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Optimising indexation arrangements under Calvo contracts and their implications for monetary policy

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Abstract

This paper investigates optimal indexation in the New Keynesian model, when the indexation choice includes the possibility of partial indexation and of varying weights on rational and lagged indexation. It finds that the Calvo contract adjusted for rationally expected indexation under both inflation and price level targeting regimes delivers the highest expected welfare under both restricted and full current information. Rational indexation eliminates the effectiveness of monetary policy on welfare when there is only price-level targeting under the current micro information. If including both wage setting and full current information, monetary policy is effective; and a price-level targeting rule delivers the highest benefits because it minimises the size of shocks to prices and thus dispersion. However, even less than full rational indexation ensures that there is very little nominal rigidity in the adapted world of Calvo contracts.

Keywords: optimal indexation, price-level target, inflation target, Calvo contracts, rational expectation, New Keynesian model.

JEL codes: E50, E52

1 Introduction

In a previous paper (Le and Minford, 2006) we showed that within the New Keynesian Model in the presence of monetary noise, the Calvo contract should be indexed for rationally expected ('rational indexation') rather than lagged inflation ('lagged indexation'), even though the latter is widely used in the literature to protect both wages and prices against movements in the general price level (Erceg et al., 2000; Christiano et al., 2005; Smets and Wouters, 2002; and Giannoni and Woodford, 2003b). That paper addressed the special case of full indexation, either rational or lagged, for both prices and wages under a now-standard central bank interest rate rule targeting inflation. In this paper, we broaden the analysis, based on the same New Keynesian model, and investigate the more general question of optimal indexation when the indexation choice includes the possibility of partial indexation and of varying weights on rational and lagged indexation. We also attempt to establish how this optimising choice would respond to the nature of monetary policy, which we restrict in the New Keynesian manner to an interest rate rule and a choice between inflation and price-level targeting. Finally we explore the implications for the choice of monetary target.

Some will reject our agenda on empirical grounds, arguing that the Calvo model with lagged indexation when accompanied by other model features fits the facts of impulse responses as suggested by Christiano, et al (2005). Clearly empirical issues are another matter entirely, not pursued here. Our sole purpose in this paper is theoretical, to find the indexation mechanism that maximises consumer welfare and to discover its implications. Nevertheless we caution against the assumption that the empirical issue is closed in this way: recent work on UK data (Minford, 2006) suggests that models with little or no nominal rigidity, as occurs under rational indexing with the Calvo model here, are rather successful in replicating impulse responses in the data when the model itself is used for identification.1

We proceed as follows. After a brief section on previous related work, we first consider a world in which the current information available to agents who are setting prices or wages is solely about their own situation ('micro' information); this boils down to them observing their own productivity shock if they are a price-setter (if they are a wage setter they would observe only their own current preference shocks but these are suppressed in this model). Thus in this world there is a lag before agents see the

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1 Preliminary results from VARs on US data (H-P-filtered or differenced) also suggest that, when identified by a model without nominal rigidity as the one here, the impulse responses in the data are not greatly at variance with the model impulse responses discussed below.
current macro outcome. We consider the implications of this world first in a linearised model with a flexible labour market, a fixed capital stock and price-setting and secondly in a full nonlinear model with investment and wage-setting added in.

Secondly we relax this information assumption and allow agents to observe all current information while they are setting prices or wages; this might occur if statistics are released rapidly or there is very efficient signal extraction from global indicators like interest rates, especially in the context of quarterly data which is our prime frame of reference. We then revisit both our linearised and our full models under this relaxed information assumption.

1.1 Previous work:

Minford, Nowell and Webb (2003); and Minford and Nowell (2003) look at wage contracts in an overlapping contracts model. In the former, they showed that contracts respond to monetary policy. The greater the persistence in shocks, the higher is indexation. The reason is that shocks would disturb prices and if they are temporary, then indexation is unhelpful, because by the time the indexation element has been spent, the shock would have disappeared. However, if the shock is permanent, indexation can be used to offset the shock’s effect on consumption. In the second paper, they expand the issue to consider the interplay of endogenous indexation and optimal monetary policy. They found that price-level rules improved welfare compared with inflation targeting rules, and in so doing reduced the degree of indexation dramatically.

These papers built on earlier work that focused on the appropriate rate of indexation in contracts going back to Gray (1976) and Fisher (1977) (see also Barro’s critique, 1977). Using the idea that labour contracts could act as insurance for workers (e.g. Azariadis, 1975, and Baily, 1974, and see Malcolmson, 1999, for a review of the contract literature) they argued that wages would not be fully indexed because it was not feasible to draw up a fully-contingent contract, expressed throughout in real terms; hence a partial indexation parameter would stand in as a contingent response to shocks.

Inflation targeting is attractive in New Keynesian models because with sticky prices variable inflation causes misallocation of resources between firms that can adjust prices and firms that cannot (Carlstrom and Fuerst, 2002). Thus inflation targeting reduces the inefficiencies associated with sticky prices. But can we do better than this? Theoretically, Carlstrom and Fuerst (2002) suggest that price targeting rules may be generally better than inflation targeting rules because they build in a backward-looking element to avoid the possibility of sunspot events that can happen under inflation targeting. Under an inflation target, past inflation misses do not affect future policy actions: there is base drift. But with a price level target, past misses must affect future policy actions because the monetary policy must get the price back to the path, so there is no base drift. In time series language price level would be $I(0)$ whereas under inflation targeting it is $I(1)$ (implying that the variance surrounding future prices rises the further in the future one looks).

There could be other reasons to have a stationary price level.

Under an inflation targeting regime the uncertainty surrounding the future price level affects the issuing of long-term bonds and wage contracts. In principle indexation can allow people to deal with real variables directly, avoiding nominal contracts. But in practice indexation is imperfect, both in timing and in exactness—Minford (2006).

Moreover, there is the practical question of the zero bound on nominal interest rates. Nowadays, when inflation is low, it has worried central banks that a serious recession could require large interest rate cuts, but the interest rate cannot go below zero at which the demand for money is indeterminate, and therefore monetary policy cannot help the economy to recover. The problem is worse if prices are falling, since then at the zero bound real interest rates remain positive. This concern has led policymakers to set inflation targets away from zero to create room for interest to fall if it needs to (Minford, 2006). However, price level targeting creates an automatic expectation of future inflation when prices fall, so lowering the real interest rates in a deflation; this could help to alleviate the zero bound.

There has been concern that price level targeting may create macroeconomic instability. Svensson (1999a) noted the consensus of earlier authors (Hall, 1984; Duguay, 1994; Bank of Canada, 1994; Fischer, 1994) that monetary policy should target prices rather inflation. This lowers long-term price variance at

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2Central banks base their policy changes on inflation projections. Then money supply can be adjusted passively by the monetary authority - supplied at whatever level is necessary to achieve the target. Changes in the money supply can be self-fulfilling because public decisions depend on what the public is expected to do, and the public, in turn, bases its behaviour on monetary actions, and there is nothing to pin down either. To avoid sunspot events, options include: a constant money growth rule; aggressive changes in interest rates in response to inflation; or basing the bulk of the response on past inflation, as in price level targeting (Carlstrom and Fuerst, 2002)
the expense of higher short-term inflation and output variance. He wrote: ‘The intuition is straightforward: in order to stabilise the price level under price-level targeting, higher-than-average inflation must be succeeded by lower-than-average inflation, higher inflation variability via nominal rigidities would then seem to result in higher output variability’. However, he pointed out that the consensus might be wrong. Given a persistent output gap, focusing on the price level rather than the inflation rate will actually reduce both inflation and output variability under discretionary policy- in the absence therefore of commitment.

Other authors have examined optimal monetary policy under commitment. For example, Smets (2000) uses a model with a Calvo-style Phillips Curve to examine the optimal horizon for bringing inflation or the price level back to their targets; he finds that optimal length becomes shorter the more forward-looking are price expectations and the steeper is the Phillips Curve. Williams (1999) evaluates a variety of rules in the FRB model in which there is a forward/backward-looking Phillips Curve and inertial pricing dynamics and finds that multi-period inflation targeting ranks highly and that price level targeting only causes minor output instability. In this commitment context, price level targeting with a suitably long horizon has little effect on the optimal trade-off, and so the long-run price level can be a anchor for policy makers. Chadha and Nolan (2002) too compare inflation and price level targeting in New Keynesian models without indexation. They show that price level targeting dominates inflation targeting under credible commitment and with sufficient inflation aversion, the inflation-targeting central bank can produce quantitatively similar results to one targeting the price level.

Here we assume full commitment to the interest rate rule in line with New Keynesian models generally; we also ignore the zero bound issue and assume no sunspots. Indexation operates with a one period lag. Thus our aim is to extend the standard New Keynesian approach to deal with the best feasible indexation.

2 RESULTS WHEN AGENTS ONLY OBSERVE MICRO INFORMATION

2.1 The linearised model with price setters only and fixed capital

In our earlier paper we set out the New Keynesian model (Canzoneri et al, 2004) characterised by optimising agents, monopolistic competition, staggered wage and price setting, capital formation and an interest rate rule, together with the addition of indexing prices and wages. In the earlier paper, we found that indexation should be rationally expected inflation, because it optimises the welfare of the representative agent in the model. It was shown analytically that as the consequence of such indexation monetary policy no longer has any effect on welfare. In this paper, we allow price- and wage-setters to choose the optimal indexation from a linear combination of lagged inflation and rationally expected inflation. This can be either partial or full indexation.

We provide a listing of the full nonlinear model in the Appendix 5.1. The following lists the linearised equations (For the detailed derivation of the equations, see Le and Minford, 2006). It is assumed that capital is exogenous and made a non-tradeable endowment resource, while the labour market is competitive. In order the equations are:

Real marginal cost

\[ \log mc_t = \log MC_t - \log P_t \}

The production function

\[ \log Y_t = \log Z_t + (1 - \nu) \log N_t \]

Ignoring government spending, the market clearing condition gives

\[ \log Y_t = \log C_t \]

An interest rate rule, without lags and with the real interest rate assumed to be set in response to inflation and the output gap, with a monetary shock:

\[ r_t = \tau \pi_t + \sigma(\log Y_t - \log Z_t) + \log M_t \]

or with the real interest rate, reacting to price level and output gap, with a monetary shock

\[ r_t = \tau \log P_t + \sigma(\log Y_t - \log Z_t) + \log M_t \]
Note that these can identically be written as rules for the nominal interest rate using $i_t = r_t + E_t \pi_{t+1}$. Note also that we treat the monetary authorities as having effective full current information; this is the standard assumption made in New Keynesian models, presumably on the grounds that the authorities in practice have good access to information about the current state of the economy, even if the isolated private agent does not as we initially assume here.

Adding in the Euler equation $\log C_t = \log C_{t+1} - r_t$ and allowing for market clearing gives us an Aggregate Demand curve:

$$\log Y_t = \frac{1}{1 - \sigma^* B^{-1}} \{ -\sigma^* \pi_t + \sigma^* \log Z_t - \sigma^* \log M_t \}$$

or

$$\log Y_t = \frac{1}{1 - \sigma^* B^{-1}} \{ -\sigma^* \log P_t + \sigma^* \log Z_t - \sigma^* \log M_t \}$$

where $\log Z_t = \rho_1 \log Z_{t-1} + \varepsilon_t$; $\log M_t = \rho_2 \log M_{t-1} + \mu_t$; $z_t$ and $\mu_t$ are i.i.d.; $B^{-1}$ is the forward operator instructing one to lead the variable but keeping the expectations data-set constant; $\sigma^* = \frac{1}{1 + \sigma}$.

The new reset price

$$\log P^*_t = \frac{1 - \alpha \beta}{1 - \alpha \beta B^{-1}} E_t \left( \frac{1 + \gamma}{1 - \nu} (\log Y_t - \log Z_t) + \log P_t - \log \tilde{P}_t \right)$$

The general price level

$$\log P_t - \log \tilde{P}_t = \alpha \left( \log P_{t-1} - \log \tilde{P}_{t-1} \right) + (1 - \alpha) \log P^*_t$$

and the price indexing formula is $\log \tilde{P}_t = k_0 \left( E^{t-1} \log P_t + k_1 \left( \log P_{t-1} - E^{t-1} \log P_t \right) \right)$, where $k_0 \in [0, 1]$ and $k_1 \in [0, 1]$. If $k_0 = 1$, then prices are fully indexed; and if $k_0 = 0$, then price is not indexed. If $k_1 = 1$, then prices are indexed to the lagged price level; and if $k_1 = 0$, then they are indexed to rationally expected general price level.

Price dispersion is

$$\log DP_t = \frac{1}{2} \sum_{i=0}^{\infty} \phi_p \alpha^i (1 - \alpha) \left[ 1 - \alpha^i \left( 1 - \alpha \right) \right] \left( \log P^*_{t-i} \right)^2$$

$$- \sum_{j=0}^{\infty} \sum_{i=0}^{\infty} \phi_p \alpha^{i+j} (1 - \alpha)^2 \log P^*_{t-i} \log P^*_{t-j}$$

In order to discuss policy within the context of this model, we evaluate expected welfare in terms of its deviation from the flex-price optimum

$$E (u_t - u_t^{\text{FLEX}}) = -E \log DP_t = \frac{1}{2} \sum_{i=0}^{\infty} \phi_p \alpha^i (1 - \alpha) \left[ 1 - \alpha^i \left( 1 - \alpha \right) \right] \text{var} \left( \log P^*_{t-i} \right)$$

$$+ \sum_{j=0}^{\infty} \sum_{i=0}^{\infty} \phi_p \alpha^{i+j} (1 - \alpha)^2 \text{cov} \left( \log P^*_{t-i}, \log P^*_{t-j} \right)$$

where $u_t^{\text{FLEX}} = \log Z_t$ under the flexible price and wage assumption.

### 2.2 Solving the model with restricted private information

Our notation is as follows: $E^{t-1} x_t = E (x_t | \Phi_{t-1})$; $E_t x_t = E (x_t | \Phi_{t-1}, \phi_t)$; $x_t^{UE} = x_t - E^{t-1} x_t$, where $\Phi_{t-1}$ is the full information set from period $t-1$ and $\phi_t$ is the limited information set available for period $t$. The plan of this section for solving the model under both price-level targeting and inflation targeting is (a) Solve for the price level $\log P_t$ using the Wold decomposition; (b) Solve for the reset price level $\log P^*_t$ using unknown parameters that were determined in (a); (c) Using (b) to find the covariances and variances of reset prices, which in turn contribute to the calculation of the expected price dispersion; (d) Expected welfare is the negative of expected price dispersion.
2.2.1 Inflation targeting rule

Here we consider the case of an interest rule that targets the inflation rate (at zero for convenience). To begin, we use equations (6), (9), (8), indexation formula and the assumption that firms have knowledge of their own micro information (productivity, prices and costs) in period \( t \) as well as the macro information of period \( (t - 1) \) to express all price related variables in terms of the future expected monetary and real shocks (for more details, see Appendix 5.2):

\[
(1 - \alpha) \log P_t^* = \log P_t - k_0 \left[ E^{t-1} \log P_t + k_1 \left( \log P_{t-1} - E^{t-1} \log P_t \right) \right] -
\]
\[
- \alpha \left\{ \log P_{t-1} - k_0 \left[ E^{t-2} \log P_{t-1} + k_1 \left( \log P_{t-2} - E^{t-2} \log P_{t-1} \right) \right] \right\}
\]
\[
= \frac{(1 - \alpha)(1 - \alpha \beta)}{1 - \alpha \beta B^{-1}} E_t \left\{ \frac{1 + \alpha}{1 - \alpha \beta} \left[ \frac{1}{1 - \alpha \beta B^{-1}} \left( -\sigma \log P_t + \sigma \log Z_t - \sigma \log M_t \right) - \log Z_t \right] \right\} (12)
\]

Collecting terms and converting the operators back into leads and lags, we obtain

\[
\log P_t - (2.5847 + 1.9028k_0k_1) \log P_{t-1} + \left( 1.8945 - 0.88889k_0 \right) E^{t-1} \log P_t -
\]
\[
- (1.75 + 3.1528k_0k_1 - 1.4915k_0) E^{t-2} \log P_{t-1} + (1.534 + 2.43k_0k_1) \log P_{t-2}
\]
\[
- 0.62k_0k_1 \log P_{t-3} - 0.62k_0 (1 - k_1) E^{t-3} \log P_{t-2} + 0.936k_0 (1 - k_1) E^{t-3} \log P_{t-1}
\]
\[
+ 0.564 (1 - k_0 + k_0k_1) E^{t-1} \log P_{t-1} + (1.8 - 1.42k_0 + 1.98k_0k_1) E^{t-1} \log P_{t+1}
\]
\[
+ (1.604 - 2.32k_0 + 2.842k_0k_1) E^{t-2} \log P_t
\]
\[
- 0.351k_0 (1 - k_1) E^{t-3} \log P_t + (-0.521 + 0.901k_0 - 0.901k_0k_1) E^{t-2} \log P_{t+1}
\]
\[
= \chi^* \left[ \left( \rho_1 - 1 \right) \left( z_t - \rho_1^2 \left( \alpha \beta + \sigma - \alpha \beta \rho_1 \right) z_{t-1} \right) - \rho_2 (1 - \alpha \beta \rho_1) (1 - \sigma \rho_1) \mu_{t-1} \right] \right] (13)
\]

Given the assumption that \( \log P_t = \sum_{i=0}^{\infty} \varepsilon_{i, z_{t-i}} + \sum_{i=0}^{\infty} \mu_{t-i} \), equation (13) can hold if and only if the sum of the LHS’s and the RHS’s coefficients on \( z_{t-1} \), on \( z_{t-2} \), ..., are equal respectively (and the same argument for \( \mu_{t-i} \)). For example these coefficients on the \( z_{t-i} \) must satisfy (Appendix 5.2 shows the equivalent for the \( \mu_{t-i} \))

\[
(z_t)
\]
\[
\varepsilon_0 = \chi^* \left( \rho_1 - 1 \right) \right] (14)
\]

\[
(z_{t-1})
\]
\[
0.564 (1 - k_0 + k_0k_1) \varepsilon_3 + \left( -1.8 + 1.42k_0 - 1.98k_0k_1 \right) \varepsilon_2 +
\]
\[
(2.8945 - 0.88889k_0 + 2.69k_0k_1) \varepsilon_1 - (2.5847 + 1.9028k_0k_1) \varepsilon_0
\]
\[
= - \chi^* \left( \rho_1 - 1 \right) \rho_1^2 \left( \alpha \beta + \sigma - \alpha \beta \rho_1 \right) \varepsilon_0 \right] (15)
\]

\[
(z_{t-2})
\]
\[
0.564 (1 - k_0 + k_0k_1) \varepsilon_4 - (2.321 - 2.322k_0 + 2.88k_0k_1) \varepsilon_3 +
\]
\[
(4.5549 - 3.2089k_0 + 5.532k_0k_1) \varepsilon_2 +
\]
\[
+ (-4.3347 + 1.4915k_0 - 5.0556k_0k_1) \varepsilon_1 + (1.534 + 2.43k_0k_1) \varepsilon_0 = 0 \right] (16)
\]

\[
(z_{t-i}, i \geq 3)
\]
\[
0.564 (1 - k_0 + k_0k_1) \varepsilon_{i+2} - (2.321 - 2.322k_0 + 2.881k_0k_1) \varepsilon_{i+1} +
\]
\[
(4.5549 - 3.56k_0 + 5.883k_0k_1) \varepsilon_i - (4.3347 - 2.4275k_0 + 6k_0k_1) \varepsilon_{i-1} +
\]
\[
(1.534 - 0.62k_0 + 3.05k_0k_1) \varepsilon_{i-2} - 0.62k_0k_1 \varepsilon_{i-3} = 0 \right] (17)
\]

The last equation is a 5th order difference equation under the real shocks and it is identical to the equation (38) in Appendix 5.2, illustrating the case of monetary shocks. Thus both equations (17) and (38) are characterised by the 5th order polynomial equation
0 = 0.564(1 - k_0 + k_0 k_1) x^5 - (2.321 - 2.322k_0 + 2.881k_0 k_1) x^4 + (4.554 - 3.56k_0 + 5.883k_0 k_1) x^3 + (-4.3347 + 2.4275k_0 - 6k_0 k_1) x^2 + (1.534 - 0.62k_0 + 3.05k_0 k_1) x - 0.62k_0 k_1 \tag{18}

We consider all possible combinations of \(k_0\) and \(k_1\) and find stable roots under each combination\(^3\). In this 5th order difference equation, the number of forward roots is two; therefore the condition for a unique saddle path is two unstable roots (looking backwards). The problem, however, is that many \((k_0, k_1)\) yield the wrong number of unstable roots. Many yield three which implies an overdetermined solution; some only have one which implies nonuniqueness. So we only report the cases that deliver the unique solution. In general, these cases will give three stable roots, called \(x_1, x_2,\) and \(x_3\); thus, the equation (18) can be reduced to

\[(1 - x_1 L) (1 - x_2 L) (1 - x_3 L) \varepsilon_i = 0 \]

and \[(1 - x_1 L) (1 - x_2 L) (1 - x_3 L) \varsigma_i = 0 \quad \text{for } i \geq 3 \tag{19}\]

for real and monetary shocks respectively. That is

\[
\varepsilon_i - (x_1 + x_2 + x_3) \varepsilon_{i-1} + (x_1 x_2 + x_1 x_3 + x_2 x_3) \varepsilon_{i-2} + x_1 x_2 x_3 \varepsilon_{i-3} = 0 \\
\varsigma_i - (x_1 + x_2 + x_3) \varsigma_{i-1} + (x_1 x_2 + x_1 x_3 + x_2 x_3) \varsigma_{i-2} + x_1 x_2 x_3 \varsigma_{i-3} = 0 \quad \text{for } i \geq 3 \tag{20}\]

The values of \(x_1, x_2,\) and \(x_3\) under each combination of \((k_0, k_1)\) are derived numerically. Table (1) reports the values of \(x_i\) and values of \(\sqrt{x_2 \times x_3}\) for all combinations of \(k_0\) and \(k_1\) that give a stable solution path (except the case of \((1, 0)\), which will be described separately—Appendix 5.3).

<table>
<thead>
<tr>
<th>((k_0, k_1)) combination</th>
<th>(x_1)</th>
<th>damping factor ((\sqrt{x_2 \times x_3}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0; all)</td>
<td>0</td>
<td>0.932623</td>
</tr>
<tr>
<td>(1, 0.2)</td>
<td>0.10047</td>
<td>0.947849</td>
</tr>
<tr>
<td>(1, 0.4)</td>
<td>0.16347</td>
<td>0.931673</td>
</tr>
<tr>
<td>(1, 0.6)</td>
<td>0.20892</td>
<td>0.925109</td>
</tr>
<tr>
<td>(1, 0.8)</td>
<td>0.24552</td>
<td>0.953922</td>
</tr>
<tr>
<td>(1, 1)</td>
<td>0.2744</td>
<td>0.945944</td>
</tr>
</tbody>
</table>

Table 1: Values of stable roots under inflation targeting

From equations (14), (15), (16), (20) and results in Table (1), we can build the system of equations and solve for \(\varepsilon_i\) under different combinations of \((k_0, k_1)\) to get the solution for the real shocks related part of the price level \(\log P_t = \sum_{i=0}^{\infty} \varepsilon_i z_{t-i}\). While from the equations in Appendix 5.2, (20) and results in Table (1), we find \(\varsigma_i\) that gives the part of price which relates to monetary shocks \(\log P_t = \sum_{i=0}^{\infty} \varsigma_i \mu_{t-i}\). However, in order to find the expected welfare, \(\log P_t^*\) must be written in terms of these unknown coefficients (the working is shown in Appendix 6.4):

\[
\log P_t^* (z_{t-i}) = \frac{-\chi' z_i + \alpha \chi' z_{t-1}}{1 - \alpha} + \frac{-k_0 k_1 (\varepsilon_0 - \varepsilon_1) + (1 - k_0) \varepsilon_1}{1 - \alpha} z_{t-1} + \\
\frac{\varepsilon_2 (k_0 k_1 + (1 - k_0)) - \varepsilon_1 (k_0 k_1 (1 + \alpha) + \alpha (1 - k_0)) + \alpha k_0 k_1 \varepsilon_0}{1 - \alpha} z_{t-2} + \\
\frac{\varepsilon_3 (k_0 k_1 + (1 - k_0)) - \varepsilon_2 (k_0 k_1 (1 + \alpha) + \alpha (1 - k_0)) + \alpha k_0 k_1 \varepsilon_1}{1 - \alpha} z_{t-3} + ... \tag{21}
\]

and

\[
\log P_t^* (\mu_{t-i}) = \frac{-k_0 k_1 (\varsigma_0 - \varsigma_1) + (1 - k_0) \varsigma_1}{1 - \alpha} \mu_{t-1} + \\
\frac{\varsigma_2 (k_0 k_1 + (1 - k_0)) - \varsigma_1 (k_0 k_1 (1 + \alpha) + \alpha (1 - k_0)) + \alpha k_0 k_1 \varsigma_0}{1 - \alpha} \mu_{t-2} + \\
\frac{\varsigma_3 (k_0 k_1 + (1 - k_0)) - \varsigma_2 (k_0 k_1 (1 + \alpha) + \alpha (1 - k_0)) + \alpha k_0 k_1 \varsigma_1}{1 - \alpha} \mu_{t-3} + ... \tag{22}
\]

\(^3k_0 = 0 : 0.2 : 1\) and \(k_1 = 0 : 0.2 : 1\)
To calculate the expected price dispersion

\[ E \log DP_t = \frac{1}{2} \sum_{i=0}^{\infty} \phi_i \alpha^i (1 - \alpha) \left[ 1 - \alpha^i (1 - \alpha) \right] \text{Var} \left( \log P_{t-i}^* \right) - \sum_{j=0}^{\infty} \sum_{i=0}^{\infty} \phi_i \phi_j \alpha^{i+j} (1 - \alpha)^2 \text{Cov} \left( \log P_{t-i}, \log P_{t-j}^* \right), \]

we need to find two components. One is the covariance which arises because of lagged terms being cross-multiplied; these terms are then the variance of the i.i.d. error times the various cross-terms. The other is the variance. We only demonstrate the calculation of these two terms under productivity shocks. The analysis under monetary shocks is analogous.

**Covariance** First, we write equation (21) as

\[ \log P_t^* (z_{t-i}) = a_0 z_1 + a_1 z_{t-1} + a_2 z_{t-2} + a_3 z_{t-3} + a_4 z_{t-4} + \ldots \]  

(23)

where

\[
\begin{align*}
a_0 &= -\frac{\gamma'}{1 - \alpha} = -\chi' \\
a_1 &= \frac{\alpha \chi'}{1 - \alpha} - \frac{\varepsilon_k k_1 \left( \varepsilon_1 - \varepsilon_0 \right)}{1 - \alpha} + \frac{\varepsilon_1 \left( k_0 k_1 \left( 1 + \alpha \right) + \alpha \left( 1 - k_0 \right) \right)}{1 - \alpha} \\
a_2 &= \varepsilon_2 \left( k_0 k_1 \left( 1 - k_0 \right) \right) - \varepsilon_1 \left( k_0 k_1 \left( 1 + \alpha \right) + \alpha \left( 1 - k_0 \right) \right) + \alpha k_0 k_1 \varepsilon_0 ; \text{etc.}
\end{align*}
\]

Second, the covariances under productivity shocks will be, for example

\[
\begin{align*}
\text{Cov}(\log P_t^*, \log P_t^*_{t-1}) &= \text{var}(z) \left[ a_0 a_1 + a_1 a_2 + a_2 a_3 + a_3 a_4 + a_4 a_5 + a_5 a_6 + \ldots \right] \\
\text{Cov}(\log P_t^*, \log P_t^*_{t-2}) &= \text{var}(z) \left[ a_0 a_2 + a_1 a_3 + a_2 a_4 + a_3 a_5 + a_4 a_6 + a_5 a_7 + \ldots \right] \\
\text{Cov}(\log P_t^*, \log P_t^*_{t-3}) &= \text{var}(z) \left[ a_0 a_3 + a_1 a_4 + a_2 a_5 + a_3 a_6 + a_4 a_7 + \ldots \right] ; \text{etc.}
\end{align*}
\]

(24)

The first few cross-terms of these covariances can be calculated explicitly, however, \( a_i \) would decay after a certain point at the rate of the dominant root, called \( \rho \), of the three stable roots. This allows us to write the equations in (24) as

\[
\begin{align*}
\text{Cov}(\log P_t^*, \log P_t^*_{t-1}) &= \text{var}(z) \left[ a_0 a_1 + a_1 a_2 + a_2 a_3 + a_3 a_4 + a_4 a_5 + a_5 a_6 + a_6 a_7 + \ldots \right] \\
\text{Cov}(\log P_t^*, \log P_t^*_{t-2}) &= \text{var}(z) \left[ a_0 a_2 + a_1 a_3 + a_2 a_4 + a_3 a_5 + a_4 a_6 + a_5 a_7 + a_6 a_8 + \ldots \right] \\
\text{Cov}(\log P_t^*, \log P_t^*_{t-3}) &= \text{var}(z) \left[ a_0 a_3 + a_1 a_4 + a_2 a_5 + a_3 a_6 + a_4 a_7 + a_5 a_8 + a_6 a_9 + \ldots \right] ; \text{etc.}
\end{align*}
\]

(25)

Therefore,

\[
-\sum_{j=0}^{\infty} \sum_{i=0}^{\infty} \text{Cov}(\log P_t^*, \log P_t^*_{t-i}) = -\frac{\alpha \phi_p \text{var}(z)}{1 + \alpha} \left[ \frac{\rho}{1 - \rho} (a_i^2 + a_j^2 + \ldots) + a_0 \left( a_1 + a_2 + \ldots + \frac{a_5}{1 - \rho} \right) + a_1 \left( a_2 + \ldots + \frac{a_4}{1 - \rho} \right) + a_2 \left( a_3 + \ldots + \frac{a_3}{1 - \rho} \right) + \ldots \right] 
\]

(26)

**Variance** The second term in the expected price dispersion formula- variance- is calculated as follows:

\[ \text{Var}(\log P_t^*) = \text{var}(z) (a_0^2 + a_1^2 + a_2^2 + a_3^2 + a_4^2 + \ldots) \]  

(27)

and

\[
\frac{1}{2} \sum_{i=0}^{\infty} \phi_i \alpha^i (1 - \alpha) \left[ 1 - \alpha^i (1 - \alpha) \right] \text{Var} \left( \log P_{t-i}^* \right) = \frac{\alpha \phi_p \text{var}(z)}{1 + \alpha} (a_0^2 + a_1^2 + a_2^2 + a_3^2 + a_4^2 + \ldots)
\]

From this point onwards, all calculation for both variance and covariance components is done numerically.

Table (2) shows the results of expected welfare under different combinations of \((k_0, k_1)\):
any shocks known at time (1975) famous irrelevance result. The intuition is that rational indexation builds into prices the effect of surprise, but not on the systematic part of the monetary rule. This is an echo of Sargent and Wallace’s New Classical one, where real output only depends on current and lagged real shocks and the monetary rule is independent of the choice of monetary policy target under the assumption of micro current information.

We obtain the equivalent results to those of inflation targeting in the two following Tables 2 and 4:

### Table 2: Expected welfare under interest rate rule targeting zero rate inflation

<table>
<thead>
<tr>
<th>$k_0$</th>
<th>$k_1$</th>
<th>$E(DP)_{prod}$</th>
<th>$E(DP)_{money}$</th>
<th>$E(DP)$</th>
<th>$E(Welfare)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0.011969955</td>
<td>-0.000499815</td>
<td>0.011470141</td>
<td>-0.011470141</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0.01006</td>
<td>0</td>
<td>0.01006</td>
<td>-0.01006</td>
</tr>
<tr>
<td>1</td>
<td>0.2</td>
<td>0.012759147</td>
<td>-1.24008E - 05</td>
<td>0.012746747</td>
<td>-0.012746747</td>
</tr>
<tr>
<td>1</td>
<td>0.4</td>
<td>0.012760099</td>
<td>-1.706E - 05</td>
<td>0.012742949</td>
<td>-0.012742949</td>
</tr>
<tr>
<td>1</td>
<td>0.6</td>
<td>0.012762261</td>
<td>-2.74937E - 05</td>
<td>0.012734767</td>
<td>-0.012734767</td>
</tr>
<tr>
<td>1</td>
<td>0.8</td>
<td>0.012770473</td>
<td>-0.000117127</td>
<td>0.012653346</td>
<td>-0.012653346</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0.012771709</td>
<td>-0.00014718</td>
<td>0.012624529</td>
<td>-0.012624529</td>
</tr>
</tbody>
</table>

### Table 3: Values of stable roots under price level targetingl

<table>
<thead>
<tr>
<th>$(k_0,k_1)$ combination</th>
<th>$x_1$</th>
<th>damping factor $(\sqrt{\sigma_2 \times \sigma_3})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0; all)</td>
<td>0</td>
<td>0.49174</td>
</tr>
<tr>
<td>(1,0.2)</td>
<td>0.86303</td>
<td>0.29339</td>
</tr>
<tr>
<td>(1,0.4)</td>
<td>0.85953</td>
<td>0.36410</td>
</tr>
<tr>
<td>(1,0.6)</td>
<td>0.93064</td>
<td>0.40279</td>
</tr>
<tr>
<td>(1,0.8)</td>
<td>0.90543</td>
<td>0.43374</td>
</tr>
<tr>
<td>(1,1)</td>
<td>0.88844</td>
<td>0.45787</td>
</tr>
</tbody>
</table>

### Table 4: Expected welfare under interest rate rule targeting price level

<table>
<thead>
<tr>
<th>$k_0$</th>
<th>$k_1$</th>
<th>$E(DP)_{prod}$</th>
<th>$E(DP)_{money}$</th>
<th>$E(DP)$</th>
<th>$E(Welfare)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0.012428407</td>
<td>-1.59348E - 05</td>
<td>0.012412473</td>
<td>-0.012412473</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0.01006</td>
<td>0</td>
<td>0.01006</td>
<td>-0.01006</td>
</tr>
<tr>
<td>1</td>
<td>0.2</td>
<td>0.012840236</td>
<td>6.59E - 07</td>
<td>0.012840895</td>
<td>-0.012840895</td>
</tr>
<tr>
<td>1</td>
<td>0.4</td>
<td>0.012898353</td>
<td>2.25E - 06</td>
<td>0.012900605</td>
<td>-0.012900605</td>
</tr>
<tr>
<td>1</td>
<td>0.6</td>
<td>0.012941003</td>
<td>5.27E - 06</td>
<td>0.012946275</td>
<td>-0.012946275</td>
</tr>
<tr>
<td>1</td>
<td>0.8</td>
<td>0.012971234</td>
<td>7.67E - 06</td>
<td>0.012978903</td>
<td>-0.012978903</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0.012991178</td>
<td>1.09E - 05</td>
<td>0.013002112</td>
<td>-0.013002112</td>
</tr>
</tbody>
</table>

### 2.2.2 Price targeting rule

Here the analysis is analogous to the one used above for the inflation target regime. Using equations (7), (8), (9), the indexation formula and the assumption that firms have knowledge of their own micro information (productivity, prices and costs) in period $t$ as well as the macro information of period $(t - 1)$, we write all the terms related to the price level as a function of real and monetary shocks. The results are identical to those for inflation targets under full rational indexation which is again the optimum. Again, monetary policy is impotent in this case.

We obtain the equivalent results to those of inflation targeting in the two following Tables 2 and 4:

### 2.3 Conclusion from the analytic with restricted private information

Under both type of monetary regime: interest rate rule with inflation target and price level target, the analytic model shows that overall the fully rational indexation for prices is always optimal. This is a confirmation of the result that was derived in our previous paper for the more restricted comparison of full rational with full lagged indexation. If agents index prices and wages rationally, expected welfare is independent of the choice of monetary policy target under the assumption of micro current information. The reason is that by allowing for rational indexation the New Keynesian Phillips Curve defaults to a New Classical one, where real output only depends on current and lagged real shocks and the monetary surprise, but not on the systematic part of the monetary rule. This is an echo of Sargent and Wallace’s (1975) famous irrelevance result. The intuition is that rational indexation builds into prices the effect of any shocks known at time $t - 1$. Whatever has happened at $t - 1$ is, in the case of the productivity shock,
built into the expected real reset price for the next period \( t \); this fixes expected real marginal cost and hence expected real output. The expected price index is then calculated as the necessary price increase that will accommodate this and the expected level of interest rates. Unexpected monetary shocks have no effect on prices because they have been pre-set in this way. Thus only lagged money shocks affect prices while only unexpected monetary shocks affect output under rational indexation in this model.

Hence with this model the monetary rule has no impact on welfare when current information is solely micro. We now turn to the full model under the same information assumption.

### 2.4 Stochastic simulation results on the full model under micro private information

We proceed to consider the stochastic simulations for expected welfare in terms of deviations from the \( \hat{\text{ex}} \)-optimum under both lagged and rational indexation when only micro information is assumed to be known at period \( t \). The aim is to relate these results to those from the analytic model above. Table (5) shows that expected welfare is maximised by rational indexation, just as in the analytic model.

<table>
<thead>
<tr>
<th>Type of shock</th>
<th>( k_0 )</th>
<th>( kw_0 )</th>
<th>( k_1 )</th>
<th>( kw_1 )</th>
<th>Welfare</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflation target</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>-0.0048</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0.8</td>
<td>0.8</td>
<td>1.1</td>
<td>-0.0269</td>
</tr>
<tr>
<td></td>
<td>0.2</td>
<td>0.2</td>
<td>1</td>
<td>1</td>
<td>-0.0162</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>-0.0162</td>
</tr>
<tr>
<td>Price target</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>-0.0048</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0.8</td>
<td>0.8</td>
<td>1.1</td>
<td>-0.0103</td>
</tr>
<tr>
<td></td>
<td>0.2</td>
<td>0.2</td>
<td>1</td>
<td>1</td>
<td>-0.0112</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>-0.0199</td>
</tr>
</tbody>
</table>

Table 5: Expected welfare for different types of indexation assuming only current information is micro; interest rule with inflation and price level targets (stochastic simulation of monetary and productivity shocks, each with standard error of 0.01).

We now find that monetary policy does, strictly speaking, have an effect on expected welfare under rational indexation. The reason for this is the introduction of wage-setting. Though prices are only affected by the productivity shock (because it alone is currently observed), the interest rate rule reacts to both inflation (or prices) and to the output gap while monetary shocks also affect the latter. This reaction alters output and so employment; with wages fixed this drives agents away from their \( \hat{\text{ex}} \)-price leisure choice, affecting their welfare. We also show in Table (6) how monetary policy choices affect expected welfare. The choice of whether to target inflation or prices is irrelevant since it is only the current price shock reaction that matters. Thus what matters in the interest rate rule is the size of the reactions to inflation or prices and to the output gap. Higher inflation or price coefficients worsen the effect of productivity shocks because they dampen price changes which means that real wages do not change as much as they should to match productivity change. Higher reactions to the output gap dampen movements in it and employment which move workers away from their \( \hat{\text{ex}} \)-price choices. What we notice is that while there are effects here, they are not at all big, because the utility function does not have much curvature in leisure. In the Calvo model the big losses arise because of price and wage dispersion. Hence we can say that effectively the results are the same as in the analytic model: full rational indexation is optimal and in this case monetary policy is (effectively) impotent.

<table>
<thead>
<tr>
<th>Full Rational indexing</th>
<th>Productivity</th>
<th>Monetary</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflation targeting</td>
<td>-0.00381</td>
<td>-0.00098</td>
<td>-0.0048</td>
</tr>
<tr>
<td>Price targeting</td>
<td>-0.00381</td>
<td>-0.00098</td>
<td>-0.0048</td>
</tr>
<tr>
<td>Stricter Inflation targeting</td>
<td>-0.00426</td>
<td>-0.00098</td>
<td>-0.00524</td>
</tr>
<tr>
<td>Stricter Price targeting</td>
<td>-0.00426</td>
<td>-0.00098</td>
<td>-0.00524</td>
</tr>
<tr>
<td>Inflation targeting (higher weight on output gap)</td>
<td>-0.00376</td>
<td>-0.00092</td>
<td>-0.00468</td>
</tr>
<tr>
<td>Price targeting (higher weight on output gap)</td>
<td>-0.00376</td>
<td>-0.00092</td>
<td>-0.00468</td>
</tr>
</tbody>
</table>

Table 6: Expected welfare for different types of indexation assuming only current information is micro; interest rule with inflation and price level targets (stochastic simulation of monetary and productivity shocks, each with standard error of 0.01).
2.5 Conclusions on case of micro current information only

What we have found in this case is that full rational indexation is the dominant strategy for private agents. This has strong implications for monetary policy. First, it is irrelevant whether the interest rate rule targets inflation or prices since only the shock to prices or inflation matters and it is the same under both rules. Second, the coefficients of the rule make no difference at all to expected welfare in the analytic model (because current prices respond to current productivity shocks only) and in the full model they make virtually no difference (since they only enter through the effect on employment whose effect on welfare is minor).

3 RESULTS WHEN AGENTS OBSERVE FULL CURRENT INFORMATION

We now turn to the case where full current information is available to private agents. This is the default assumption made in New Keynesian models. As we noted earlier the justification presumably lies in the overlap between the length of time in which prices and wages are not changed at all- a quarter- and the production of current macro information by statistics offices and the private sector itself. In the course of three months price and wage setters may well be fairly well informed about what is going on in that quarter so that the assumption of full knowledge may be a close approximation. At any rate we now explore the implications of this assumption within the full model. Under full information the analytic model becomes too complex to solve under indexation schemes which vary the weights on lagged and rational indexation; we confine ourselves to some insights from the analytic model as far as we can take it.

We check the stochastic simulation under the assumption of full information being available in period t (Table 7). The pair \((k0, kw0)\) show the weights on lagged and rational indexation in indexation formulas for prices and wages respectively, while \((k1, kw1)\) shows whether prices and wages are partially or fully indexed. Our stochastic simulations are done for 100 sets of 40 overlapping shocks- with both productivity and monetary shocks. Similarly to Minford and Nowell (2003), we treat each period outcome as a stochastic experiment of equal likelihood. We ignore the discount rate in calculation of the expected welfare. Firstly, in each set, in the first period, it runs for the first shock and records the welfare of this period. The first period values are then used as the base values for the next period simulation and so on. Then, we have 4000 observations which we average to get expected welfare. This process repeats for each \((k0, kw0, k1, kw1)\), where values of these parameters all belong to the interval of \([0, 1]\) and they move with a step of 0.2. Finally, we compare all the expected welfare values to find the weighting scheme that gives the maximum expected welfare.

Table 7 reports the results of stochastic simulations on the full model, showing the optimal indexation scheme under our two shocks to productivity and money.

<table>
<thead>
<tr>
<th></th>
<th>k0</th>
<th>kw0</th>
<th>k1</th>
<th>kw1</th>
<th>Best expected welfare</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflation targeting</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>-0.00572</td>
</tr>
<tr>
<td>Price targeting</td>
<td>0</td>
<td>0</td>
<td>0.8</td>
<td>1</td>
<td>-0.00096</td>
</tr>
<tr>
<td>Stricter price target</td>
<td>0</td>
<td>0</td>
<td>0.8</td>
<td>1</td>
<td>-0.00034</td>
</tr>
</tbody>
</table>

Shocks assumed to be both monetary and productivity each with standard error of 0.01.

Table 7: Optimal index weighting scheme under inflation and price level targeting with full current information assumption

We note that:

1. lagged indexation does not have any weight in the optimal indexation scheme.
2. monetary policy is effective on welfare; as we move from inflation to price targeting and then to stricter price targeting expected welfare improves.
3. the extent of price indexation also responds endogenously to this change in monetary policy: it drops somewhat. While full rational indexation is best under inflation targeting, price indexation drops to only 80% (though still on the rational index) as price level targeting is introduced.

Let us consider these points in turn.

1. To understand why lagged indexation does not enter the optimal indexation scheme, we refer back to our earlier paper where we showed that lagged indexation created an additional correlation between lagged price surprises and lagged prices: this tends to raise the variability of accumulated reset prices on
balance. Hence the optimal indexation scheme only has rational indexation in it. The explanation can be briefly described as follows: the reset price under lagged indexation is given as the reset price under rational indexation plus an extra term

\[ \log P_t^* \text{ (lagged)} = \log P_t^* \text{ (rational)} - \left( \frac{\mu_{t-1} - \alpha \mu_{t-2}}{1 - \alpha} \right) \]

where \( \mu_{t-1} = \log P_{t-1} - E_{t-1} \log P_t \). The reset price under the rational expectation is only a function of the current and lagged productivity shocks. The term \(- \left( \frac{\mu_{t-1} - \alpha \mu_{t-2}}{1 - \alpha} \right)\) in the reset price under lagged indexation contains all past productivity and monetary shocks. The difference between expected welfare under rational indexation and lagged indexation is divided into two parts. The first part consists all the terms \((q)\) that are not related to lagged productivity shock \((z_{t-1})\), such as all the productivity shocks that occur before or at period \((t - 2)\) and all monetary shocks. These shocks are the ones that do not enter the reset price expression under rational indexation. The expected welfare of this part is 
\[ -\phi_p \alpha (1 - \alpha \varphi q) \] which must worsen the expected welfare under lagged indexation compared with rational indexation. The second part of the expected welfare consists only of terms in \(z_{t-1}\) which are hence correlated with welfare under rational indexation. This part is
\[ -\phi_p \alpha (Y^2 + (\psi_0 + \alpha Y'\gamma)^2 + (1 - \alpha) \gamma' (\psi_0 + \alpha Y')) \var(\gamma) \]
which turns out for the calibrated values of the model to improve expected welfare compared with rational indexation. In aggregate the comparison between the resulted expected welfare levels under lagged and rational indexation depends on the magnitudes of the two parts above. The simulation results suggest that the first part dominates the second, so that lagged indexation lowers expected welfare by introducing additional lagged shocks into the reset price.

(2) Monetary policy is now effective on expected welfare because full information causes current monetary and productivity shocks to affect both reset prices and wages and the interest rate rule modifies these effects through its reaction coefficients. We can demonstrate this conclusion in the analytic model with its assumptions of no labour market, no capital accumulation, and only price setting. reset price. This model’s solution for the reset price under full rational indexation is:

\[ \log P_t^* = 1.5134 z_t - 1.0141 z_{t-1} - 1.4095 \mu_t + 0.7659 \mu_{t-1} \]

Here the monetary shock and its lagged value join the productivity shock and its lagged value in affecting the reset price and so expected welfare.

Moving from an inflation to a price-level target has the effect of increasing the response of real interest rates to price shocks and so dampening these. The reason is that price-level targeting effectively raises the real interest rate, \( r_t \), response to a price shock because \( r_t = R_t - E_t \pi_{t+1} \), which is \( r_t = \text{rule} - (E_t P_{t+1} - P_t) \). Under inflation targeting, the last term in the real interest rate equation is small or zero so the real interest, \( r_t \), is just the rule; but under price-level targeting, \( E_t P_{t+1} \) is small or zero, so the rule becomes \( \text{rule} + P_t \). Therefore, a price-level target stops prices moving as much as they do under an inflation target. This reduces the variance of the reset price and also that of both of the real wage and of employment which additionally enters the welfare function in the full model. We can see the effect of the reduced reset price variance from simulations of the analytic model under full current information and full rational indexation - Table (8) below

<table>
<thead>
<tr>
<th>Inflation target</th>
<th>Price target</th>
</tr>
</thead>
<tbody>
<tr>
<td>Productivity</td>
<td>Monetary</td>
</tr>
<tr>
<td>-0.001363</td>
<td>-0.001026</td>
</tr>
</tbody>
</table>

Table 8: Expected welfare for rational indexation under interest rule with inflation and price level targets, assuming full current information (stochastic simulation of monetary and productivity shocks, each with standard error of 0.01)

(3) We turn last to why indexation is sensitive to monetary policy. We can understand this in terms of the alteration less than full rational indexation creates in the reset price, \( p_t^* \). Using the reset price equation which under full rational indexation can be written as function of unexpected current and lagged price changes (Le and Minford, 2006, equation (49)):

\[ (1 - \alpha) \log P_t^* = \log P_t^{UE} - \alpha \log P_{t-1}^{UE}, \]

\[ q_{t-1} = \beta q_{t-1} \]
If we now deviate from full rational indexation by reducing the indexation to \((1 - k)\) this gives us instead
\[
(1 - \alpha) \log P_t^* = \log P_t^{UE} + kE_{t-1} \log P_t - \alpha \log P_{t-1}^{UE} - \kappa \alpha E_{t-2} \log P_{t-1},
\]
(31)

We can see that this creates a potential correlation between \(E_{t-1} \log P_t\) and \(\log P_{t-1}^{UE}\). Suppose there is a shock to the price level, then under price-level targeting there is a commitment to remove some or all of this shock from next period’s price level; thus write \(E_{t-1} \log P_t = \rho(1 - \beta) \log P_{t-1}^{UE}\) where \(\rho\) is the model-generated persistence in prices and \(\beta\) is the extent of its removal by the price-targeting rule. It follows that the variance of \(\log P_t^*\) which enters expected welfare will equal \((\frac{1}{1 - \alpha})^2[\text{Var} \log P_t^{UE}](1 + [\kappa \alpha (1 - \beta)]^2 + \{\alpha^2 + [k \rho (1 - \beta)]^2 - 2 \kappa \alpha \rho (1 - \beta))\)). The difference of this from the variance at \(k = 0\) is \((\frac{1}{1 - \alpha})^2[\text{Var} \log P_t^{UE}][k \alpha \rho (1 - \beta)]^2 + [k \rho (1 - \beta)]^2 - 2 \kappa \alpha \rho (1 - \beta))\). For this difference to be negative for positive \(\{k, \rho (1 - \beta)\}\) we require that \(2 \alpha > \rho (1 - \beta)(1 + \kappa \alpha^2)\). Price-level targeting generates a value of \(\beta\) close to unity, hence reducing the right hand side to close to zero- the stricter the closer to zero. However, inflation-targeting tends to induce a positive serial correlation, \(\rho_\alpha\), between rates of inflation; thus the serial correlation between price levels is \(1 + \rho_\alpha = \rho (1 - \beta)\) here.

So what we find is that Calvo persistence produces a reason to bias indexation away from full in order to induce a helpful correlation offsetting the persistence. Nevertheless it remains optimal to use rational indexation in preference to lagged, essentially because the latter introduces unnecessary extra correlations which tend to raise the variance of \(\log P_t^*\).

What are the impulse responses to a monetary shock under the three monetary regimes we have identified? The charts show them in turn for inflation targeting, price targeting and stricter price targeting, the latter two with endogenously slightly lower than full (rational) indexation. What we see is that they have none of the supposed hallmark properties of New Keynesian models: there is little persistence, no ‘bump shape’ in either inflation or output, but rather there is a brief moving shock oscillation followed by virtually no residual effect at all. Price level targeting increases stability, the stricter the greater the increase. As for the productivity shock (figure ??) we see a rather similar effect to that of a monetary shock superimposed on the steady declining effect of declining productivity on output and consumption. There is plainly no nominal rigidity to speak of in these effects; there is solely an effect of the Calvo mechanism causing relative prices to move in response to both shocks because only a minority of price and wage setters are able to change their relative price currently in response to a current shock.

4 CONCLUSION

We conclude that the Calvo contract adjusted for rationally expected indexation under both monetary regimes - inflation and price level targeting- delivers the highest expected welfare. This holds under both information assumptions though under full information price-level targeting tends to induce slightly less than full indexation because price shocks are less persistent. Rational indexation eliminates the effectiveness of monetary policy on welfare when there is only price-setting under only current micro information. However in the broader context of wage-setting and full current information, both of which are standard in New Keynesian models, monetary policy regains effectiveness; and the class of interest rate rules that delivers the highest benefits are those which target the price level as strictly as possible, the reason being that this policy minimises the size of shocks to prices and hence of wage and price dispersion. However rational indexation even when less than full ensures that there is very little nominal rigidity in this adapted world of Calvo contracts. It may be replied that in this case Calvo contracts lose their empirical attractiveness, implying that rational indexation should not be adopted for empirical reasons. This may or may not be the case and should be the subject of further work- initial indications are that models with little nominal rigidity may perform rather better empirically than has been realised. The key points we are trying to make in this paper are first that there is a strong theoretical case for such indexing; second that its effect is to deprive New Keynesian models of their central supposed features; and third that it strengthens the case for price level targeting.
Figure 1: Dynamic paths after an unexpected 0.01 rise in interest rate under different monetary regimes

Figure 2: Dynamic paths after an unexpected 0.01 rise in productivity under different monetary regimes
5 APPENDIX

5.1 Non-linear model

Consumption

\[ C_t = \frac{1}{P_t \lambda_t} \]  

(A1)

Capital constraint

\[ K_t = (1 - \delta)K_{t-1} + I_t - \frac{1}{2} \psi \left[ \frac{I_t}{K_{t-1}} - \delta \right]^2 K_{t-1} \]  

(A2)

\[ \frac{R_t}{W_t} = \frac{\nu}{1 - \nu} \frac{N_t(f)}{K_t(f)} \]  

(A3)

Marginal cost

\[ MC_t = \frac{1}{\nu \nu (1 - \nu)^{(1 - \nu)}} \frac{R_t W_t^{1-\nu}}{Z_t} \]  

(A4)

\[ \lambda_t P_t = \xi_t - \xi_t \psi \left[ \frac{I_t}{K_{t-1}} - \delta \right] \]  

(A5)

\[ \xi_t = \beta E_t \left\{ \lambda_{t+1} R_{t+1} + \zeta_{t+1} \left[ 1 - \delta - \frac{1}{2} \psi \left[ \frac{I_{t+1}}{K_t} - \delta \right]^2 + \psi \left[ \frac{I_{t+1}}{K_t} - \delta \right] \right] \lambda_{t+1} \right\} \]  

(A6)

Indexation formulas for:

Price

\[ \bar{P}_t = (\alpha' P_{t-1} + (1 - \alpha') E^{t-1} \log P_t)^{\alpha''} \]  

(32)

Wage

\[ \bar{W}_t = (\omega' W_{t-1} + (1 - \omega') E^{t-1} \log W_t)^{\omega''} \]

Price setting behaviour:

\[ P_t = \mu_P \frac{P B_t}{P A_t} \]  

(A7)

\[ P B_t = \alpha P E_t P B_{t+1} + \lambda_j \frac{MC_t}{P_t} \left( \frac{P_t^{\phi} P_t}{P_{t-1}^{\phi}} \right) \bar{P}_t Y_t \]  

(A8)

\[ P A_t = \alpha P E_t P A_{t+1} + \lambda_t \left( \frac{P_t^{\phi}}{P_{t-1}^{\phi}} \right) \bar{P}_t Y_t \]  

(A9)

Aggregate price

\[ \left( \frac{P_t}{P_{t-1}} \right)^{1-\phi_p} = (1 - \alpha) \left( \frac{P_t}{P_{t-1}} \right)^{(1-\phi_p)} + \alpha \left( \frac{P_t}{P_{t-1}} \right) \]  

(A10)

Aggregate output

\[ Y_t = \frac{Z_t K_t^{\nu} N_t^{1-\nu}}{D P_t} \]  

(A11)

Price dispersion

\[ D P_t = (1 - \alpha) \left( \frac{P_t}{P_{t-1}^{\phi} (f/P_t)} \right)^{\phi_p} + \alpha \left( \frac{P_t}{P_{t-1}^{\phi}} \right)^{\phi_p} \left( \frac{P_t}{P_{t-1}^{\phi}} \right) ^{-\phi_p} D P_{t-1} \]  

(A12)

Market clearing condition

\[ Y_t = C_t + I_t + G_t \]  

(A13)

Wage setting behaviour

\[ W_t^{(1+\phi_w) \chi} = \mu_w \frac{W B_t}{W A_t} \]  

(A14)
\[ WB_t = \omega \beta E_t WB_{t+1} + N_{t}^{1+\chi} \left( \frac{W_t}{W_t} \right) \phi_w(1+\chi) \]  
\[ (A15) \]

\[ WA_t = \omega \beta E_t WA_{t+1} + \lambda_t \bar{P}_t N_t \left( \frac{W_t}{W_t} \right) \phi_w \]  
\[ (A17) \]

Aggregate wage
\[ \left( \frac{W_t}{W_t} \right)^{1-\phi_w} = (1 - \omega)W_t^{(1-\phi_w)} + \omega \left( \frac{W_{t-1}}{W_{t-1}} \right)^{1-\phi_w} \]  
\[ (A18) \]

Welfare
\[ U_t = E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left[ \log C_t - \frac{1}{(1+\chi)} AL_t \right], \]  
\[ (A19) \]

Average disutility of work
\[ AL_t = N_t^{1+\chi} DW_t \]  
\[ (A20) \]

Wage dispersion
\[ DW_t = (1 - \omega) \left( \frac{W_t^*(0)\bar{W}_t}{W_t} \right)^{-\phi_w(1+\chi)} + \omega \left( \frac{W_{t-1}}{W_{t-1}} \right)^{-\phi_w(1+\chi)} \frac{DW_{t-1}}{W_{t-1}} \]  
\[ (A21) \]

Euler equation
\[ E_t \lambda_{t+1}^{t+1} = E_t \Delta_{t,t+1} = \frac{1}{1+i} \]  
\[ (A22) \]

Monetary policy
\[ i_t = 0.222 + 0.82i_{t-1} + 0.35552\pi_t + 0.032384 (output\ gap)_t + \epsilon_{i,t}, \]  
\[ (A23) \]

or
\[ i_t = 0.222 + 0.82i_{t-1} + 0.35552\log P_t + 0.032384 (output\ gap)_t + \epsilon_{i,t}, \]  
\[ (A24) \]

Following Canzoneri et al (2004) we define the output gap as the log deviation of output from its steady state level.

### 5.2 Inflation targeting rule under restricted information assumption

#### 5.2.1 Equation

Equation (12) is rewritten as

\[ \log P_t - k_0 (1 - k_1) E^{t-1} \log P_t - (k_0k_1 + \alpha) \log P_{t-1} + \alpha k_0 (1 - k_1) E^{t-2} \log P_{t-1} + \alpha k_0 k_1 \log P_{t-2} \]
\[ = \frac{(1 - \alpha) (1 - \alpha \beta) \sigma^* \chi^* (B^{-1} - 1) \log Z_t - (1 - \alpha) (1 - \alpha \beta) \sigma^* \chi^* E^{t-1} \log M_t}{(1 - \alpha \beta E^{-1} B^{-1}) (1 - \sigma E^{t-1} B^{-1})} \]
\[ - \frac{(1 - \alpha) (1 - \alpha \beta) [\sigma^* \chi^* - (1 - \sigma B^{-1}) + k_0 (1 - k_1) (1 - \sigma E^{t-1} B^{-1})] E^{t-1} \log P_t}{(1 - \alpha \beta E^{-1} B^{-1}) (1 - \sigma E^{t-1} B^{-1})} \]
\[ + \frac{(1 - \alpha) (1 - \alpha \beta) [\sigma^* \chi^* - k_0 k_1 (1 - \sigma E^{t-1} B^{-1})] \log P_{t-1}}{(1 - \alpha \beta E^{-1} B^{-1}) (1 - \sigma E^{t-1} B^{-1})} \]  
\[ (33) \]

where \( \frac{1+\chi}{1-\sigma} = \chi^* \).
5.2.2 LHS of equation

Take all the price related variables to the RHS of this equation, we get the equation (34) in the text. The RHS of the latter equation is in the text, and assuming \( \rho_1 = \rho_2 \), the derivation of the LHS is as following:

\[
(1 - \alpha) (1 - \alpha \beta) \chi^* \sigma^* \left( \frac{E_t (1 - B^{-1}) (-\log Z_t) (1 - \alpha \beta E^{-1} B^{-1}) (1 - \sigma^* E^{-1} B^{-1})}{(1 - \alpha \beta B E^{-1} B^{-1}) (1 - \sigma^* B E^{-1} B^{-1})} - E^{-1} \log M_t \right)
\]

\[
= (1 - \alpha) (1 - \alpha \beta) \chi^* \sigma^* \left( \frac{(\rho_1 - 1) (1 - \alpha \beta E^{-1} B^{-1}) (1 - \sigma^* E^{-1} B^{-1}) \log Z_t}{(1 - \alpha \beta \rho_1) (1 - \sigma^* \rho_1)} - \rho_2 \log M_{t-1} \right)
\]

\[
= (1 - \alpha) (1 - \alpha \beta) \chi^* \sigma^* \left( \frac{(\rho_1 - 1) (1 - \alpha \beta E^{-1} B^{-1} - \sigma^* E^{-1} B^{-1} + \alpha \beta E^{-1} B^{-2}) \log Z_t}{(1 - \alpha \beta \rho_1) (1 - \sigma^* \rho_1)} \right)
\]

\[
= \chi^* \left[ (\rho_1 - 1) \left( 1 - \alpha \beta E^{-1} B^{-1} - \sigma^* E^{-1} B^{-1} + \alpha \beta E^{-1} B^{-2} \right) \log Z_t \right]
\]

\[
= \chi^* \left[ \left( \frac{\rho_1 - 1}{1 - \rho_1} \right) \left( 1 - \alpha \beta \rho_1 \right) \left( 1 - \sigma^* \rho_1 \right) \log M_{t-1} \right]
\]

\[
5.2.3 \text{Collect unknown coefficients}
\]

Multiplying both RHS and LHS by \((1 - \rho_1 L)\), we get equation (13) in the text. Given that \( \log P_t = \sum_{i=0}^{\infty} \varepsilon_i z_{t-i} + \sum_{i=0}^{\infty} \varepsilon_i \mu_{t-i} \), we determine the unknown coefficients on \( \mu_{t-i} \) as follows

\[
\mu_0 = 0
\]

\[
(\mu_{t-1})
\]

\[
0.564 (1 - k_0 + k_0 k_1) \varepsilon_3 + (-1.8 + 1.42 k_0 - 1.98 k_0 k_1) \varepsilon_2 + (2.8945 - 0.8889 k_0 + 2.69 k_0 k_1) \varepsilon_1 - (2.5847 + 1.9028 k_0 k_1) \varepsilon_0 = -0.0933
\]

\[
(\mu_{t-2})
\]

\[
0.564 (1 - k_0 + k_0 k_1) \varepsilon_4 + (-2.321 - 2.322 k_0 + 2.881 k_0 k_1) \varepsilon_3 + (4.5549 - 3.2089 k_0 + 5.532 k_0 k_1) \varepsilon_2 + (-4.3347 + 1.4915 k_0 - 5.0556 k_0 k_1) \varepsilon_1 + (1.534 + 2.43 k_0 k_1) \varepsilon_0 = 0
\]

\[
(\mu_{t-i}, i \geq 3)
\]

\[
0.564 (1 - k_0 + k_0 k_1) \varepsilon_{i+2} - (2.321 - 2.322 k_0 + 2.881 k_0 k_1) \varepsilon_{i+1} + (4.5549 - 3.56 k_0 + 5.883 k_0 k_1) \varepsilon_i - (4.3347 - 2.4275 k_0 + 6 k_0 k_1) \varepsilon_{i-1} + (1.534 - 0.62 k_0 + 3.05 k_0 k_1) \varepsilon_{i-2} - 0.62 k_0 k_1 \varepsilon_{i-3} = 0
\]

5.3 Solution for \( \log P^*_t \) under Rational indexation

It is a brief description of solution for \( \log P^*_t \), for the details, see Le and Minford (ibid.). From the equations (9) and the assumption of rational expectation, the reset price is

\[
\log P^*_t = E^{-1} \log P^*_t + \log P^{*UE}_t = \log P^{*UE}_t - \frac{\alpha}{1 - \alpha} \log P^{UE}_{t-1}
\]

where

\[
\log P^{UE}_t = (1 - \alpha) \log P^{*UE}_t
\]
and from equation of the general price level and the assumption that firms have knowledge of their own micro information in period t as well as the macro information of period (t - 1):

\[ \log P_t^{UE} = \log P_t - E_t^{-1} \log P_t^{*} \]

\[ = (1 - \alpha \beta) \sum_{i=0}^{\infty} (\alpha \beta)^i \left( \frac{1 + \chi}{1 - \nu} \right) \left[ E_t^{i-1} \log Y_{t+i} - E_t \log Z_{t+i} \right] \]

\[ - (1 - \alpha \beta) \sum_{i=0}^{\infty} (\alpha \beta)^i \left( \frac{1 + \chi}{1 - \nu} \right) \left[ E_t^{i-1} \log Y_{t+i} - E_t^{-1} \log Z_{t+i} \right] \]

\[ = (1 - \alpha \beta) \left( \frac{1 + \chi}{1 - \nu} \right) \frac{1}{1 - \alpha \beta \beta_1} (-z_t) \]

Hence the renewed price is rewritten as

\[ \log P_t^{*} = \log P_t^{UE} - \alpha \log P_t^{UE} = \chi \frac{(1 - \alpha L)}{(1 - \alpha)} (-z_t) , \]

And the expected welfare under the rational indexation for prices is

\[ E(u_t - u_t^{FLEX}) = -\phi \alpha \chi^{''} \text{var}(z) \]

This solution is identical under two monetary policy rules, that targets inflation and price level.

### 5.4 General Solution for \( \log P_t^{*} \)

From equation (9)

\[
\log P_t^{*} = \frac{1}{1 - \alpha} \left[ \log P_t - \log \hat{P}_t - \alpha \left( \log P_{t-1} - \log \hat{P}_{t-1} \right) \right]
\]

\[ = \frac{1}{1 - \alpha} \left\{ \log P_t - k_0 \left( E_t^{-1} \log P_t + k_1 \left( \log P_{t-1} - E_t^{-1} \log P_t \right) \right) \right\} \]

\[ = \frac{1}{1 - \alpha} \left\{ -\alpha \left[ \log P_{t-1} - k_0 \left( E_t^{-1} \log P_{t-1} + k_1 \left( \log P_{t-2} - E_t^{-2} \log P_{t-1} \right) \right) \right] \right\} \]

\[ = \frac{1}{1 - \alpha} \left\{ E_t^{-1} \log P_t + \log P_t^{UE} - \alpha \left( E_t^{-2} \log P_{t-1} + \log P_t^{UE} \right) \right\} \]

\[ = \frac{1}{1 - \alpha} \left\{ E_t^{-1} \log P_t - k_0 E_t^{-1} \log P_{t-1} \right\} \]

where \( P_t^{UE} = (1 - \alpha) \left( 1 - \alpha \beta \right) \left( \frac{1 + \chi}{1 - \beta} \right) \frac{1}{1 - \alpha \beta \beta_1} (-z_t) = -\chi' z_t \), and it does not depend on any monetary shocks.

Therefore, consider only productivity shocks, equation (44) can be rewritten as

\[
\log P_t^{*} (z_{t-1}) = \frac{1}{1 - \alpha} \left\{ [z_\chi' z_t + \alpha \chi' z_{i-1}] + (1 - k_0) E_t^{-1} \log P_t - \alpha (1 - k_0) E_t^{-2} \log P_{t-1} \right\}
\]

\[ - k_0 k_1 v_{t-1} + \alpha k_0 k_1 v_{t-2} \]

given that the term \( \frac{1}{1 - \alpha} [z_\chi' z_t + \alpha \chi' z_{i-1}] \) is equal to \( \log P_t^{*} \) under rational indexation. This equation in term can be written using Wold decomposition:

\[
\log P_t^{*} (z_{t-1}) = \frac{1}{1 - \alpha} \left\{ [z_\chi' z_t + \alpha \chi' z_{i-1}] + (1 - k_0) E_t^{-1} \log P_t - \alpha (1 - k_0) E_t^{-2} \log P_{t-1} \right\}
\]

\[ - k_0 k_1 (z_{t-1} - E_t^{-1} \log P_t) + \alpha k_0 k_1 (z_{t-2} - E_t^{-2} \log P_{t-1}) \]

\[ = \frac{1}{1 - \alpha} \left\{ [z_\chi' z_t + \alpha \chi' z_{i-1}] + (1 - k_0) \sum_{i=0}^{\infty} \xi_{i-1} \right\} - \alpha (1 - k_0) \xi_1 \xi_{i-1} \xi_{i-2} \xi_{i-3} \right\}

\[ = \frac{1}{1 - \alpha} \left\{ [z_\chi' z_t + \alpha \chi' z_{i-1}] + (1 - k_0) \xi_{t-1} \right\} - \alpha (1 - k_0) \xi_1 \xi_{i-1} \xi_{i-2} \xi_{i-3} \right\}

\[ = \frac{1}{1 - \alpha} \left\{ [z_\chi' z_t + \alpha \chi' z_{i-1}] + (1 - k_0) \frac{\xi_{t-1}}{1 - \alpha} \right\} - \alpha (1 - k_0) \frac{\xi_1}{1 - \alpha} \xi_{i-1} \xi_{i-2} \xi_{i-3} \right\}

\[ = \frac{1}{1 - \alpha} \left\{ [z_\chi' z_t + \alpha \chi' z_{i-1}] + (1 - k_0) \frac{\xi_{t-1}}{1 - \alpha} \right\} - \alpha (1 - k_0) \frac{\xi_1}{1 - \alpha} \xi_{i-1} \xi_{i-2} \xi_{i-3} \right\}

\[ + \alpha k_0 k_1 \xi_{t-2} + \frac{1}{1 - \alpha} \frac{\xi_3 (k_0 k_1 + (1 - k_0) \xi_1) (1 + \alpha) + \alpha (1 - k_0) + \alpha k_0 k_1 \xi_1}{1 - \alpha} z_{t-3} + ... \]
Assume that there are only monetary shocks \( \mu_{t-i} \), then equation (44) is

\[
\log P_t^* (\mu_{t-i}) = \frac{1}{1 - \alpha} \left\{ \begin{array}{l}
(1 - k_0) E^{t-1} \log P_t - \alpha (1 - k_0) E^{t-2} \log P_{t-1} \\
- \frac{k_0 k_1}{1 - \alpha} \log P_{t-1} - E^{t-1} \log P_t + \alpha k_0 k_1 (\log P_{t-2} - E^{t-2} \log P_{t-1})
\end{array} \right\}
\]

\[
= \frac{1}{1 - \alpha} \left\{ \begin{array}{l}
(1 - k_0) \sum_{i=1}^{\infty} \Sigma_{t-i} - k_0 k_1 \left( \sum_{i=2}^{\infty} \Sigma_{t-i} - \sum_{i=1}^{\infty} \Sigma_{t-i} \right) \\
+ \alpha k_0 k_1 (\sum_{i=2}^{\infty} \Sigma_{t-i} - \sum_{i=1}^{\infty} \Sigma_{t-i}) - \alpha (1 - k_0) \sum_{i=1}^{\infty} \Sigma_{t-i} \right\}
\]

\[
= \frac{-k_0 k_1 (\Sigma_0 - \Sigma_1) + (1 - k_0) \Sigma_1}{1 - \alpha} z_{t-1} + \frac{\Sigma_2 (k_0 k_1 + (1 - k_0)) - \Sigma_1 (k_0 k_1 (1 + \alpha) + \alpha (1 - k_0)) + \alpha k_0 k_1 \Sigma_0}{1 - \alpha} z_{t-2} + \frac{\Sigma_3 (k_0 k_1 + (1 - k_0)) - \Sigma_2 (k_0 k_1 (1 + \alpha) + \alpha (1 - k_0)) + \alpha k_0 k_1 \Sigma_1}{1 - \alpha} z_{t-3} + \ldots
\]

(47)

6 Bibliography

References


