k_t^\alpha + r_{t+1}^i b_t^{i,d} + r_{t+1}^n b_t^{n,d} + r_{t+1}^m m_t^d \quad \text{Budget constraint when old} \quad (\text{E3})$$

$$\ln A_t = (1-\rho) \ln A_{mean} + \rho \ln A_{t-1} + e_t \quad \text{Productivity} \quad (\text{E4})$$

$$m_t^d = \theta c_{t,Y} \quad \text{CIA constraint} \quad (\text{E5})$$

$$r_{t+1}^m \equiv 1/(1+\pi_{t+1}) \quad \text{Real return on money balances} \quad (\text{E6})$$

$$r_{t+1}^n = R_t r_{t+1}^m \quad \text{Real return on nominal bonds} \quad (\text{E7})$$

$$r_{t+1}^i = r_t \times \left[ \frac{(1+\pi_{t+1}^{ind})}{(1+\pi_{t+1})} + v_{t+1} \right] \quad \text{Real return on indexed bonds} \quad (\text{E8})$$

$$c_{t,Y}^{-\delta} = E_t \left( c_{t+1,O}^{-\delta} \left( (1+\theta)r_{t+1}^n - \theta r_{t+1}^m \right) \right) \quad \text{Euler equation for nominal bonds} \quad (\text{E9})$$

$$c_{t,Y}^{-\delta} = E_t \left( c_{t+1,O}^{-\delta} \left( (1+\theta)r_{t+1}^i - \theta r_{t+1}^m \right) \right) \quad \text{Euler equation for indexed bonds} \quad (\text{E10})$$

$$c_{t,Y}^{-\delta} = E_t \left( c_{t+1,O}^{-\delta} \left( (1+\theta)\alpha A_{t+1} k_t^{\alpha-1} - \theta r_{t+1}^m \right) \right) \quad \text{Euler equation for capital} \quad (\text{E11})$$

$$g_t = \tau^j \varpi + m_t^s - r_t^m m_{t-1}^s + b_t^{i,s} - r_t^i b_{t-1}^{i,s} + b_t^{n,s} - r_t^n b_{t-1}^{n,s} \quad \text{Govt. budget constraint} \quad (\text{E12})$$

$$E(g_t) = g^* \quad \text{Long run government spending target (implies } \tau^j \text{)} \quad (\text{E13})$$

$$\ln(m_t^{s,IT} / m_{t-1}^{s,IT}) = 30 \times \pi + \sum_{i=1}^{30} \varepsilon_{i,t} + \ln(c_{t,Y} / c_{t-1,Y}) - \pi_t \quad \text{IT money rule} \quad (\text{E14})$$

$$\ln(m_t^{s,PLT} / m_{t-1}^{s,PLT}) = 30 \times \pi + \varepsilon_{30,t} - \varepsilon_{30,t-1} + \ln(c_{t,Y} / c_{t-1,Y}) - \pi_t \quad \text{PLT money rule} \quad (\text{E15})$$

$$U^{society} = \frac{1}{T} E \left[ \sum_{t=1}^T (u_{t,Y}(c_{t,Y}) + u_{t,O}(c_{t,O})) \right] \quad \text{Social welfare} \quad (\text{E16})$$

$$\pi_t^{ind,IT} = 30 \times \pi + \sum_{i=1}^{30} \varepsilon_{i,t}^{ind} \quad \text{Inflation rate to which index bonds are linked (IT)} \quad (\text{E17})$$

$$\pi_t^{ind,PLT} = 30 \times \pi + \varepsilon_{30,t}^{ind} - \varepsilon_{30,t-1}^{ind} \quad \text{Inflation to which index bonds are linked (PLT)} \quad (\text{E18})$$

$$c_{t,Y}^{-\delta} = E_t(c_{t+1}^{-\delta}) \quad \text{Total bond supply rule} \quad (\text{E19})$$

$$b_t^s = b_t^{i,s} + b_t^{n,s} \quad \text{Total bond supply definition} \quad (\text{E20})$$

$$m_t^d = m_t^s \quad \text{Money market equilibrium} \quad (\text{E21})$$

$$b_t^{n,d} = b_t^{n,s} = (1-a)b_t^s \quad \text{Market-clearing in nominal bonds} \quad (\text{E22})$$

$$b_t^{i,d} = b_t^{i,s} = ab_t^s \quad \text{Market-clearing in indexed bonds} \quad (\text{E23})$$

$$c_{t,Y} + c_{t,O} + k_t + g_t = \varpi + A_t k_{t-1}^\alpha \quad \text{Market-clearing in goods} \quad (\text{E24})$$