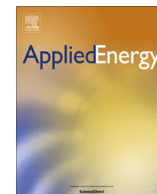




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Steady state analysis of gas networks with distributed injection of alternative gas[☆]

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HIGHLIGHTS

- A steady-state analysis method for gas networks was developed.
- This method is used for gas networks with distributed injection of alternative gas.
- A gas network with injection of upgraded biogas and hydrogen was simulated.
- Results show the impact on pressure and gas quality in the network.

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ABSTRACT

A steady state analysis method was developed for gas networks with distributed injection of alternative gas. A low pressure gas network was used to validate the method. Case studies were carried out with centralized and decentralized injection of hydrogen and upgraded biogas. Results show the impact of utilizing a diversity of gas supply sources on pressure distribution and gas quality in the network. It is shown that appropriate management of using a diversity of gas supply sources can support network management while reducing carbon emissions.

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1. Introduction

The future role of gas in the UK energy mix has become an increasingly debated issue [1,2]. It is evident that to meet the statutory carbon emission targets the use of natural gas needs to decline over time [3]. However, most recently published future scenarios [4] expect that natural gas will continue to play a pivotal role in the transition to a low carbon energy system.

The UK Government strategy for low carbon heat [5] identifies opportunities to decarbonize parts of the gas network by using

renewable gas. Government incentives are already in place for developers of anaerobic digesters to inject upgraded biogas into the natural gas grid [6]. There are proposals to inject hydrogen produced from renewable sources in the natural gas network [7]. This would allow the very large transport and storage capacities of the existing gas infrastructure to be used for indirect electricity transport and storage [8]. A number of other new sources of gas are also anticipated to be injected into the gas distribution grid: i.e. biomass gasification products, shale gas, coal bed methane. However, the impact of using dissimilar gas supply sources in the existing natural gas system needs to be carefully investigated.

The current gas quality standards are based on the quality of gas sourced from the UK continental shelf (UKCS) [10]. This has traditionally been the primary source of supply for Britain. However, over the next few decades, it may be necessary to assess the compatibility of the gas supply system to operate with a diversity of gas sources. Some of these new gas sources are likely to be geographically clustered which could have significant implications for managing the distribution network, both locally and at a network wide level.

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Modeling and simulation allows the study of gas network operation by means of mathematical models of gas flow in pipes. If it is assumed that mathematical models are adequate, simulation will obtain a detailed knowledge of the real properties of the network under varying operational conditions. Simulations can be carried out in steady and unsteady states. The scope of this work is on simulating gas networks in steady state. Steady state is a snapshot of gas network operation where the parameters characterizing the flow of gas are independent of time.

Steady state analysis of gas networks is usually used to compute nodal pressures and pipe flows for given values of source node pressures and gas consumption [10]. Newton-nodal, newton loop and Hardy Cross methods are widely used [10–15].

Traditional methods of modeling and simulating gas networks assume a gas mixture with a uniform composition to be transported via the network [10,11]. Methods for simulation of gas networks considering a diversity of alternative gas injections have not been reported. The method proposed advances the conventional method used for steady state analysis, and allows studying the impact of injecting alternative gas supplies (E.g. Hydrogen, biogas) at different locations on a given network. The model can support decision making on the allowable amount and content of alternative gas in distribution grids. A steady state analysis method was developed for gas networks with distributed injection of alternative gas. Two approaches for gas demand formulation are compared. A case study is used to demonstrate the applicability of the method to analyze the impact of using an alternative gas mixture (High hydrogen, upgraded biogas) and distributed injection of alternative gas (hydrogen, upgraded biogas) on the steady state gas flow parameters. The results of the case study are discussed and the performance of the model compared to traditional methods of gas network simulation.

2. Impact of injecting alternative gases in the natural gas grid

Injection of alternative gases in a gas grid has an impact both at appliance level and at network level. According to [16], a gas appliance is adjusted to function properly at the “normal test pressure” for the given type of gas and must then operate satisfactorily, without additional adjustment, within specified limits of appliance inlet pressure. Utilizing a diversity of supply sources may lead to stronger variations in gas composition. Amidst these variations, appliance burners have to perform satisfactorily without readjustment on fuel gases that vary considerably in their combustion characteristics from the gas mixture on which the initial adjustment was made. Extensive research has been undertaken for ways to predict the interchangeability of one gas with another [17–20]. It was recognized that correlation of heating values and specific gravities alone was insufficient, and a third factor, namely flame characteristics which depend on chemical composition, must be included. However, formulas and indexes based only on the two former factors are widely used because of their simplicity. The Wobbe index is widely used in Europe, together with a measured or calculated flame speed factor for assessing interchangeability [16]. Wobbe index is defined as

$$\text{Wobbe Index} = \frac{\text{Gross Calorific Value}}{\sqrt{\text{Specific Gravity}}} \quad (1)$$

This number is proportional to the heat input to a burner at a constant pressure [11].

Different components of a gas system, such as underground and over ground storage, gas turbines and engines, domestic and industrial appliances, compressors, and valves, are usually designed to transport and operate on natural gas with a consistent quality. The tolerance level of these components to gas mixture

composition can vary. For example an admixture of up to 10% hydrogen in volume of natural gas is possible in parts of the natural gas system whereas the limit drops to 2% if a natural gas vehicles refuelling system is connected (due to steel tanks in natural gas vehicles) [7]. Therefore, at present it is not possible to specify limiting values for alternative gas injections which would be valid for all parts of the gas infrastructure.

Variations in gas composition may also have an impact on temperature and pressure changes in the pipelines [20,21]. The flow of a fluid in a pipe is governed by the Navier–Stokes equation [22]. If steady state is assumed and the effect of gravity on the fluid flow is neglected, the relationship reduces to an equation which is usually used for the calculation of pressure drop dp along a length dx of a pipeline with internal diameter D (Darcy’s equation) [10,21],

$$dp = -0.5 \times \frac{\rho v^2 \lambda \times dx}{D} \quad (2)$$

ρ is the mass density of the gas at the pressure and temperature of the pipeline, v is the average velocity of the gas and λ is the friction factor.

Integration of this equation gives the pressure drop over a finite length of the pipeline. The friction factor λ depends on the Reynolds number and the pipe roughness. Reynolds number is given by

$$\text{Re} = \frac{\rho v D}{\eta} \quad (3)$$

where η is the dynamic viscosity of the gas mixture at the pressure and temperature of the pipeline.

If η is taken as constant (most gas distribution systems operate in partial turbulent region) and the temperature is assumed constant along a given pipeline, the pressure drop is a function of the mass density and the average velocity of the gas in the pipe. In normal situations the average velocity of gas flow to supply a given energy demand can be calculated using the calorific value of gas. However, when the gas mixture composition varies from that of natural gas (due to the use of dissimilar gas supply sources), the volume flow (therefore velocity) of gas to transport the same amount of energy will vary. The combined effect of mass density of the new gas mixture and the velocity of gas required to meet the energy demand will affect the pressure drop in gas pipelines.

Therefore, network analysis with injection of alternative gases needs to consider the variability of gas composition in different parts of the network, and its impact on the state of the network. Several studies have initiated methods for tracking the calorific value of gas in gas distribution grids [23], however to our knowledge none have considered the simulation of gas networks in steady state.

Previous work on assessing the impact of alternative gas supply sources on the existing gas network has focused on the durability and safety aspects of gas system components to different gas mixtures (e.g. hydrogen tolerance levels in components of the gas system) [7,20]. The impact of gas mixture properties on gas pipe flow and thereby gas network operation and management have not been investigated in detail. Such a method would require assessing the properties of each injected gas and admixtures of the injected gas with natural gas at each node in the network where two or more flows combine. These properties, for example specific gravity, calorific value combined with quantities of gas injections will impact on the consequent flow pattern and the pressure delivery of the gas system. The method would enable to gain valuable insights to the allowable quantities and types of alternative gas in different load conditions of the network such that

- (a) the tolerance levels of the gas system to different gas admixtures is not compromised and

(b) suitable pressures to ensure the safe operation of gas appliances is delivered.

3. Method of steady state analysis with distributed injection of alternative gas

3.1. Steady state analysis problem in gas networks

A single pressure tier in gas distribution i.e. low pressure or medium pressure network is modeled. A typical gas network within a single pressure tier may consist one or more natural gas infeed sites [11], distributed alternative gas supply sites, gas loads and pipelines. Compressors and pressure regulator valves are not modeled as they usually represent the interface between two pressure tiers.

A directed graph is used as an efficient way to represent and model a gas network [10]. The pipelines are represented by branches (also called edges or arcs). The interconnection points of pipelines, gas loads and sources are represented by nodes (or vertices).

The problem of simulation of gas networks in steady state is to compute the value of node pressures and the value of gas flows in individual pipes for known source pressures, source gas mixture composition and gas load demand. The pressure at the nodes and the flow rates in the pipes must satisfy the pipe flow equation and must meet the gas load demand while satisfying the first and second Kirchoff's laws.

A summary of the gas network steady state analysis problem is shown in Table 1

3.2. Formulation of the steady state equations

In the proposed method, a set of algebraic equations, equal in number to the state variables to be calculated are formulated using the gas pipe flow equations and Kirchoff's first law applied at nodes. The following section describes the formulation of steady state equations for network analysis.

3.2.1. Gas load demand

Energy demand at a node depends on the gas appliances connected to that particular location in the network. In conventional gas network analysis methods, gas flow demand, driven by appliance pressure regulator valves (at standard temperature and pressure conditions (STP)) is used as a proxy to energy demand (usually in [m³/h]) [10]. This is suitable, when the gas composition across the network is uniform and the gas flow demand is directly proportional to the combustion energy demand.

$$H_{load} \propto Q_{load} \quad (4)$$

where H_{load} – Energy demand, Q_{load} – Gas flow rate demand.

The combustion energy required at a gas node i , can be calculated as,

$$H_{load,i} = Q_{load,i} \times GCV_i \quad (5)$$

where GCV – Gross Calorific Value of gas.

However, in an area with multiple supply sources, the composition of gas mixture across the network may vary. In order to perform an accurate analysis which meets the energy demand, the gas flow demand needs to be calculated depending on the gas mixture composition delivered at each node. Therefore, the proposed method of solution computes the gas flow demand considering the calorific value of the gas mixture at the load node. The method of calculating the gas composition is described in Section 3.2.4. For comparison, the conventional method of specifying the gas flow demand is also simulated i.e. gas flow demand calculated assuming the calorific value of natural gas. The method for calculating the specific density and calorific value for a gas mixture at STP are as specified in the European Standard EN ISO 6976:2005.

3.2.2. Distributed gas supply sources

Distributed gas supply sources are modeled as gas flow injections at specified nodes. The gas flow rate injected at a gas node i , can be calculated as,

$$Q_{source,i} = (-1) \times \frac{H_{source,i}}{GCV_{source,i}} \quad (6)$$

where Q_{source} – Gas flow rate injected, H_{source} – Gaseous energy injection rate, GCV – Gross calorific value of the supply source.

For gas nodes where both a demand and distributed injection exist, from Eqs. (5) and (6) the net gas load at gas node i , can be written as

$$H_{net\ demand,i} = H_{load,i} - H_{supply,i} \quad (7)$$

$$Q_{net\ demand,i} \left(\frac{m^3}{s} \right) = \frac{H_{load,i} - H_{source,i} \left(\frac{kJ}{s} \right)}{GCV_{node} \left(\frac{kJ}{m^3} \right)} \quad (8)$$

For distributed gas supply nodes, the net gas load is negative, and for demand nodes it is positive.

3.2.3. Pipe flow formulation

The general flow equation for steady-state gas flow is derived as [10]

$$Q_n = \sqrt{\frac{\pi^2 R_{air} T_n}{64 p_n}} \sqrt{\frac{[(p_1^2 - p_2^2) - \frac{2p_{av} S g h}{Z R_{air} T}]}{f S L T Z}} D^5 \quad (9)$$

where Q_n – pipe volume flow in Standard Temperature and Pressure (STP); p_1 – pressure at pipe starting node; p_2 – pressure at pipe end node; D – Diameter of pipe; f – friction factor; S – Specific gravity; L – length of pipe, R_{air} – Density of air at STP, T_n – Temperature at STP, p_n – Pressure at STP, p_{av} – average pressure in pipe, g – gravitational acceleration, h – difference in elevation at pipe starting node and end node; T – Temperature of gas; Z – compressibility of gas.

A number of assumptions for simplification are applied in the derivation of the general flow equation for network analysis, which are [10]

1. Steady flow.
2. Isothermal flow due to heat transfer with the surroundings through the pipe wall.

Table 1
Summary of the gas load flow problem.

Node type	No of nodes (Total nodes = N)	Quantities specified	State variables to be calculated
Main natural gas source node	N_s	Pressure, gas mixture composition	–
Alternative gas injection node	N_I	Gas injection (Volume flow), gas mixture composition	Pressure
Load node	N_L	Gas load demand (Energy demand)	Pressure
Pipe intersection nodes	$N - N_s - N_I - N_L$	–	Pressure

3. Negligible kinetic energy change in the pipe.
4. Constant compressibility of the gas over the length of the pipe.
5. Validity of Darcy friction loss relationship.
6. Constant friction coefficient along the pipe length.

Several simplified flow equations are used in the gas industry. The main differences are on the expression assumed for the friction factor. The pipe flow equation is reduced to a functional relationship [10] between gas flow, pressure drop in pipes, pipe dimensions, average temperature and characteristics of gas. The pipe parameters and average temperature of gas is assumed to be constant for a given simulation. The change of temperature in the gas at injection is neglected. The following simplified equations are used in the model for the case of low pressure and medium pressure networks.

For low pressure networks (<75 mbar gauge), Lacey's equation is used [10,11]

$$Q_n = 5.72 \times 10^{-4} \sqrt{\frac{(p_1 - p_2)D^5}{fSL}} \quad (10)$$

where Q_n – pipe volume flow in STP; p_1 – pressure at pipe starting node; p_2 – pressure at pipe end node; D – Diameter of pipe; f – friction factor; S – Specific gravity; L – length of pipe.

Where value of f is determined by the Unwin's low pressure formula

$$f = 0.0044 \left(1 + \frac{12}{0.276D} \right) \quad (11)$$

For medium pressure networks (0.75–7 bar gauge), Polyflo equation is used [10,11]

$$Q_n = 7.57 \times 10^{-4} \times \frac{T_n}{p_n} \sqrt{\frac{(p_1^2 - p_2^2)D^5}{fSLT}} \quad (12)$$

where T_n – Temperature at STP; p_n – Pressure at STP, T – Temperature of gas.

Where value of f is determined by

$$\sqrt{\frac{1}{f}} = 5.338(Re)^{0.076}E \quad (13)$$

where Re – Reynolds number; E – efficiency factor for the pipe.

3.2.4. Nodal formulation

According to Kirchhoff's first law, the algebraic sum of the gas flows at any node is zero. Assuming perfect mixing i.e. when the mixing of gases create no chemical reaction or state difference in the constituent gases, this means the gas flow demand at any node is equal to the sum of branch flows into and out of the node. Therefore, at any gas node i ,

$$\sum Q_{in,i} - \sum Q_{out,i} = Q_{net\ demand,i} \quad (14)$$

where $Q_{in,i}$ – Incoming volume flows to node i , $Q_{out,i}$ – Outgoing volume flows from node i , $Q_{net\ demand,i}$ – Net gas demand at node i .

Eq. (14) is expressed as,

$$\sum_{j=1}^m a_{ij}Q_j = Q_{net\ demand,i} \quad j = 1, \dots, N \quad (15)$$

where m – number of branches, N – total number of nodes, a_{ij} element from row i and column j in the branch nodal incidence matrix (Branch nodal incidence matrix is a matrix representation of branches and their connections to nodes in a directed graph [10]); Q_j – volume flow rate in branch j ; $Q_{net\ demand,i}$ – Net gas demand at node i as formulated in Eq. (8).

Eq. (15) in matrix form is,

$$AQ = Q_{net\ demand} \quad (16)$$

where A – Branch nodal incidence matrix; Q – Branch flow rate vector; $Q_{net\ demand}$ – Nodal gas demand vector.

Due to the diversity of supply sources in the network, specific gravity of gas mixtures flowing into the node may be dissimilar. To calculate the specific gravity of gas mixture flowing out of a node and also to any demand, an equation for mass continuity at each node is written. At any gas node i , this is

$$\sum (Q_{in,j} \times S_{in,j}) - \left(\sum Q_{out,i} \right) \times S_{out,i} = (Q_{net\ demand,i}) \times S_{out,i} \quad (17)$$

where $Q_{in,i}$ – Incoming volume flows to node i ; $S_{in,j}$ – Specific gravity of incoming gas mixture; $Q_{out,i}$ – Outgoing volume flows from node i ; $S_{out,j}$ – Specific gravity of outgoing gas mixture; $Q_{net\ demand,i}$ – Net gas demand at node i .

$$S_{out,i} = \frac{\sum Q_{in,i} \times S_{in,i}}{\left[\sum Q_{out,i} + Q_{net\ demand,i} \right]} \quad (18)$$

Specific gravity effect the volume flow for a given pressure difference across the pipe (Eq. (9)). Therefore to accurately calculate the volume flow rate in a pipe, specific gravity of the gas mixture needs to be determined. An algorithm developed for computing the specific density of the gas mixture at each node/pipe for a given set of nodal pressures is shown in Fig. 1. As the first step, an order for the analysis of branch flows need to be established considering node pressures. This sequence ensures the composition of incoming gas flows to a particular branch are always known. The second step performs a mass flow balance at each node according to the sequence established and thereby progressively computes the specific gravity, gas mixture composition and pipe flow rate at each branch.

The pressure drops in any branch J , are related to nodal pressures as follows

$$\Delta P_j = \sum_{i=1}^N -a_{ji}P_i \quad j = 1, \dots, m \quad (19)$$

where ΔP_j – pressure drop in branch j ; m – number of branches; N – total number of nodes; a_{ji} – element from row j and column i in the branch nodal incidence matrix; P_i – pressure at node i .

In matrix form Eq. (19) is expressed as

$$\Delta P = -A^T P \quad (20)$$

where ΔP – vector of pressure drops in branches; A – branch nodal incidence matrix; P – vector of nodal pressure.

According to pipe flow Eqs. (10) and (11), branch flow rate is a function of the pressure drop and specific gravity of the gas (Friction factor is a function of constant pipe parameters and flow rate). Therefore, branch flow vector Q is expressed as

$$Q = \varphi(\Delta P, S) \quad (21)$$

$$Q = \varphi(-A^T P, S) \quad (22)$$

where Q – vector of branch flow rate; A – branch-nodal incidence matrix; P – vector of nodal pressure; S – vector of specific gravity of gas at each node; ε – to indicate a functional relationship.

Substituting for Q in Eq. (16) and $Q_{net\ demand}$ with Eq. (8)

$$\frac{H_{load,j} - H_{source,j}}{GCV_{node}} = A\varphi(-A^T P, S) \quad (23)$$

By removing the equation at main source node (where the pressure is known) Eq. (23) is rearranged as follows,

$$0 = A_1\varphi(-A^T P, S) - \frac{H_{net\ demand}}{GCV} \quad (24)$$

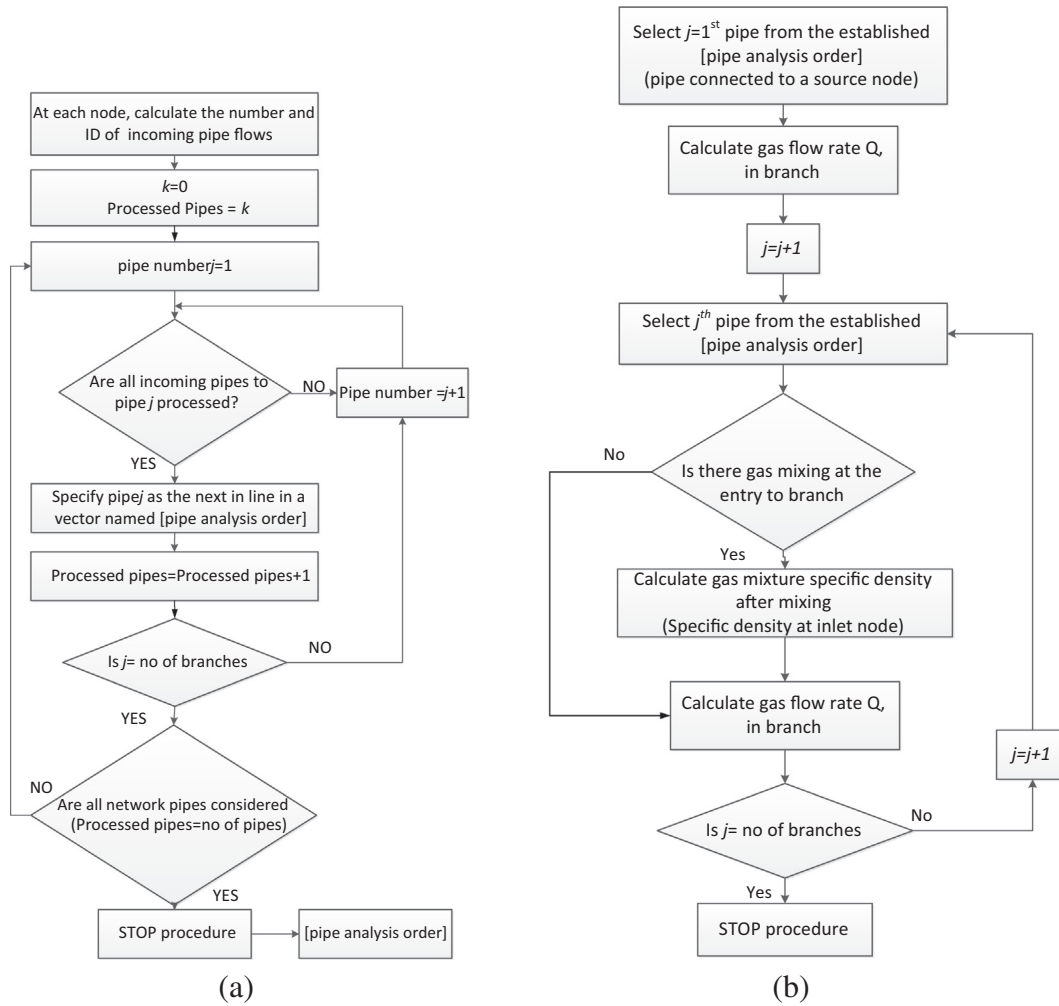


Fig. 1. (a) Algorithm for establishing a sequence for analysis of nodal flows. (b) Algorithm for progressive calculation of gas specific density and pipe flow.

where A_1 – reduced branch nodal incidence matrix; A – branch nodal incidence matrix; P – vector of nodal pressure; S – vector of specific gravity of gas at each node; $H_{net\ demand}$ – Net energy demand; GCV – Gross calorific value.

3.3. Solution method

The method proposed solve a set of non-linear Eqs. (24) formulated at each node. An initial approximation for the node pressures are iteratively corrected using the ‘Newton–Raphson’ method. At each iteration the specific gravity and calorific value at each node and branch flows are calculated using the algorithm illustrated earlier, external to the ‘Newton–Raphson’ correction.

At each iteration the left hand side of the Eq. (22) is not equal to zero. The pressures are initially only approximations of their true values and the flows calculated from these pressures are not balanced at each node. The imbalance at each node is a function of all nodal pressures (except the fixed source pressures) and is denoted as f .

The set of nodal error functions is represented by

$$F(P) = \begin{bmatrix} f_1(p_1, p_2, \dots, p_N) \\ f_2(p_1, p_2, \dots, p_N) \\ \dots \\ f_N(p_1, p_2, \dots, p_N) \end{bmatrix} \quad (25)$$

where F – vector of nodal error functions; f_i – nodal error function at node i , p_1, \dots, p_N – nodal pressures. The nodal error function for node i is expressed as,

$$f_i = \sum_{i=1}^N a_{j,i} \varphi(-A^T P, S) - \frac{H_{net\ demand,i}}{GCV_i} \quad (26)$$

And in matrix form

$$F(P) = A_1 \varphi(-A^T P, S) - \frac{H_{net\ demand}}{GCV} \quad (27)$$

The Newton nodal method solves the set of Eq. (27) iteratively until the nodal errors, $F(P)$ are less than a specified tolerance.

The iterative scheme for correcting the approximations to the nodal pressures is given in [10].

If the correction to be applied to an initial guess of nodal pressure vector is δP_1 , the calculated pressure for next iteration is calculated as

$$P_1^{k+1} = P_1^k + (\delta P_1)^k \quad (28)$$

The term δP_1 is calculated from the Taylor series expansion,

$$J^k (\delta P_1)^k = -[F(P_1)]^k \quad (29)$$

The matrix J is the nodal Jacobian matrix and is given by

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial P_1} & \frac{\partial f_1}{\partial P_2} & \frac{\partial f_1}{\partial P_3} & \dots & \frac{\partial f_1}{\partial P_N} \\ \frac{\partial f_2}{\partial P_1} & \frac{\partial f_2}{\partial P_2} & \frac{\partial f_2}{\partial P_3} & \dots & \frac{\partial f_2}{\partial P_N} \\ \dots & \dots & \dots & \dots & \dots \\ \frac{\partial f_N}{\partial P_1} & \frac{\partial f_N}{\partial P_2} & \frac{\partial f_N}{\partial P_3} & \dots & \frac{\partial f_N}{\partial P_N} \end{bmatrix} \quad (30)$$

The flow chart of the method is shown in Fig. 2.

4. Case study

Fig. 3 shows a gas network used to test and validate the model performance. A study example was designed to resemble an actual low pressure gas network.

The network was designed meshed and connected to the main gas supply via Node 1. The pressure at Node 1 was held constant at 75 mbar in all cases. An extended radial branch from Node 7 (Node 7–Node 9–Node 10–Node 11) was used to represent the critical consumer with a minimum pressure requirement. Vertices No 2, 3, 4, 5, 6, 8 are aggregated demand nodes. The length and diameter data of pipes are included in the Appendix. The energy demand at each node is shown in Table 2.

The case studies are described in Table 3. Initially, a reference case was established by simulating the network with conventional natural gas as the only source of supply at Node 1. The 2nd and 3rd case studies simulated the gas network operation with a high hydrogen natural gas mixture (10%) and an upgraded biogas mixture as the only source of supply at Node 1. The 4th and 5th case studies considered distributed injection of 200 kJ/s of energy in form of hydrogen and upgraded biogas at Node 12 while maintaining the main natural gas supply at Node 1.

The case studies were formulated in two different ways, Method A) by formulating the energy demand assuming natural gas as the only source

- the gas flow demand is calculated as a flow rate demand assuming natural gas supply.

Method B) by formulating the energy demand considering the variations in gas composition supplied

- the gas flow demand is calculated considering the calorific value of the gas delivered at each node.

The case studies were designed to demonstrate the impact of gas mixture composition and the injection of distributed gas injection on network steady state parameters.

The different gas mixture compositions used for simulations are shown in Table 4

5. Results

5.1. The reference case

Table 5 shows the gas load flow results for the reference case, including the pressures at each node, flow rates at each branch and the no of iterations for solution.

Minimum pressure observed under steady state is 23.4 mbar at Node 11. Minimum flow rate in a branch is observed in the branch

Table 2
Nodal energy demand and source pressure for the case study (Reference case).

Node number	Energy demand (kJ/s)	Natural gas flow demand (m ³ /h)	Pressure (mbar)
1 (SourceNode)	0	0	75
2	2500	219	–
3	2200	192	–
4	2000	175	–
5	2600	228	–
6	1800	157	–
7	500	43.8	–
8	2350	206	–
9	550	48	–
10	475	42	–
11	350	30	–

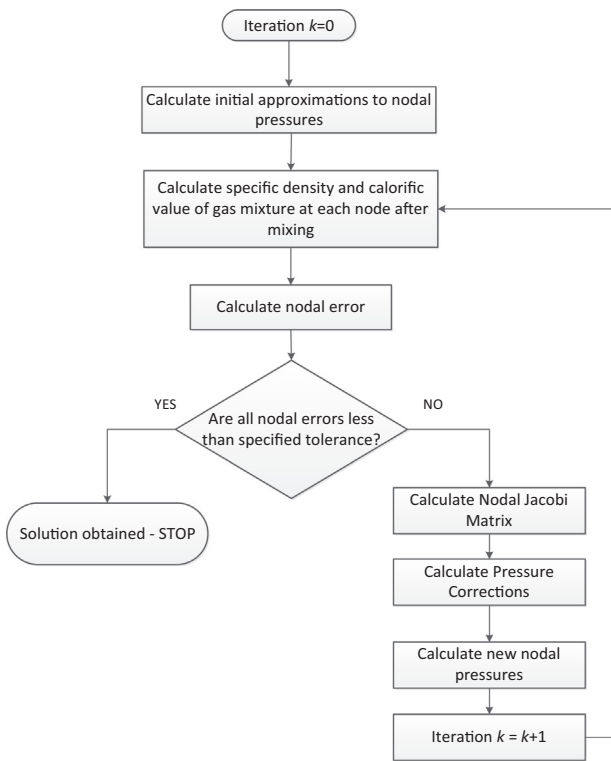


Fig. 2. Flowchart for the Newton method.

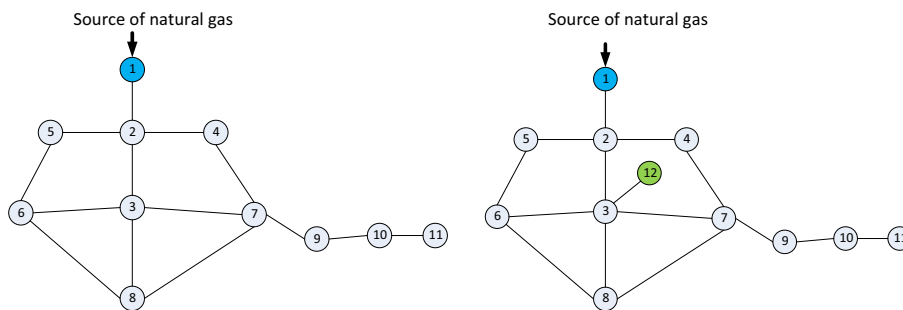


Fig. 3. Case study network (a) reference network. (b) Network with distributed injection at Node 12.

Table 3
Case studies.

Case study number	Description	Method of formulation	
		A	B
1	Conventional natural gas supply at Node 1	✓	
2	10% hydrogen in the natural gas blend supplied at Node 1	✓	✓
3	Upgraded biogas supplied at Node 1	✓	✓
4	200 kJ/s Hydrogen injected at Node 12	✓	✓
5	200 kJ/s upgraded Biogas injected at Node 12	✓	✓

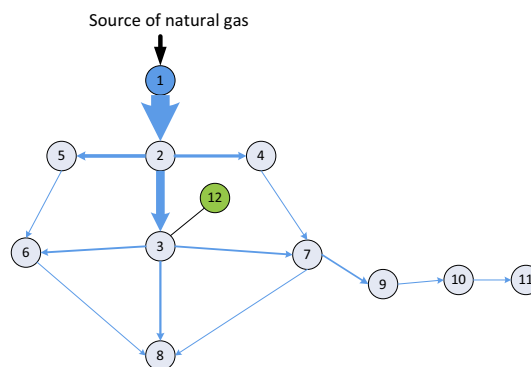
Table 4
Molar fractions of gases in mixtures used for case studies.

	CH ₄	C ₂ H ₆	C ₃ H ₈	C ₄ H ₁₀	CO ₂	N ₂	H ₂	Other	GCV (MJ/m ³)	SG	Wobbe (MJ/m ³)
Natural gas	0.9	0.06	0.01	0.001	0.005	0.02	–	–	41.04	0.6048	52.77
High hydrogen (10% hydrogen)	0.81	0.054	0.009	0.0009	0.0045	.018	0.1	–	37.06	0.545	50.2
Upgraded biogas (High CH ₄ content)	0.94	–	–	–	0.025	0.025	0.005	0.005	37.40	0.58	49.1
Hydrogen injection	–	–	–	–	–	–	1	–	12.75	0.0696	48.3

Table 5
Gas load flow results for the reference case.

Node	Pressure (mbar)	Branch	From–To	Flow rate (m ³ /h)
1	75	1	1–2	1344
2	66.09	2	2–3	627.37
3	46.68	3	2–4	233.10
4	46.95	4	2–5	264.47
5	41.45	5	3–6	139.91
6	38.40	6	3–7	132.10
7	39.30	7	3–8	162.39
8	37.39	8	5–6	36.41
9	28.15	9	4–7	57.67
10	24.14	10	6–8	18.43
11	23.42	11	7–8	25.31
		12	7–9	120.61
		13	9–10	72.36
		14	10–11	30.70

No of iterations for solution: 6
Maximum nodal error: 0.01 m³/h

**Fig. 4.** Network flow pattern of the reference case (Width of the arrow is proportional to flow rate).

from Node 6 to Node 8 of 18.4 m³/h. Fig. 4 shows the gas flow pattern in the low pressure network.

5.2. Impact of an alternative gas mixture in gas mains

Case studies 2 and 3 were simulated to analyze the impact of varying the main gas supply source mixture composition on the steady state of the network. Simulations were performed for 2 diverse gas mixture compositions as earlier stated. Results of the two methods are compared (see Fig. 5).

It should be noted that conventional methods of steady state analysis is capable of performing the simulations for case study 2 and 3 by adjusting the specific gravity, calorific value and other properties of the gas mixture. The results presented serve as an introduction to the impacts of using an alternative gas mixture as the supply source and provide a basis for comparison of results in Sections 5.3 and 5.4.

Fig. 5 shows the pressure gradient diagram from source to critical consumer in different case studies. Table 6 shows the nodal gas pressure results for case studies 2 and 3.

The impact of gas mixture composition on the steady state nodal pressure is evident from Fig. 5. When using the high hydrogen gas mixture, Method A shows an increase in steady state nodal pressures relative to the reference case. This is due to the lower

specific gravity of the high hydrogen gas mix (0.54) compared to conventional natural gas (0.6). As the gas flow demand remains unchanged from the reference case in Method A, the pressure drop in each branch is reduced due to lower specific gravity of the gas mixture (Eq. (9)). The critical consumer, Node 11, sees a 20% increase in pressure compared to the reference case due to a 10% reduction in the specific gravity of the gas mix.

A similar explanation can be given to the case with upgraded biogas where the specific gravity of the gas mixture is 3% less than the reference case. Node 11 observes a 7.9% increase in the pressure delivered.

However, the high hydrogen gas mixture and upgraded biogas both have a lower calorific value compared to the reference natural gas mixture (Table 4). Therefore maintaining the same flow rate as the reference case does not guarantee meeting the energy demand. Table 7 shows the energy received at nodes in each case compared to the reference case (using Method A). Fig. 6 shows the unmet energy demand when using method A, in case 2 and case 3.

When employing Method B to formulate the problem, calorific value of the gas mixture at load is taken into account. The gross calorific value of high hydrogen natural gas and upgraded biogas is 9.6% and 8.8% less than the reference natural gas mix. Therefore to meet the same energy demand, gas flow rate at each gas load increases proportionately. Consequently, in both cases employing Method B, a decrease in the steady state nodal pressures are observed compared to the reference case. Node 11 observes an 11% and 35% reduction in pressure compared to the

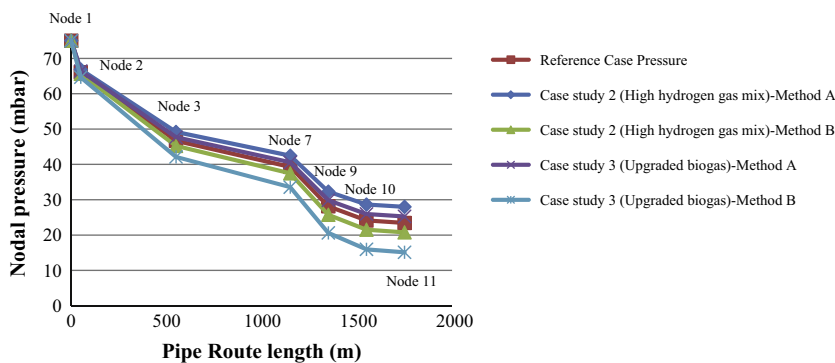


Fig. 5. Pressure gradient plot (Node 1 to Node 11) for alternative gas mixture at source (Case studies 2 and 3, both methods A and B).

Table 6 Case study results: nodal pressure for cases 2 and 3.

Node No.	Pressure (mbar)				
	Ref	High hydrogen gas mixture		Upgraded biogas mixture	
		Method A	Method B	Method A	Method B
1	75	75	75	75	75
2	66.09	66.88	65.63	66.41	64.65
3	46.68	49.18	45.22	47.70	42.12
4	46.95	49.43	45.50	47.96	42.43
5	41.45	44.42	39.72	42.66	36.05
6	38.40	41.64	36.52	39.72	32.52
7	39.30	42.46	37.42	40.59	33.53
8	37.39	40.71	35.45	38.74	31.34
9	28.16	32.30	25.74	29.84	20.62
10	24.14	28.64	21.53	25.97	15.96
11	23.42	27.99	20.77	25.28	15.13

Table 7 Energy delivered in case studies 2 and 3 (Method A).

Node Number	Actual energy demand (kJ/s)	High hydrogen case	Upgraded biogas case
1 (Source Node)	0		
2	2500	2257.80	2278.22
3	2200	1986.87	2004.83
4	2000	1806.25	1822.57
5	2600	2348.12	2369.34
6	1800	1625.62	1640.31
7	500	451.56	455.64
8	2350	2122.34	2141.52
9	550	496.72	501.21
10	475	428.98	432.86
11	350	316.09	318.95

the network. This shows that, the source gas mixture composition used and the method of formulating the problem have an impact on the final solution.

5.3. Impact of distributed supply source injection

The next set of case studies simulates a distributed supply source injection at Node 12. Node 1 remains the main source of gas supply (Natural gas) maintained at 75 mbar. Injection of hydrogen and upgraded biogas at Node 12 is considered. A constant energy content of 200 kJ/s is injected in each case.

Tables 8 and 9 show the results for steady state simulation formulated using both methods A and B, with distributed hydrogen injection and upgraded biogas injection at Node 12.

Unlike the case studies discussed, distributed injection of an alternative supply source changes the gas mixture composition unevenly in different parts of the network. The variation in gas mixture composition depends on the load distribution. Fig. 7 shows the gas flow pattern with distributed injection of alternative gas supply source at Node 12 (Cases 4 and 5).

When Method A is used, the gas flow demand is the same as the reference case flow demand (based on natural gas calorific value). However, due to the uneven gas mixture composition at demand nodes imbalances in energy supply and demand occur. Fig. 8 shows the unmet energy demand in the cases 4 and 5 when using Method A.

Injected hydrogen and upgraded biogas does not affect the gas mixture at Nodes 1, 2, 4 and 5, due to the flow pattern in the network (Fig. 7). Therefore, when using Method A, gas flow demand is accurately calculated by assuming natural gas composition in those nodes. The gas mixture received at the rest of the nodes is of a varied composition. Thus, maintaining the same flow rate as the reference case does not guarantee meeting the specified energy

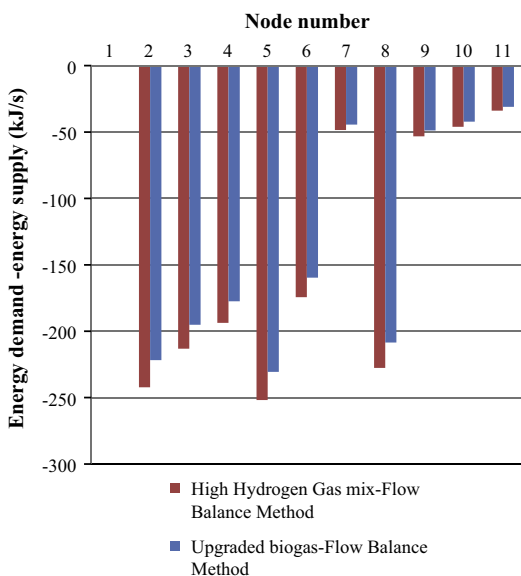


Fig. 6. Unmet energy demand for case 2 and 3 when using Method A.

reference case for high hydrogen gas mix and upgraded biogas (Fig. 5). Two opposing effects on the pressure drop calculation occur. A higher flow rate in gas pipes increases the pressure drop compared to the reference, while a lower specific gravity of the gas mix reduces it (Eq. (9)). The combined effect in these cases is an increase in pressure drops thus lower nodal pressures across

Table 8
Gas load flow results for hydrogen injection at Node 12.

Node	Method A		Method B		Branch	From-To	Method A	Method B
	Pressure (mbar)	Wobbe index	Pressure (mbar)	Wobbe index			Flow rate (m ³ /h)	Flow rate (m ³ /h)
1	75	52.77	75.00	52.77	1	1–2	1288	1292
2	66.82	52.77	66.32	52.77	2	2–3	584.93	588.62
3	49.95	51.63	47.83	51.67	3	2–4	226.83	227.28
4	48.69	52.77	47.37	52.77	4	2–5	256.72	257.27
5	43.60	52.77	41.92	52.77	5	3–6	145.31	144.93
6	41.72	51.82	39.08	51.88	6	3–7	137.09	136.72
7	42.62	51.94	40.02	51.99	7	3–8	166.02	165.76
8	40.99	51.68	38.08	51.73	8	5–6	28.66	29.21
9	32.11	51.94	28.54	51.99	9	4–7	51.40	51.85
10	28.32	51.94	24.40	51.99	10	6–8	16.08	16.25
11	27.64	51.94	23.66	51.99	11	7–8	24.03	24.11
12	50.00	48.33	47.88	48.33	12	7–9	120.61	120.61
					13	9–10	72.36	72.36
					14	10–11	30.70	30.70
					15	12–3	56.47	56.47

Table 9
Gas load flow results for upgraded biogas injection at Node 12.

Node	Method A		Method B		Branch	From-To	Method A	Method B
	Pressure (mbar)	Wobbe index	Pressure (mbar)	Wobbe index			Flow rate (m ³ /h)	Flow rate (m ³ /h)
1	75.00	52.77	75.00	52.77	1	1–2	1325	1326
2	66.32	52.77	66.32	52.77	2	2–3	612.13	613.33
3	47.76	52.66	47.77	52.66	3	2–4	231.28	231.50
4	47.45	52.77	47.44	52.77	4	2–5	262.29	262.56
5	42.05	52.77	42.03	52.77	5	3–6	141.45	141.64
6	39.32	52.69	39.30	52.69	6	3–7	133.57	133.76
7	40.24	52.70	40.21	52.70	7	3–8	163.38	163.69
8	38.37	52.67	38.34	52.67	8	5–6	34.23	34.51
9	29.10	52.70	29.03	52.70	9	4–7	55.85	56.07
10	25.09	52.70	25.01	52.70	10	6–8	17.79	17.91
11	24.37	52.70	24.29	52.70	11	7–8	24.96	25.05
12	47.80	48.98	47.82	48.98	12	7–9	120.61	120.84
					13	9–10	72.36	72.50
					14	10–11	30.70	30.76
					15	12–3	19.25	19.25

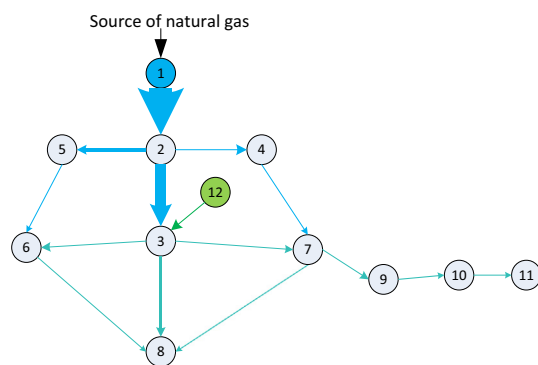


Fig. 7. Gas flow pattern with distributed supply source injection (cases 4 and 5).

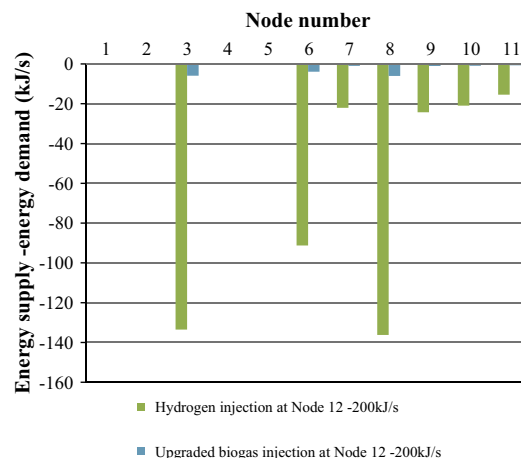


Fig. 8. Unmet energy demand at gas load nodes in case studies 4 and 5 (Method A).

demand. An energy content of 200 kJ/s when converted to volume flow rate is 57 m³/h and 19.25 m³/h for hydrogen and upgraded biogas. Thus, the volume flow injected in terms of hydrogen is three fold compared to the injection of upgraded biogas. The relatively greater unmet energy demand in case of hydrogen injection is a combined effect of a larger volume flow injection and the relatively low energy density of hydrogen (less than 1/3 of natural gas). Upgraded biogas is comparatively closer to natural gas in terms of energy density and specific gravity. Therefore the unmet

energy demand in the case of upgraded biogas injection is comparatively lower.

When Method B is used the volume flow demand is calculated depending upon the gas mixture composition received at the particular node. Therefore, the energy demand is met even though the gas composition may vary in different parts of the network.

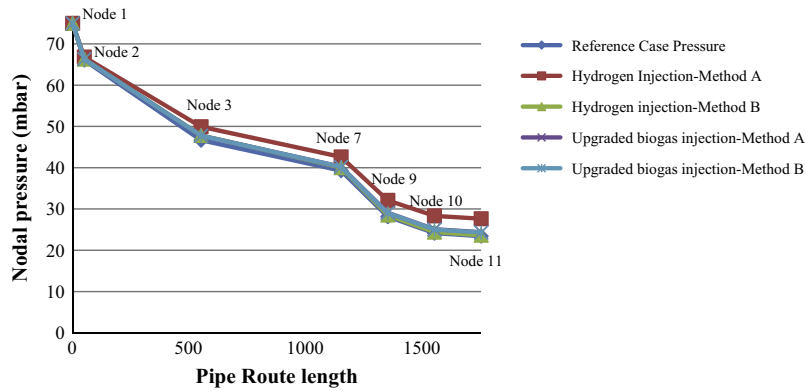


Fig. 9. Pressure profile plot for distributed injection case (Node 1 to Node 11).

The impact of distributed gas injection on nodal pressure for case studies 4 and 5 is shown in Fig. 9. Nodal pressure across the network increase compared to the reference case due to distributed injection and reduced gas flow from main supply source. Therefore distributed injections in these cases are supporting the pressure management of the network.

5.4. Impact on gas network regulations

The current regulatory framework for gas network operation specifies the content and characteristics of the gas permitted to be transported and injected in the UK gas mains. At present, regulation limits the maximum allowable hydrogen content to less than 0.1% (by volume) and a Wobbe index range between 47.2 and 51.41 MJ/m³ [9]. Under these conditions, distributed injection is restricted to upgraded bio methane from anaerobic digesters which is similar in composition to natural gas. There are several research and demonstration projects [20,24] that propose relaxing some of the tight requirements in gas content and characteristics specified in regulations. The limits on hydrogen content and the Wobbe index are of particular interest. Therefore, it is the authors' view that the need for methods to analyze the impact of distributed injection of alternative fuels will become significant.

Fig. 10 is a scatter plot of the gas mixture properties received at gas nodes in all case studies analyzed. In all case studies, the gas delivered to nodes remains within regulatory Wobbe index limits.

The calorific value variation and the specific gravity variations are also shown in Fig. 10. The parameters vary in a narrow range near the normal value. Therefore, the impact on appliance performance is considered acceptable from a network analysis perspective.

6. Discussion

The research work presented extends the conventional method of steady state simulation of gas networks to a more comprehensive analysis that considers the distributed injection of new supply sources. The model has shown good convergence characteristics. In all case studies, the number of iterations required to reach an error tolerance of 0.01 (m³/h) was less than 12. However, gas network models in real life usually simulate a much larger number of nodes and branches [12,23,24]. Therefore, further studies need to be undertaken to test the performance of the numerical solution technique employed in more complex networks.

The results show that, the two different methods of formulating gas demand presented have an impact on the final solution.

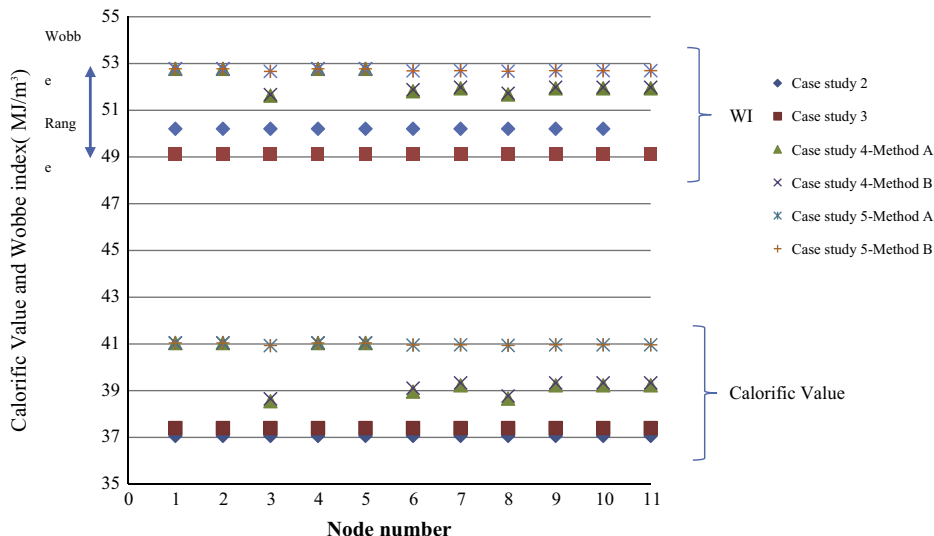


Fig. 10. Calorific value and Wobbe index (WI) of gas mixtures for all case studies.

Conventional network analysis methods use flow rate as a proxy to energy at gas demand nodes. As the case studies with hydrogen and upgraded biogas injection shows, gas flow rate alone is no longer sufficient to ensure the supply of energy demand. The case studies show a relatively small variation in gas flow properties. However, increasing the diversity in gas supplies may require a revision of gas safety regulations and appliances operating in a wider Wobbe index range [19,20]. Discussed research being carried out in realizing these appliances for future applications.

An important consideration in the model is the influence of gas mixture properties on gas pipe flow. According to Eq. (9), specific gravity is the main intrinsic property of a gas mixture that affects gas flow for a given pressure drop across a pipe. When supply sources with dissimilar gas composition are used, gas mixture properties (i.e. specific gravity) in each pipe section may vary. Therefore, the model calculates gas admixture properties at each node in the network where two or more flows combine. Pipe volume flow (at standard temperature and pressure) has an inverse square root relationship to the specific gravity of the gas admixture. It was shown that the type of gas injected can have a significant impact on the final result of steady state pressure delivery in the network. In Section 5.2 with high hydrogen content in natural gas, the pressure at a node varied between +20% and –35% from a reference natural gas system for methods A and B. This highlights the importance of the method of gas network analysis that considers dissimilar gas properties of new supply sources.

The composition of natural gas may vary due to the diversity of supply sources used in the UK i.e. LNG from Qatar, North sea gas, European gas imports. The model can account for this by adjusting the different gas mixture fractions in natural gas. The gas fractions are used in the calculation of calorific value and specific density of natural gas. It is also used for calculating the composition of gas flows in each branch as mentioned in 3.2. The model is also capable of simulating more than one source of distributed injection. Depending on the demand distribution, the locations and quantity of supply sources, the gas flow pattern and pressure profiles will change. If sufficient local resources are available, parts of the gas network may be controlled in islanded operation. However, an anticipated challenge in connecting distributed supply sources is the management of the gas network during low demand seasons and potential reverse flows. Network simulations should be undertaken to study the operating conditions of the gas network in diverse seasonal demand scenarios.

Results for distributed injection with alternatives to natural gas shows an impact on gas pressure and the quality (Wobbe index, Calorific value, specific gravity) delivered to final consumers. If managed appropriately these variations may be tolerated by the appliances. Conclusions cannot be made by simply an analysis of steady state. Further experimental work on the impact of possible variations in gas composition and pressure input, on appliance performance and network reliability has to be tested before introducing in scale. If, however proven that certain alternative gas injections can be accepted without major complications to network operation, it will help to loosen a number of tight regulations in place. For example, if it was found that a higher fraction of hydrogen or lower quality biomethane can be tolerated by local gas networks without serious concerns, it would improve the economic viability of many projects that would otherwise not be feasible. This is particularly the case for many anaerobic digester installations where the specifications for upgrading biogas require very high standards of scrubbing to rid of the CO₂ and other pollutants. Relieving such inflexible regulations will allow more renewable gas to be utilized.

Also, in a future scenario where low gas demand is prevalent, local gas supply sources can supply the majority of the demand while only using mains supply during emergencies. Gas billing can be adjusted to the calorific value of gas delivered to individual consumers by using measurements and analysis tools [23]. A detailed analysis of the gas network will be required in such a transformation and the method developed could be of support.

7. Conclusions and future work

In the near future, studies of gas networks will need to take account the impact of utilizing a diversity of gas supply sources. This research work presents a model developed for the steady state analysis of gas networks with centralized and decentralized alternative gas sources. Two methods of formulating the problem are compared. A low pressure gas network was used to test and validate the model. Several case studies simulated the centralized and decentralized injection of hydrogen and upgraded biogas in the supply of energy demand. Results show the impact of using different gas supply sources on the pressure and gas quality delivered to different parts of the network. The method of calculating the gas flow demand is shown to have an impact on the final simulation results. Therefore further work is required to understand the implications of each method of formulating the gas flow problem. Case studies show, that if managed appropriately distributed supply of gas sources could support network management, while also reducing gas import dependence. Simulations will need to be complemented by extensive experimental work to recognize implications on appliance performance and network management of such transformation. The ability to utilize the existing gas system infrastructure to supply renewable gases will support the economics of the low carbon transition.

Acknowledgments

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All data created during this research is available by contacting the corresponding author.

Appendix A

Table A.

Table A
Network pipe data.

Branch	From-To	Pipe Length (m)	Pipe Diameter (mm)
1	1–2	50	160
2	2–3	500	160
3	2–4	500	110
4	2–5	500	110
5	3–6	600	110
6	3–7	600	110
7	3–8	500	110
8	5–6	600	80
9	4–7	600	80
10	6–8	780	80
11	7–8	780	80
12	7–9	200	80
13	9–10	200	80
14	10–11	200	80

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