Algorithm XXX: DAESA — a Matlab Tool for Structural Analysis of Differential-Algebraic Equations: Software

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DAESA, Differential-Algebraic Equations Structural Analyzer, is a MATLAB tool for structural analysis of differential-algebraic equations (DAEs). It allows convenient translation of a DAE system into MATLAB and provides a small set of easy-to-use functions. DAESA can analyze systems that are fully nonlinear, high-index, and of any order. It determines structural index, number of degrees of freedom, constraints, variables to be initialized, and suggests a solution scheme. The structure of a DAE can be readily visualized by this tool. It can also construct a block-triangular form of the DAE, which can be exploited to solve it efficiently in a block-wise manner.

Categories and Subject Descriptors: I.6.7 [Computing methodologies]: Simulation support systems; G.4 [Mathematical software]: Matlab, Algorithm design and analysis

Additional Key Words and Phrases: Differential-algebraic equations, structural analysis, modeling

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1. INTRODUCTION

We present DAESA, Differential-Algebraic Equations Structural Analyzer, a MATLAB tool for structural analysis (SA) of differential-algebraic equations (DAEs). It allows convenient translation of a DAE into MATLAB and provides (currently 18) easy-to-use functions for determining the structural index, the number of degrees of freedom (henceforth referred to as the DOF), the constraints and a solution scheme, and for visualizing the structure of the DAE. DAESA can also construct a block-triangular form (BTF), which can be exploited for efficient solution in a block-wise fashion.

Our package is applicable to DAE systems of the general form

\[ f_i(t, x_j \text{ and derivatives of them}) = 0, \quad i = 1, \ldots, n, \]  

where \( t \) is the independent variable, and the \( x_j(t) \) are \( n \) state variables. The formulation (1) includes high-order systems and systems that are nonlinear in leading deriv-
tives. Furthermore, (??) includes systems of ordinary differential equations (ODEs) and pure algebraic systems.

DAESA performs analysis similar to that of the C++ solver DAETS [?, ?]. However, DAETS is not suitable for rapid investigation of DAEs, as it requires C++ knowledge and compiling the user code. The goal of this work is to produce a light-weight, easy-to-use tool with convenient facilities for rapidly exploring a DAE's structure. The present tool is based on Pryce's SA [?] and recent developments by the authors on improving this analysis using block-triangularization of the DAE. The choice of MATLAB is due to its ubiquity and ease-of-use, as well as its operator overloading, which is central to our implementation.

We do not define the terminology we use here, nor present the underlying theory—for this see the companion paper [?] and the references therein.

This article is organized as follows. Section ?? gives an overview of DAESA. Section ?? presents several examples of analyzing DAEs. Section ?? investigates the performance of this package and the relative amount of work of various parts of it. Conclusions are given in Section ??.

2. OVERVIEW OF DAESA

DAESA exploits MATLAB's operator overloading to process a DAE given by a user-supplied function for evaluating the \( f_i \) in (??). In particular, it extracts the signature matrix \( \Sigma \) and determines for each equation if it is quasilinear in the leading derivatives in the sense explained in the companion paper, see also [?].

After the signature matrix is constructed, DAESA finds out if the problem is structurally well-posed, and if so, finds a highest-value transversal (HVT) and solves a linear assignment problem to calculate the offsets of the problem, and then determines its structural index and DOF. Since it knows the structure of the analyzed DAE, DAESA reduces it to block triangular form (BTF), finds local offsets, and determines block by block quasilinearity. Based on the offsets and linearity information, DAESA deduces which variables and derivatives of them need to be initialized and what the constraints are.

This package provides functions for reporting the constraints, initialization summary, and a solution scheme, and functions for displaying the original structure of the DAE, as well as for displaying coarse and fine BTFs of the DAE structure.

The DAESA package builds around three classes: sigma, qla, and SAd ata. The signature matrix is obtained by executing the function defining the DAE with objects of the class sigma. The quasilinearity analysis is carried out by executing this function with objects of the class qla. In both cases, the processing of the DAE is done through operator overloading of the arithmetic operators and the elementary functions. The SA is performed by the function daeSA. It returns an object of the class SAd ata, which encapsulates all the data obtained from the SA. Each of the remaining DAESA functions takes an object of this class as a parameter and extracts from it the data it needs. This mechanism (of the main function returning an object, and the remaining functions querying this object) ensures simple and consistent function interfaces.

Remark. The structural index computed by DAESA is an upper bound on the differentiation index, and often they are the same. Although successful on many problems

Footnote 1: Concepts explained in the companion paper are typeset in slanted font on first occurrence.
of interest, the underlying SA theory (and DAESA) may fail to determine the correct structural, and therefore differentiation index on some problems; see e.g. [?; ?].

We could “certify” that the SA is successful, if the system Jacobian J could be shown to be non-singular at a consistent point [?]. The present tool does not compute consistent points and does not evaluate J: it performs symbolic-type analysis of DAEs. We plan to incorporate the evaluation of J in a future version of DAESA.

3. DAESA EXAMPLES

In this section, we illustrate some of the capabilities of DAESA. Namely, we investigate the SA of DAESA on a simple DAE, the chemical Akzo Nobel problem [?] (§??). We also present results from analyzing a DAE arising from modeling a distillation column (§??), and show how DAESA reports structurally ill-posed (SIP) problems (§??). For details, see the DAESA user guide [?], which is also part of the distribution of this package.

3.1. Simple DAE: Chemical Akzo Nobel

We show DAESA’s SA on the chemical Akzo Nobel problem, an index-1 DAE. The equations for this problem are given in §5.1 of the companion paper. The DAESA function for evaluating them is displayed in Figure ??.

```matlab
function f = akzonobel(t,y)
k1 = 18.7; k2 = 0.58; k3 = 0.09; k4 = 0.42;
K = 34.4; klA = 3.3; CO2 = 0.9; H = 737;
Ks = 115.83;
r1 = k1*y(1)^4*sqrt(y(2));
r2 = k2*y(3)*y(4);
r3 = k2/K*y(1)*y(5);
r4 = k3*y(1)*y(4)^2;
r5 = k4*y(6)^2*sqrt(y(2));
Fin = klA*(CO2/H - y(2));
f(1) = -Dif(y(1),1) - 2.0*r1 + r2 - r3 - r4;
f(2) = -Dif(y(2),1) - 0.5*r1 - r4 - 0.5*r5 + Fin;
f(3) = -Dif(y(3),1) + r1 - r2 + r3;
f(4) = -Dif(y(4),1) - r2 + r3 - 2.0*r4;
f(5) = -Dif(y(5),1) + r2 - r3 + r5;
f(6) = Ks*y(1)*y(4) - y(6);
end
```

Fig. 1. DAESA function for evaluating the chemical Akzo Nobel problem

Structural analysis is performed by the call daeSA(@akzonobel,6). The structure of a DAE, and its coarse and fine BTFs, are displayed graphically by the showStruct function. In Figure ??, we show the fine BTF for this problem as produced by the call showStruct(sadata,’disptype’,’fineblocks’): it decomposes into six fine blocks.

The solution scheme, for computing derivatives of the solution, is produced by printSolScheme, the ‘compact’ form of whose output is on the left of Figure ??.

This says a solution may be determined by giving initial values (IVs) for $y_1$, $y_2$, $y_3$, $y_4$ and $y_5$ at stage $-1$. Then using these IVs, at stage $k = 0$ we solve six linear scalar equations, one per fine block. More detail is given by the ‘full’ form of the output, summarized in
Solution scheme for 'akzonobel' problem

Initialization summary:
y_1, y_2, y_3, y_4, y_5

\begin{align*}
  k = -1: & \{1\} : y_5 \\
          & \{1\} : y_4 \\
          & \{1\} : y_3 \\
          & \{1\} : y_2 \\
          & \{1\} : y_1 \\
  k = 0: & \{[6]\} : y_6 \\
          & \{[5]\} : y_5' \\
          & \{[4]\} : y_4' \\
          & \{[3]\} : y_3' \\
          & \{[2]\} : y_2' \\
          & \{[1]\} : y_1'
\end{align*}

\begin{tabular}{|c|c|c|c|}
\hline
k & \text{Give } y_1, y_2, y_3, y_4, y_5 \text{ as initial values} & \text{solve for} & \text{using} \\
\hline
\hline
\hline
k = -1 & \text{initial values} & \text{computed} \\
\hline
k = 0 & \begin{align*}
  f_6 &= 0 \\
  f_5 &= 0 \\
  f_4 &= 0 \\
  f_3 &= 0 \\
  f_2 &= 0 \\
  f_1 &= 0
\end{align*} & y_5 & y_4 & y_3 & y_2 & y_1 & y_6 \\
\hline
\end{tabular}

Fig. 2. Fine BTF and solution scheme for the chemical Akzo Nobel problem

the inset: e.g., it says one solves \( f_5 = 0 \) for \( y_5' \), using the values of \( y_1 \) to \( y_4 \) given as IVs, and the previously computed value of \( y_6 \).

The computations at stage \( k > 0 \) are deduced from stage \( k = 0 \). That is, we solve first \( f_6^{(k)} = 0 \) for \( y_6^{(k)} \) and then solve \( f_i^{(k)} = 0 \) for \( y_i^{(k+1)} \) in the order \( i = 5, 4, 3, 2, 1 \)—all linear, and using previously computed values for the derivatives of the solution.

In [?], this problem is classified as nonlinear, requiring IVs for all variables and their first derivatives. Without the information produced by our BTF it would be regarded as fully coupled, needing solution of a non-linear system of size six to determine values for \( y_i^{(k+1)} \), \( i = 1, \ldots, 5 \), and \( y_6^{(k)} \) at stage \( k = 0, 1, \ldots \).

3.2. Chemical engineering application: distillation column

Systems of DAEs are commonly used in chemical engineering to describe mass/energy balances and constitutive relations, such as mass/energy transfer rates, reaction kinetics, thermodynamic properties and relations and control laws. Depending on the assumptions used to construct the model, a DAE system with an index exceeding one is possible. Then typically an index reduction is performed, and the resulting system is simulated with a standard, index-1 DAE solver (e.g., SUNDIALS [?] or DASSL [?]).

From a modeler’s perspective, one might be interested in determining the indices of equations that need to be differentiated to reduce to a lower-index DAE. These are

\[ \{ i \mid c_i > 0, \ i = 1, \ldots, n \}, \]

where \( c_i \) is the global offset of equation \( i \), and they can be readily obtained by the getOffsets function of DAESA.

In this section, we show results of DAESA’s SA on a separation model\( ^2 \) for a distillation column with a partial reboiler and total condenser. The present model is an index-2 system of 129 equations. We refer to this system as DISTCOL. In [?], it is reduced to index-1 using dummy derivatives [?], resulting in 173 equations, and then simulated with SUNDIALS [?]. We refer to this system as DISTCOLDD. Their structures are shown in Figure ??.

\( ^2 \)This model is provided by Ian Washington of the Department of Chemical Engineering of McMaster University.

![Diagram](image.png)

Fig. 4. Distillation model: structure of index-2 and index-1 formulations. Offsets, and equations and variable indices are not displayed for problems of size \( n > 40 \).
3.2.1. Coarse BTF. In Figure ??(a) and (b), we show the coarse BTFs of DISTCOL and DISTCOLDD, respectively. Consider DISTCOL. With the getBTF function, we determine that there are 14 blocks of size 1 and one block of size 115. Using getQL data, we determine that blocks 1:1 and 3:117 are non-quasilinear. The rest are quasilinear. For
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3.2.2. Fine BTF. Figure ??(a) and (b) show the fine BTFs of DISTCOL and DISTCOLDD, respectively. DISTCOL has 52 blocks of size 1, of which 13 are non-quasilinear, and 11 quasilinear blocks of size 7. Obviously, the 115×115 block (from the coarse BTF) has decomposed into smaller blocks. For DISTCOLDD, we have 85 blocks of size 1, of which 13 are non-quasilinear, 11 non-quasilinear blocks of size 3, and 11 quasilinear blocks of size 5. Either way, the BTF shows great potential for speeding up numerical solution.

3.3. Troubleshooting

DAESA can report missing equations and/or variables in problem formulation, and can produce a diagnostic BTF, indicating over- and under-determined parts of the system.

3.3.1. Reporting missing equations and/or variables. In the MOD2PEND problem from the companion paper, we remove the third equation, apply daeSA, and then call showStruct; see Figure ??(a). Then we remove the occurrences of variable μ and apply these two functions; see Figure ??(b). The function showStruct displays missing equations and/or variables using shading.

![Fig. 7. Displaying the structure of two ill-posed problems](image)

For large problems, one may not be able to easily find missing equation and/or variable indices from the displayed structure. The function getMissing produces indices of equations and variables that are missing in the problem. For example, calling

```matlab
[meqn, mvar] = getMissing(sadata)
```

on the second DAE (with missing equation 3 and variable 6) gives \( \text{meqn}=3, \text{mvar}=6 \).

### 3.3.2 Diagnosing SIP problems.

When a DAE does not have missing equations or variables but is nevertheless SIP, we can use \texttt{getBTF} to identify under-, over-, and well-determined subsystems. As an example, the function in Figure ?? defines a SIP prob-

```matlab
function f = illPosed3(t, z)
    x1 = z(1); x2 = z(2); x3 = z(3);
    x4 = z(4); x5 = z(5); x6 = z(6);
    f(1) = x1 + x2 + x5 + x6;
    f(2) = x1^3 + x2 + x5 + x6;
    f(3) = x5 * x6;
    f(4) = -x5^3 + x6^4;
    f(5) = x1 + x2 + x3 + x4 + x5 + x6^3;
    f(6) = x5 + x6;
end
```

Fig. 8. Example of a SIP problem

```matlab
sadata3 = daeSA(illPosed3, 6);
[pe, pv, cb, fb] = getBTF(sadata3);
showStruct(sadata3, 'disptype', 'blocks');
```

produces the output in Figure ?? The output vectors \( \text{pe} \) and \( \text{pv} \) give the permutation of equations and variables, respectively; \( \text{cb} \) is empty; and \( \text{fb} \) specifies the boundaries of the under-, well- and over-determined blocks. Here, we have a structurally under-determined equation \( 0 = f_5 \) in variables \( x_3 \) and \( x_4 \), and a structurally over-determined set of equations \( 0 = f_3, f_4, f_6 \) in variables \( x_5 \) and \( x_6 \).

---

**ILLPOSED3: Diagnostic BTF**

<table>
<thead>
<tr>
<th>Indices of Equations</th>
<th>Indices of Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 5 1 2 3 4 6</td>
<td>0 0 0 0 0 0</td>
</tr>
</tbody>
</table>

**Fig. 9.** Output and diagnostic BTF of the SIP problem from Figure ??
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4. PERFORMANCE

In this section we study the performance of the main function, daeSA. We apply it on a fully dense problem and five sparse problems. We describe these problems in §4.1. and then present timing and profiling data in §4.2. and §4.3., respectively.

4.1. Problems used

4.1.1. Dense DAE. The Layne Watson exponential cosine curve example [?] is a standard benchmark problem for demonstrating the ability of continuation methods to solve systems of nonlinear equations. The task is to find a fixed point of the nonlinear function \( g : \mathbb{R}^n \rightarrow \mathbb{R}^n \) defined by

\[
g_i(x) = g_i(x_1, \ldots, x_n) = \exp \left( \cos \left( i \cdot \sum_{k=1}^{n} x_k \right) \right), \quad i = 1, \ldots, n.
\]

Using DAETS [?], we sought a fixed point of the parameterized problem \( x = \lambda g(x) \), that is a root of

\[
f(\lambda, x) = x - \lambda g(x) = 0. \tag{2}
\]

When \( \lambda = 0 \), equation (2) has the trivial solution \( x = 0 \). We tracked solutions to the desired root at \( \lambda = 1 \). This was implemented in DAETS using arc-length continuation. Namely, we rewrote the system (2) as \( n \) equations in \( n+1 \) unknowns:

\[
f(y) = 0, \tag{3}
\]

where \( y = (x; \lambda) \), and using a new independent variable \( s \), we added to (2) the equation \( ||dy/ds||_2 = 1 \), that is

\[
\sum_j y_j'^2 - 1 = 0. \tag{4}
\]

Combining (2) and (3), we have an index-1, dense, non-quasilinear system of \( n + 1 \) equations and variables. We refer to the resulting systems as LW.

There is no nontrivial block structure, either coarse or fine, as shown in Figure 4.1, for \( n = 11 \).

4.1.2. Sparse DAEs. The problems below consist of \( P \) chained pendula, where each pendulum is of size 3. The resulting systems are of size \( n = 3P \), and each pendulum is quasilinear. The aim is to produce parameterized problems of similar sparsity patterns but differing significantly in index and block structure.

MULTIPEND. The first and the \( i \)th pendula (\( 2 \leq i \leq P \)) are

<table>
<thead>
<tr>
<th>first pendulum</th>
<th>( i )th pendulum</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 0 = x_1'' + \lambda_1 x_1 )</td>
<td>( 0 = x_i'' + \lambda_i x_i )</td>
</tr>
<tr>
<td>( 0 = y_1'' + \lambda_1 y_1 - G )</td>
<td>( 0 = y_i'' + \lambda_i y_i - G )</td>
</tr>
<tr>
<td>( 0 = x_1^2 + y_1^2 - L^2 )</td>
<td>( 0 = x_i^2 + y_i^2 - (L + c\lambda_{i-1})^2 ).</td>
</tr>
</tbody>
</table>

The state variables of the \( i \)th pendulum (\( i \geq 1 \)) are \( x_i, y_i, \) and \( \lambda_i; G > 0 \) is gravity, \( L > 0 \) is the length of the first pendulum, and \( c \) is a constant. A system of \( P \) pendula has index \( 2n/3 + 1 = 2P + 1 \).
The coarse and fine BTFs are the same. We display in Figure 10 the fine BTF for $P = 4$.

**MULTIPENDA.** This problem is obtained by replacing the third equation in each pendulum, except in the first one, by

$$0 = x_i'' + y_i'' - (L + cx_{i-1})^2.$$

The index is 3, for any $P$. There are $P$ coarse blocks of size 3, and each such block except the top-left one is decomposed into 3 fine blocks of size 1.

The coarse and fine BTFs for $P = 4$ are displayed in Figure ??, ??.

**MULTIPENDB.** The pendula are

- first pendulum
- $i$th pendulum, $2 \leq i \leq P - 1$
- $P$th pendulum

$$0 = x_i'' + \lambda_i x_i + cx_{i+1} \quad 0 = x_i'' + \lambda_i x_i + cx_{i+1} \quad 0 = x_{P-1}'' + \lambda_{P-1} x_{P-1}$$

$$0 = y_i'' + \lambda_i y_i - G \quad 0 = y_i'' + \lambda_i y_i - G \quad 0 = y_{P-1}'' + \lambda_{P-1} y_{P-1} - G$$

$$0 = x_i^2 + y_i^2 - L^2 \quad 0 = x_i^2 + y_i^2 - (L + cx_{i-1})^2 \quad 0 = x_{P-1}^2 + y_{P-1}^2 - (L + cx_{P-1})^2$$

The index is $P + 2$, for any $P$. Here there is only one coarse block, which decomposes into $P$ fine blocks of size 3.

The coarse and fine BTFs for $P = 4$ are displayed in Figure ??, ??.

**MULTIPENDC.** We replace the first equation in each pendulum $i < P$ in MULTIPENDA by

$$0 = x_i'' + \lambda_i x_i + cx_{i+1}.$$

The index is $P + 2$, for any $P$. There is one coarse block, which does not decompose into fine blocks (figure omitted).
MULTIPENDA: Coarse BTF
Size 12, structural index 3, DOF 11
Shaded: structural nonzeros in system Jacobian J
Boxed: positions that contribute to det(J)
Indices of Variables
Indices of Equations

MULTIPENDA: Fine BTF
Size 12, structural index 3, DOF 11
Shaded: structural nonzeros in system Jacobian J
Boxed: positions that contribute to det(J)
Indices of Variables
Indices of Equations

MULTIPENDA: Coarse BTF
Size 12, structural index 6, DOF 8
Shaded: structural nonzeros in system Jacobian J
Boxed: positions that contribute to det(J)
Indices of Variables
Indices of Equations

MULTIPENDA: Fine BTF
Size 12, structural index 6, DOF 8
Shaded: structural nonzeros in system Jacobian J
Boxed: positions that contribute to det(J)
Indices of Variables
Indices of Equations

Fig. 11. MULTIPENDA: coarse and fine BTFs

MULTIPENDA. Now we replace the first equation in each pendulum $i < P$ in MULTIPENDA by

$$0 = x''_i + \lambda_i x_i + c x''_{i+1}.$$
The index is $P/2 + (9 - (-1)^P)/4$. For any $P$, there is one coarse block. However, if $P$ is odd, there is one fine block, and if $P$ is even, there are $3P$ fine blocks. In Figure ??, we show the fine BTFs for $P = 3$ and $P = 4$ respectively.

![MULTIPENDD: Fine BTF](image)

Fig. 13. MULTIPENDD: fine BTFs

4.2. CPU time vs. size

The results in this subsection are produced with MATLAB R2013a on a Linux server with 16 CPUs and 64GB of RAM. The CPUs are Intel Xeon E7-8870 2.4GHz. The implementation of daesa does not take advantage of the parallel capabilities of MATLAB; that is, it is a serial implementation.

Figure ?? plots CPU time versus $n$ for the LW and pendulum problems for $n = 600 : 600 : 9600$, and separately for $n = 9600 : 600 : 18000$, ??, Denoting by $t_i$ the CPU time corresponding to size $n_i$, we find the constants in $\alpha n^\beta$ using a least-squares fit of $\log \alpha + \beta \log n_i = \log t_i$ and plot these fits.

For dense problems, we might expect $O(n^3)$ work due to the complexity for finding a HVT; here it is more like $O(n^2)$. For the sparse pendulum problems, the CPU time behaves more like $O(n^{5.5})$.

4.3. Work breakdown

We profile the execution of daesa on the above problems, using $P = 10, 100, 500, 1000$ and 3000 pendula, with problem sizes $n = 3P$. We use the same $n$'s for the LW problem.

In Figure ??, we display the fraction of time used for computing $\Sigma$, HVT, offsets, and performing quasilinearity analysis (QLA) for the LW and pendulum problems. For the remaining problems, the plots are very similar to Figure ??, and we omit them here.
For the LW problem, as $n$ increases, the computing time is dominated by the time for finding a HVT, Figure ?? For the remaining problems, most the time is in computing $\Sigma$ and finding QLA information, the latter being the dominating contribution, Figure ??

Fig. 14. CPU time versus $n$. The constants in the legends are the computed $\beta$'s in $\alpha n^\beta$.

Fig. 15. Profiling daeSA. The fractions of the total time are denoted as: $\Sigma$ for computing the $\Sigma$ matrix; HVT for finding HVT(s); $O$ for computing canonical offsets $c$ and $d$; QLA for performing quasilinearity analysis; Rest the rest of daeSA.

5. CONCLUSION AND FUTURE WORK

The PhD thesis of P. Bunus [?] clearly stated the goal of building software to analyze DAEs generated by modeling languages, to diagnose errors in model construction and to make numerical solution more efficient. He attacked it using methods based on sparsity, but the relation of sparsity to DAE block structure was then insufficiently understood.

Much progress has been made since then in such structural analysis software; we believe the DAESA package currently goes furthest towards Bunus’s goal. It is at present
unique in being based on a systematic theory for DAEs that can be fully implicit and of arbitrary order and index. Its features:

— report structural index and DOF;
— report solution scheme plus information about initial values and constraints;
— display structure and BTF graphically;

are present in other systems, but not in so far-reaching a way. We believe no other system exploits fine BTF to minimize the number of initial values required for numerical solution.

DAESA’s abilities to diagnose ill-posedness, and report under- and over-determined parts of a system, potentially go beyond what is currently available in terms of suggesting ways of correcting them. However, this needs considerable further development.

As for performance, tests on a large number of problems of various sizes and sparsities show DAESA is well able to analyze DAEs of size up to 18,000. Though the current version exploits sparsity only partially, the expected $n^3$ dependence on problem size has not materialized. The tests reported here give an indication where to concentrate effort to improve the algorithms. We are developing faster methods to compute the offsets, while future work will focus on reducing the time for QLA and computing the $\Sigma$ matrix; cf. Figure ??.

Further developments of this tool include implementation of the computation of the system Jacobian, which will provide a check for the success of the SA. That is, if this Jacobian is nonsingular (up to roundoff), the SA has succeeded. We have a prototype implementation, which will be incorporated in a future version of DAESA.

A complete DAE solver (with the capabilities of DAETS) in MATLAB would enable researchers and practitioners to use it readily within existing MATLAB code and to experiment within this problem-solving environment. Building such a solver requires a tool for computing Taylor coefficients (TCs). A suitable candidate is the ADTAYL [?] package. It is designed to compute TCs for an explicit function, given TCs of its argument(s), but it is inefficient for computing TCs for the solution to an ODE, let alone a DAE. We are working on extending it to compute such coefficients. Finding consistent initial point, stepsize control, and projections on each integration step can be handled as in DAETS.

An exciting direction for research and implementation is parallelizing the algorithms for DAE solving based on Pryce’s SA. The DAETS solver can handle efficiently a few hundred equations, but would be unacceptably slow for thousands of equations. For large and sparse problems, one could take advantage of the BTFs in two ways. First, we could use the coarse BTF to integrate the DAE system in a block-wise manner, similarly to how block-triangular linear systems are solved. Then we could pipeline the integration of these subproblems, where the integrations advance asynchronously in time (companion paper). Second, we could use the fine BTF to generate TCs for smaller subproblems, versus computing TCs for the whole system. As a result, we need to solve smaller linear systems, while in the original method (and DAETS), we solve $n \times n$ systems [?; ?].

In its present form, DAESA is a collection of MATLAB functions, so that its facilities are invoked at the MATLAB command line or within code. We are developing a GUI interface to DAESA that will allow interactive study of DAEs. This will allow zooming into the structure of large matrices and identifying lines of code that lead to a given pattern in the signature or system Jacobian matrix.
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