The impact of information sharing, random yield, correlation, and lead times in closed loop supply chains

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Abstract

We investigate the impact of advance notice of product returns on the performance of a decentralised closed loop supply chain. The market demands and the product returns are stochastic and are correlated with each other. The returned products are converted into “as-good-as-new” products and used, together with new products, to satisfy the market demand. The remanufacturing process takes time and is subject to a random yield. We investigate the benefit of the manufacturer obtaining advance notice of product returns from the remanufacturer. We demonstrate that lead times, random yields and the parameters describing the returns play a significant role in the benefit of the advance notice scheme. Our mathematical results offer insights into the benefits of lead time reduction and the adoption of information sharing schemes.

Keywords: Supply chain management, closed loop supply chain, information sharing, random yield, lead time.
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1. Introduction

Due to growing concerns with environmental issues, collection and recycling systems for post-consumer products have been developed in many countries. Return rates for polyethylene terephthalate (PET) bottles, for example, are increasing year by year in many countries. The 2012 rates in Europe, Japan and the USA are 52% (Petcore 2013), 90.4% (CPBR 2014) and 30.8% (Napcore 2013), respectively. At the same time, many companies have been developing new remanufacturing processes. Suntory, one of the largest food and beverage companies in Japan, has developed bottle-to-bottle mechanical recycling technology that enables the company to produce PET bottles solely from reused resin (Suntory 2013).

This world-wide environmental movement is mainly driven by the sustainability ethic (Welle 2011), but the impact of the recycling system on the dynamics of the supply chain is not well understood.

Akçalı and Çetinkaya (2011) argue closed loop supply chains (CLSCs) are generally acknowledged to be more complex than traditional supply chains due to a number of factors: Both the demands and the product returns must be forecasted and incorporated into replenishment decisions. The demand and return may be correlated to each other. Two different lead times are present, the manufacturing lead time and the remanufacturing lead time. In many practical situations, the returned products will also be variable in quality, resulting in a remanufacturing process with a random yield.

It is often advocated that to improve supply chain performance, information should be shared between players. The value of such information sharing in traditional supply chains is well recognized (see Gavirneni, Kapuscinski and Tayur 1999; Lee, So and Tang 2000). However, there is little research that addresses information sharing in CLSCs.

This research investigates the impact of the remanufacturer providing advance notice of the product returns on the performance of the manufacturer in a decentralised CLSC. We focus on the stochastic and dynamic performance of the supply chain. The lead times, the degree of correlation between the demand and the product returns, and the random yield of the remanufacturing process are all incorporated into a mathematical model to investigate the benefit of advance notice via a variance analysis. With a constant lead time, the returns are converted into “as-good-as-new” products that are used alongside newly manufactured items to satisfy market demand. To cope with the uncertainty in demand and returns the manufacturer must forecast them both. However, the returns are already known to the external remanufacturer and this information could be shared in an advance notice scheme. We demonstrate that both the remanufacturing and the manufacturing lead times, the remanufacturing yield, the parameters of the return process, and the advanced notice scheme can have a significant impact on the manufacturer's performance.

As we progressed through our study we became aware that certain knowledge from our understanding of traditional supply chains does not hold true for CLSC. For example, the rule-of-thumb that “reducing lead time improves the dynamic performance of a traditional supply chain” does not always hold true in our CLSC. We were also surprised to learn that higher returns can sometimes reduce supply chain performance.

This paper is organized as follows. Section 2 provides a literature review. Section 3 defines our CLSC model. Section 4 deduces managerial properties from an analysis of the production quantities and inventory levels. Section 5 presents insights from a numerical exploration. Section 6 concludes. Some proofs are provided in the appendices, and summarises some numerical experiments for verification.

2. Literature review

Using some approximations when necessary, Ketzenberg, van der Laan and Teunter (2006) presented two analytical models for quantifying the value of information in the CLSC: a one-period model and a multi-period model. Information on the market demand, the returns and the remanufacturing yield was shared and its impact investigated. Assuming a capacitated CLSC, Ketzenberg (2009) investigated the value of sharing demand, returns, yield and capacity utilization information. Costs were quantified using a simulation study. It was shown that information regarding capacity utilization leads to the largest average benefit, though no type of information is dominant.

De Brito and van der Laan (2009) investigated the impact of imperfect information on the forecast of lead time demand in a remanufacturing setting. Inventory cost was used to quantify the consequences of imperfect information. Based on an analysis of four different forecasting methods, they concluded that the most informed forecasting method does not always result in the least cost. Flapper, Gayon and Vercraene (2012) considered imperfect advance return information and inventory cost using a Markov decision formulation. A random return lead time was assumed in a model with finite capacity but no correlation existed between demand and returns. They concluded that advance return information can reduce inventory cost by up to 5%, and this was affected by the expected return lead time.

The importance of considering delays in a system is well recognized (Forrester 1961). Flapper, Ferrer and Ketzenberg (2004) and Gayon and Vercraene (2012) suggested that it might be reasonable to assume that lead times affect the value of information sharing. Assuming that both lead times were stochastic, van der Laan, Salomon and Dekker (1999) numerically investigated the impact of lead times. Poisson distributions were used to represent demand

and return processes. It was found that a longer remanufacturing lead time resulted in a cost reduction, though longer manufacturing lead times always resulted in a cost increase. Inderfurth and van der Laan (2001) also supported this finding. Despite this theoretical support, Guide (2000) found that 60% of remanufacturing executives were under pressure to reduce remanufacturing lead times.

It is widely recognised, that demand and product returns are correlated with each other (Akçalı and Çetinkaya 2011). This correlation assumption is intuitively understandable, as part of the demand eventually becomes the input into the remanufacturing process (Akçalı and Çetinkaya 2011). Van der Laan et al. (1999) and Ketzenberg, van der Laan and Teunter (2006) modelled correlation between demand and the product returns with product returns that were a random function of the demand. Mitra (2012) assumed that product returns were a fraction of the demand plus a random term.

Practically it is common to have a random yield in the remanufacturing process as the quality of the return products is understandably varied (Guide 2000). Ferrer and Ketzenberg (2004), Ketzenberg, van der Laan and Teunter (2006) and Ketzenberg (2009) used a Bernoulli process to represent a remanufacturing process with random yields. Yano and Lee (1995) suggested that one advantage of using the Bernoulli process was its simplicity, but this approach forbids the specification of yield variability. Akçalı and Çetinkaya (2011) suggested that only a few studies incorporate a random yield assumption. This rarity is probably due to the analytical complexity introduced by this feature.

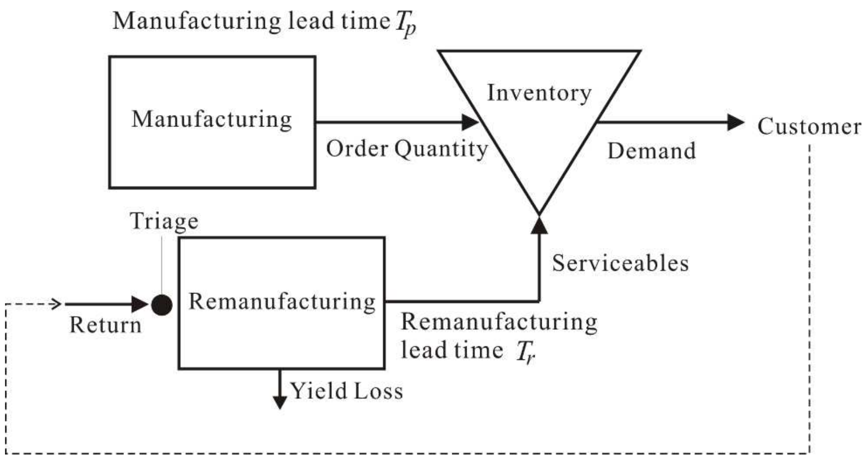

Figure 1. Schematic of material flow

Our research considers the impact of the value of advance notice on the dynamic and stochastic performance of a decentralised CLSC. The demand and the returns are stochastic and cross-correlated. In our model, the lead times of the manufacturer and the remanufacturer and the random yields in the remanufacturing process are considered. We characterize the variances of the serviceable products, the net stock levels and the production orders without specifying their probability distribution functions (PDFs). To the best of our knowledge, there are no previous studies that simultaneously consider the value of information, the impact of lead times and the random yield in a CLSC setting with correlated demands and returns in such a way. Interested readers can find a comprehensive review of recent CLSC literature in Akçalı and Çetinkaya (2011) and Govindan, Soleimani and Kannan (2015).

3. Model

Figure 1 shows a schematic of our decentralised CLSC model. It is a periodic review system where both the manufacturer and the remanufacturer employ the same review period. The manufacturer uses an order-up-to policy (Hosoda and Disney 2006) to determine its production quantity. Both the manufacturing and the remanufacturing processes have unlimited capacity. We assume that there is no difference between remanufactured and new products in terms of quality. This assumption may not be as restrictive as it first seems. For example, Suntory (2013) makes bottles made from both recycled PET resin and petroleum-based resources for the same soft drink product and the customer is not aware of any difference. A push policy is assumed to operate at the remanufacturer. Once returns are available, the remanufacturing process starts immediately and the remanufactured products are subsequently shipped to the manufacturer without delay. The push policy is appropriate in our decentralised setting and fits well with the ethics of sustainability and common industrial practice.

The random yield is modelled using a stochastically proportional yield model (Hening and Gerchak 1990). This model is appropriate when the system is subject to material variations (Yano and Lee 1995), and it has previously been used in a remanufacturing study by Tao, Zhou and Tang (2012). The yield is identified at the beginning of the remanufacturing process in what is generally called a “triage” process.

3.1 Market demand and returns

It is assumed that both the market demands (D_t) and the product returns (R_t) are white noise processes. This white noise assumption is widely used in much of the CLSC literature (e.g. Ketzenberg, van der Laan and Teunter 2006; Ketzenberg 2009). The mean of D_t and R_t are μ_D and μ_R, respectively. The correlation between $D_{t-\tau}$ and R_t is captured by the correlation coefficient θ, $|\theta| \leq 1$, where τ is a time delay over which the correlation acts and is a non-negative integer. ε_t is an identically and independently distributed (i.i.d.) random variable with a mean of zero and a standard deviation of σ_ε. ζ_t is another zero mean i.i.d. random variable with a standard deviation of $\sigma_\zeta = k\sigma_\varepsilon$ where k is a non-negative scale factor. ε_t and ζ_t are independent. The demands and the product returns are given by

$$\begin{gathered} D_t = \mu_D + \varepsilon_t, \\ R_t = \mu_R + \theta k \, \varepsilon_{t-\tau} + \sqrt{1-\theta^2}\zeta_t, \end{gathered} \tag{1}$$

where the correlation between $D_{t-\tau}$ and R_t becomes θ, see Appendix A. It should be noted that we do not model the correlation between the "satisfied demand" and the returns. Also, there is no correlation between $D_{t-\tau-x}$ and R_t when x is a nonzero integer. This assumption might not be the most general representation but, to the best of our knowledge, this is the first research which explicitly incorporates correlation between demands and returns in the literature. The correlation coefficient, $|\theta| \leq 1$. If demand and the return are independent each other we set $\theta = 0$. If larger (smaller) demands eventually results in larger (smaller) returns then there is likely to be positive correlation between demand and returns, $0 < \theta \leq 1$. If a collect-and-return process shares a limited logistics capacity with the delivery of new product to customers there could be negative correlation between demand and returns, $-1 \leq \theta < 0$; as when the requirements for delivering new (returned) products is high there is less logistics capacity available to collect returns (new products).

Appendix A shows that the standard deviations of $D_t = \sigma_\varepsilon$ and $R_t = k\sigma_\varepsilon$, respectively. If k is greater than unity, the standard deviation of R_t becomes larger than the standard deviation of D_t. It is assumed that $\mu_D \gg \mu_R$, as in van der Laan et al. (1999), since practically the product returns are a portion of the demand. This assumption might not hold at the very end of a product life cycle or when a new version/edition of the product is introduced. However, in our Suntory example, as the returned bottles are mechanically destroyed and reformed into a new bottle, this factor is not an issue.

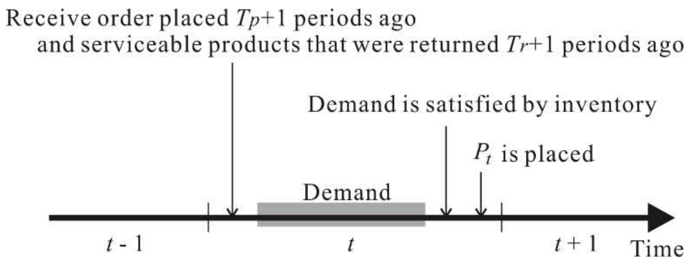

Figure 2. Sequence of events at manufacturer

3.2 Sequence of events

At the beginning of time period t, the remanufacturer observes the total number of units R_t that have been returned from the market place. All of the returns are then pushed into the remanufacturing process. The remanufacturing process is not capacitated but it is subject to a random yield. If the remanufacturer receives R_t at t, the quantity of serviceable goods the remanufacturer actually processes is $\Xi(R_t) = \xi_t R_t \leq R_t$ due to the random yield. It is assumed that the value of $\Xi(R_t)$ is recognised by the remanufacturer at time t, and the yield distribution does not depend on time t or the quantity of R_t. When R_t is realised, ξ_t is also identified, as in Ketzenberg (2009). The expected yield $\bar{\xi}$ ($= E[\xi_t]$), mean returns μ_R , and the remanufacturing lead time T_r, are known by the manufacturer. Remanufactured products are then pushed into the manufacturer's inventory at the beginning of period $t + T_r + 1$ in order to partially satisfy the market demand D_{t+T_r+1}.

The manufacturer's lead-time is T_p. At the beginning of period t, the manufacturer receives a quantity of brand-new goods from its production line equal to $P_{t-(T_p+1)}$, the order placed in period $t - (T_p + 1)$ in addition to the remanufactured products from the remanufacturer. The market demand D_t is then observed and satisfied from the on-hand inventory. If the manufacturer does not have sufficient on-hand inventory to fill the demand, the unmet demand is backlogged. At the end of period t, the manufacturer places a production order P_t to meet the future demand, taking into account the expected future product return rate. Figure 2 illustrates the sequence of events. Note that the manufacturer makes his production decision after he has received product from the remanufacturer. The manufacturer's net stock level at the end of period t, follows

$$NS_t = NS_{t-1} + \Xi\left(R_{t-(T_r+1)}\right) + P_{t-(T_p+1)} - D_t. \tag{2}$$

3.3 *The ordering policy*

Let IP_t^+ denote the manufacturer's inventory position the moment after the production order P_t is determined. IP_t^+ is the net stock level at time t plus the sum of open manufacturing orders, $IP_t^+ = NS_t + \sum_{i=0}^{T_p} P_{t-1}$. The value of IP_t^+ is known to the manufacturer, since all information is local. Hosoda and Disney (2012) showed that in a traditional supply chain setting, regardless of the ordering policy used, $NS_{t+T_p+1} = IP_t^+ - \sum_{i=1}^{T_p+1} D_{t+i}$ always exists. However in our CLSC it is necessary to incorporate the incoming pipeline inventory (WIP) that the remanufacturer will send to the manufacturer during the interval $(t, t+T_p+1]$. PIR_t is the pipeline inventory, the products currently being remanufactured that have successfully cleared triage. Let $FPIR_t$ represent the future pipeline inventory at time t. Consequently, we have the following relationship:

$$NS_{t+T_p+1} = IP_t^+ - \sum_{i=1}^{T_p+1} D_{t+i} + PIR_t + FPIR_t, \tag{3}$$

where

$$IP_t^+ = NS_t + P_{t-T_p} + \cdots + P_t,$$

$$PIR_t = \begin{cases} \sum_{i=T_r-T_p}^{T_r} \Xi(R_{t-i}), T_r \geq T_p \\ \sum_{i=0}^{T_r} \Xi(R_{t-i}),\ \ T_r < T_p \end{cases}$$

and

$$FPIR_t = \begin{cases} 0, T_r \geq T_p \\ \sum_{i=1}^{T_p-T_r} \Xi(R_{t+i}), T_r < T_p\ . \end{cases}$$

Note that the manufacturer does not know PIR_t when there is no advance notice scheme. In the absence of advance notice, the manufacturer must use the expected value of PIR_t, $\widehat{PIR}_t$, to determine P_t. Hence, the advance notice of product returns will influence manufacturing performance. When $T_r < T_p$, $FPIR_t$ contains information which will only be known in the future; the actual value of $FPIR_t$ is unknown at time period t. Therefore, the manufacturer must forecast the value of $FPIR_t$. This implies that the magnitude of the relationship between T_r and T_p will also influence the ordering policy.

With knowledge of (2), the following relationship between IP_t^+ and IP_{t-1}^+ can be obtained

$$IP_t^+ = IP_{t-1}^+ - D_t + \Xi\left(R_{t-(T_r+1)}\right) + P_t.$$

P_t can then be written as

$$P_t = D_t - \Xi\left(R_{t-(T_r+1)}\right) + IP_t^+ - IP_{t-1}^+. \tag{4}$$

From (3), we may obtain another form of IP_t^+,

$$IP_t^+ = \sum_{i=1}^{T_p+1} D_{t+i} - PIR_t - FPIR_t + NS_{t+T_p+1}. \tag{5}$$

As the manufacturer cannot observe IP_t^+ the expected value of IP_t^+ must be used instead,

$$E[IP_t^+] = \widehat{D} - \widehat{PIR_t} - \widehat{FPIR}_t + TNS, \tag{6}$$

where

$$\widehat{D} = E\left[\sum_{i=1}^{T_p+1} D_{t+i}\right] = \left(T_p + 1\right)\mu_D,$$
$$\widehat{PIR_t} = E[PIR_t], \widehat{FPIR}_t = E[FPIR_t], TNS = \mathrm{E}[NS_{t+T_p+1}],$$

and the target net stock (TNS) level is a time invariant constant predetermined to minimise inventory holding and backlog cost. If the distribution of the inventory is known, the TNS may be identified using standard newsvendor techniques and the inventory costs become linear functions of the standard deviation of the inventory levels, Brown (1963). However, the distribution of the inventory level is difficult to determine due to the non-linear impact of the random yield. This means numerical approaches are required to allocate costs. For this reason we have elected to judge performance based solely on the variance of inventory and capacity levels.

From (4) and (6) we may obtain the OUT replenishment policy for our CLSC,

$$P_t = D_t - \Xi\left(R_{t-(T_r+1)}\right) + (E[IP_t^+] - E[IP_{t-1}^+]).$$

Note that the values of $E[IP_t^+]$ and $E[IP_{t-1}^+]$ depend upon the availability of the advance notice scheme. We have the following two cases in our setting: 1) advance notice is not available (case N) and 2) advance notice is available (case A). Further note that P_t can be negative, indicating that the sum of the on-hand inventory and the pipeline inventory is higher than the target order-up-to level. In such a case, the excess inventory will stay there until being used as part of a future replenishment. This assumption is called the costless return assumption (see Dong and Lee 2003; Hosoda and Disney 2009). However, this costless return assumption is not as restrictive as it appears, especially when $\mu_D \gg \mu_R$.

3.4 Case N: No advance notice

In this case information about the returns (R_t) and the yield (ξ_t) is not shared. This implies the manufacturer does not know the value of $\Xi(R_t)$. The expected value $E[\Xi(R_t)] = \bar{\xi}\mu_R$ must be used instead. The estimated values of $\widehat{FPIR}_t$ and $\widehat{PIR}_t$ for the manufacturer then becomes

$$\widehat{FPIR}_t = E[FPIR_t] = \begin{cases} 0,\ T_r \geq T_p \\ \left(T_p - T_r\right)\bar{\xi}\mu_R, T_r < T_p, \end{cases} \tag{7}$$

and

$$\widehat{PIR}_t = E[PIR_t] = \begin{cases} \left(T_p + 1\right)\bar{\xi}\mu_R,\ T_r \geq T_p \\ (T_r + 1)\bar{\xi}\mu_R, T_r < T_p. \end{cases} \tag{8}$$

From (6), (7) and (8), we can see that $E[IP_t^+] = E[IP_{t-1}^+]$ and P_t reduces to

$$P_t^N = D_t - \Xi(R_{t-(T_r+1)}) \tag{9}$$

Note that the manufacturer knows only the value of $\Xi(R_{t-(T_r+1)})$; the values of $R_{t-(T_r+1)}$ and ξ_t are unknown.

3.5 Case A: Advance notice.

In this case information about the returns R_t and the random yield ξ_t is shared with the manufacturer. It is also assumed that the manufacturer is proficient at analysing time series and is able to obtain the values of $\{\varepsilon_t, \varepsilon_{t-1}, ...\}$, θ, k and τ as well as μ_D, μ_R and $\bar{\xi}$ from the historical time series of D_t and R_t. In this setting, $\widehat{FPIR}_t$ and $\widehat{PIR}_t$ become

$$\widehat{FPIR}_t = \begin{cases} 0,\ T_r \geq T_p \\ (T_p - T_r)\bar{\xi}\mu_R,\ T_p > T_r \wedge \tau = 0 \\ (T_p - T_r)\bar{\xi}\mu_R + \bar{\xi}\theta k \sum_{i=1}^{\tau} \varepsilon_{t+1-i},\ T_p - T_r \geq \tau \geq 1 \\ (T_p - T_r)\bar{\xi}\mu_R + \bar{\xi}\theta k \sum_{i=1}^{T_p - T_r} \varepsilon_{t-\tau+i}, \tau > T_p - T_r > 0, \end{cases}$$

and

$$\widehat{PIR}_t = \sum_{i=(T_r - T_p)^+}^{T_r} \Xi(R_{t-i}).$$

The formula for P_t^A then depends on the values of T_p, T_r and τ:

$$P_t^A = \begin{cases} D_t - \Xi\left(R_{t-(T_r - T_p)}\right), T_r \geq T_p \\ D_t - \Xi(R_t),\ T_p > T_r \wedge \tau = 0 \\ D_t - \Xi(R_t) + \bar{\xi}\theta k(\varepsilon_{t-\tau} - \varepsilon_t),\ T_p - T_r \geq \tau \geq 1 \\ D_t - \Xi(R_t) + \bar{\xi}\theta k\left(\varepsilon_{t-\tau} - \varepsilon_{t-(\tau - T_p + T_r)}\right), \tau > T_p - T_r > 0. \end{cases} \tag{10}$$

Having defined the replenishment policies, the next section derives expressions for the variance of the production and net stock levels.

4. Variance analysis

The variance expressions shown in this section are obtained without specific assumptions of the distribution of D_t, R_t or ξ_t. We use $V[x]$ to denote the variance of x.

4.1 Case N: The closed loop supply chain with no advance notice

When no advance notice is given, P_t is given by (9), and its variance is

$$V[P^N] = \sigma_\varepsilon^2 + V[\Xi(R)], \tag{11}$$

where Appendix B shows $V[\Xi(R)] = \bar{\xi}^2 k^2 \sigma_\varepsilon^2 + V[\xi](\mu_R^2 + k^2 \sigma_\varepsilon^2)$. In the right hand side of (3), the manufacturer knows only the locally available information, IP_t^+. Hence, $V[NS^N]$ can be written as

$$V[NS^N] = \mathrm{E}\left[\left(PIR_t + FPIR_t - \sum_{i=1}^{T_p+1} D_{t+i} - E\left(PIR_t + FPIR_t - \sum_{i=1}^{T_p+1} D_{t+i}\right)\right)^2\right]$$

$$= \begin{cases} (T_p+1)(\sigma_\varepsilon^2 + V[\Xi(R)]) - 2\bar{\xi}\theta k(T_p - T_r - \tau)\sigma_\varepsilon^2, \ T_p - T_r \geq \tau \\ (T_p+1)(\sigma_\varepsilon^2 + V[\Xi(R)]), \text{otherwise.} \end{cases} \tag{12}$$

4.2 Case A: The CLSC with advance notice

The ordering policy in this case is described by four formulae, see (10). Fortunately, the variance of P_t^A reduces to the following two expressions.

$$V[P^A] = \begin{cases} V[P^N] - 2\bar{\xi}\theta k \sigma_\varepsilon^2, T_p - T_r \geq \tau \\ V[P^N], \text{otherwise.} \end{cases} \tag{13}$$

By following a similar method for the case N, we may obtain an expression for $V[NS^A]$;

$$V[NS^A] = \mathrm{E}\left[\left(PIR_t + FPIR_t - \sum_{i=1}^{T_p+1} D_{t+i} - E\left(PIR_t + FPIR_t - \sum_{i=1}^{T_p+1} D_{t+i}\right)\right)^2\right]$$

$$= \begin{cases} (T_p+1)\sigma_\varepsilon^2, T_r \geq T_p \\ (T_p+1)\sigma_\varepsilon^2 + (T_p - T_r)V[\Xi(R)] - \tau\bar{\xi}^2\theta^2 k^2 \sigma_\varepsilon^2 - 2\bar{\xi}\theta k(T_p - T_r - \tau)\sigma_\varepsilon^2, T_p - T_r \geq \tau \\ (T_p+1)\sigma_\varepsilon^2 + (T_p - T_r)\left(V[\Xi(R)] - \bar{\xi}^2\theta^2 k^2 \sigma_\varepsilon^2\right), \tau > T_p - T_r > 0. \end{cases}$$

The following insights can be obtained from the variance expressions.

Property 1. When the return and the yield information is shared, the variance of the net stock levels reduces (*i.e.* $V[NS^A] < V[NS^N]$).

This is intuitively understandable, as the advance information reduces the uncertainty in the system and the net stock levels can be more tightly controlled. This property means the advance notice scheme allows the manufacturer to reduce his inventory-related costs.

Property 2. A CLSC with i.i.d. demands and returns generates bullwhip (*i.e.* $V[P^N] > \sigma_\varepsilon^2, V[P^A] > \sigma_\varepsilon^2$).

This property shows that the bullwhip behaviour of the CLSC is different to a traditional supply chain where the variance of the production orders is equal to the variance of the demand for the OUT policy under i.i.d. demand and minimum mean squared error forecasting (Lee, So and Tang 2000). This result suggests a CLSC is more likely to experience bullwhip than a traditional supply chain.

Property 3. Sharing return and the yield information reduces the variance of the production order (*i.e.* $V[P^A] < V[P^N]$), *if and only if* $T_p - T_r \geq \tau$ *and* θ *is positive.*

This suggests that advance notice of the product returns enables the manufacturer to reduce the bullwhip effect. However, this desirable outcome occurs only in a limited set of circumstances. For example, if D_t and R_t are mutually independent (that is, $\theta = 0$), then $V[P^A] = V[P^N]$. This may lead us to the conclusion that the advance notice scheme does not influence the bullwhip effect. When $T_p - T_r \geq \tau$ and $\theta < 0$, the variance of the production order increases when information is shared. Therefore, if the reduction of the bullwhip effect is a major concern, we should be careful when using an advance notice scheme. Managers should pay attention to the values of $\{\theta, T_p, T_r, \tau\}$.

If there is flexibility in the choice of values for T_p and T_r, Properties 4–5 are useful.

Property 4. When information about the returns and the yield is shared, the variance of the net stock levels ($V[NS^A]$) *decreases in* T_r *if and only if* $T_p - T_r \geq \tau \wedge \theta < V[\Xi(R)]/(2\bar{\xi}k\sigma_\varepsilon^2)$.

When $T_p - T_r \geq \tau$, differentiating $V[NS^A]$ with respect to T_r yields $\partial V[NS^A]/\partial T_r = 2\bar{\xi}\theta k\sigma_\varepsilon^2 - V[\Xi(R)]$. Therefore $\partial V[NS^A]/\partial T_r$ becomes negative if $\theta < V[\Xi(R)]/(2\bar{\xi}k\sigma_\varepsilon^2)$.

Property 5. When information about the returns and the yield is shared and $\tau > T_p - T_r > 0$, the variance of the net stock levels ($V[NS^A]$) decreases in T_r. Increasing the value of T_r (until $T_r = T_p$) reduces the value of $V[NS^A]$.

Property 5 is proved by noticing that $(T_p - T_r)\left(V[\Xi(R)] - \bar{\xi}^2\theta^2k^2\sigma_\varepsilon^2\right) \geq 0$, when $\tau > T_p - T_r > 0$.

Properties 4 and 5 produce a practically useful insight; under certain conditions, a longer remanufacturing lead time (T_r) can decrease the net stock variance of the manufacturer. For example, if demands and returns are independent of each other (i.e. $\theta = 0$, which is always less than $V[\Xi(R)]/(2\bar{\xi}k\sigma_\varepsilon^2)$), $T_p > T_r$ and $\tau = 0$, longer remanufacturing lead times decrease the inventory variance. In a traditional supply chain, it is known that longer lead times increase net stock variance (Lee, So and Tang 2000; Chen et al. 2000; Hosoda and Disney 2006). Our results indicate that such an insight obtained from a supply chain without returns is not valid in our CLSC. Using a numerical analysis, van der Laan, Salomon and Dekker (1999) also found this phenomenon could be observed when $T_p > T_r$. Inderfurth and van der Laan (2001) reported similar findings to Properties 4 and 5, although the settings and assumptions used in their model were different from ours. We have provided validation of Inderfurth and van der Laan's (2001) lead time paradox by mathematically establishing and characterising its existence, albeit in a very different CLSC.

Interestingly, the lead time paradox can be observed even when the advance notice is not available to the manufacturer. Equation (12) suggests that when $T_p - T_r \geq \tau$, the variance of the net stock levels ($V[NS^N]$) decreases in T_r if θ is negative.

Property 6. When $T_r > T_p$, $V[NS^A]$ is independent of T_r.

Property 6 suggests that when an advanced noticed scheme is available a shorter remanufacturing lead time (T_r) does not decrease the net stock variance when $T_r > T_p$. Therefore managers should think carefully about investing in capability to reduce T_r as this may not reduce inventory costs. Indeed, as $V[NS^A] = (1 + T_p)\sigma_\varepsilon^2$, they should focus efforts on reducing T_p. Property 6 occurs because the advance notice scheme allows one to remove all the uncertainty associated with the returns and the remanufacturing process.

Finally, our variance expressions reveal that irrespective of the availability of advance notice, the following two fundamental trade-off issues exist in CLSCs.

Property 7. Except in $V[NS^A]$ when $T_r \geq T_p$, the production and the net stock variances are increasing in μ_R.

It is obvious from the variance expressions that when μ_R increases, $V[\Xi(R)]$ will increase, which could result in lower supply chain performance. Companies should be careful about increasing the average return rate, μ_R. A similar phenomenon was identified by van der Laan et al. (1999) who concluded that it may be unwise to remanufacture all returned products. These findings suggest that whilst larger values of μ_R are preferable for the environment, lower values of μ_R enhance the dynamic performance of the supply chain. $V[NS^A]$ is independent of μ_R only when $T_r \geq T_p$.

Property 8. The production and the net stock variances are increasing in the mean of the random yield $\bar{\xi}$ and / or its variance $V[\xi]$. A single exception is $V[NS^A]$ when $T_r \geq T_p$.

$V[\Xi(R)]$ increases in $\bar{\xi}$ and $V[\xi]$. If the mean yield $\bar{\xi}$ increases but $V[\xi]$ remains constant, the production and the net stock variances increase. This suggests that a more effective remanufacturing process may lead to lower supply chain performance.

Properties 4, 5 and 6, lead to the following managerial insights. To reduce $V[NS^A]$, managers should ensure that $T_r \geq T_p$. In addition, as a longer lead time may generate additional costs (for example WIP costs), reducing T_r to meet the condition $T_r \geq T_p$ is preferable. This will naturally result in $T_r = T_p$, a setting that resolves the worrying trade-off revealed by Properties 7 and 8 since when $T_p = T_r$, $V[NS^A]$ is independent of μ_R, $\bar{\xi}$ and $V[\xi]$.

5. Numerical example for uniformly distributed yields

In this section we will conduct a numerical investigation to verify our mathematical insights. We assume that the demand is normally distributed and that the remanufacturing yield ξ_t is uniformly distributed between $0 \leq a \leq b \leq 1$. The PDF of a uniformly distributed random variable is given by

$$f(\xi) = \begin{cases} \dfrac{1}{b-a}, a \leq \xi \leq b \\ 0, \xi < a \ \vee \xi > b \end{cases}$$

giving an average yield of $\bar{\xi} = (a+b)/2$ with a variance of $V[\xi] = (b-a)^2/12$. The impact of this uniform distribution assumption on the PDF of the production orders is shown in Appendix C. Through extensive simulation we have observed that the difference between the actual PDF and the normal PDF with a matched mean and variance becomes indistinguishable when the variance of the random yield is small. In this situation an investigation of costs based on newsvendor techniques that exploits only the first and second moments will be quite accurate.

Assume now that the following expression is a good indicator of the value of advance notice on the inventory cost,

$$\widehat{\Delta} = \frac{\sqrt{V[NS^N]} - \sqrt{V[NS^A]}}{\sqrt{V[NS^N]}} \times 100,$$

and, unless otherwise stated, the following values are present: $\mu_D = 100$, $\mu_R = 50$, $\sigma_\varepsilon = 1$, $k = 1$, $T_p = 5$, $T_r = 1$, $\tau = 2$, $\theta = 0.7$, $a = 0$ and $b = 1$.

Using Mathematica we have created Figure 3 which illustrates the value of $V[\Xi(R)]/(2\bar{\xi}k\sigma_\varepsilon^2)$, when $0 < k \leq 4$, $0 \leq a \leq 1$ and $b = 1$. Since $|\theta| \leq 1$, we observe that one of the required conditions for the property 4, $\theta < V[\Xi(R)]/(2\bar{\xi}k\sigma_\varepsilon^2)$, is met in almost all cases. Only when the value of a is quite high (say $a > 0.9$) and the value of k is relatively small ($k < 2$) does such a condition not hold.

Figure 4 illustrates the impact of T_r and μ_R on $\widehat{\Delta}$. The graph on the left-hand side shows that when $T_r \geq T_p$ (= 5, in this case), $\widehat{\Delta}$ is maximized and independent of T_r. The graph on the right-hand side of Figure 4 illustrates the impact of μ_R on $\widehat{\Delta}$. The value of μ_R varies from 10 to 90. Figure 4 shows that $\widehat{\Delta}$ is increasing in μ_R and is affected by T_r, but $\widehat{\Delta}$ becomes less sensitive to μ_R as μ_R increases. Overall, Figure 4 suggests that increasing the remanufacturing lead time T_r, or the mean returns μ_R, results in higher benefits from the advance notice scheme. Also, the advance notice is most valuable when $T_r \geq T_p$ and μ_R is large. Note however that a longer T_r or larger μ_R may increase other costs, such as WIP or remanufacturing costs which are not captured by our objective function.

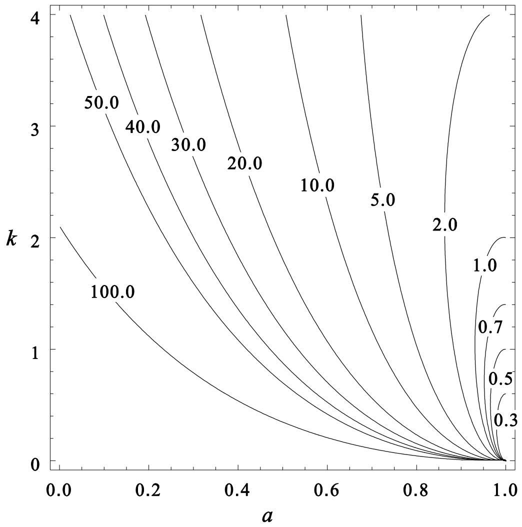

Figure 3. Value of $\frac{\boldsymbol{V[\Xi(R)]}}{\boldsymbol{2\bar{\xi}k\sigma_\varepsilon^2}}$ *when* $\mathbf{0 < k \leq 4}$, $\mathbf{0 \leq a \leq 1}$ *and* $\mathbf{b = 1}$

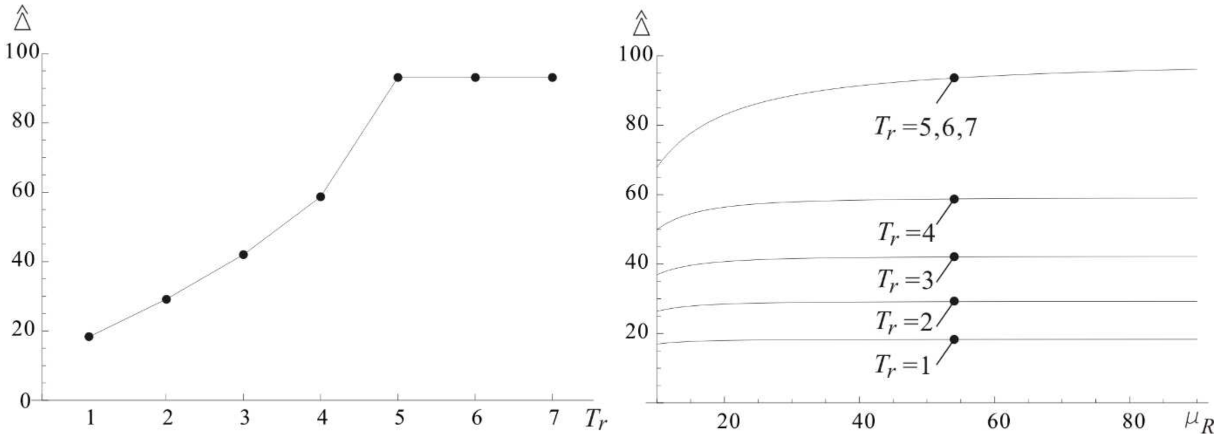

Figure 4. Impact of $\boldsymbol{T_r}$ *and* $\boldsymbol{\mu_R}$ *when* $\boldsymbol{T_p = 5}$

Consider now the impact of $\bar{\xi}$ as we increase the value of a from zero to unity and hold $b = 1.0$. Note that in this setting, $\bar{\xi}$ is increasing in a (since $\bar{\xi} = (a + b)/2$) but $V[\xi]$ becomes smaller (since $V[\xi] = (b - a)^2/12$), see Figure 5. It is shown that the impact of a or $\bar{\xi}$ is largely dependent on the value of the scale factor k and the correlation factor θ, particularly when a or $\bar{\xi}$ is large. High values of a imply higher values of $\bar{\xi}$ and smaller values of $V[\xi]$ which together result in high values of $\widehat{\Delta}$ when the demand and the returns are highly correlated (e.g. $\theta \geq 0.8$). We can also see that the values of a (or $\bar{\xi}$), k and θ have almost no impact on $\widehat{\Delta}$ when a or $\bar{\xi}$ is small (e.g. a < 0.4 or $\bar{\xi}$ < 0.7). Figure 5 also indicates that there is a benefit to the advance notice scheme even when the value of a is small (that is when $\bar{\xi}$ is small and $V[\xi]$ is high). This implies that a high yield is not required to benefit from the advance notice scheme. Indeed higher values of a could reduce the value of the advance notice, particularly when θ is small.

Figure 6 illustrates the situation when $T_p = T_r = 5$. Note that under the condition $T_r \geq T_p$, $\widehat{\Delta}$ is independent of θ. The value of $\widehat{\Delta}$ in Figure 6 is almost always better than in Figure 5. Only when $a = 1$, $\theta = 1$ and $k = 1$ will these two values become equal. This indicates that irrespective of the value of θ, increasing T_r up to five (so that it equals T_p in this example) yields better performance. Figure 6 shows that $\widehat{\Delta}$ is decreasing in both a and $\bar{\xi}$. Both Figures 5 and 6 indicate that improving the mean and the variance of the yield may reduce the value of the advance notice scheme. It should be noted that when $T_p = T_r$, $V[NS^A]$ is independent of a, $\bar{\xi}$, and θ since $V[NS^A] = (T_p + 1)\sigma_\varepsilon^2$. Thus, the decreasing trend of $\widehat{\Delta}$ in a and $\bar{\xi}$ in Figure 6 is simply because $V[NS^N]$ is decreasing in a and $\bar{\xi}$.

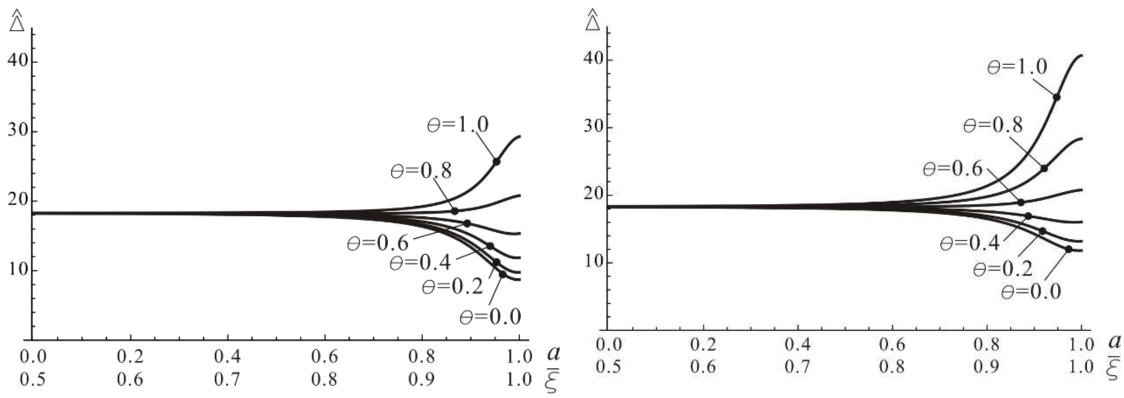

Figure 5. Impact of a, $\bar{\xi}$ and θ on $\widehat{\Delta}$ when $k = 1$ (left) and $k = \sqrt{2}$ (right)

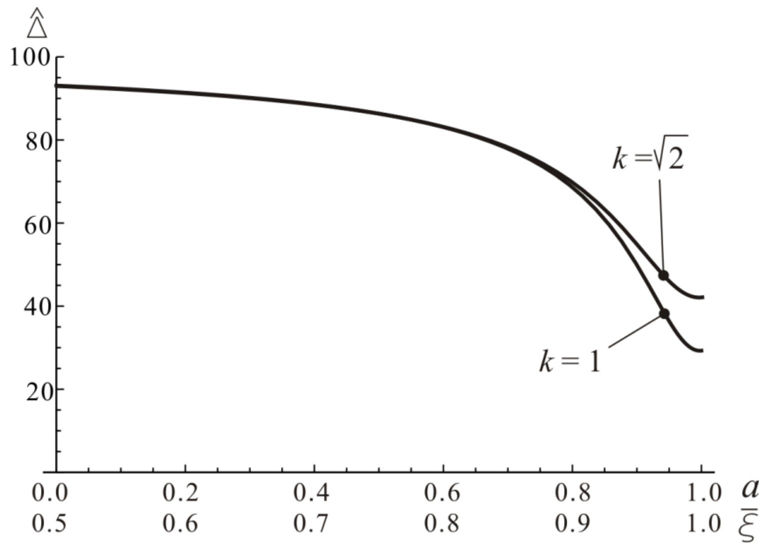

Figure 6. Impact of a and $\bar{\xi}$ on $\widehat{\Delta}$ when $T_p = T_r = 5$, $k = 1$ and $k = \sqrt{2}$

Figure 7 illustrates the impact of τ (the abscissa) and $T_p - T_r$ (the ordinate) on $\widehat{\Delta}$ (the numbers in the figure). We can see that the advance notice scheme provides the largest benefit when $T_r \geq T_p$, irrespective of τ. This suggests that the manufacturer will obtain a benefit from reducing his lead time T_p to T_r, but not from reducing it further.

6. Conclusions

Using a mathematical model and a numerical study, we have investigated the benefit of an advance notice scheme and its dependence on lead times, random yield and correlation between demand and returns. We have shown that sharing return and yield information may be beneficial to the manufacturer. In certain scenarios, however, the production variance could increase, although the net stock variance decreases as the result of the advance notice scheme. We found that longer remanufacturing lead times T_r may reduce inventory variance. This is a rediscovery and mathematical validation of the lead time paradox first identified by van der Laan, Salomon, and Dekker (1999) and then investigated by Inderfurth and van der Laan (2001). Our model considers a somewhat different setting to these previous studies, suggesting that the lead time paradox may be quite common in CLSCs. We have also shown that increasing the returns and the yields could have a negative impact on the system. This might be an interesting topic for future research.

Our findings yield the following general guidelines for managers. Advance notice of returns allow tighter control of inventories, especially when two lead times are equal and both are minimised. CLSCs with advance notice and two identical and minimised lead times not only reduce inventory variance but also can avoid the lead time paradox and the fundamental trade-off between the volume of return and dynamic supply chain performance.

Finally, research limitations should be mentioned. The findings shown in the research may not be applicable to other settings. For example, we considered a decentralised, push system. In a centralised system, an inventory of remanufacturable products at remanufacturer could be held to allow the remanufacturer to exploit a pull policy in order to achieve a more efficient supply chain. This is a different scenario that requires a different model and may result in different findings.

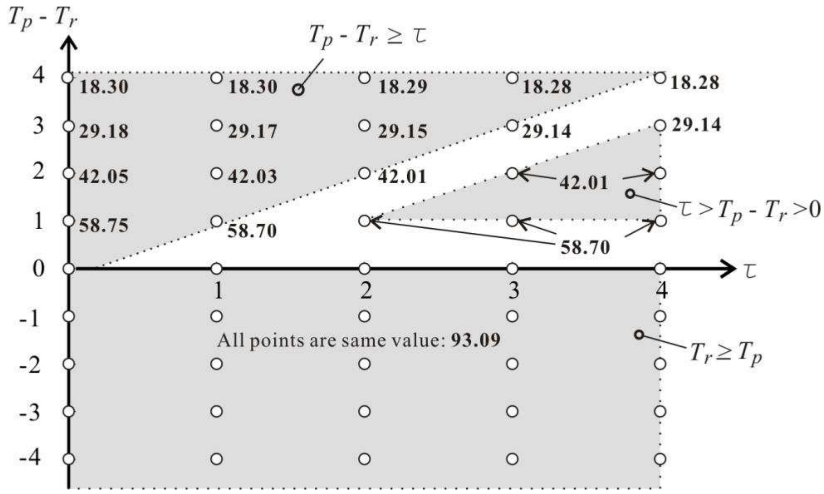

Figure 7. Impact of T_r *and* τ *on* $\widehat{\Delta}$ *when* $T_p = 5$, $1 \leq T_r \leq 9$ *and* $0 \leq \tau \leq 4$

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Appendix A: Variance of the returns and the correlation between the demand and returns

The variance of R_t and the correlation coefficient between $D_{t-\tau}$ and R_t are identified herein. The variance of a random variable X is the expected value of its squared deviations from the mean; $E[(X-\mu)^2]$ where $\mu = E[X]$. As $\sigma_\zeta = k\sigma_\varepsilon$, the variance of R_t is

$$V[R] = \mathrm{E}[(R_\mathrm{t} - \mu_R)^2] = E\left[\left(\theta k \varepsilon_{t-\tau} + \sqrt{1-\theta^2}\zeta_t\right)^2\right] = \theta^2 k^2 \sigma_\varepsilon^2 + (1-\theta^2)k^2\sigma_\varepsilon^2 = k^2\sigma_\varepsilon^2.$$

Using the covariance of $D_{t-\tau}$ and R_t, the correlation coefficient, $cov(D_{t-\tau}, R_t)$, is given by

$$\frac{cov(D_{t-\tau}, R_t)}{\sigma_\zeta \sigma_\varepsilon} = \frac{E\left[\varepsilon_{t-\tau}\left(\theta k \varepsilon_{t-\tau} + \sqrt{1-\theta^2}\zeta_t\right)\right]}{\sigma_\zeta \sigma_\varepsilon} = \frac{\theta k \sigma_\varepsilon^2}{k\sigma_\varepsilon^2} = \theta.$$

Appendix B: Variance of the remanufacturing yield

To obtain the variance of the remanufacturing yield, $\Xi(R)$, we note that the variance of a random variable X is equal to $V[X] = E[X^2] - E[X]^2$. This leads us to

$$V[\Xi(R)] = E[\Xi(R_t)^2] - E[\Xi(R_t)]^2 = E[\mu_R^2 \xi_t^2 + \theta^2 k^2 \xi_t^2 \varepsilon_{t-\tau}^2 + (1-\theta^2)\xi_t^2 \zeta_t^2] - \bar{\xi}^2 \mu_R^2.$$

Since $E[X^2] = V[X] + E[X]^2$, $E[\xi_t^2]$ can be written as $V[\xi] + \bar{\xi}^2$, which yields the final expression of $V[\Xi(R)]$:

$$V[\Xi(R)] = \bar{\xi}^2 k^2 \sigma_\varepsilon^2 + (\mu_R^2 + k^2 \sigma_\varepsilon^2) V[\xi].$$

This result suggests that $V[\Xi(R)]$ is increasing in $\bar{\xi}$, k^2, σ_ε^2, μ_R and $V[\xi]$. Note that the levels $\bar{\xi}$ and μ_R influence the variance in this non-linear system. This does not happen in linear systems.

Appendix C: Verification of the normal distribution assumption

Consider the following numerical scenario. Let the mean demand $\mu_D = 20$ with a variance $\sigma_\varepsilon^2 = 2$, and mean returns $\mu_R = 10$ with a variance of $k^2\sigma_\varepsilon^2 = 4$, implying the scale parameter $k = \sqrt{2}$. Assume that both the returns and the demand are normally distributed. The demand and the returns are correlated with a correlation coefficient of $\theta = 0.5$ and a correlation lag parameter of $\tau = 3$. Consider the case where the minimum of the uniformly distributed random yield $a = 0.1$, the maximum $b = 0.9$ and the lead times $T_p = 2$ and $T_r = 4$.

The results from simulating the system for 100,000 periods in Excel are summarised with a frequency plot of the production orders, see Figures 8 and 9. We have also plotted a normal distribution with the same mean and variance as the relevant system state. Although the first two moments are identical, the PDF is not completely captured. This is because the multiplication of the returns by the remanufacturing yield creates a non-linear system that is very difficult to characterise fully.

The most extreme error in the frequency plot can be seen in the yield (Figure 8) for the case when $a = \{0.1, 0.7\}$ and $b = 0.9$. This is the source of the non-linearity in the model. When the variance of the yield reduces (when $a = 0.7$ and $b = 0.9$ are used for the boundaries of the uniformly distributed yield), the normal approximation becomes more accurate.

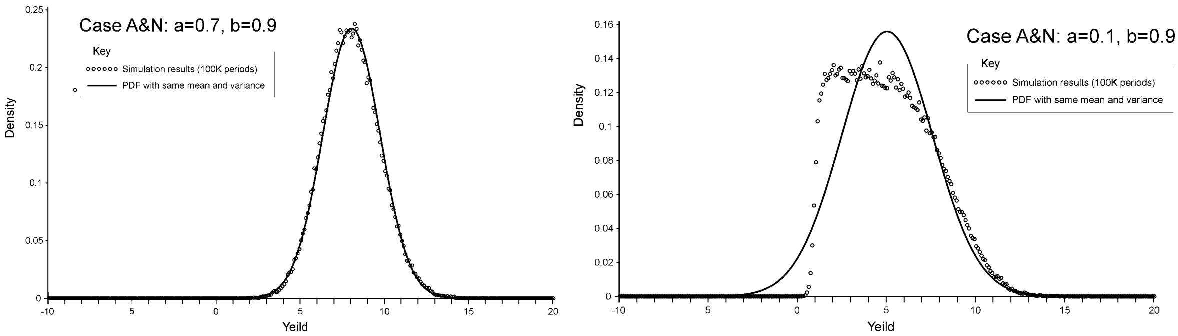

Figure 8. The actual density of the yield verses a normally distributed approximation based on the first two moments

The PDF of the orders becomes more normal than the yield PDF, and we can again see that the smaller yield variances induce a better fit to the normal distribution, as shown in Figure 9.

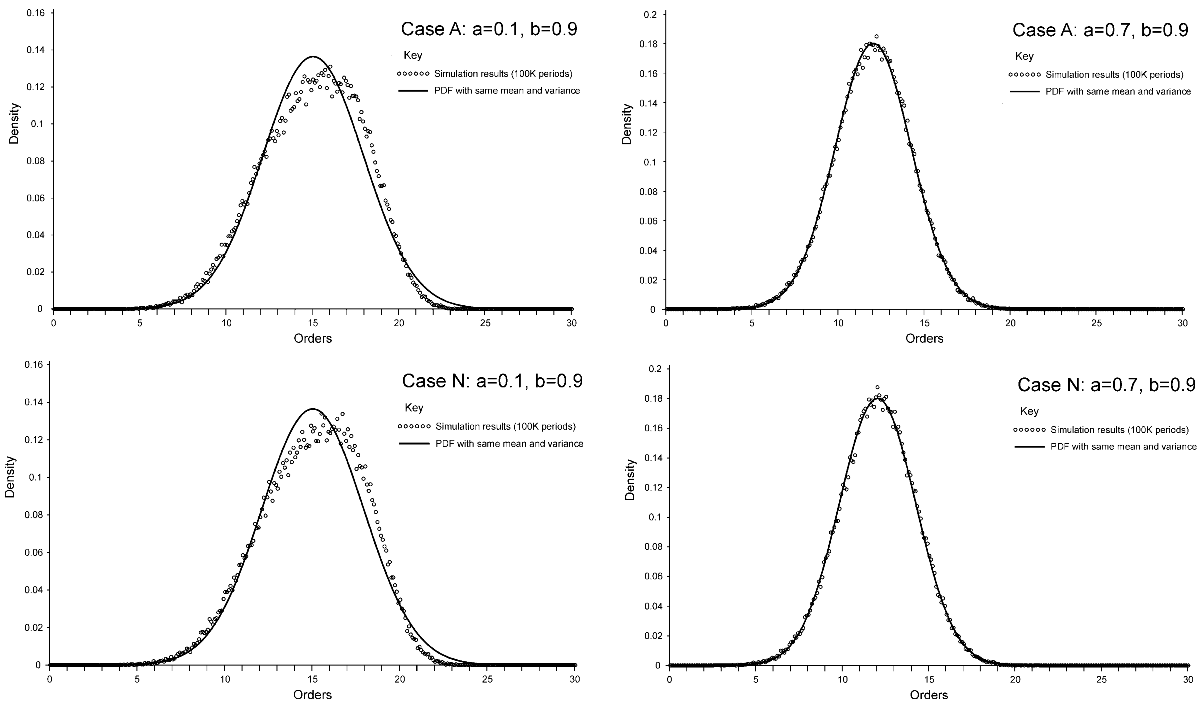

Figure 9. The actual density of the orders verses a normally distributed approximation based on the first two moments

The inventory PDFs are shown in Figure 10. The dominant factor determining normality now seems to be whether there is advance notice or not. Advance notice also has a significant impact on reducing the inventory variance. We can also see that when the yield has a reduced variance, then the first and second moments better describe the density of the inventory levels.

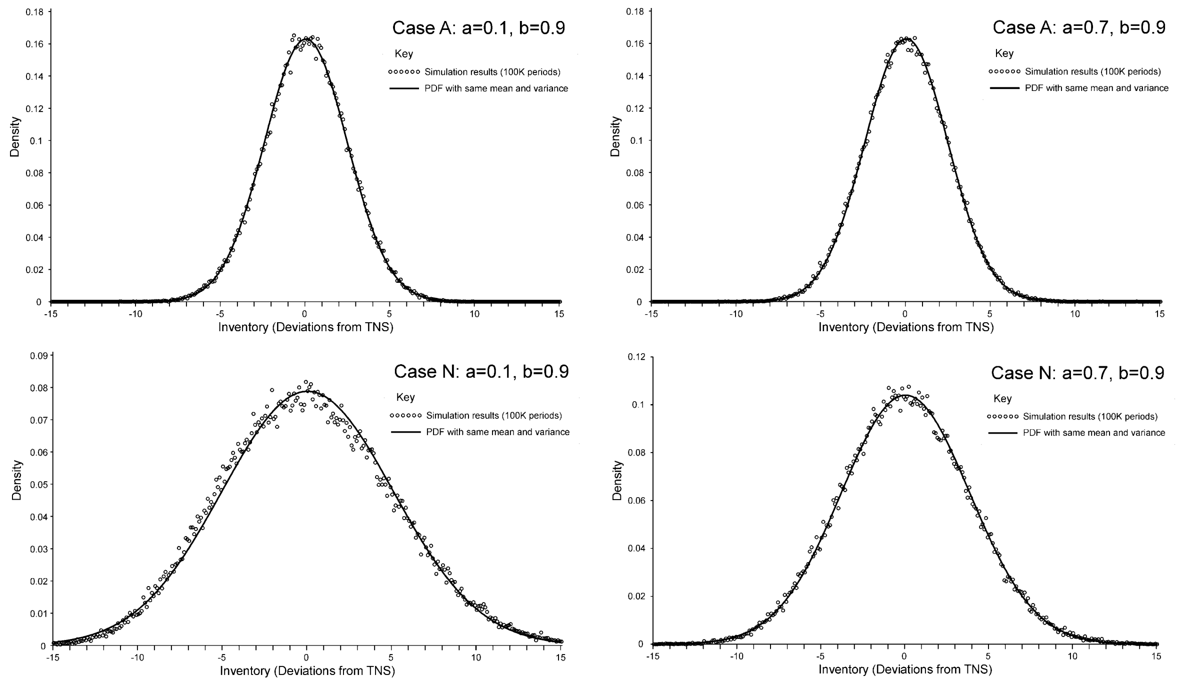

Figure 10. The actual density of the inventory levels verses a normally distributed approximation based on the first two moments