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I hope that you find the briefings to be interesting, stimulating and relevant.

Professor Martin Kitchener
Dean, Cardiff Business School

Supply Chain Chaos: Introduction

Supply chains are complex dynamic systems that contain many forms of nonlinearities and physical constraints.

This briefing shows, for the first time, that the nonnegative constraint on orders in supply chains can cause chaos.

Chaos costs, but chaos can be eliminated by careful design of replenishment rules.
What is an inventory system?

An inventory system is concerned with the flow of materials or finished goods along the supply chain. The goal of inventory control is to ensure smooth and even flow of product through the supply chain whilst maintaining availability without holding excessive inventory.

Material flow in supply chain is driven by demand and decisions and so there are two components in the inventory system.

1. An objective component involves the physical movement of goods – transportation, accumulation and consumption.

2. A subjective component includes perceptions and decisions such as forecast and ordering.

As these aspects are all inter-related, a “system” exists.

What is an ordering policy?

An ordering/production policy is a procedure or algorithm to determine the quantity of product to manufacture or distribute and how operations managers control inventory.

The ordering policy should follow some basic rules: order more when you have less; order more when you need more. In the language of systems, these are “negative feedback” and “positive feed-forward” rules.

When an order is placed, it takes time to produce and distribute products during which new demand may have occurred. There are also two kinds of inventories that a manager needs to think of when determining ordering quantities: inventory on hand (orders that have arrived) and inventory on order (the open orders, the orders placed but that have not yet arrived). See Fig. 1.

Consciously or not, a supply manager frequently assigns different levels of importance (or weights) to these two kinds of inventories. When thinking about designing ordering policies, it is usually about setting the relative weights when making replenishment decisions.

Fig. 1
Control mechanism in inventory systems

Replenishment decision

Open orders

Inventory on hand

Shipment

Arrival

Consumption
Why do we need to design an ordering policy?

The physical aspects of supply chains are usually considered to be predetermined. The design of the ordering policy is vital as it influences how an inventory system behaves and performs. Therefore it must be designed carefully. Many supply managers still do not have a clear procedure to follow when making such decisions due to the variety of situations they have to deal with.

In many cases it is also necessary to build an automated inventory system, where inventory levels are monitored, replenishment orders quantities are decided, and orders are issued automatically. One such case is supermarkets, where thousands of different products need to be managed to avoid excess inventory and stock-out. Here an ordering policy is not suggested, but required.

How is an ordering policy designed?

Answer: math.

The real world is almost always too complex to analyse. In this regard, it’s important to find a compromise between simplicity and reality, so that the problem is both significant and solvable. This process is called modelling.

Establishing the model is the first step before specifying which kind of performance is desirable. That is to say, aims and objectives must be set up.

For example, either reducing inventory related cost or reducing ordering fluctuations can be objectives. Sadly they tend to contradict each other and they need to be carefully balanced.

There are several techniques that can be applied when designing replenishment policies.

- **Observe** - when there is a need to know whether a certain policy meets some basic criteria.
- **Compare** - when there are a few alternative policies on hand.
- **Tune** - to find the best parameter values in a given system.
- **Optimise** – to identify the best policy among all possible policies that could exist.
What new discoveries have been made?

Finding the right balance between tractability and relevance is not easy. In previous studies, nonnegative constraints on the variable of orders have been discarded. But this assumption is unrealistic. Negative orders mean goods are returned to, and accepted by, suppliers who are often reluctant to do so. To this end, nonnegative constraint was added to the orders.

To analyse this model a new weapon is necessary – the theory of nonlinear dynamic systems. **New dynamical behaviours are exposed with this new technique, which could never be discovered with old theory.**

There is, (see Fig. 2):

- **Convergence**, meaning that if demand becomes constant, then orders will eventually be constant.
- **Divergence**, where orders will diverge drastically from the desired level.
- **Periodical and quasi-periodical fluctuations**, where the orders go up and down with the same pattern and the same pace, creating the phenomenon known as rogue seasonality and economic cycles.
- **Chaos**, where the system would behave so randomly and so sensitively, that even the slightest change causes an unpredictable future.

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**Fig. 2**

*Dynamic behaviours observed in nonlinear inventory system model*

- Convergent
- Periodic
- Quasi-periodic
- Chaotic
- Divergent

Oscillation patterns in FR system

(a) Convergent
(b) Periodic
(c) Quasi-periodic
(d) Chaotic
(e) Divergent

Supply chain cost

Low

High

Orders

Time
New discoveries continued

It is possible to determine when such behaviour will exist and that knowledge can be used to properly design replenishment rules (see Fig. 3 and Fig. 4). Also delicate structures known as “sausages” have been discovered (see Fig. 5).

However, that is not the end of story. Order fluctuations are expensive in supply chains – it creates capacity losses, overtime and labour idling. When designing an ordering policy, it is very important to maintain smooth orders over time. It is important then to know the magnitude of the maximum and minimum ordering quantities.

The minimum order quantity is zero as orders are nonnegative. The maximum order quantity is harder to determine but it has been possible to calculate maximum possible ordering quantity for a given replenishment system design. Fig. 6 and Fig. 7 show, through bifurcation diagrams, how the theoretical curves (brown) perfectly envelope and stratify the different density of the orders (orange dots).
Conclusions

By careful design of replenishment rules, chaos can be avoided in supply chains.

We need to properly select feedback parameters and match them to the lead times.
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