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Optimal Tax Policy and Wage Subsidy in an Imperfectly Competitive Economy

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Abstract:

In an imperfectly competitive economy with direct and indirect taxes, the first best wage subsidy overcompensates workers and provides the incentive to misreport working hours. We show that in the second best optimum where the government cannot use a wage subsidy, the optimal policy is to tax labour income at a zero rate. This policy is optimal because it minimizes the incentive to misreport working hours.

JEL Codes: D42, E62, H21, H30.

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1 Introduction

It is well known that in an imperfectly competitive economy the first best tax policy involves a lump sum tax and subsidies to the monopoly distorted returns to labour and capital (e.g. Judd (1997), Guo and Lansing (1999)). We consider two restrictions on this first best outcome. We assume that lump sum taxes are not available, which motivates the standard Ramsey (1927) problem. In addition, because wage subsidies are typically associated with misreporting of working hours, we consider a Ramsey problem that does not permit wage subsidy in the imperfectly competitive sector. We find that with these two restrictions, in a steady state and in the initial period, optimal labour income tax rate in the imperfectly competitive sector is zero.

In an imperfectly competitive economy, profit seeking investment results in over accumulation of capital and a suboptimal level of working hours. In order to push the private return to factors up to their socially optimal level, the best policy should be one that subsidizes the returns to these factors. This is one of the key findings of both Judd (1997) and Guo and Lansing (1999). However, if the government has a consumption tax in the scheme, the first best tax policy (which involves a subsidy to the monopoly distorted private returns to factors) overcompensates workers, which in turns leads to misreporting of working hours.

Typically, the disincentive effect of a wage subsidy is stronger if the government uses a consumption tax. This is because generally government spending is large relative to the difference between consumption and wage income. Expressed as a share of GDP, the average difference between consumption and wage income in the US is approximately 0.14, while average government expenditure’s share in GDP is approximately 0.23\(^2\). The present value of private and government consumption must equal the present value of wage and profit income plus the value of initial assets. Since the consumption-income difference is generally small relative to the government expenditure, a combination of wage subsidy and a consumption tax results in a very large tax/subsidy rate. A large wage subsidy is not good since it provides strong incentives to misreport the hours worked. In addition, a large consumption tax rate may well lead to unreported barters.

\(^2\)This is based on the 1947-2006 quarterly series of Gross Domestic Product (GDP), Personal Consumption Expenditure (PCEC), Government Expenditures (GCE) and Wages and Salary Accruals (WASCUR), from Federal Reserve Bank of St. Louis, Economic Data-FREDII.
The government can implement the first best policy as in Judd (1997) and Guo and Lansing (1999) by setting the consumption tax rate equal to zero only if it can implement a lump sum tax (or an equivalent). If there is no lump sum tax, the government faces the second best problem, and cannot set the consumption tax rate equal to zero. In this paper we consider a model where lump sum taxes or any equivalent of a lump sum tax are not available and the government cannot use a wage subsidy in the imperfectly competitive sector. We show that the second best policy in such a setting involves a zero labour income tax in the imperfectly competitive sector. This policy is optimal because it minimizes the incentive to misreport working hours, which in turns brings the working hours back to the socially optimal level3.

2 The Decentralized Economy

There are two production sectors: sector $y$ (the competitive sector) produces final goods (the numeraire), and sector $z$ (the monopoly sector) produces a continuum of intermediate goods. The technologies are:

\[ y_t = \left( \int_0^1 z_{jt}^{1-\sigma} \, dz \right)^{\frac{1}{1-\sigma}} n_t^{1-\nu}; \quad \nu \in (0, 1); \sigma \in (0, 1) \]  \hspace{1cm} (1)

\[ z_{jt} = k_{jt}^{\alpha} n_t^{1-\alpha}; \quad \alpha \in (0, 1) \]  \hspace{1cm} (2)

where $n$ is working time, $k$ is capital and $z_j$ is the level of intermediate good $j \in [0, 1]$. Let $p_j$ denote the relative price of intermediate good $j$. Firms in the monopoly sector exploit the demand for intermediate goods and make positive economic profits $\pi$. In a symmetric equilibrium, intermediate goods price, profits, and factor incomes are $p_t = \nu (y_t)^{1-\sigma} z_t^{-\sigma} (n_{yt})^{(1-\sigma)/(1-\nu)}$; $\pi_t = (\sigma \nu) y_t$; $w_{yt} n_{yt} = (1-\nu) y_t$; $w_{zt} n_{zt} = (1-\alpha) \nu (1-\sigma) y_t$, and $r_t k_t = \alpha \nu (1-\sigma) y_t$. The economy’s resource constraint is:

\[ 0 = k_t^{\alpha \nu} n_t^{(1-\sigma)n_{yt}^{1-\nu}} + (1-\delta) k_t - c_t - g_t - k_{t+1}; \quad \delta \in (0, 1) \]  \hspace{1cm} (3)

3Our study is also motivated by Coleman II (2000), who construct an optimal taxation problem in a perfectly competitive economy, and show that the US economy could attain maximum welfare gains from switching to Ramsey policy if the government is prohibited to use wage subsidy.
where $c_t$ is private consumption, and $g_t = \bar{g} > 0$ is exogenously given government expenditure. The government’s budget constraints (with symmetry) is:

$$0 = \tau_{ct} c_t + \tau_{yt} w_{yt} n_{yt} + \tau_{zt} w_{zt} n_{zt} + \tau_{kt} (r_t k_t + \kappa \pi_t) + R_t^{-1} b_{t+1} - b_t - g_t$$

where the tax rates are $\tau_{kt}, \kappa \tau_{kt}, \tau_{ct}, \tau_{st}, s \in \{y, z\}$, for capital income, profits (with $\kappa \in [0, 1]$), private consumption and labour income, respectively, and $R_t$ is rate of return on bonds $b_t$.

The households derive utility from consumption and leisure, and their utility maximization problem is:

$$\max_{\{c_t, n_{yt}, n_{zt}, k_{t+1}, b_{t+1}\}} \sum_{t=0}^{\infty} \beta^t u (c_t, 1 - n_{yt} - n_{zt})$$

s.t. 

$$0 = (1 - \tau_{yt}) w_{yt} n_{yt} + (1 - \tau_{zt}) w_{zt} n_{zt} + [(1 - \tau_{kt}) r_t + 1 - \delta] k_t + b_t + (1 - \kappa \tau_{kt}) \pi_t$$

$$- (1 + \tau_{ct}) c_t - k_{t+1} - R_t^{-1} b_{t+1}$$

where $\beta \in (0, 1)$, and $u(.)$ satisfies standard regularity assumptions. With $\xi_t \equiv (1 + \tau_{ct})^{-1}$, the symmetric equilibrium conditions are (3), (4), transversality conditions, prices and profit, and

$$- u_{n_{st}}(t) = u_c(t) \xi_t (1 - \tau_{st}) w_{st}; \quad s \in \{y, z\}$$

$$\frac{u_c(t) \xi_{t+1}}{u_c(t+1) \xi_t} = R_t = \beta \left[(1 - \tau_{kt+1}) r_{t+1} + 1 - \delta\right]$$

With $\bar{r} \equiv (1 - \tau_k) r$ and $\bar{w}_z \equiv (1 - \tau_z) w_z$, in the symmetric equilibrium cost of capital and labour in the monopoly sector are given by $\bar{r} = (1 - \sigma) (1 - \tau_k) MP_k$ and $\bar{w}_z = (1 - \sigma) (1 - \tau_z) MP_{nz}$. In order to push the private return to these factors up to their socially optimal levels, the first best policy should subsidize these income and use a lump sum tax to raise the revenue required to finance the government expenditure and the subsidies.
However, with consumption taxation in the scheme the first best policy that replicates the socially optimal outcome would actually *overcompensate* labour income in the monopoly sector. If the government can use a lump sum tax, the policy that generates the first best outcome includes a large lump sum tax, \( c = 0 \), \( y = z \), and \( k = \frac{1 - \sigma}{1 - \sigma} \). The first best capital subsidy pushes up the private return to capital such that \( e = \text{MP}_k \); but if \( \tau > 0 \), the wage subsidy in the monopoly sector is higher than what is required to push the private return up to the socially optimal return, i.e. with this subsidy in the scheme, wage in the monopoly sector is overcompensated.

### 3 The Ramsey Problem

Our assumption that lump sum taxes are not available motivates the standard optimal taxation problem, which we characterize as the Ramsey (1927) problem. We use the primal approach to derive the conditions that characterize the Ramsey allocation. Then we look for the taxes that can implement the second-best wedges. Given the current framework, in the standard Ramsey problem the government chooses the allocation \( \{c_t, n_{yt}, n_{zt}, k_{t+1}, b_{t+1}\}_{t=0}^\infty \) that maximizes welfare subject to the resource constraint (3), and an implementability constraint that ensures that the resulting taxes, allocations and prices are consistent with the decentralized equilibrium. The implementability constraint corresponding to the current model is:

\[
0 = \sum_{t=0}^\infty \beta^t [u_c(t)c_t + u_{ny}(t)n_{yt} + u_{nz}(t)n_{zt} - u_c(t)\xi_t(1 - \kappa\tau_{kt})\pi_t] - \Omega(c_0, n_{y0}, n_{z0}, \tau_{k0}, \xi_0) \tag{8}
\]

where using (6), (7) and equilibrium prices and profits,

\[
(1 - \kappa\tau_{kt})\pi_t = \begin{cases} 
\sigma \nu (1 - \kappa) k_t^{\alpha \nu} n_{zt}^{(1-\alpha)\nu} n_{yt}^{1-\nu} + k_t\kappa \sigma \frac{1}{\alpha(1-\sigma)} \left[ \frac{u_c(t-1)\xi_{t-1}}{\beta u_c(t)\xi_t} \right] - (1 - \delta) ; & \text{for } t \geq 1 \\
(1 - \kappa\tau_{k0}) \sigma \nu k_0^{\alpha \nu} n_{z0}^{(1-\alpha)\nu} n_{y0}^{1-\nu} ; & \text{for } t = 0
\end{cases}
\tag{9}
\]

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4The socially optimal outcome is a solution to the planner’s problem of choosing allocations to maximize welfare subject to (3). The first best policy is the policy that replicates the socially optimal allocations in the current setting.
and, $\Omega (c_0, n_{y0}, n_{z0}, \tau_{k0}, \xi_0) \equiv u_c (0) \xi_0 \left\{ (1 - \tau_{k0}) \alpha \nu (1 - \sigma) \frac{w_0}{k_0} + 1 - \delta \right\} k_0 + b_0$.

Since we assume that the government faces a restriction in using wage subsidy in the monopoly sector, we add a non-negative wage taxation constraint to the Ramsey problem. Substituting for equilibrium wage in (6) for the monopoly sector, we derive the constraint that characterizes non-negative wage taxation in the monopoly sector:

$$0 \leq \xi_t (1 - \alpha) \nu (1 - \sigma) k_t^{\alpha \nu} n_{zt}^{(1-\alpha)\nu-1} n_{yt}^{1-\nu} + \frac{u_{nz} (t)}{u_c (t)}$$

(10)

For fixed $g_t, \tau_{k0}, \tau_{c0}, k_0 > 0$, and $b_0$, the Ramsey problem is the government’s problem of choosing allocations and $\{\xi_t\}_{t=1}^\infty$ that maximizes welfare subject to constraints (3), (8), (9) and (10). Let $\eta \geq 0$, $\{\chi_t\}_{t=0}^\infty$ and $\{\mu_t\}_{t=0}^\infty$ denote the multipliers for (8) (with (9)), (3) and (10), respectively, and denote the Pseudo utility function as $G (c_t, n_{yt}, n_{zt}, k_t, \xi_t, \eta)$, such that

$$G(.) \equiv u (c_t, 1 - n_{yt} - n_{zt}) + \eta \left[ u_c (t) c_t + u_{ny} (t) n_{yt} + u_{nz} (t) n_{zt} - u_c (t) \xi_t (1 - \kappa_{kt}) \pi_t \right]$$

where $(1 - \kappa_{kt}) \pi_t$ is given by (9).

Assume that the instantaneous utility is separable in consumption and labour and linear in labour. The first order conditions for $t \geq 1$ corresponding to the Ramsey problem are:

$$0 = G_c (t) - \chi_t - \mu_t \frac{u_{nz} (t) u_{cc} (t)}{\{u_c (t)\}^2}$$

(11a)

$$0 = G_{ny} (t) + (1 - \nu) k_t^{\alpha \nu} n_{zt}^{(1-\alpha)\nu-1} n_{yt}^{1-\nu} \left[ \chi_t + \mu_t \xi_t \nu \frac{(1 - \alpha) (1 - \sigma)}{n_{zt}} \right]$$

(11b)

$$0 = G_{nz} (t) + \nu (1 - \alpha) k_t^{\alpha \nu} n_{zt}^{(1-\alpha)\nu-1} n_{yt}^{1-\nu} \left[ \chi_t + \mu_t \xi_t \frac{(1 - \sigma) [\nu (1 - \alpha) - 1]}{n_{zt}} \right]$$

(11c)

$$0 = \chi_t - \beta \left[ G_k (t+1) + \chi_{t+1} \left\{ \nu \alpha k_{t+1}^{\alpha \nu-1} n_{zt+1}^{(1-\alpha)\nu-1} n_{yt+1}^{1-\nu} + 1 - \delta \right\} + \mu_{t+1} \xi_{t+1} \left[ (1-\alpha)(1-\sigma) \nu^2 k_{t+1}^{\alpha \nu-1} n_{zt+1}^{(1-\alpha)\nu-1} n_{yt+1}^{1-\nu} \right] \right]$$

(11d)

$$0 = G_\xi (t) + \mu_t \nu (1 - \alpha) (1 - \sigma) k_t^{\alpha \nu} n_{zt}^{(1-\alpha)\nu-1} n_{yt}^{1-\nu}$$

(11e)
and the Kuhn-Tucker condition is:

\[ \xi_t (1 - \alpha) \nu (1 - \sigma) k_t^{\alpha \nu} n_z^{(1-\alpha)\nu-1} n_y^{1-\nu} + \frac{u_{n_2}(t)}{u_c(t)} \geq 0, \text{ with equality if } \mu_t > 0 \]  

(12)

which, together with (8), (9), (3) for all \( t \) and the first order conditions for \( t = 0 \) characterize the Ramsey equilibrium. Notice that the derivative of the Pseudo utility function with respect to \( \xi_t \) is:

\[ G_{\xi}(t) = -\eta \left[ \xi_t u_c(t) \frac{(1 - \kappa \tau_{kl}) \pi_t u_c(t) + u_c(t-2)}{\alpha (1-\sigma)\beta} \left\{ k_{t-1} \frac{u_c(t-1)}{u_c(t-1)\xi_t} - k_t \frac{u_c(t-1)\xi_{t-1}}{u_c(t)\xi_t} \right\} \right] \]  

(13)

where \((1 - \kappa \tau_{kl}) \pi_t\) is given by (9). In a steady state, (13) and (11e) together imply:

\[ 0 = \eta [(1 - \kappa \tau_{k}) \pi u_c] - \mu (1 - \alpha) \nu (1 - \sigma) k^{\alpha \nu} n_z^{(1-\alpha)\nu-1} n_y^{1-\nu} \]  

(14)

**Proposition 1** In an imperfectly competitive economy, if the government is not permitted to subsidize wage in the imperfectly competitive sector, in period 0 and in a steady state the optimal labour income tax in the imperfectly competitive sector is zero.

**Proof.** Consider (14), which implies \( \mu > 0 \). Thus in a steady state the non-negativity constraint (10) is satisfied with equality, and \( \tau_z = 0 \). The first order condition to the Ramsey problem with respect to \( \xi_0 \) is:

\[ 0 = -\eta (1 - \kappa \tau_{k0}) \pi_0 u_c(0) + \mu_0 (1 - \alpha) \nu (1 - \sigma) k_0^{\alpha \nu} n_2^{(1-\alpha)\nu-1} n_{y0}^{1-\nu} - \eta R_0 k_0 u_c(0) \]  

(15)

which implies \( \mu_0 > 0 \), i.e. at \( t = 0 \) the non-negativity constraint (10) is satisfied with equality, and \( \tau_{z0} = 0 \). □

The transition to this steady state is however not necessarily characterized by \( \mu_t > 0 \) for all \( t \geq 1 \). Although \( \mu_0 > 0 \) and \( \mu > 0 \), starting at \( t = 1 \) many paths of tax rates can achieve \( \tau_z = 0 \) and all these paths may be consistent with the equilibrium behavior of tax payers at the optimal allocations. Some of these paths may have \( \tau_z > 0 \) along the transition, which is explained by the term \( \left\{ k_{t-1} \frac{u_c(t-2)}{u_c(t-1)\xi_t} - k_t \frac{u_c(t-1)\xi_{t-1}}{u_c(t)\xi_t} \right\} \) in (13).
4 Concluding Remarks

In an imperfectly competitive economy, over accumulation of capital induces significant loss in welfare (because of profit seeking motive), but more capital per worker increases the real wage. Since the first best tax policy involves a subsidy to wage, with consumption taxation in the scheme the large subsidy rate overcompensates labour income and provides incentives to misreport working hours. We show that with a restriction on the use of wage subsidy, the government’s optimal choice is to set wage tax equal to zero in the long run and in the initial period which minimizes the incentive to misreport working hours.

References


