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**Cardiff Economics
Working Papers**

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E2011/3

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ISSN 1749-6101
January 2011, updated March 2011

Optimal indexation of government bonds and monetary policy[1]

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Jan 2011, updated March 2011

Preliminary Draft

Abstract
Using an overlapping generations model in which the young save for old age using indexed and nominal government bonds, this paper investigates how optimal indexation is influenced by monetary policy. In order to do so, two monetary policies with markedly different long run implications are examined: inflation targeting and price-level targeting. Optimal indexation differs significantly under the two regimes. Under inflation targeting, long-term inflation uncertainty is substantial due to base-level drift in the price level. Nominal bonds are thus a poor store of value and optimal indexation is relatively high (76 per cent). With price-level targeting, by contrast, long-term inflation uncertainty is minimal because the price level is trend-stationary. This makes nominal bonds a better store of value compared to indexed bonds, reducing optimal indexation somewhat (26 per cent). Importantly for these results, the model captures two imperfections of indexation (indexation bias and lagged indexation) that are calibrated to the UK case.

Keywords: optimal indexation, government bonds, inflation targeting, price-level targeting.

JEL Classification: E52, E58.

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[1] The author gratefully acknowledges help and guidance from his supervisors, Professor Patrick Minford and Dr. Panayiotis Pourpourides. The author also thanks the ESRC for funding this research.

1. Introduction

This paper investigates the link between optimal indexation and monetary policy. The motivation for studying this issue can be traced back to the seminal papers on optimal indexation by Fisher (1975) and Gray (1976). Gray investigated indexation of wage contracts in the face of real and nominal shocks. She showed that optimal indexation depends on the variances of real and nominal disturbances, increasing with the nominal-to-real volatility ratio. Therefore, indexation of wages should fall under monetary policy regimes that reduce nominal volatility – a prediction that appears to be borne out by the data (Holland, 1986; Amano, Ambler and Ireland, 2007).[2] The impact of monetary policy on optimal indexation of wages has been investigated more recently by Minford, Nowell and Webb (2003) and Amano, Ambler and Ireland (2007). These authors study optimal indexation under inflation targeting (IT) and price-level targeting (PLT), motivated by the theoretical finding that PLT reduces nominal volatility substantially at medium- and long-term horizons. Consistent with Gray (1976), they find that optimal indexation is lower under PLT than IT because nominal volatility is reduced over the wage-contracting horizon.

Fischer (1975) uses a portfolio approach to study the demand for price-level-indexed bonds. In his model, households receive income from human capital and choose an optimal portfolio consisting of equity, nominal bonds and indexed bonds. Fischer shows that if the real return on human capital is uncorrelated with inflation, then consumers will strictly prefer indexed bonds over nominal bonds, because the former are a perfect store of purchasing power. However, this dominance results breaks down if inflation is correlated with the real return on human capital (or, more generally, with other sources of income), since households can diversify consumption risk by holding at least some nominal bonds. Moreover, the optimal demands for indexed and nominal bonds depend on the extent of inflation risk, thus positing a link between optimal indexation and monetary policy as in the case of wage indexation.

A second important finding from a portfolio approach to indexed bonds is that full indexation is not optimal if indexation is imperfect, because nominal bonds will diversify consumption risk if the correlation between inflation and the 'indexation error' is sufficiently small. The importance of modelling imperfect indexation to the price level has recently been emphasised in the context of private debt by Meh, Quadrini and Terajima (2009). They develop a model in which financial contracts are imperfectly indexed to inflation because nominal prices are observed with delay, as in Jovanovic and Ueda (1997). Contracting results from entrepreneurs entering into debt contracts with financial intermediaries in order to finance investment, and the extent of indexation is determined endogenously as part of an optimal incentive-compatible contract. Since indexation is imperfect, only partial indexation to prices is optimal, and the optimal degree of indexation increases with the magnitude of price-level uncertainty.

[2] For example, Ambler, Amano and Ireland report that the proportion of wage settlements with cost-of-living clauses in Canada fell from 22 per cent between 1978 and 1989 to around 10 per cent between 1995 and 1999.

One criticism that can be levelled at the portfolio approach to indexed bonds is that it ignores the necessity that the government explicitly finance issuance of debt,[3] thus failing to provide an equilibrium solution to the optimal indexation problem. When the government must finance bond issuance subject to its budget constraint, nominal bonds are useful if the government is required to balance its budget in each period (Levhari and Liviatan, 1976), or if inflation is correlated with the tax burden (Bohn, 1988). This literature, however, focuses only upon necessary conditions for partial indexation to be optimal. In the current paper, the optimal share of indexed government debt is computed directly. Moreover, the model relaxes the oft-made but implausible assumptions that (i) indexed bonds are perfectly indexed, and (ii) bond risk-premia are equal to zero.[4]

The key feature of the model is that optimal indexation of government debt is determined endogenously in response to monetary policy. Since the government takes into account consumers' first-order optimality conditions when choosing the optimal level of indexation, it effectively solves an optimal commitment Ramsey problem (see Ljungqvist and Sargent, 2000). Motivated by recent literature in the area of optimal wage indexation, the alternative monetary policies considered in this paper are IT and PLT. Indexation in the model is subject to two distinct imperfections – indexation bias and lagged indexation. The presence of imperfect indexation in the model is crucial. Indeed, as optimal indexation in the model minimises consumption volatility across old generations, issuing nominal bonds is desirable only if indexed bonds are a risky asset.

The model consists of overlapping generations of consumers and a long-lived government. Consumers hold money balances, productive capital and indexed and nominal government bonds.[5] Each period in the model lasts 30 years and consumers live for two periods, namely, youth and old age. Monetary policy is implemented by the government through IT and PLT money supply rules, and aggregate uncertainty is introduced into the model via real shocks to productivity and various nominal disturbances, including money supply shocks. The government is the monopoly supplier of both bonds and money, and meets an exogenous long run government spending target by taxing young consumers.

The main finding from the model is that optimal indexation is significantly lower under PLT than IT (26 per cent vs. 76 per cent). The reasoning is as follows. Long-term inflation uncertainty is substantial under IT because of base-level drift: even if the central bank misses its inflation target by only a small percentage in each year, these misses can accumulate and become quite large after 30 years. Consequently, nominal bonds are a poor store of value compared to indexed bonds and optimal indexation is relatively high. Under PLT, by contrast,

[3] Campbell and Shiller (1996) provide a useful discussion of the impact of introducing indexed bonds on government financing costs (within the context of the US economy).
[4] Bond risk-premia are zero in linear or log-linearised models. Alternatively, it sometimes assumed that marginal utility is linear so that consumers are risk-neutral (e.g. Bohn, *op. cit.*).
[5] Productive capital was first introduced into the overlapping generations model by Diamond (1965), who extended the standard life-cycle model (see Samuelson, 1958) from partial to general equilibrium.

the price level is returned to its target path following inflationary shocks. Past deviations from the inflation target therefore do not accumulate over the long-term, reducing inflation volatility by an order of magnitude and making nominal bonds a somewhat better store of value compared to indexed bonds.

In order to investigate the source of the reduction in optimal indexation under PLT, the indexation differential is decomposed into 'indexation bias' and 'indexation lag' components. This decomposition reveals that around nine-tenths of the reduction in indexation under PLT is due to the indexation lag. The substantive reduction due to the indexation lag arises because its contribution to real return volatility is the same under IT and PLT, as the indexation lag length is invariant to a change in monetary policy regime. Hence, when inflation risk is reduced under PLT, real return volatility falls more sharply on nominal bonds than on indexed bonds, providing an incentive for substitution towards nominal bonds. The long-term contracting horizon in the model (30 years) is crucial for this result because inflation volatility is reduced markedly under PLT at a long horizon (Dittmar, Gavin and Kydland, 1999; Bordo, Dittmar and Gavin, 2007).

On the other hand, indexation bias plays a relatively small role in reducing optimal indexation by ensuring that, even in the absence of an indexation lag, it is optimal for some nominal bonds to be issued. Conditional on the presence of nominal bonds in consumer portfolios, PLT reduces optimal indexation relative to IT because it dilutes the positive correlation between the real return on nominal bonds and the real return on money balances, thus reducing consumption covariance risk associated with holding nominal bonds. The real return correlation falls under PLT because expected inflation varies over time. The reasoning is that whilst a nominal bond compensates consumers for *anticipated* inflation fluctuations, money balances do not – so that there is a wedge between the co-movement in returns. This contrasts with the IT case where a nominal bond is effectively money plus a constant 'mark-up' for expected inflation, so that the real return correlation is perfect (i.e. +1).

The result that optimal indexation of government bonds can vary substantially across monetary policy regimes has potentially important policy implications. Firstly, models that do not endogenise the extent of indexation in response to monetary policy are vulnerable to the Lucas critique (Lucas, 1976) and may give rise to seriously misleading results in policy analyses or forecasting exercises. Secondly, as the optimal indexation results in this paper arise from comparing IT and PLT policies, they have important implications for central banks like the Bank of Canada that are considering switching from IT to PLT and are interested in evaluating the performance of these two regimes in simulated models of the economy.[6] Finally, the willingness of governments to issue indexed bonds may be influenced considerably by monetary policy, highlighting a potentially important interaction between fiscal policy and monetary policy. Whilst the results in this paper provide intuition for why

[6] The Bank of Canada is currently conducting a review of price-level targeting in anticipation of its next policy agreement with the Government in 2011. The review was announced in Bank of Canada (2006).

governments might issue both indexed and nominal government bonds, they cannot explain fully the prevalence of low levels of indexation in developed economies.

The paper proceeds as follows. Section 2 sets out the model, including the monetary policy rules under IT and PLT; Section 3 discusses model calibration and the simulation methodology employed; Section 4 discusses the optimal indexation problem; and Section 5 presents simulation results. Finally, Section 6 concludes and discusses policy implications.

2. The Model

The model is an overlapping generations (OLG) model of life-cycle saving in which consumers hold money balances, capital, and indexed and nominal government bonds. Consumers have homogenous preferences and live for two periods of 30 years: in the first they are 'young' and receive an exogenous endowment income; in the second they are the retired 'old' who receive the proceeds from their savings in youth. Population growth is set equal to zero for simplicity[7] and, without loss of generality, each generation is assumed to have a constant size of one.[8]

Aggregate uncertainty is introduced into the model via real productivity shocks and various nominal disturbances, including money supply shocks. Although 'fiat money' is a popular way of justifying money holdings in OLG models (see e.g. McCandless and Wallace, 1991), this approach is not theoretically convincing because fiat money must offer the same return as non-monetary assets to have value, implying deflation if these assets offer real returns. Money is instead introduced by a cash-in-advance constraint, an approach taken by a number of recent contributions that investigate optimal monetary policy in OLG economies (e.g. Michel and Wigniolle, 2005; Gahvari, 2007). Monetary policy takes the form of IT and PLT money supply rules. In response to monetary policy, young consumers demand indexed and nominal bonds, money balances and capital. Capital is used to produce output that is consumed in old age. As such, capital is a hedge against inflation – a view long held by theoretical economists and an implication of the Fisher equation (see Bodie, 1976).

The government is responsible for implementing monetary policy and sets the total bond supply and the mix between indexed and nominal bonds (through individual bond supplies). The total bond supply is set to ensure optimal consumption smoothing (in expected terms) for each generation – along the lines of the standard OLG model where government bonds are 'net wealth' (see Barro, 1974; Minford and Peel, 2002). The mix between indexed and nominal bonds is chosen to maximise social welfare, subject to consumers' first-order

[7] Constant population growth would introduce an additional parameter (the population growth rate) but would not change model dynamics or, therefore, the optimal indexation results.
[8] There is no loss of generality because the focus throughout is on per-capita values. All model equations would be left unchanged if generations had a constant size greater than one and were populated by homogenous consumers. The only difference is that per-capita values would need to be multiplied by the constant generation size in order to get economy-wide aggregates.

conditions for saving, the money supply rule in place, and a long run government spending target.[9] Since indexed and nominal bonds are priced to rule out arbitrage via consumers' first-order conditions, all indexation shares in the range [0,1] are feasible equilibria. In effect, the government's problem is to select the equilibrium from this feasible set that maximises social welfare (under IT and PLT).

The model is solved using a second-order approximation in Dynare++ (Julliard, 2001). This point is crucial since a linear approximation would eliminate risk-premia in the returns on indexed and nominal bonds. More generally, it is well-known that linear approximation can lead to an inaccurate social welfare ranking of alternative policies (in this case, alternative indexation shares) because it ignores the impact of uncertainty on the stochastic means of endogenous variables in the model (Kim and Kim, 2003; Schmitt-Grohé and Uribe, 2004).

2.1 The consumer problem

Consumers live for two periods of 30 years and have constant relative risk aversion (CRRA) preferences over consumption:

$$u_t = u_{t,Y}(c_{t,Y}) + E_t u_{t+1,O}(c_{t+1,O}) \tag{1}$$

where $u_{t,Y}(c_{t,Y}) \equiv c_{t,Y}^{1-\delta}/(1-\delta)$ is utility in youth and $u_{t+1,O}(c_{t+1,O}) \equiv c_{t+1,O}^{1-\delta}/(1-\delta)$ is utility in old age. Consumption in period *t* when young is denoted $c_{t,Y}$ and consumption in period $t+1$ when old is denoted $c_{t+1,O}$.[10]

The budget constraint of young agents can be expressed in real terms as follows:

$$c_{t,Y} + b_t^{i,d} + b_t^{n,d} + m_t^d + k_t = \varpi(1-\tau^j) \tag{2}$$

where ϖ is a young consumer's constant real endowment income;[11] $b_t^{i,d} \equiv B_t^{i,d}/P_t$ is real demand for indexed bonds; $b_t^{n,d} \equiv B_t^{n,d}/P_t$ is real demand for nominal bonds; and $m_t^d \equiv M_t^d/P_t$ is real demand for money balances. Note that uppercase values are nominal and P_t is the aggregate price level. Capital in real terms is given by k_t, and τ^j for $j \in (IT, PLT)$ is the constant rate of income tax.

[9] Government spending is used up in projects that have no direct effect on consumption or utility.
[10] Consumers do not discount consumption in old age, as is often assumed in models with overlapping generations. Examples from the literature that use this assumption include Champ and Freeman (1990) and Brazier, Harrison, King and Yates (2006).
[11] A constant endowment is specified for simplicity. Including endowment shocks will not affect the optimal indexation results so long as such shocks are orthogonal.

Following Artus (1995), consumers' demand for money arises from a cash-in-advance (CIA) constraint which states that real monetary savings are a fraction $0 < \theta < 1$ of consumption when young: [12]

$$m_t^d \geq \theta \times c_{t,Y} \tag{3}$$

As shown in Appendix A, the CIA constraint will bind with strict equality if the monetary return on nominal bonds exceeds one. Intuitively, since money is a perfect store of nominal value, an optimising consumer will not hold monetary savings in excess of the proportion θ required by the CIA constraint if nominal bonds pay a higher return. The CIA constraint is taken to be strictly binding, i.e. $m_t^d = \theta c_{t,Y}$. [13]

Capital is used to produce output in old age via a production function that exhibits diminishing returns. The depreciation rate on capital is 100 per cent; hence capital lasts for only one period.[14] Since the amount of output produced using capital depends on productivity, capital is a claim to an uncertain amount of real output in old age.

Output in old age is given by the following production function:

$$y_{t+1,O} = A_{t+1} k_t^{\alpha} \qquad\qquad 0 < \alpha < 1 \tag{4}$$

where α is the elasticity of output with respect to capital.

Productivity A_t follows an AR(1) process in logs:

$$\ln A_t = (1-\rho)\ln A_{mean} + \rho \ln A_{t-1} + e_t \qquad 0 < \rho < 1 \tag{5}$$

where the productivity innovation e_t is an IID-Normal random variable with mean zero and variance σ_e^2.

Consumption in old age consists of output produced using capital and savings income from holding money and bonds. Real consumption by the old is therefore given by

$$\begin{aligned} c_{t+1,O} &= A_{t+1} k_t^{\alpha} + r_{t+1}^i b_t^{i,d} + r_{t+1}^n b_t^{n,d} + r_{t+1}^m m_t^d \\ &= A_{t+1} k_t^{\alpha} + (a r_{t+1}^i + (1-a) r_{t+1}^n) b_t^d + r_{t+1}^m m_t^d \end{aligned} \tag{6}$$

where $a \equiv b_t^{i,d} / b_t^d$ is the share of indexed bonds in consumers' bond portfolios; $b_t^d \equiv b_t^{i,d} + b_t^{n,d}$ is total demand for government bonds in real terms; $r_{t+1}^m \equiv 1/(1+\pi_{t+1})$ is the

[12] Cited by Crettez, Michel and Wigniolle (1999). This constraint is interpreted as a 'cash-in-advance' specification in the OLG literature.
[13] Note that the nominal (or money) return on nominal bonds was greater than one in all simulations.
[14] Given that each period lasts 30 years, the assumption of full depreciation is empirically plausible. See Nadiri and Prucha (1996) and studies cited therein.

gross real return on money balances held from youth to old age; and $\pi_t \equiv (P_t / P_{t-1} - 1)$ is the rate of inflation in period *t*. The real returns on indexed bonds and nominal bonds, r^i_{t+1} and r^n_{t+1} respectively, are explained in detail below.

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Indexed bonds pay an *ex ante* riskless gross real return r_t that is endogenously determined.[15] However, due to indexation bias and lagged indexation, the *ex post* real return on an indexed bond will in general differ from this riskless return. In particular, the *ex post* real return on an indexed bond held from period t to period $t+1$ is given by

$$r^i_{t+1} = r_t \times \left[\frac{(1+\pi^{ind}_{t+1})}{(1+\pi_{t+1})} + v_{t+1} \right] \tag{7}$$

where π^{ind}_t is the biased rate of inflation to which indexed bonds are linked, π_t is the true rate of inflation, and v_t is a Gaussian 'white noise' innovation whose variance σ^2_v is based on the indexation lag length.

The first term in square brackets reflects indexation bias: its value will deviate from one if 'true' and 'biased' inflation are not equal. Indexation is biased because the price index used for indexation differs from the true one that defines consumers' standard of living. In the UK, for example, index-linked gilts are indexed to the Retail Prices Index (RPI), whereas the Retail Prices Index excluding mortgage interest payments (RPIX) may better reflect the inflation rate faced by the majority of pensioners (i.e. 'old generations') who do not make mortgage repayments (Leceister, O'Dea and Oldfield, 2008). The extent of indexation bias depends on the relative variances of true and biased inflation and the correlation between the two inflation rates.

The second term in square brackets captures the impact of lagged indexation on the *ex post* real return received on indexed bonds. The indexation lag is motivated by the presence of data publication and collection lags, which are responsible for indexation occurring with a lag in practice. The magnitude of the indexation lag on government bonds differs across countries. For example, the large majority of outstanding index-linked gilts in the UK are indexed to the RPI with an 8-month lag (DMO, 2010), whereas indexed bonds in the US and Canada are linked to the Consumer Prices Index (CPI) with a 3-month lag. The indexation lag is modelled by a white noise innovation because this provides a simple way to capture volatility arising from lagged indexation when the indexation lag length is small relative to the holding period. The innovation v_t is exogenous and invariant to monetary policy, reflecting the assumption that the indexation lag on indexed bonds is not affected by a shift in monetary policy regime.

[15] The return r_t ensures that the market for indexed bonds clears.

Nominal bonds pay a riskless nominal return R_t. The *ex post* real return on nominal bonds is certain but for inflation risk and is given by

$$r_{t+1}^n = R_t /(1+\pi_{t+1}) = R_t \times r_{t+1}^m \tag{8}$$

where the nominal return R_t is endogenously determined.[16]

Finally, the initial old are endowed with m_0 units of real money balances, an initial stock of government debt $b_0 = b_0^i + b_0^n$, and capital k_0; their corresponding level of consumption is $c_{1,O}$. In model simulations, these initial values are set equal to the deterministic steady-state values. Trivially, the utility of the initial old is given by

$$u_{1,O} = \frac{c_{1,O}^{1-\delta}}{1-\delta} \tag{9}$$

2.2 Consumers' first-order conditions

Consider the following Lagrangian:

$$L_t \equiv E_t \begin{bmatrix} \frac{1}{1-\delta}\left(c_{t,Y}^{1-\delta} + c_{t+1,O}^{1-\delta}\right) + \lambda_{t,Y}(\varpi(1-\tau^j) - m_t^d - b_i^{i,d} - b_t^{n,d} - k_t - c_{t,Y}) \\ + \mu_t(m_t^d - \theta c_{t,Y}) + \lambda_{t+1,O}(A_{t+1}k_t^\alpha + r_{t+1}^i b_t^{i,d} + r_{t+1}^n b_t^{n,d} + r_{t+1}^m m_t^d - c_{t+1,O}) \end{bmatrix} \tag{10}$$

where $\lambda_{t,Y}$ ($\lambda_{t+1,O}$) is the Lagrange multiplier on young (old) consumers' budget constraints, and μ_t is the Lagrange multiplier on the CIA constraint.

First-order conditions are as follows:

$$c_{t,Y} : c_{t,Y}^{-\delta} = \lambda_{t,Y} + \theta\mu_t \tag{11}$$

$$c_{t+1,O} : \lambda_{t+1,O} = c_{t+1,O}^{-\delta} \tag{12}$$

$$b_t^{i,d} : \lambda_{t,Y} = E_t\left(\lambda_{t+1,O} r_{t+1}^i\right) \tag{13}$$

$$b_t^{n,d} : \lambda_{t,Y} = E_t\left(\lambda_{t+1,O} r_{t+1}^n\right) \tag{14}$$

$$m_t^d : \lambda_{t,Y} = E_t\left(\lambda_{t+1,O} r_{t+1}^m\right) + \mu_t \tag{15}$$

$$k_t : \lambda_{t,Y} = E_t\left(\lambda_{t+1,O}\alpha A_{t+1}k_t^{\alpha-1}\right) \tag{16}$$

[16] In particular, R_t ensures that the market for nominal bonds clears.

Substituting out the Lagrange multipliers on budget constraints when young and old gives the following consumption Euler equations for indexed bonds, nominal bonds, and capital respectively:

$$c_{t,Y}^{-\delta} = E_t(c_{t+1,O}^{-\delta} r_{t+1}^i) + \theta\mu_t \tag{17}$$

$$c_{t,Y}^{-\delta} = E_t(c_{t+1,O}^{-\delta} r_{t+1}^n) + \theta\mu_t \tag{18}$$

$$c_{t,Y}^{-\delta} = E_t(c_{t+1,O}^{-\delta} r_{t+1}^k) + \theta\mu_t \tag{19}$$

where $r_{t+1}^k \equiv \alpha A_{t+1} k_t^{\alpha-1}$ is the real return on capital.

The Lagrange multiplier on the cash-in-advance constraint is given by

$$\mu_t = E_t\left(c_{t+1,O}^{-\delta}(r_{t+1}^i - r_{t+1}^m)\right) = E_t\left(c_{t+1,O}^{-\delta}(r_{t+1}^n - r_{t+1}^m)\right) = E_t\left(c_{t+1,O}^{-\delta}(r_{t+1}^k - r_{t+1}^m)\right) \tag{20}$$

where the multiple equalities follow from the absence of arbitrage opportunities across assets. Intuitively, Equation (20) states that, absent uncertainty, a sufficient condition for the CIA constraint to be strictly binding (i.e. $\mu_t > 0 \;\; \forall t$)[17] is that money be rate of return dominated by other assets.

Substituting out for the Lagrange multiplier, the consumption Euler equations can be written in the following form:

$$c_{t,Y}^{-\delta} = E_t\left(c_{t+1,O}^{-\delta}\left((1+\theta)r_{t+1}^i - \theta r_{t+1}^m\right)\right) \tag{21}$$

$$c_{t,Y}^{-\delta} = E_t\left(c_{t+1,O}^{-\delta}\left((1+\theta)r_{t+1}^n - \theta r_{t+1}^m\right)\right) \tag{22}$$

$$c_{t,Y}^{-\delta} = E_t\left(c_{t+1,O}^{-\delta}\left((1+\theta)r_{t+1}^k - \theta r_{t+1}^m\right)\right) \tag{23}$$

Equations (21) to (23) show that the CIA constraint gives rise to an additional term $\theta(r_{t+1}^h - r_{t+1}^m)$, for $h \in \{i, n, k\}$, on the right hand side of the consumption Euler equations for asset holdings. The intuition behind this additional term is that reducing consumption when young by one unit has a knock-on effect via the CIA constraint of reducing money holdings by θ units, because money holdings are proportional to consumption. This reduction in money holdings makes available an extra θ units of endowment income for purchases of indexed bonds, nominal bonds, or capital (young consumers are indifferent between all three at the margin). Consequently, consumers receive an additional return $\theta \times r_{t+1}^h$ in old age whilst losing $\theta \times r_{t+1}^m$ from the reduction in money balances.

[17] See Hodrick, Kocherlakota and Lucas (1991).

2.3 Government

The government finances real spending g_t by taxing young consumers, printing money and issuing indexed and nominal government bonds. The government budget constraint is thus given by

$$g_t = \tau^j \varpi + m_t^s - r_t^m m_{t-1}^s + b_t^{i,s} - r_t^i b_{t-1}^{i,s} + b_t^{n,s} - r_t^n b_{t-1}^{n,s} \tag{24}$$

where $\tau^j \varpi$ is revenue from taxing young consumers' endowment incomes, $b_t^{i,s}$ ($b_t^{n,s}$) is the real supply of indexed (nominal) bonds issued by the government in period *t*, and m_t^s is the real money supply in circulation in period *t*.

The government sets the income tax rate on young consumers' endowment incomes τ^j (where $j \in (IT, PLT)$) in order to achieve a long run target level of real government spending of $E(g_t) = g^*$ and controls the money supply in the economy via money supply rules.[18] The total bond supply $b_t^s \equiv b_t^{i,s} + b_t^{n,s}$ is set to ensure that the marginal utility of consumption in youth is equated with the expected marginal utility of consumption in old age, or $c_{t,Y}^{-\delta} = E_t(c_{t+1,O}^{-\delta})$. This policy ensures perfect consumption-smoothing in expected terms for each generation, thereby increasing lifetime utility as in the standard OLG model in which government bonds are 'net wealth' (Barro, 1974).[19] [20] Individual bond supplies are constrained to be non-negative, so that $b_t^{i,s} \geq 0$ and $b_t^{n,s} \geq 0$ for all *t*.

The division of the total bond supply between indexed and nominal bonds, as defined by the indexation share $a \in [0,1]$, is chosen by the government to maximise social welfare, taking into account consumers' first-order conditions, the money supply rule in place, and the necessity of meeting the long run government spending target. The optimal indexation problem is dealt with formally in Section 4.

2.4 Monetary Policy

The major difference between IT and PLT is that the former implies base-level drift in the price level, whilst the latter prevents base-level drift. To allow for this difference, the 30-year (i.e. one period) money supply rules under IT and PLT are derived from a *yearly horizon*. With this approach, equilibrium inflation in the model reflects the presence of base-level drift

[18] Hence the government has risk-neutral preferences over government spending.

[19] The 'net wealth' result was first demonstrated formally by Barro, but he argues against government bonds being net wealth because introducing a bequest motive into the OLG model resurrects the Ricardian equivalence proposition.

[20] In a model without uncertainty, the government can set the total bond supply so that all generations enjoy perfect consumption smoothing *ex post*, thereby maximising lifetime utility for all generations. See Minford and Peel (2002) for a simple example.

under IT, and its absence under PLT. Since the derivation of 30-year money supply rules from yearly ones is long-winded, the details are presented in Appendix B.

Given the 30-year horizon embedded in the model, monetary policy does not respond directly to output or productivity. Furthermore, since the government can commit to money supply rules, no time-inconsistency or credibility issues arise in relation to monetary policy. The money supply rules given below are stated in terms of the nominal money supply (which is non-stationary), but the money supply is converted back into real terms in order to solve the model in Dynare++.

The IT money supply rule

The nominal money supply rule under IT takes the following form:

$$\ln(M_t^{s,IT} / M_{t-1}^{s,IT}) = 30 \times \pi + \sum_{i=1}^{30} \varepsilon_{i,t} + \ln(c_{t,Y} / c_{t-1,Y}) \tag{25}$$

where π is the annual inflation target and the $\varepsilon_{i,t}$ s are Gaussian white noise money supply innovations in year *i* of period *t* with variance σ^2.

Notice that the 30-year money supply innovation is simply the sum of the yearly money supply innovations that accumulate in each period due to base-level drift. In the absence of money supply innovations, Equation (25) implies perfect stabilisation of inflation at the inflation target.

Money market equilibrium (i.e. $M_t^d = M_t^s$, where $M_t^d = P_t m_t^d$) implies that inflation under IT is given by[21]

$$\pi_t^{IT} = 30 \times \pi + \sum_{i-1}^{30} \varepsilon_{i,t} \tag{26}$$

Therefore, expected inflation is equal to the 30-year inflation target, and the inflation variance is thirty times the yearly money supply innovation variance:

$$E_t \pi_{t+1}^{IT} = 30 \times \pi \tag{27}$$

$$\text{var}(\pi_t^{IT}) = 30\sigma^2 \tag{28}$$

Intuitively, expected inflation is equal to the inflation target because the government makes a fully credible commitment to an IT money supply rule. The inflation variance is thirty times the yearly innovation variance because of base-level drift: money supply innovations cause

[21] To arrive at this expression, take the first difference of the natural log of (nominal) money demand and use the approximation $\pi_t \approx \ln P_t - \ln P_{t-1}$. Then set money demand equal to money supply and solve for inflation.

inflation to deviate from target in each year, and over 30 years these innovations accumulate, with each one adding to long-term inflation uncertainty.

The PLT money supply rule

The nominal money supply rule under PLT is given by

$$\ln(M_t^{s,PLT} / M_{t-1}^{s,PLT}) = \ln(P_t^* / P_{t-1}^*) + \varepsilon_{30,t} - \varepsilon_{30,t-1} + \ln(c_{t,Y} / c_{t-1,Y}) \tag{29}$$

where P_t^* is the target price level and $\varepsilon_{30,t}$ is the money supply innovation in year 30 of period *t*.

In the absence of money supply innovations, Equation (29) implies perfect stabilisation of the price level at target. However, the price level will deviate from its target value when there are money supply innovations. The presence of a lagged money supply innovation in Equation (29) reflects the response of the PLT money supply rule to the price-level deviation in the previous period – a response that is necessary to return the price level to its target path.

It assumed that the target log price level under PLT increases at the target rate of inflation under IT: [22]

$$\ln P_t^* = p_0 + (30 \times \pi)t \tag{30}$$

where p_0 is the initial target price level.

The money supply rule in Equation (30) can therefore be written as follows:

$$\ln(M_t^{s,PLT} / M_{t-1}^{s,PLT}) = 30 \times \pi + \varepsilon_{30,t} - \varepsilon_{30,t-1} + \ln(c_{t,Y} / c_{t-1,Y}) \tag{31}$$

In contrast to the IT case, the 30-year money supply rule contains only two yearly money supply innovations, and these are spaced apart by 30 years. The reasoning is as follows: innovations that occur in years 1-29 are offset in the following year in order to bring the price level back to its target path. For instance, a shock in year 29 will be offset in year 30, the last year of the current period. However, the innovation in year 30 of each period cannot be offset until year 1 of the next period. Hence the innovations $\varepsilon_{30,t}$ and $\varepsilon_{30,t-1}$ enter the money supply rule. The first is the innovation in year 30 of the current period, and the second is the innovation from year 30 of the previous period (which is then offset in year 1 of the current period).

[22] The rate of inflation implied by the target price path is assumed to be equal to the inflation target to ensure direct comparability of IT and PLT. With this assumption, IT and PLT are identical in the absence of money supply innovations and PLT can be interpreted as 'average inflation targeting'.

Money market equilibrium implies that inflation under PLT is given by

$$\pi_t^{PLT} = 30 \times \pi + \varepsilon_{30,t} - \varepsilon_{30,t-1} \tag{32}$$

Hence expected inflation is state-contingent, and the 30-year inflation variance is two times the yearly innovation variance:

$$E_t \pi_{t+1}^{PLT} = 30 \times \pi - \varepsilon_{30,t-1} \tag{33}$$

$$\text{var}(\pi_t^{PLT}) = 2\sigma^2 \tag{34}$$

Both of these results have been discussed in the PLT literature (e.g. Svensson, 1999; Minford, 2004). First, expected inflation varies because past deviations from the target price path are subsequently offset, and rational agents take this into account when forming their inflation expectations. Second, the 30-year inflation variance is 15 times lower under PLT since inflation depends on only 2 yearly money supply innovations, compared to 30 under IT. The reason is that yearly deviations from the inflation target are offset and hence do not accumulate to increase long-term inflation uncertainty.

In order to make the differences between IT and PLT concrete, Panels (a) and (b) of Figure 1 report impulse responses of inflation to a period-*t* money supply innovation. As the yearly money supply innovation variance has not yet been calibrated, the innovation was normalised to one in the IT case and scaled accordingly in the PLT case. The differences between IT and PLT are clear: the initial impact is somewhat smaller under PLT because of the lower (30-year) money supply innovation variance; and the inflationary shock is reversed in the following period under PLT but is treated as a bygone under IT.

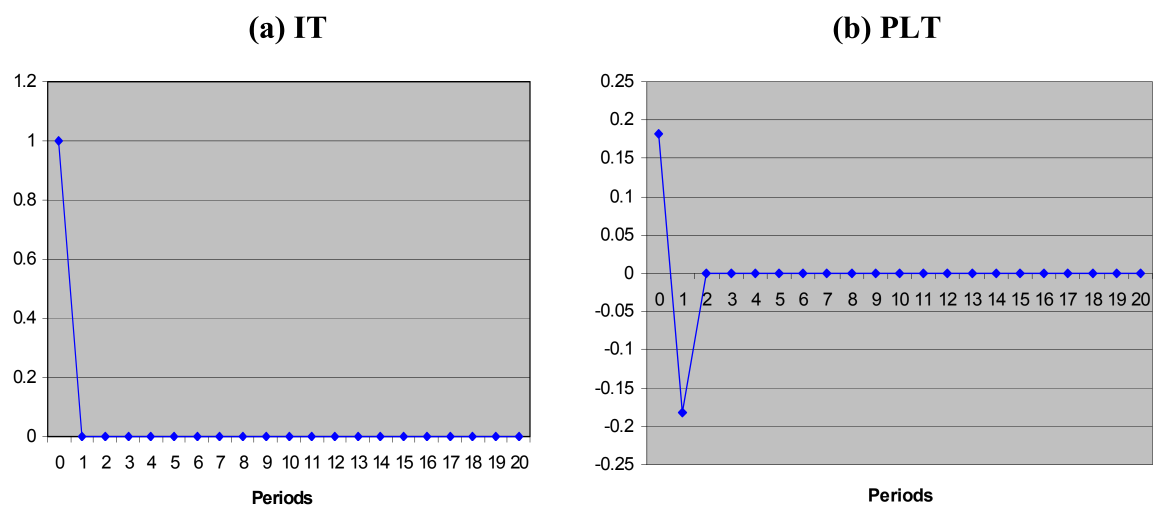

Fig. 1 – Inflation impulse responses to a money supply innovation

Finally, the biased inflation rate to which indexed bonds are linked is given by an exogenous process that has the same functional form as true inflation. In particular, the long run mean is set equal to the 30-year inflation target, and inflation responds only to current innovations under IT but to current and past innovations under PLT. As a result, the 30-year variance for biased inflation is also 15 times lower under PLT than IT.

The biased inflation rate used for indexation is given by

$$\left.\begin{aligned} \pi_t^{ind,IT} &= 30 \times \pi + \sum_{i=1}^{30} \varepsilon_{i,t}^{ind} && \text{under IT} \\ \pi_t^{ind,PLT} &= 30 \times \pi + \varepsilon_{30,t}^{ind} - \varepsilon_{30,t-1}^{ind} && \text{under PLT} \end{aligned}\right\} \tag{35}$$

where $\varepsilon_{i,t}^{ind} \sim N\left(0, \sigma_{ind}^2\right)$, and σ_{ind}^2 is the yearly innovation variance to biased inflation.

The $\varepsilon_{i,t}^{ind}$ s are assumed to be serially-uncorrelated. They are, however, contemporaneously cross-correlated with innovations to true inflation, with the strength of the correlation reflecting the extent of indexation bias. Both the cross-correlation between innovations and the innovation variances for true and biased inflation are estimated using UK data (see Section 3).

2.5 Social welfare

The government maximises the unconditional expectation of social welfare – that is, the average across all possible histories of shocks (see Damjanovic, Damjanovic and Nolan, 2008). The unconditional welfare criterion was first proposed by Taylor (1979) and has been used in numerous papers in the monetary policy literature since, including Rotemberg and Woodford (1998), Clarida, Gali and Gertler (1999), and Schmitt-Grohé and Uribe (2007).[23]

Given consumers' lifetime utility function (see Equation (1)) and the utility of the initial old, average lifetime utility across T generations is given by

$$\begin{aligned} U^T &= \frac{1}{T}\left[u_{1,O}(c_{1,O}) + \sum_{t=1}^{T} u_t\right] \\ &= \frac{1}{T}\left[u_{1,O}(c_{1,O}) + \sum_{t=1}^{T-1}\left(u_{t,Y}(c_{t,Y}) + E_t u_{t+1,O}(c_{t+1,O})\right) + u_{T,Y}(c_{T,Y})\right] \\ &= \frac{1}{T}\left[\sum_{t=1}^{T} u_{t,Y}(c_{t,Y}) + u_{1,O}(c_{1,O}) + \sum_{t=1}^{T-1} E_t u_{t+1,O}(c_{t+1,O})\right] \end{aligned} \tag{36}$$

[23] Examples of OLG models in which monetary policy is evaluated using an unconditional social welfare criterion include Brazier, Harrison, King and Yates (2006) and Kryvtsov, Shukayev and Ueberfeldt (2007).

Social welfare is defined as the unconditional expectation of this expression, or

$$U^{society} \equiv E\left[U^T\right] = \frac{1}{T} E\left[\sum_{t=1}^{T} \left(u_{t,Y}(c_{t,Y}) + u_{t,O}(c_{t,O})\right)\right] \tag{37}$$

2.6 Steady state and market-clearing conditions

The model's deterministic steady state and market-clearing conditions are presented in Appendix C, and Appendix D gives a full model listing.

3. Model calibration

3.1 Money supply rules and biased inflation

In order to make the money supply rules operational, the yearly inflation target and money supply innovation variance were estimated using UK inflation over the IT period. The stochastic process for biased inflation was calibrated in the same way. The Retail Prices Index excluding mortgage interest payments (RPIX) was chosen as the measure of 'true' inflation and the Retail Prices Index (RPI) as the 'biased' measure, with the sample period running from 1997Q3 to 2010Q2.[24] The RPIX was chosen as the measure of true inflation because it excludes mortgage interest payments, which are not faced by the majority of pensioners in the UK (Leicester, O'Dea and Oldfield, 2008). It also includes council tax and housing costs, both of which are relatively more important costs for pensioners that are excluded from the Consumer Prices Index (CPI). Given that indexed bonds in the UK are linked to the Retail Prices Index (RPI), the stochastic process for biased inflation was calibrated using the RPI.

Although the inflation target in the UK was changed from 2.5 per cent for the RPIX to a 2 per cent target for the CPI in December 2003, the adjustment was based on historical experience with the intention of ensuring that there was no material change in monetary policy strategy (King, 2004).[25] As such, this event was not treated as a structural break in the sample. In concordance with this treatment, the Quandt-Andrews and Chow breakpoint tests were unable to reject the null hypothesis of no breakpoint. Figure 2 shows quarterly RPI and RPIX inflation over the sample period. The RPI and RPIX track each other well, but there are some non-trivial deviations in the first half of the sample period and also in the last eight quarters.

[24] The Bank of England was assigned an inflation target soon after 'Black Wednesday' in 1992, but was not given full operational independence until May 1997.
[25] The main argument cited in favour of the shift to the CPI was international comparability.

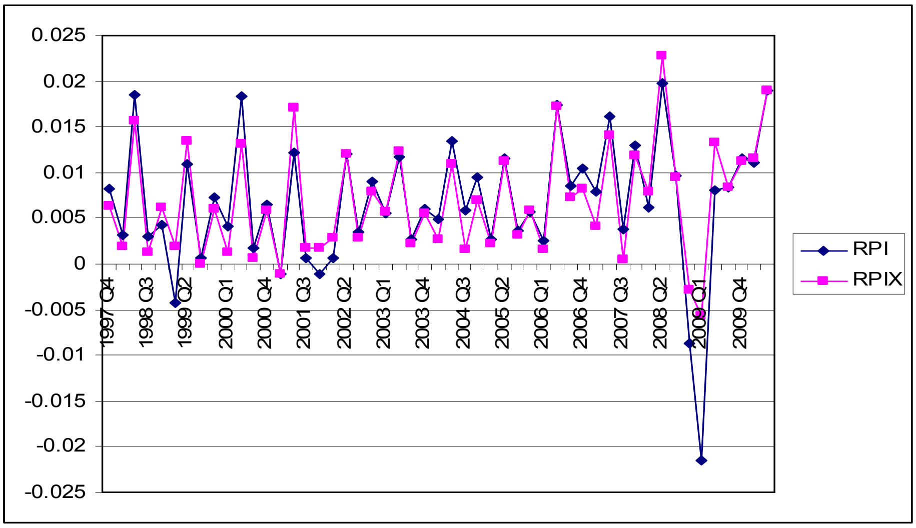

Fig. 2 – RPI and RPIX over the sample period

The following regression was estimated at a quarterly frequency q:

$$\pi_q^l = c + \varepsilon_q^l \tag{38}$$

where π_q^l, $l \in \{RPIX, RPI\}$, is inflation in quarter q and ε_q^l is the regression residual to inflation.

The estimation results from this regression and test statistics are shown in Table 1. The estimate for constant term c gives the mean quarterly rate of inflation over the sample period. The value of 0.007 for both the RPIX and RPI implies mean annual inflation of 0.028 ($= 4 \times 0.007$), or 2.8 per cent, which is close to the annual RPIX target of 2.5 per cent that was the focus of UK monetary policy from March 1997 to December 2003. The difference between mean quarterly inflation and the quarterly rate implied by the annual target of 2.5 per cent was not statistically significant at the 5 per cent level, so the yearly inflation target was set at $\pi = 0.025$, or 2.5 per cent per year.

The variances of yearly innovations to the money supply and biased inflation were estimated using the regression residuals. In particular, based on the estimated quarterly innovation variances, yearly innovation variances were calculated under the assumption that there is a unit root in the price level – as is implied Dickey-Fuller unit root tests on the RPIX and RPI (see Table 1).[26] [27] These yearly variances were used to calibrate the money supply rules and the stochastic processes for biased inflation, as shown in Table 2. Notably, the null hypothesis that the RPIX innovation is normally distributed could not be rejected by the Jarque-Bera test (see penultimate row of Table 1). However, normality of the RPI innovation was rejected.

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[26] An intercept and trend were included in the test regression.

[27] PLT is assumed to offset inflationary shocks at a yearly horizon. Hence, the quarterly (but not yearly) price level should follow a random walk.

Table 1 – RPIX and RPI regression results, 1997Q3-2010Q2

Parameter/test	RPIX	RPI
c (s.e.)	0.007 (0.001)	0.007 (0.001)
Quarterly inflation standard deviation, $sd(\varepsilon_q^l)$	0.0060	0.0073
Implied yearly standard deviation	0.012 $\left(\sqrt{4} \times 0.006\right)$	0.015 $\left(\sqrt{4} \times 0.0073\right)$
Dickey-Fuller unit-root test on log price index (prob. value)	-1.706 (0.73)	-1.380 (0.85)
Jarque-Bera test on ε_q^l (prob. value)	1.837 (0.40)	30.58 (0.00)
RPI-RPIX inflation correlation, i.e. $corr(\pi_q^{RPI}, \pi_q^{RPIX}) = corr(\varepsilon_q^{RPI}, \varepsilon_q^{RPIX})$	0.89	

RPI and RPIX inflation are strongly positively correlated, with a correlation coefficient of 0.89. This correlation was taken as the contemporaneous correlation between innovations to true inflation and biased inflation, and was therefore used as a basis for calibrating the covariance between innovations to actual and biased inflation (see the final row of Table 2).[28] Overall, the results suggest a relatively small indexation bias: innovations to RPIX and RPI inflation are closely correlated and their variances are similar. For completeness, Table 2 lists calibrated values in the money supply rules and the stochastic processes for biased inflation.

Table 2 – Calibrated values in money supply rules and biased inflation

Model parameter	Role in the model	Calibrated value
$30 \times \pi$	Inflation target over 30 years	0.75
$\mathrm{var}(\varepsilon_{i,t})$	Yearly money supply innovation variance	1.44×10^{-4}
$\mathrm{var}(\varepsilon_{i,t}^{ind})$	Yearly biased inflation innovation variance	2.13×10^{-4}
$\mathrm{cov}(\varepsilon_{i,t}, \varepsilon_{i,t}^{ind})$	Yearly covariance between innovations	1.56×10^{-4} $\left(= 0.89 \times sd(\varepsilon_{i,t}) \times sd(\varepsilon_{i,t}^{ind})\right)$

3.2 Calibrating stochastic productivity

When calibrating the stochastic process for productivity, it is important to take into account the 30-year horizon of the OLG model. In order to do so, a typical quarterly calibration from the real business cycle (RBC) literature is extended over a 30-year horizon.

[28] Note that for correlated random variables X and Y, $\mathrm{cov}(X,Y) = corr(X,Y) \times sd(X) \times sd(Y)$.

Consider an AR(1) process for log productivity at a quarterly horizon *q*:

$$\ln A_q = (1-\rho_q)\ln A_{q,mean} + \rho_q \ln A_{q-1} + e_q \qquad 0 < \rho_q < 1 \tag{39}$$

where e_q is an IID-Normal productivity innovation with mean zero and variance σ_q^2.

By substituting repeatedly for lagged productivity terms, productivity over a 30-year (i.e. 120-quarter) horizon can be obtained as follows:

$$\ln A_q = (1-\rho_q^{120})\ln A_{q,mean} + \rho_q^{120} \ln A_{q-120} + \sum_{j=0}^{119} \rho_q^j e_{q-j} \tag{40}$$

Therefore, productivity in any given period *t* is given by

$$\ln A_t = (1-\rho)\ln A_{mean} + \rho \ln A_{t-1} + e_t \tag{41}$$

where $\ln A_{mean} \equiv (1-\rho_q^{120})\ln A_{q,mean}/(1-\rho)$, $\rho \equiv \rho_q^{120}$ and $e_t \equiv \sum_{j=0}^{119} \rho_q^j e_{q-j}$.

Equation (41) is used as basis for calibrating the stochastic productivity process in the OLG model. Many papers in the RBC literature (e.g. King and Rebelo, 2000) use quarterly calibrations of productivity in which the autoregressive parameter is slightly below one and the innovation standard deviation is less than 0.008. Gavin, Keen and Pakko (2009) set the innovation standard deviation at 0.005, consistent with the lower volatility of output in the 'Great Moderation' period. The calibration here uses the same standard deviation as their paper (i.e. $\sigma_q = 0.005$), and an autocorrelation coefficient of $\rho_q = 0.996$ that is consistent with the bulk of the RBC literature. Consequently, the calibrated 30-year productivity process has a first-order autocorrelation of 0.618 and an innovation standard deviation of 0.04398.[29] Finally, steady-state productivity was set equal to 0.75. The calibration of the productivity process is summarised in Table 3.

Table 3 – Calibration of stochastic productivity

Model parameter	Role in the model	Calibrated value
ρ	Persistence in productivity at a 30-year horizon	0.618
σ_e	Productivity innovation standard deviation (at a 30-year horizon)	0.04398
A_{mean}	Steady-state level of productivity	0.75

[29] In particular, $\sigma_e = \sqrt{(1-0.996^{240})(1-0.996^2)^{-1}} \times 0.005$. This expression uses the fact that $\mathrm{var}(e_t) = (1+\rho_q^2+\rho_q^4+...+\rho_q^{238})\sigma_q^2 = (1-\rho_q^{240})\sigma_q^2/(1-\rho_q^2)$.

3.3 Calibrating the indexation lag

The random innovation v_t is used to proxy for the impact of an indexation lag on the *ex post* real return on indexed bonds. In order to calibrate its standard deviation, a number of points should be considered. First, given the specification of the real return on indexed bonds, it should have the same units as the term $(1+\pi_t^{ind})/(1+\pi_t)$ which it appears in brackets alongside. Hence v_t is interpreted as the impact of the indexation lag, in percentage points, on the inflation-indexed component of an indexed bond. Second, the variance of v_t should reflect the volatility of the inflation rate to which indexed bonds are linked, measured over a horizon defined by the length of the indexation lag. Given that the indexation lag on the majority of outstanding index-linked gilts in the UK is 8 months, this variance was estimated using the rate of RPI inflation measured over a three-quarter horizon.[30]

The following regression was estimated:

$$\Delta\pi_{q:q-3} = c + \varepsilon_{q:q-3} \tag{42}$$

where $\Delta\pi_{q:q-3}$ is the differential between RPI inflation in quarter q and RPI inflation in quarter q-3, and $\varepsilon_{q:q-3}$ is the regression residual (and the empirical counterpart to v_t).

Table 4 – RPI indexation lag regression results

Parameter/test	Result
c (s.e.)	0.0002 (0.0017)
Standard deviation of the residual, i.e. $sd(\varepsilon_{q:q-3})$	0.0121
Jarque-Bera test on $\varepsilon_{q:q-3}$ (prob. value)	10.80 (0.005)

Table 4 shows the regression results. The constant term is insignificant – offering support to the assumption that v_t is mean zero – but the Jarque-Bera test marginally rejects the assumption that the residual is normally-distributed at the 1 per cent significance level. The regression residual standard deviation is equal to 0.0121, or 1.2 per cent. The variance for v_t was therefore calibrated at $\sigma_v^2 = 0.0121^2 = 0.000146$.

[30] Using 3 quarters (9 months) meant that the same quarterly RPI data could be used in estimation throughout the paper.

3.4 Model parameter calibration

Table 5 summarises the calibration of the remaining parameters in the model. The CIA coefficient θ, the share of money holdings in consumption when young, was calibrated to roughly match UK data. In particular, notes and coins in circulation amount to 3 to 4 per cent of annual UK GDP over the past decade (ONS Financial Statistics, 2010), with total household consumption accounting for around 65 per cent of GDP (ONS Blue Book, 2010). On this basis, θ was set equal to 0.10.

The coefficient of relative risk aversion was set equal to 3. This value lies in the mid-range of calibrated values considered plausible in the literature. It is higher than a standard RBC calibration of unity, but somewhat lower than values typically used in the open-economy literature that attempts to match exchange rate volatility and persistence (e.g. Chari, Kehoe and McGrattan, 2002; Kocherlakota and Pistaferri, 2007), or the literature that attempts to resolve asset-pricing puzzles by appealing to relatively levels of high risk aversion (e.g. Bansal and Yaron, 2004). The value of 3 is close to the estimate of 3.5 reached by Tödter (2008) using US stock return data from 1926 to 2002.

The long run target level of government spending was set equal to 20 per cent of steady-state GDP. This long run target is similar to the level of UK government expenditure as a percentage of GDP (ONS Blue Book, 2010). The parameter α, the elasticity of output with respect to capital, was set at 0.375, which lies in the mid-range of calibrated values in the RBC literature.[31] Young consumers' endowment income was set so that steady-state GDP (or aggregate income) was equal to 2. As a result, consumption levels, government spending and asset holdings can be interpreted as proportions of GDP after division by 2.

Table 5 –Calibration of model parameters

Model parameter	Role in the model	Calibrated value
θ	Proportion of consumption when young held as money balances	0.10
δ	Coefficient of relative risk aversion	3
ϖ	Endowment income of young consumers	1.641
g^*	Long run government spending target	0.40
α	Elasticity of output produced in old age with respect to capital	0.375

[31] Papers in the RBC literature typically combine capital and labour in a Cobb-Douglas production function, so that α is the share of capital income in output. Many papers in this literature set $\alpha \approx 1/3$, but some papers use somewhat higher calibrations (e.g. Perli and Sakelleris, 1998; King, Plosser and Rebelo, 1988).

3.5 Deterministic steady-state

The deterministic steady-state values of key variables under the baseline calibration are reported in Table 6. [32]

Table 6 – Key variables at steady-state

Model variable	Steady-state value
$c_{t,Y}$	0.730
$c_{t,O}$	0.730
$b_t^d (= b_t^s)$	0.343
$m_t^d (= m_t^s)$	0.073
k_t	0.140
π_t	0.75

Note: Steady-state GDP is equal to 2

Aggregate consumption accounts for 73 per cent of steady-state GDP (which approximately matches developed economies) and is split equally between consumption by young and old generations. Money holdings are 3.7 per cent of GDP (i.e. 0.073/2), which is similar to the UK share of notes and coins in GDP over the past decade. Steady-state inflation is equal to the 30-year inflation target of 0.75, or a 75 per cent increase in prices over a 30-year horizon. Since there is full depreciation of capital, investment is given by the level of capital holdings. Steady-state investment is thus 7 per cent of GDP, with the remaining 20 per cent of GDP accounted for by government spending. Steady-state capital holdings are 41 per cent of bond holdings, which is similar to the average ratio of investment to government bonds in the UK over the past decade (ONS Blue Book, 2010; ONS Financial Statistics, 2010).

3.6 Solving the model

The model is solved using second-order approximation in Dynare++ (Julliard, 2001). It is important to use non-linear approximation methods for two reasons. First, linearizing the model would remove covariance risk, thus eliminating risk-premia in the returns on indexed and nominal bonds. Second, when comparing social welfare across alternative monetary policy regimes, linear approximation can easily lead to an inaccurate ranking of policies because it neglects the impact of second-order terms on the stochastic means of endogenous variables in the model. For instance, Kim and Kim (2003) present a simple two-agent economy in which linearization leads to the spurious conclusion that autarky delivers higher social welfare than full risk sharing. In the model at hand, spurious conclusions regarding optimal indexation could be drawn if linear approximation methods were employed.

[32] Since steady-state returns are equalised across indexed and nominal bonds, the deterministic steady-state is invariant to the indexation share a.

In order to solve for the optimal indexation shares under IT and PLT, a method akin to grid search was employed. The model was first simulated for indexation shares from 0 to 1, with social welfare recorded for each simulation. Based on preliminary analysis, the searchable range was then narrowed down in order to identify the optimal indexation shares exactly. For each indexation share, social welfare was calculated using 1000 simulations of 5000 periods each, with the simulation seed randomly chosen in each simulation.

4. Optimal indexation

The government chooses the mix of indexed and nominal bonds to maximise social welfare subject to its budget constraint, its long run target for government spending, consumers' first-order conditions for optimal saving, the money supply rule, and the model's other equilibrium conditions. Consequently, the policy being studied is a Ramsey policy: the government can commit and takes into account the optimal reactions of consumers when making its optimal indexation choice.

Social welfare is given by Equation (37), but that expression is cumbersome to work with analytically. Hence consider the following equation:

$$U^{society} = E\big(u(c_{t,Y}) + u(c_{t,O})\big) \tag{43}$$

This expression arises if the utility of the initial old is excluded from social welfare (or if the limit of Equation (37) is taken as the number of generations *T* tends to infinity). The reason is that all generations, except the initial old, are *ex ante* homogenous and hence have the same long run average level of utility. Given that the model is solved using a second-order perturbation method, the optimal indexation problem faced by the government can then be formulated using a second-order Taylor expansion of Equation (43) around unconditional mean consumption levels.

In particular, the optimal indexation problem can be stated as follows:

$$\max_{a} U^{society} \approx \left(\frac{\left(Ec_{t,Y}\right)^{1-\delta} + \left(Ec_{t,O}\right)^{1-\delta}}{1-\delta} \right) - \frac{1}{2}\left(\left|U^{society}_{c_Y c_Y}\right| \mathrm{var}(c_{t,Y}) + \left|U^{society}_{c_O c_O}\right| \mathrm{var}(c_{t,O}) \right) \tag{44}$$

subject to

$E(g_t) = g*$ and $g_t = \tau^j\, \varpi + m_t^s - r_t^m m_{t-1}^s + b_t^{i,s} - r_t^i b_{t-1}^{i,s} + b_t^{n,s} - r_t^n b_{t-1}^{n,s}$;
the IT or PLT money supply rule; and the other model equations listed in Appendix D;

where $U^{society}_{c_Y c_Y} = -\delta\left(Ec_{t,Y}\right)^{-(1+\delta)}$ $\left(U^{society}_{c_O c_O} = -\delta\left(Ec_{t,O}\right)^{-(1+\delta)}\right)$ is the second derivative of the social welfare function with respect to $c_{t,Y}$ ($c_{t,O}$), evaluated at $Ec_{t,Y}$ ($Ec_{t,O}$).

In order to gain some intuition for the factors driving optimal indexation, consider a first-order Taylor expansion of the first term on the right hand side of Equation (44) around the deterministic steady-state.[33]

Using this approximation results in the following social welfare criterion:

$$\max_{a} U^{society} \approx c_O^{1-\delta}\left(Ec_{t,Y} + Ec_{t,O} + \frac{2\delta \times c_O}{1-\delta} \right) - \frac{1}{2}\left(\left| U_{c_Y c_Y}^{society} \right| \mathrm{var}(c_{t,Y}) + \left| U_{c_O c_O}^{society} \right| \mathrm{var}(c_{t,O}) \right) \qquad (45)$$

where $Ec_{t,Y} + Ec_{t,O}$, the average level of aggregate consumption, is approximately invariant to the indexation share.

The invariance of the average level of aggregate consumption to the indexation share can be seen by summing the budget constraints of young and old generations to arrive at the goods market-clearing condition, or $c_{t,Y} + c_{t,O} + k_t + g_t = \varpi + A_t k_{t-1}^{\alpha}$. Taking the unconditional expectations operator through this condition gives $Ec_{t,Y} + Ec_{t,O} = \varpi - g^* + E\left(A_t k_{t-1}^{\alpha} - k_t\right)$, which is approximately invariant to the indexation share a since capital is a pure real asset whose return is uncorrelated with real bond returns. The key to the invariance result is that the government must meet its long run government spending target of g^*, so that the long run average level of government spending Eg_t is independent of the indexation share.

For instance, supposing that nominal bonds have a higher risk premium than indexed bonds (as is the case under the baseline calibration), a marginal reduction in indexation will, *ceteris paribus*, increase average consumption by old generations. However, a marginal reduction in indexation will reduce average government spending (by exactly the same amount), because the average cost of issuing government debt has risen. Therefore, in order to meet the long run government spending target g^*, the tax rate on young generations would need to increase. The consequent reduction in average consumption by young generations will offset the increase that accrues to old generations, so that the average level of aggregate consumption is unchanged.[34]

Given that the first term on the right hand side of Equation (45) is invariant to the indexation share, the government is effectively minimising a loss function in consumption volatility – that is, optimal indexation is driven by a consumption insurance motive.

[33] This approximation is employed only to provide intuition for the results that follow. When the model was simulated the full expression for social welfare was evaluated.

[34] Of course, a similar argument applies for an increase in indexation.

Thus, using notation employed by Woodford (2003), the optimal indexation problem can be expressed as follows:

$$\min_{a} U^{society} \approx \frac{1}{2}\left(\left|U^{society}_{c_Y c_Y}\right| \mathrm{var}(c_{t,Y}) + \left|U^{society}_{c_O c_O}\right| \mathrm{var}(c_{t,O})\right) \quad + \quad t.i.p. \tag{46}$$

where $t.i.p.$ stands for 'terms independent of policy'.

The key term in Equation (46) is given by $(1/2)\left|U^{society}_{c_O c_O}\right| \mathrm{var}(c_{t,O})$. The reasoning is as follows. First, the consumption variance across young generations is somewhat smaller than the consumption variance across old generations, since consumption volatility for the young arises only indirectly through small portfolio substitution effects due to fluctuations in assets' expected returns, whilst consumption by the old is impacted directly by *ex post* shocks to asset returns. Second, consumption volatility across old generations depends directly on the indexation share, whilst the indexation share has only a small indirect impact on consumption volatility across young generations.[35]

In Appendix E it is shown that under reasonably general conditions (which are satisfied by the baseline calibration), the driving term $(1/2)\left|U^{society}_{c_O c_O}\right| \mathrm{var}(c_{t,O})$ will be minimised by choosing the indexation share so that the consumption variance across old generations is (approximately) minimised, or

$$\frac{\partial\, \mathrm{var}(c_{t,O})}{\partial a} \approx 0 \tag{47}$$

Equation (47) can therefore be used to derive an approximate expression for the optimal indexation share.

The key terms in the consumption variance across old generations are given by[36]

$$\mathrm{var}(c_{t,O}) \approx \mathrm{var}(y_{t,O}) + \mathrm{var}(r_t b^s_{t-1}) + \mathrm{var}(r^m_t m^d_{t-1}) + 2\,\mathrm{cov}(r_t b^s_{t-1}, r^m_t m^d_{t-1}) \tag{48}$$

where $y_{t,O} \equiv A_t k^{\alpha}_{t-1}$ is output produced by old generations and $r_t \equiv a r^i_t + (1-a) r^n_t$ is the overall bond portfolio return.

[35] In fact, consumption volatility across young generations is independent of the indexation share under IT because expected inflation is constant. Under PLT, however, expected inflation is time-varying, so the young undertake substitution between money and non-monetary assets. As a result, consumption volatility across young generations is not independent of the indexation share, though numerically the impact of the indexation share on volatility is trivial.

[36] Note that since capital is a claim to real output, its real return is uncorrelated with indexed and nominal bond returns, and the real return on money balances.

Differentiating Equation (48) with respect to the indexation share and setting the result equal to zero gives an approximate expression for the optimal indexation share a^*.

Appendix F shows that this expression is as follows:

$$a^* \approx \frac{\mathrm{var}(r_t^n b_{t-1}^s) + \mathrm{cov}(r_t^n b_{t-1}^s, r_t^m m_{t-1}^d) - \mathrm{cov}(r_t^i b_{t-1}^s, r_t^m m_{t-1}^d) - \mathrm{cov}(r_t^i b_{t-1}^s, r_t^n b_{t-1}^s)}{\mathrm{var}(r_t^i b_{t-1}^s) + \mathrm{var}(r_t^n b_{t-1}^s) - 2\,\mathrm{cov}(r_t^i b_{t-1}^s, r_t^n b_{t-1}^s)} \qquad (49)$$

Intuitively, the optimal indexation share is (i) increasing in the return variance on nominal bonds, (ii) decreasing in the return variance on indexed bonds, and (iii) increasing (decreasing) in the extent to which the real returns on nominal (indexed) bonds and money covary. Notice also that full indexation will not, in general, be optimal (unless real returns on indexed and nominal bonds are themselves strongly positively correlated), since holding nominal bonds will diversity consumption risk in old age. All four of these predictions are confirmed by the simulation results in the next section.

5. Simulation results

Panel (a) of Figure 3 shows how social welfare varies with the indexation share under IT, and Panel (b) shows the corresponding variation in consumption volatility across old generations. An indexation share of 76 per cent maximises social welfare. As expected, the optimal indexation share is driven by consumption volatility across old generations. A relatively high indexation share is optimal under IT because long-term inflation volatility is substantial, so that nominal bonds are a relatively poor store of value compared to indexed bonds. Indeed, the simulated real return variance on nominal bonds is almost two-and-a-half times as high as on indexed bonds (see Table 8).

Despite this substantial return volatility differential, it is optimal for consumers to hold some nominal bonds in their portfolios for diversification reasons, as there is only a weak correlation between real returns on indexed and nominal bonds.[37] The result that optimal indexation is relatively high under IT is consistent with the findings of Minford, Nowell and Webb (2003) and Amano, Ambler and Ireland (2007) in the context of optimal wage indexation.

[37] In fact, there is a slightly negative correlation between bond returns. The reason is that unanticipated inflation will tend to reduce the real return on nominal bonds but increase the real return on indexed bonds, since biased inflation will typically 'overshoot' true inflation due to its higher variance.

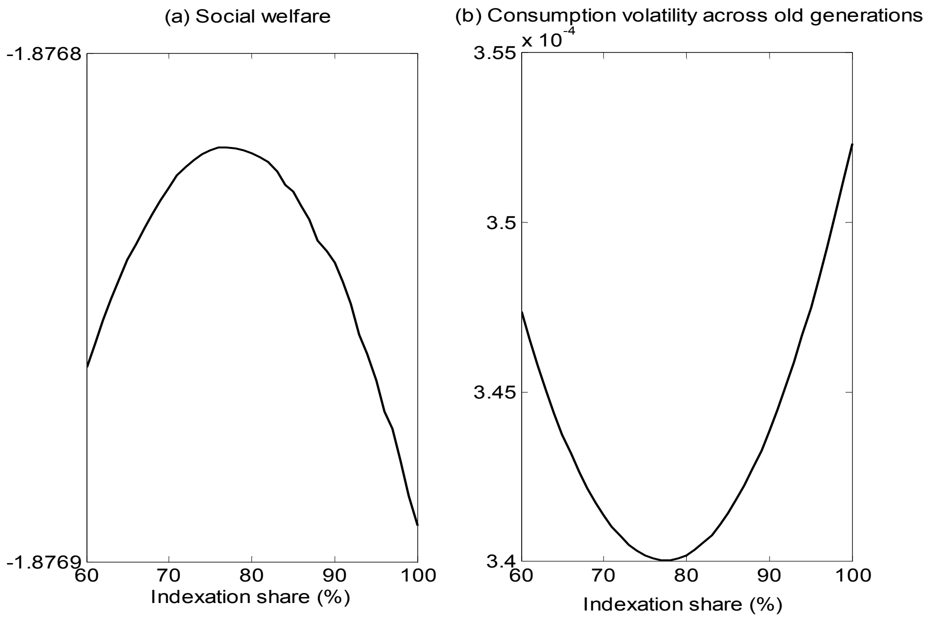

Fig. 3 – Optimal indexation under IT

Figure 4 shows the impact of the indexation share on social welfare and consumption volatility under PLT. Optimal indexation is somewhat lower than under IT at 26 per cent (see Panel (a)), indicating that it is optimal for almost three-quarters of bond holdings to be in the form of nominal bonds. The reasoning for this result can be seen from Panel (b), which shows that consumption volatility across old generations is minimised at an indexation share close to 26 per cent. Hence nominal bonds become a relatively better store of value than in the IT case, enabling old generations' real consumption risk to be reduced by substitution towards nominal bonds and away from indexed bonds.

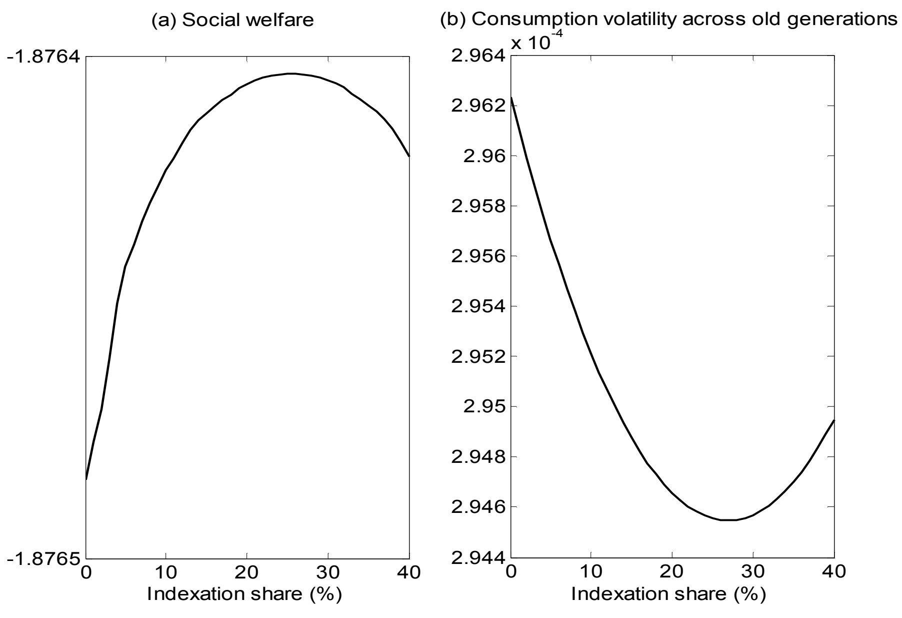

Fig. 4 – Optimal indexation under PLT

The IT and PLT optimal indexation results are summarised in Table 7, which reports the indexation shares that maximise social welfare and also the indexation shares at which consumption volatility across old generations is minimised. The optimal indexation shares do not coincide exactly with the ones that minimise consumption volatility because the result that minimising consumption volatility maximises social welfare holds only as an approximation (which was invoked analytically to provide intuition). These simulation results suggest, however, that the approximation is a reasonably good one.

Table 7 – Optimal indexation shares under IT and PLT

Monetary policy	**Optimal indexation share**	**Indexation share at which** $\mathrm{var}(c_{t,O})$ **is minimised**
IT	76%	77%
PLT	26%	27%

There are two factors driving the substantial reduction in optimal indexation under PLT. First, the reduction in (long-term) inflation uncertainty under PLT benefits holders of nominal bonds disproportionately, because real return volatility on nominal bonds is driven purely by inflation risk, whereas indexed bonds are also impacted by the *indexation lag* – a source of real return volatility that remains unchanged under PLT. As a result, real return volatility falls more sharply on nominal bonds than indexed bonds, giving consumers a consumption-insurance incentive to substitute towards nominal bonds. The approximate formula for the indexation share in Equation (49) confirms that a reduction in the nominal-to-indexed return variance ratio will reduce optimal indexation.[38] The marked reduction in this ratio under PLT can be seen clearly from Table 8. The standard deviation on indexed bonds is approximately halved from 230 basis points under IT to 120 under PLT, but the standard deviation of the return on nominal bonds falls to less than one-fifth of its IT value, from 360 to 70 basis points.

Table 8 – Real return standard deviations

Monetary policy	**Indexed Bonds**	**Nominal bonds**
IT	230	360
PLT	120	70

Notes: Figures are in basis points and are rounded

Second, the lower indexation share under PLT is driven by *indexation bias*. The reasoning is as follows. With consumers holding both indexed and nominal bonds in their portfolios for diversification reasons, covariance risk between bond returns and the real return on money affects consumption volatility in old age, and hence optimal indexation. Nominal bonds perform relatively better under PLT in terms of this covariance risk, because the real return

[38] The easiest way to obtain this result is to divide the numerator and denominator of Equation (49) by $\mathrm{var}(r_t^n b_{t-1}^s)$ and then solve for the optimal share of nominal bonds, $1-a*$.

on nominal bonds is strongly positively correlated with the real return on money balances under IT, but only weakly so under PLT. There is thus an additional diversification motive for holding nominal bonds under PLT: nominal bonds will tend to pay a relatively low return when the real return on money is high, therefore stabilising consumption in old age.

Table 9 – Real return correlations between assets

Correlation	r^i, r^n	r^i, r^m	r^i, r^k	r^n, r^k	r^n, r^m
IT	-0.11	-0.11	0	0	1.00
PLT	-0.03	-0.04	0	0	0.08

Note: Figures are rounded to two decimal places

Table 9 shows that the nominal bonds-money real return correlation falls substantially from a perfect positive correlation of 1 under IT to only 0.08 under PLT, whilst other return correlations are largely unchanged.[39] That a lower correlation between the real return on nominal bonds and the real return on money will reduce optimal indexation can be seen formally from the approximate expression in Equation (49). The lower correlation under PLT can be explained by the fact that expected inflation becomes time-varying. This has the effect of 'diluting' the positive correlation between the real return on nominal bonds and the real return on money balances, because nominal bonds provide insurance against *anticipated fluctuations* in inflation, whilst money balances do not.[40]

In order to investigate the source of the reduction in optimal indexation under PLT, the indexation differential was decomposed into indexation bias and indexation lag components:

$$a^{IT} - a^{PLT} = \underbrace{\Delta a^{IT} - \Delta a^{PLT}}_{\text{Indexation lag diff.}} + \underbrace{a^{IT}_{\text{no lag}} - a^{PLT}_{\text{no lag}}}_{\text{Indexation bias diff.}} \tag{50}$$

where, for $j \in (IT, PLT)$, a^j is the fully optimal indexation share; $a^j_{\text{no lag}}$ is the optimal indexation share in the absence of lagged indexation; and $\Delta a^j \equiv a^j - a^j_{\text{no lag}}$ is the differential in optimal indexation due solely to the presence of an indexation lag.

Table 10 reports the results from the decomposition of IT-PLT indexation differential.

[39] There is a perfect positive correlation under IT because expected inflation is constant, so that a nominal bond is equivalent to money plus a constant nominal 'mark-up' for expected inflation.
[40] Under PLT, a nominal bond is equivalent to money plus a time-varying nominal 'mark-up' that captures fluctuations in expected inflation. Since innovations to inflation are serially-uncorrelated, expected inflation need not be strongly correlated with actual inflation – hence explaining the relatively weak positive correlation.

Table 10 – Decomposition of the indexation differential

Indexation share/differential	IT	PLT
Optimal indexation share, a^j	76	26
IT-PLT differential, $a^{IT} - a^{PLT}$	50	
Optimal with biased indexation only, $a^j_{\text{no lag}}$	82	76
IT-PLT indexation bias differential	6	
IT-PLT indexation lag differential	44	

Only 6 per cent of the indexation differential between IT and PLT is due to indexation bias, with the remaining 44 per cent due to lagged indexation. The impact of the indexation lag is substantial because long-term inflation risk is reduced by an order of magnitude under PLT. This has the effect of reducing return volatility on nominal bonds disproportionately because real return risk on nominal bonds results solely from inflation risk, whilst indexed bonds are also subject to risk resulting from the indexation lag (which is unchanged under PLT). On the other hand, the role played by indexation bias in the IT-PLT indexation differential is relatively small. Intuitively, as money holdings are small, the reduced correlation between nominal bond returns and the return on money balances has relatively little impact on consumption volatility, or, therefore, on the optimal indexation share. Moreover, the extent of indexation bias captured in the model is relatively small since true and biased inflation are strongly positively correlated and have similar variances.

6. Conclusions and policy implications

An important finding from past literature is that optimal indexation of wage and debt contracts depends crucially on the extent of nominal volatility over the contracting horizon. Motivated by this literature, this paper investigated the link between optimal indexation of government bonds and monetary policy, with a particular focus on long-term nominal volatility. In order to do so, the paper set out an overlapping generations model in which each period lasts 30 years and young consumers save for old age using indexed and nominal government bonds whose real payoffs are vulnerable to long-term inflation risk. Consumers also hold money and productive capital in their portfolios. The key feature of the model is that indexation of government bonds is determined endogenously in response to monetary policy as part of an optimal commitment Ramsey policy implemented by the government.

In terms of monetary policy, two policies with drastically different long-term implications were considered: inflation targeting and price-level targeting. Monetary policy is characterised by a large degree of long-term nominal uncertainty under inflation targeting due to base level drift, but nominal uncertainty is minimal under price-level targeting, because the price level is trend-stationary. Past literature has reached the conclusion that optimal indexation increases with nominal volatility (e.g. Gray, 1976; Meh, Quadrini and Terajima, 2009). This literature therefore predicts that optimal indexation will be lower under price-level targeting than inflation targeting, as found in the context of wage indexation by Minford, Nowell and Webb (2003) and Amano, Ambler and Ireland (2007). The primary aim of this paper was determine whether this same conclusion holds with respect to optimal indexation of government debt. Crucially, and in contrast to past literature, the model set out in this paper requires that the government explicitly finance issuance of bonds when (i) indexed bonds are imperfectly linked to inflation, and (ii) bond risk-premia arise endogenously in response to monetary policy.

The main finding from the model was that optimal bond indexation is substantially lower under price-level targeting. The reasoning runs as follows. Long-term inflation uncertainty is substantial under inflation targeting because of base-level drift: even if the central bank misses its inflation target by only a small percentage in each year, these misses can accumulate and become large after 30 years. Consequently, nominal bonds are a relatively poor store of value as compared to indexed bonds. Optimal indexation is therefore relatively high under inflation targeting at 76 per cent. Under price-level targeting, by contrast, past deviations from the inflation target do not accumulate over time, so that long-term inflation volatility falls by an order of magnitude. Nominal bonds therefore become a relatively better store of value as compared to indexed bonds, enabling consumption risk in old age to be reduced via substitution towards nominal bonds and away from indexed bonds. Given the substantial reduction in inflation risk, optimal indexation falls rather sharply to 26 per cent. Crucially for these results, the model captures two imperfections in indexation that are calibrated to the UK case, *viz.* indexation bias and lagged indexation.

In order to investigate the source of the reduction in optimal indexation under price-level targeting, the indexation differential was decomposed into 'indexation bias' and 'indexation lag' components. This decomposition revealed that most of the reduction in optimal indexation under price-level targeting was due to the indexation lag, and only around one-tenth due to indexation bias. The substantial reduction due to the indexation lag arises because its impact on indexed bonds' real return volatility is common to both monetary policy regimes; hence, when inflation risk is reduced under price-level targeting, real return volatility on nominal bonds falls disproportionately compared to indexed bonds, giving consumers an incentive to substitute from indexed bonds to nominal bonds. The reduction due to the indexation lag is substantial because inflation volatility falls markedly under price-level targeting as a consequence of the 30-year contracting horizon captured by the model.

Indexation bias plays only a small role in reducing optimal indexation. Indeed, the impact from indexation bias arises only indirectly through money – the other nominal asset in consumers' portfolios. Specifically, price-level targeting dilutes the positive correlation between the real return on nominal bonds and the real return on money because, in contrast to inflation targeting, expected inflation fluctuates in response to past shocks, and nominal bonds protect consumers' savings from *anticipated inflation fluctuations* whilst money does not. As a result, consumption risk in old age is reduced when consumers hold a higher proportion of nominal bonds in their portfolios. The small reduction in optimal indexation due to indexation bias can be explained by the fact that consumers' holdings of money balances are relatively low, so that fluctuations in the real return on money are not a major source of consumption risk.

The result that optimal indexation of government bonds can vary substantially with monetary policy has a number of important policy implications. Firstly, models that do not endogenise indexation of government debt in response to monetary policy are vulnerable to the Lucas critique and may give rise to seriously misleading results. As such, endogenising indexation in a microfounded way is an important task for future research (Ambler, 2009). Secondly, as the results in this paper arise from comparing inflation targeting and price-level targeting policies, there are potential implications for central banks like the Bank of Canada that are considering switching from inflation targeting to price-level targeting in the future. Most notably, endogenising indexation in quantitative models used for policy analysis may make price-level targeting more or less desirable vis-à-vis inflation targeting. Lastly, the extent to which governments are prepared to issue indexed bonds may be influenced considerably by monetary policy. The model in this paper can explain why it might be optimal for governments to issue both indexed and nominal government debt, but cannot explain fully the current low levels of indexation in developed economies.

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Appendix A – Proof that the CIA constraint binds with strict equality when $R_t > 1$

In this appendix it is shown that the CIA constraint is strictly binding if the gross monetary return on a nominal bond exceeds the gross return on money of one. The Lagrangian from the main text can be used to derive this result, with allowance made for the possibility that the CIA constraint may not hold with strict equality. Consequently, the Lagrangian will additionally give rise to Kuhn-Tucker conditions relating to the Lagrange multiplier on the CIA constraint.

Proposition: The CIA constraint binds with strict equality when $R_t > 1$

Proof.
From the main text the first-order conditions for indexed bonds, nominal bonds and money holdings are as follows:

$$c_{t,Y}^{-\delta} = E_t\left(c_{t+1,O}^{-\delta} r_{t+1}^{i}\right) + \theta\mu_t \tag{A1}$$

$$c_{t,Y}^{-\delta} = E_t\left(c_{t+1,O}^{-\delta} r_{t+1}^{n}\right) + \theta\mu_t \tag{A2}$$

$$c_{t,Y}^{-\delta} = E_t\left(c_{t+1,O}^{-\delta} r_{t+1}^{m}\right) + (1+\theta)\mu_t \tag{A3}$$

where μ_t is the Lagrange multiplier on the CIA constraint.

The Kuhn-Tucker conditions associated with μ_t are summarised in the following equation:

$$\left\{\mu_t \geq 0 \quad \text{and} \quad \mu_t(m_t - \theta c_{t,Y}) = 0\right\} \tag{A4}$$

where the second equation, the complementary slackness condition, implies that the CIA constraint will be strictly binding if $\mu_t > 0$ for all *t*.

Using Equations (A2) and (A3), the Lagrange multiplier μ_t will strictly positive iff

$$E_t\left(c_{t+1,O}^{-\delta} r_{t+1}^{n}\right) > E_t\left(c_{t+1,O}^{-\delta} r_{t+1}^{m}\right) \qquad \forall t \tag{A5}$$

Substitution of the real return on nominal bonds into Equation (A5) gives

$$R_t \times E_t\left(c_{t+1,O}^{-\delta} r_{t+1}^{m}\right) > E_t\left(c_{t+1,O}^{-\delta} r_{t+1}^{m}\right) \qquad \forall t \tag{A6}$$

Dividing Inequality (A6) by $E_t\left(c_{t+1,O}^{-\delta} r_{t+1}^{m}\right)$ yields the following necessary condition for the nominal interest rate:

$$R_t > 1, \qquad \forall t \tag{A7}$$

Finally, notice that $E_t\left(c_{t+1,O}^{-\delta} r_{t+1}^{n}\right) = E_t\left(c_{t+1,O}^{-\delta} r_{t+1}^{i}\right)$, such that Inequality (A7) ensures that holding indexed bonds is also strictly preferred to holding money, i.e.

$$E_t\left(c_{t+1,O}^{-\delta} r_{t+1}^{i}\right) > E_t\left(c_{t+1,O}^{-\delta} r_{t+1}^{m}\right), \quad \forall t \quad \text{iff} \quad R_t > 1 \qquad \textbf{Q.E.D.} \tag{A8}$$

Appendix B – Derivations of the IT and PLT money supply rules from a yearly horizon

Inflation targeting

Consider the following *yearly IT money supply rule* that aims at a constant inflation target and is subject to exogenous monetary innovations in each year *i*:

$$\ln M_i^{s,IT} = \ln M_{i-1}^{s,IT} + \pi + \varepsilon_i + \ln c_{i,Y} - \ln c_{i-1,Y} \tag{B1}$$

where π is the yearly inflation target and ε_i is an IID-normal money supply innovation with mean zero and variance σ^2.

To derive a 30-year money supply rule from this yearly specification, substitute repeatedly for the lagged money supply term on the right-hand side of Equation (B1) until the following 30-year money supply rule is reached:

$$\ln M_i^{s,IT} = \ln M_{i-30}^{s,IT} + 30\times\pi + \sum_{j=0}^{29}\varepsilon_{i-j} + \ln c_{i,Y} - \ln c_{i-30,Y} \tag{B2}$$

This equation states that the 30-year growth rate of the nominal money supply has three components: a 30-year inflation target $30\times\pi$; the sum-total of 30 separate yearly money supply innovations; and the 30-year rate of growth of consumption by the young.

Given that each period *t* lasts 30 years, Equation (B2) implies that the money supply rule in any period *t* can be represented in the following form:

$$\ln M_t^{s,IT} - \ln M_{t-1}^{s,IT} = 30\times\pi + \sum_{i=1}^{30}\varepsilon_{i,t} + \ln c_{t,Y} - \ln c_{t-1,Y} \tag{B3}$$

where, for ease of exposition, the Gaussian white noise money supply innovations have been re-indexed from years 1 to 30, and the time subscript indicates that all 30 innovations belong to period *t*.

Price-level targeting

Consider the following *yearly PLT money supply rule* that aims at a target yearly (log) price level of $\ln P_i^* = p_0 + \pi\times i$ in each year *i*:

$$\ln M_i^{s,PLT} = \ln M_{i-1}^{s,PLT} + \pi + \varepsilon_i - \varepsilon_{i-1} + \ln c_{i,Y} - \ln c_{i-1,Y} \tag{B4}$$

where π is the constant yearly inflation target that is consistent with the target price path, and ε_i is an IID-Normal innovation with mean zero and variance σ^2 (exactly as in the IT case).

To derive the implied money supply rule over a 30-year horizon, substitute for the lagged money supply term on the right hand side of Equation (B4) until the following expression is reached:

$$\ln M_i^{s,PLT} = \ln M_{i-30}^{s,PLT} + 30 \times \pi + \varepsilon_i - \varepsilon_{i-30} + \ln c_{i,Y} - \ln c_{i-30,Y} \tag{B5}$$

Given that each period *t* lasts 30 years, Equation (B5) implies a period *t* money supply rule of the form

$$\ln M_t^{s,IT} - M_{t-1}^{s,IT} = 30 \times \pi + \varepsilon_{30,t} - \varepsilon_{30,t-1} + \ln c_{t,Y} - \ln c_{t-1,Y} \tag{B6}$$

where again the money supply innovations have been indexed to reflect the year in which they occur, and the *t* subscript indicates the period to which the innovations belong.

Appendix C: Deterministic steady-state and market-clearing conditions

Deterministic steady state[41]

$$c_Y + b^{i,d} + b^{n,d} + m^d + k = \varpi(1-\tau^j), \qquad \text{for } j \in (IT, PLT) \tag{C1}$$

$$c_O = Ak^{\alpha} + r^i b^{i,d} + r^n b^{n,d} + r^m m \tag{C2}$$

$$R = (1+\pi^{ss})r^n \tag{C3}$$

$$r^i = \frac{(1+\pi^{ind})}{(1+\pi^{ss})}r \tag{C4}$$

$$r^m = 1/(1+\pi^{ss}) \tag{C5}$$

$$g = \tau^j \varpi + (1-r^i)b^{i,s} + (1-r^n)b^{n,s} + m^d \pi^{ss}/(1+\pi^{ss}) \tag{C6}$$

$$\pi^{ss} = 30 \times \pi \tag{C7}$$

$$\pi^{ind} = \pi^{ss} \tag{C8}$$

$$m^s = m^d \tag{C9}$$

$$m^d = \theta c_Y \tag{C10}$$

$$b^{i,d} = b^{i,s} = a \times b^s \tag{C11}$$

$$b^{n,d} = b^{n,s} = (1-a) \times b^s \tag{C12}$$

$$b^d = b^{i,d} + b^{n,d} = b^s = \frac{\varpi(1-\tau^j) - (1+r^m)m^d - (1+Ak^{\alpha})k}{1+r} \tag{C13}$$

(from bond supply rule, $c_Y^{-\delta} = c_O^{-\delta}$ in SS)

$$c_Y^{-\delta} = c_O^{-\delta}\left((1+\theta)r^i - \frac{\theta}{1+\pi^{ss}}\right) \tag{C14}$$

$$c_Y^{-\delta} = c_O^{-\delta}\left((1+\theta)r^n - \frac{\theta}{1+\pi^{ss}}\right) \tag{C15}$$

$$r^i = r^n = r = \frac{1+\theta+\pi^{ss}}{(1+\pi^{ss})(1+\theta)} \quad \text{(implied by the previous two equations)} \tag{C16}$$

$$\alpha A k^{\alpha-1} = r^n = r^i \qquad \text{(from the Euler equations for capital and bonds)} \tag{C17}$$

$$A = A_{mean} \tag{C18}$$

[41] π^{ss} denotes the steady-state rate of inflation.

Market-clearing conditions

A monetary equilibrium in the OLG economy is a set of allocations $\left\{c_{t,Y}, c_{t,O}, b_t^{i,d}, b_t^{i,s}, b_t^{n,d}, b_t^{n,s}, k_t, m_t^d, m_t^s, g_t, \pi_t, r_t^n, R_t, r_t^m, \tau^j\right\}_{t=1}^T$ with the following properties for all *t*:

(1) Allocations $\left\{c_{t,Y}, c_{t+1,O}, b_t^{i,d}, b_t^{n,d}, k_t, m_t^d\right\}_{t=1}^T$ solve the maximisation problem of a young consumer born at time *t*;

(2) The goods, money and bond markets clear:

$$\varpi + A_t k_{t-1}^{\alpha} = c_{t,Y} + c_{t,O} + g_t + k_t \tag{C19}$$

$$m_t^d = m_t^s \tag{C20}$$

$$b_t^{i,d} = b_t^{i,s} \tag{C21}$$

$$b_t^{n,d} = b_t^{n,s} \tag{C22}$$

(3) The government's budget constraint and long run government spending target are satisfied:

$$g_t = \tau^j \ \varpi + m_t^s - r_t^m m_{t-1}^s + b_t^{i,s} - r_t^i b_{t-1}^{i,s} + b_t^{n,s} - r_t^n b_{t-1}^{n,s} \tag{C23}$$

$$E(g_t) = g^* \tag{C24}$$

Appendix D: Model listing

$$u_t = \frac{c_{t,Y}^{1-\delta}}{1-\delta} + E_t \frac{c_{t+1,O}^{1-\delta}}{1-\delta}$$ Lifetime utility of generation t (D1)

$$c_{t,Y} + b_t^{n,d} + b_t^{i,d} + m_t^d + k_t = \varpi(1-\tau^j)$$ Budget constraint when young (D2)

$$c_{t+1,O} = A_{t+1}k_t^{\alpha} + r_{t+1}^i b_t^{i,d} + r_{t+1}^n b_t^{n,d} + r_{t+1}^m m_t^d$$ Budget constraint when old (D3)

$$\ln A_t = (1-\rho)\ln A_{mean} + \rho \ln A_{t-1} + e_t$$ Productivity (D4)

$$m_t^d = \theta c_{t,Y}$$ CIA constraint (D5)

$$r_{t+1}^m \equiv 1/(1+\pi_{t+1})$$ Real return on money balances (D6)

$$r_{t+1}^n = R_t r_{t+1}^m$$ Real return on nominal bonds (D7)

$$r_{t+1}^i = r_t \times \left[\frac{(1+\pi_{t+1}^{ind})}{(1+\pi_{t+1})} + v_{t+1} \right]$$ Real return on indexed bonds (D8)

$$c_{t,Y}^{-\delta} = E_t\left(c_{t+1,O}^{-\delta}\left((1+\theta)r_{t+1}^n - \theta r_{t+1}^m\right)\right)$$ Euler equation for nominal bonds (D9)

$$c_{t,Y}^{-\delta} = E_t\left(c_{t+1,O}^{-\delta}\left((1+\theta)r_{t+1}^i - \theta r_{t+1}^m\right)\right)$$ Euler equation for indexed bonds (D10)

$$c_{t,Y}^{-\delta} = E_t\left(c_{t+1,O}^{-\delta}\left((1+\theta)\alpha A_{t+1}k_t^{\alpha-1} - \theta r_{t+1}^m\right)\right)$$ Euler equation for capital (D11)

$$g_t = \tau^j\ \varpi + m_t^s - r_t^m m_{t-1}^s + b_t^{i,s} - r_t^i b_{t-1}^{i,s} + b_t^{n,s} - r_t^n b_{t-1}^{n,s}$$ Government budget constraint (D12)

$$E(g_t) = g^*$$ Long run government spending target (implies τ^j) (D13)

$$\ln(m_t^{s,IT} / m_{t-1}^{s,IT}) = 30 \times \pi + \sum_{i=1}^{30} \varepsilon_{i,t} + \ln(c_{t,Y} / c_{t-1,Y}) - \pi_t$$ IT money rule (D14)

$$\ln(m_t^{s,PLT} / m_{t-1}^{s,PLT}) = 30 \times \pi + \varepsilon_{30,t} - \varepsilon_{30,t-1} + \ln(c_{t,Y} / c_{t-1,Y}) - \pi_t$$ PLT money rule (D15)

$$U^{society} = \frac{1}{T} E\left[\sum_{t=1}^{T}\left(u_{t,Y}(c_{t,Y}) + u_{t,O}(c_{t,O})\right)\right]$$ Social welfare (D16)

$$\pi_t^{ind,IT} = 30 \times \pi + \sum_{i=1}^{30} \varepsilon_{i,t}^{ind}$$ Inflation rate to which index bonds are linked (IT) (D17)

$$\pi_t^{ind,PLT} = 30 \times \pi + \varepsilon_{30,t}^{ind} - \varepsilon_{30,t-1}^{ind}$$ Inflation to which index bonds are linked (PLT) (D18)

$c_{t,Y}^{-\delta} = E_t\left(c_{t+1}^{-\delta}\right)$	Total bond supply rule	(D19)
$b_t^s = b_t^{i,s} + b_t^{n,s}$	Total bond supply definition	(D20)
$m_t^d = m_t^s$	Money market equilibrium	(D21)
$b_t^{n,d} = b_t^{n,s} = (1-a)b_t^s$	Market-clearing in nominal bonds	(D22)
$b_t^{i,d} = b_t^{i,s} = ab_t^s$	Market-clearing in indexed bonds	(D23)
$c_{t,Y} + c_{t,O} + k_t + g_t = \varpi + A_t k_{t-1}^{\alpha}$	Market-clearing in goods	(D24)

Appendix E – An approximate first-order condition for the optimal indexation problem

In the main text it is argued that the indexation share a will be chosen to approximately solve the following problem:

$$\min_{a} \frac{1}{2}\left|U^{society}_{c_O c_O}\right| \mathrm{var}(c_{t,O}) \qquad + \qquad t.i.p. \tag{E1}$$

where $\left|U^{society}_{c_O c_O}\right| \equiv \delta\left(Ec_{t,O}\right)^{-(1+\delta)}$ is the absolute value of the second derivative of the social welfare function with respect to $c_{t,O}$ (evaluated at its unconditional mean) and $t.i.p.$ stands for 'terms independent of policy'.

The first-order condition for this problem is given by

$$\frac{1}{2}\left(Ec_{t,O}\right)^{-2(1+\delta)}\left[\frac{\partial\,\mathrm{var}(c_{t,O})}{\partial a}\times\left(Ec_{t,O}\right)^{1+\delta} - (1+\delta)\left(Ec_{t,O}\right)^{\delta}\,\mathrm{var}(c_{t,O})\times\frac{\partial Ec_{t,O}}{\partial a}\right] = 0 \tag{E2}$$

Hence the optimal indexation share will satisfy the following equation:

$$\frac{\partial\,\mathrm{var}(c_{t,O})}{\partial a}\times\left(Ec_{t,O}\right)^{1+\delta} = (1+\delta)\left(Ec_{t,O}\right)^{\delta}\,\mathrm{var}(c_{t,O})\times\frac{\partial Ec_{t,O}}{\partial a} \tag{E3}$$

Rearranging Equation (E3) for $\partial\,\mathrm{var}(c_{t,O})/\partial a$ yields

$$\frac{\partial\,\mathrm{var}(c_{t,O})}{\partial a} = (1+\delta)\left[\frac{\mathrm{var}(c_{t,O})}{Ec_{t,O}}\right]\times\frac{\partial Ec_{t,O}}{\partial a} \tag{E4}$$

Hence iff $\mathrm{var}(c_{t,O})/Ec_{t,O} \approx 0$, the first-order condition for the optimal indexation share can be approximated by[42]

$$\frac{\partial\,\mathrm{var}(c_{t,O})}{\partial a} \approx 0 \tag{E5}$$

Thus, if the variance of consumption is small relative to its mean, the optimal indexation share will approximately minimise consumption volatility across old generations.

[42] Note that $\partial Ec_{t,O}/\partial a = E[(r^i_t - r^n_t)b^s_{t-1}]$ will be close to zero given that indexed and nominal bonds are priced to give equivalent expected utility.

Appendix F – Deriving an approximate expression for the optimal indexation share

In this appendix, an approximate expression for the optimal indexation share is derived by minimising the consumption variance across old generations. As noted in the main text, the key terms in this variance are given by

$$\operatorname{var}(c_{t,O}) \approx \operatorname{var}(y_{t,O}) + \operatorname{var}(r_t b_{t-1}^s) + \operatorname{var}(r_t^m m_{t-1}^d) + 2\operatorname{cov}(r_t b_{t-1}^s, r_t^m m_{t-1}^d) \tag{F1}$$

where $y_{t,O} \equiv A_t k_{t-1}^{\alpha}$ is output produced by old generations and $r_t \equiv a r_t^i + (1-a) r_t^n$ is the overall return on old generations' bond portfolios.

Since $r_t \equiv a r_t^i + (1-a) r_t^n$, Equation (F1) can be written in terms of the indexation share as follows:

$$\begin{aligned}\operatorname{var}(c_{t,O}) \approx{}& \operatorname{var}(y_{t,O}) + a^2 \operatorname{var}(r_t^i b_{t-1}^s) + (1-a)^2 \operatorname{var}(r_t^n b_{t-1}^s) + \operatorname{var}(r_t^m m_{t-1}^d) \\ & 2a(1-a)\operatorname{cov}(r_t^i b_{t-1}^s, r_t^n b_{t-1}^s) + 2a\operatorname{cov}(r_t^i b_{t-1}^s, r_t^m m_{t-1}^d) + 2(1-a)\operatorname{cov}(r_t^n b_{t-1}^s, r_t^m m_{t-1}^d)\end{aligned} \tag{F2}$$

Minimising Equation (F2) with respect to the indexation share a gives following first-order condition:

$$\frac{\partial \operatorname{var}(c_{t,O})}{\partial a} = 2\begin{bmatrix} a\operatorname{var}(r_t^i b_{t-1}^s) - (1-a)\operatorname{var}(r_t^n b_{t-1}^s) + (1-2a)\operatorname{cov}(r_t^i b_{t-1}^s, r_t^n b_{t-1}^s) \\ +\operatorname{cov}(r_t^i b_{t-1}^s, r_t^m m_{t-1}^d) - \operatorname{cov}(r_t^n b_{t-1}^s, r_t^m m_{t-1}^d) \end{bmatrix} = 0 \tag{F3}$$

Solving Equation (F3) for the optimal indexation share a^* gives the expression reported in the main text, i.e.

$$a^* \approx \frac{\operatorname{var}(r_t^n b_{t-1}^s) + \operatorname{cov}(r_t^n b_{t-1}^s, r_t^m m_{t-1}^d) - \operatorname{cov}(r_t^i b_{t-1}^s, r_t^m m_{t-1}^d) - \operatorname{cov}(r_t^i b_{t-1}^s, r_t^n b_{t-1}^s)}{\operatorname{var}(r_t^i b_{t-1}^s) + \operatorname{var}(r_t^n b_{t-1}^s) - 2\operatorname{cov}(r_t^i b_{t-1}^s, r_t^n b_{t-1}^s)} \tag{F4}$$