FISCAL POLICY SUSTAINABILITY AND INFLATION IN THE UK

AN EMPIRICAL INVESTIGATION

A thesis presented
by

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Abstract

This thesis collects three interrelated chapters of empirical work, which are connected to each other in the sense of testing UK fiscal policy. The first chapter investigates whether the UK fiscal policy was consistent with an intertemporal budget constraint for a long period of time from 1955 to 2006. I find evidence of sustainability with three structural breaks, respectively occurring in the early 1970s, early 1980s and late 1990s. UK fiscal policy has been sustainable throughout the sample period except from 1973-1981 when a non-Ricardian regime applied. For the remaining periods correction of fiscal disequilibrium occurs through adjustments in public revenue rather than expenditure. Finally, I find evidence of non-linear fiscal adjustment, with UK authorities not reacting to relatively small deficits; but correcting exceedingly large deficits and any temporary surpluses relatively fast. The second chapter investigates whether the Fiscal Theory of the Price Level can deliver a reasonable explanation for UK inflation in the 1970s, a period in which the government greatly increased public spending without raising taxes and money growth was entirely endogenous. The implied model of inflation is tested in two ways: for its trend using cointegration analysis and for its dynamics using the method of indirect inference. I find that it is not rejected. I also find that the model's errors indicate omitted dynamics which merit further research. Finally, when the model is extended to the data for the output gap and interest rates it is rejected for these further two variables. With a normalised Mahalanobis Distance of 5.1 overall and 3.3 for the data variances, this rejection is not so catastrophic that some re-specification could not possibly repair the model. But, perhaps not surprisingly, it indicates that the rather simple set-up of the model, while well able to capture the wide fluctuations of inflation in this unusual policy environment, cannot capture the behaviour of output and interest rates.

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Introduction

The Greek crisis has reminded the world of the importance of budget sustainability. For the UK, too, with an annual budget deficit close to Greek levels, the risk of bankruptcy cannot be easily ignored. While the recent large cut in government budget expenses has eased most fears of its bankruptcy there is still concern that its public debt may not be sustainable. In this thesis I focus on the historical behaviour of UK fiscal policy, check for its sustainability over different periods, and examine one particular episode, the 1970s, in some detail where sustainability appears to have been lacking.

The first chapter of this thesis tests for fiscal policy sustainability in the UK for the period 1955-2006. I address a number of issues in testing the intertemporal budget constraint. First, I model the long-run relationship between government revenue and expenditure using cointegrating test by identifying the structural shifts endogenously where the break dates are specified by the standard Chi Square statistics. Second, I consider each regime separately and compute Error Correction Models to see how budget disequilibrium is restored viz, government spending and/or taxes adjustment. Finally, the recent studies of government IBC show increasing evidence of non-linear fiscal adjustment. I test for non-linearity and provide estimates of dynamic non-linear ECM of fiscal adjustment. In short, I find evidence of multiple structural breaks in the path of public finances and UK fiscal policy is sustainable over the time period examined. However, the results are different across regimes.

In particular, I find a non-Ricardian fiscal policy regime for UK 1970s, where fiscal disequilibrium was restored by neither government expenditure nor taxes. Under such a regime, with monetary policy accommodative, inflation must adjust to ensure that the intertemporal budget constraint on fiscal policy is satisfied - the Fiscal Theory of the Price Level (FTPL). The aim of the second chapter is to investigate empirically whether the FTPL can deliver a reasonable explanation for UK inflation in the 1970s.

The policy environment of the 1970s appears to satisfy the FTPL's requirements. UK government attempted to control inflation using statutory wage and price controls. They therefore thought that no money supply rule was required to pin down inflation and the Bank of England (which was not independent at this time) would simply have been told to set money supply equal to the demand for money by consumers, thus making money supply endogenous. Any inflation rate would be consistent with such monetary policies, so that it was not pinned down by monetary policy in such a regime. At the same time, fiscal policies to reduce unemployment were pursued, so that fiscal policy was not influenced the usual Ricardian concern to balance tax and spending. Therefore, in order for the government's budget constraint to be satisfied under these conditions, inflation must adjust and this was what pins down inflation, basically inflation jumps to whatever level is needed to satisfy the government budget constraint.

The FTPL implies a relationship between the trend in the level of inflation and trends in fiscal variables. This relationship can be tested by cointegration analysis in a familiar way. Indirect inference, less familiarly, can be used to evaluate the model's dynamics by checking whether its simulated dynamic behaviour is consistent with the data. The data

under indirect inference is described by some time-series equation. The model's simulated behaviour implies a range of time-series behaviour depending on the shocks hitting it; this range can be described by the parameters of the same time-series equation that fits the data. I can derive the implied statistical joint distribution for the parameters of this equation and test whether the parameters of the time-series equation from the data lie jointly within this distribution at some confidence level.

In the third chapter I have extended the empirical tests of the model for the 1970s episode to the data for the output gap and interest rates. In the FTPL framework, inflation is determined by the fiscal conditions as above. Output and interest rates are determined by inflation interacting with the IS curve and the Phillips curve. The model is solved recursively and tested by the method of indirect inference. What I find is that while the model was not rejected for inflation, which is recursively prior determined, it is rejected for these other two variables.

The plan of the thesis is as follows. In the first chapter I assess whether UK public finances were sustainable for the period 1955-2006. In the second chapter I investigate empirically whether the Fiscal Theory of the Price Level (FTPL) can deliver a good explanation for UK inflation in the 1970s where the period monetary policy loses control on inflation. In the third chapter, I extend the model to the data for the output gap and interest rates and test it against the data. Finally, I draw the overall conclusions.

Chapter 1
UK Fiscal Policy Sustainability, 1955-2006

1.1 Introduction

In recent months fiscal policy sustainability has returned to the forefront of policy debate. This follows the significant increase in many countries' public debt caused by the economic downturn following the global credit crunch and government-sponsored banking rescue plans. A country's fiscal policy is sustainable when its intertemporal government budget constraint is met, implying that the stock of outstanding public debt is offset by expected future primary surpluses. Sustainable fiscal policy excludes the possibility of ponzi games where the government systematically services the cost of existing debt exclusively by issuing new one. Investors' willingness to hold the government's outstanding bonds depends on the latter's perceived ability to generate future surpluses by reducing excessive spending and/or increasing public revenue. Doubts regarding this ability will cause the government difficulties in marketing its debt (Quintos, 1995) and, after a critical threshold is surpassed, lead to a non-Ricardian, fiscal-dominance regime where the IBC is met through higher inflation rate reducing the real value of outstanding bonds, as suggested by the Fiscal Theory of the Price Level (see e.g. Leeper (1991), Woodford (1996, 1998a and 1998b)).

A country whose public finances have been hit particularly hard by the global financial crisis is the UK. In 2009, the UK deficit to GDP ratio reached the level of 11.5%,

the highest among G7 members. This significant worsening of the UK's fiscal outlook, and continuing concerns regarding the UK's banking system fragile state, have raised concerns about the sustainability of the UK's triple A credit rating, causing a lively debate on the optimal extend and speed of fiscal adjustment. Existing studies suggest that the UK has a sound record in correcting fiscal imbalances, both historically (see e.g. Ahmed and Rogers, 1995) as well in recent years (see Considine and Gallagher, 2008). Given the increased current focus on fiscal policy, empirical evidence regarding the sustainability of UK budget finances is timelier than ever.

In this chapter I revisit the question of UK fiscal policy sustainability from 1955 to the year preceding the onset of the fiscal downturn, 2006. Compared to existing studies, my analysis provides four distinct features. First, I test for fiscal policy sustainability accounting for structural shifts in UK fiscal policy, identified using tests for endogenous structural breaks. Second, I assess the sustainability of UK fiscal policy for each of the endogenously identified fiscal regimes. Third, I test whether deviations from the path of sustainable fiscal dynamics are corrected through adjustments in government revenue and/or expenditure. Finally, I test for non-linear adjustment in UK fiscal policy taking into account the endogenously structural shifts.

The main findings can be summarized as follows: First, the UK fiscal policy has been sustainable over the period under examination. Second, it has been subject to three structural breaks, respectively located in the early 1970s, early 1980s and late 1990s. These dates coincide with important shifts in UK fiscal policy, with the first break moving government finances away from sustainability and the remaining two towards it. Third, fis-

cal policy was sustainable during all fiscal regimes, except from 1973-1981 when a non-Ricardian regime applied. Fourth, correction of deviations from fiscal sustainability has been taking place through adjustment of public revenue rather than expenditure. Finally, I find evidence of non-linearities in UK fiscal policy, with the UK government not reacting to relatively small deficit values; but correcting exceedingly large deficits and any temporary surpluses relatively fast.

Overall, my findings confirm the status of the UK government as a historically sound sovereign borrower; and suggest a fundamentally sound UK fiscal position at the eve of the credit crunch crisis. Given, however, the depth of the ensuing banking crisis and worsening of the UK's fiscal outlook, this does not leave any room for fiscal complacency. Having said so, the findings suggest is that in the coming years of fiscal consolidation UK authorities will more likely than not enjoy the markets' confidence in their historical ability to restore sustainability, even in the face of large fiscal shocks such as the present one. Within the current environment of increased risk aversion, and as the EMU sovereign debt crisis has amply demonstrated, such market credibility will be a significant advantage at the disposal of UK authorities striving to maintain sustainable fiscal dynamics.

The rest of the chapter is organised as follows: section 1.2 discusses the theoretical model of government IBC; section 1.3 reviews the relevant literature on testing government sustainability constraint; section 1.4 discusses the data; section 1.5 lists out the linear test on government budget sustainability; section 1.6 presents the results on linear error correction model; section 1.7 analyses the issue of non-linear fiscal adjustment; section 1.8 concludes.

1.2 Analytical framework

The government's budget constraint can be written as (Hakkio and Rush, 1991)

$$b_t = (1 + r_t)b_{t-1} + G_t - R_t \tag{1.1}$$

where, b_t denotes the real stock of government outstanding public debt, r_t denotes the real interest rate, G_t denotes the real government expenditure net of interest and R_t is the real tax revenues (all variables are expressed in real term).

Since equation (1.1) holds for every period, taking expectation of it and solving for government real debt b_t recursively yields

$$b_t = \sum_{\tau=t+1}^{T} \left[\prod_{j=1}^{\tau} \frac{(R_\tau - G_\tau)}{(1 + r_{t+j})} \right] + \left[\prod_{j=t}^{T-1} \frac{b_T}{(1 + r_j)} \right] \tag{1.2}$$

Assuming constant interest rates equation (1.2) can be simplified as

$$b_t = \sum_{\tau=t+1}^{T} \frac{(R_\tau - G_\tau)}{(1 + r)^{\tau-t}} + \left[\frac{b_T}{(1 + r)^{T-t}} \right] \tag{1.3}$$

which implies that the present-value government budget constraint is given by

$$b_t = \sum_{\tau=t+1}^{\infty} \frac{(R_\tau - G_\tau)}{(1 + r)^{\tau-t}} + \lim_{T\to\infty} \left[\frac{b_T}{(1 + r)^{T-t}} \right] \tag{1.4}$$

Equation (1.4) is the standard intertemporal government budget constraint. It states that the real public outstanding stock of debt b_t must be financed by its primary surpluses. The transversality condition ($\lim_{T\longrightarrow\infty} \left[\frac{b_T}{(1+r)^{T-t}} \right] = 0$ implies the government solvency condition) must be satisfied at all times. This rules out ponzi games, i.e. the possibility of bubble financing of government expenditure. If the transversality condition holds, the

present-value budget constraint of the government can be rewritten as

$$b_t = \sum_{\tau=t+1}^{\infty} \frac{(R_\tau - G_\tau)}{(1+r)^{\tau-t}} \tag{1.5}$$

Equation (1.5) indicates that the necessary and sufficient condition for fiscal policy to be sustainable is the real interest rate to be lower than the growth rate of public debt, given by the deficit. If the real growth rate is larger than the real interest rate, the transversality condition is sufficient, but not necessary, for fiscal policy to be sustainable (Domar 1944).

1.3 Related literature

Existing studies on fiscal policy sustainability mainly address three questions. The first, and main one, is whether fiscal policy is sustainable or not. The second is whether fiscal policy involves structural breaks. Finally, the third is whether fiscal adjustment involves non-linearities. A basic concept in this literature is the government's intertemporal budget constraint (IBC), given by equation (1.4) above.

The empirical analysis on whether fiscal policy of government is sustainable can be grouped into two categories. The first applies unit root tests on government deficit and/or discounted debt series. Unit roots are interpreted as evidence of unsustainable fiscal dynamics. Hamilton and Flavin (1986) assume a constant real interest rate and test the following relationship

$$\lim_{T\to\infty} \left[\frac{b_T}{(1+r)^T} \right] = A_0 = 0 \tag{1.6}$$

against alternative

$$\lim_{T\to\infty} \left[\frac{b_T}{(1+r)^T} \right] = A_0 > 0 \tag{1.7}$$

If one substitutes equation (1.7) into (1.4) leads to

$$b_t = \sum_{\tau=t+1}^{T} \frac{(R_\tau - G_\tau)}{(1+r)^{\tau-t}} + A_0 (1+r)^t \tag{1.8}$$

Hamilton and Flavin (1986) argue that a sufficient condition for the validity of the IBC is the stationarity of the primary deficit, i.e. $A_0 = 0$ in equation (1.8). Their empirical findings reject the hypothesis of unit root in US real deficit and real stock of debt. On this basis, they conclude that the U.S. fiscal policy satisfied the IBC over the period from 1960-1984. Trehan and Walsh (1988) argue that the only necessary and sufficient condition for the IBC to be met is for the deficit series inclusive of interest payments to be statinonary. They conclude that US public finances were sustainable over the period 1890-1986. Kremers (1989) applies unit root tests on government debt-to-GNP and interest-to-GNP ratios. He finds US fiscal policy to be sustainable for most of the inter- and post-war period but not for the period after 1981. Wilcox (1989) introduces stochastic real interest rates. He argues that the IBC may be satisfied even if the level of the primary debt is non-stationary; and the sufficient condition for sustainability is for the discounted value of public debt to converge to zero. Using this criterion, he finds US fiscal policy to be unsustainable for the post-1974 period.[1]

The second category applies tests for cointegration between public deficit and debt or, more frequently, government expenditure and government revenue. Haug (1991) tests

[1] Other studies adopting this approach include Féve and Hénin (2000) and Uctum and Wickens (2000). Féve and Hénin (2000) use semi-annual data and test for fiscal policy sustainability for G7 countries, concluding that a unit root cannot be rejected for Germany, France, Italy and Canada. Uctum and Wickens (2000) use annual data over the period 1965-1994 testing for fiscal sustainability in the US and eleven EU countries. They conclude that only Denmark, the Netherlands, Ireland and France were on a sustainable fiscal path.

for cointegration between real government debt and real surplus using quarterly US data over the period 1960-1986. He finds evidence of cointegration suggesting sustainable US fiscal policy. MacDonald (1992) provides a similar analysis for the period 1951-1984. Using monthly data, he reaches the opposite conclusion. On the other hand, Hakkio and Rush (1991) test for cointegration between US real per capita government revenue and expenditure using quarterly data for the period 1950 to 1988. They focus directly on government expenditures and revenues where they rewrite equation (1.4) in term of total expenditure

$$TG_t = r_t b_{t-1} + G_t = R_t + \sum_{\tau=t}^{T} \frac{(\Delta R_\tau - \Delta G_\tau)}{(1+r)^{\tau-t-1}} + \lim_{T\to\infty} \left[\frac{\Delta b_T}{(1+r)^T} \right] \tag{1.9}$$

where TG_t denotes the real total government spending on goods and services, transfer payments, and the interest on the debt. G_t denotes the real government expenditure net of interest and R_t is the real tax revenues. If both R_t and G_t are assumed to be I(1) processes, then ΔR_t and ΔG_t are stationary. If both R_t and G_t follow random walks with drift

$$R_t = \alpha_1 + R_{t-1} + \varepsilon_{1t} \tag{1.10}$$

$$G_t = \alpha_2 + G_{t-1} + \varepsilon_{2t} \tag{1.11}$$

In this case, equation (1.9) can be rewritten as

$$TG_t = \alpha + R_t + \lim_{T\to\infty} \left[\frac{\Delta b_T}{(1+r)^T} \right] + \varepsilon_t \tag{1.12}$$

where,

$\alpha = \sum_{\tau=t}^{T} \frac{(\alpha_1-\alpha_2)}{(1+r)^{\tau-t-1}} = \left(\frac{1+r}{r}\right)(\alpha_1 - \alpha_2)$

and $\varepsilon_t = \left(\frac{1+r}{r}\right)(\varepsilon_{1t} - \varepsilon_{2t})$.

Assuming the limit term in equation (1.12) goes to zero then the hypothesis of public debt sustainability can be written as

$$R_t = \alpha + \beta TG_t + \varepsilon_t \tag{1.13}$$

Hakkio and Rush (1991) assume stochastic real interest rates and argue that for fiscal policy to be sustainable public revenue and expenditure should be cointegrated with $\beta = 1$. Using the entire sample period, they find these conditions to be met. However, they find US fiscal policy not to be sustainable following 1964, with evidence of non-cointegration being particularly strong during the period 1976-1988. Using the same cointegration methodology, Ahmed and Rogers (1995) conclude that UK fiscal policy is sustainable over the period spanning over two centuries. Corsetti and Roubini (1991) provide a similar analysis for selected EMU countries finding that their government finances do not satisfy the IBC.[2]

Tests of fiscal policy sustainability based on cointegration tests are subject to biased inference in case the underlying cointegrating relationship is subject to structural breaks. Hakkio and Rush (1991), MacDonald (1992) and Haug (1995) address structural instability by choosing the break dates exogenously. By contrast, Quintos (1995) uses tests determining the break dates endogenously. She also introduces the concepts of strong- and weak-form fiscal policy sustainability. Her definitions encompass and extend previous definitions. In view of the generality of her approach I adopt it for my own econometric investigation below. Strong-form sustainability is equivalent to the sustainability definition used by Hamilton and Flavin's (1986) and Hakkio and Rush (1991). Under weak-form

[2] Other studies using a cointegration framework to test the validity of the IBC in Europe include Bravo and Silvestre (2002) and Afonso and Rault (2010) for eleven and fifteen EU countries respectively. Both studies reach mixed results with regards to the validity of the IBC in their sample countries.

sustainability the limit term in equation (1.4) converges to zero but at rate lower compared to the strong-form sustainability case. Furthermore, under weak-form sustainability the limit term in equation (1.4) converges to zero faster when government revenue and expenditure are cointegrated rather than when they are not. Weak form sustainability implies that the deficit and undiscounted debt series may be mildly explosive, in which case an unpredictable shock may put public finances into an unsustainable path. As a result, under weak-form sustainability the government may face difficulties marketing its debt and be obliged to pay higher interest rates to service it. In terms of equation (1.13) fiscal policy is weak-form sustainable if $0 < \beta < 1$, irrespective of whether R_t and TG_t are cointegrated or not; weak-form sustainable if $\beta = 1$ and R_t and TG_t are non-cointegrated; strong form sustainable if $\beta = 1$ and R_t and TG_t are cointegrated; and non-sustainable if $\beta = 0$.

To see these, Quintos (1995) rewrites the equation (1.4) in first differenced

$$\Delta b_t = \sum_{\tau=t+1}^{\infty} \frac{(\Delta R_\tau - \Delta G_\tau)}{(1+r)^{\tau-t}} + \lim_{T\to\infty} \left[\frac{\Delta b_T}{(1+r)^{T-t}} \right] \tag{1.14}$$

For equation (1.14) to converge to a stable solution operating as a binding fiscal constraint, the last term should convergence to zero, i.e.

$$\lim_{T\to\infty} \left[\frac{\Delta b_T}{(1+r)^{T-t}} \right] = 0 \tag{1.15}$$

Assume the interest rates are constant at r, under the condition that Δb_T is stationary, the evolution of the limit term in (1.15) behaves like

$$E_t \left[\lim_{T\to\infty} e^{-kT} \right] = 0 \tag{1.16}$$

where k is a constant. If Δb_T is non-stationary, then equation (1.15) behaves like the following process

$$E_t\left[\lim_{T\to\infty} e^{-kT}\sqrt{T}\right] = 0 \tag{1.17}$$

Quintos shows that the stationarity of Δb_T is the sufficient condition for the term in (1.15) to go to zero. In addition, if government revenue is not cointegrated with government expenditure then the term in (1.16) goes to zero faster than that in (1.17), where (1.16) and (1.17) is the 'strong' and 'weak' condition for deficit sustainability respectively.

Quintos' (1995) tests of fiscal policy sustainability are based on equation (1.18) below

$$\Delta b_t = (1-\beta)TG_t - \alpha - \varepsilon_t \tag{1.18}$$

Quintos shows that if $0 < \beta < 1$ then Δb_t is I(1) regardless of whether ε_t is stationary or not. Hence, the necessary and sufficient conditions for Δb_t to be stationary are $\beta = 1$ and ε_t is stationary. Quintos applies her methodology to US quarterly data covering the period 1947-1992. She concludes that the US fiscal policy is weakly sustainable despite a negative structural break in the early 1980s causing non-cointegration after 1980.

Arghyrou and Luintel (2007) use Quintos's methodology to test for fiscal policy sustainability in four heavily indebted EMU countries, namely Greece, Ireland, Italy and the Netherlands. They find that the introduction of the Maastricht Treaty in 1991 has caused a structural break towards sustainability; and that fiscal policy at the eve of the Euro's introduction in 1999 was strong-form sustainable in Ireland and weak-form sustainable in the rest of their sample countries. Finally, they find evidence of non-linear fiscal adjustment for their sample countries, which is consistent with the findings of Bohn (1998), Cipollini

(2001), Sarno (2001), Arestis et al (2004), Bajo-Rubio et al (2004, 2006), Chortareas et al (2008), Considine and Gallagher (2008) and Cipollini et al (2009). These studies model the dynamics of the discounted public debt series or the cointegrating vector between public revenue and expenditure in a number of different countries using variants of threshold autoregressive (TAR) models. The intuition underlying these non-linear models is that fiscal adjustment takes place more rapidly when budget deficits or the stock of outstanding debt exceed certain critical thresholds beyond which they are considered exceedingly large.

1.4 Data

For the econometric investigation I use data for UK total managed public expenditure inclusive of interest payments on outstanding public debt and total public revenue excluding seignorage. The data source is the UK Office of National Statistic (ONS) data bank. The data frequency is quarterly and covers the period 1955Q1-2006Q1. I calculate real government revenue R_t and real government expenditure TG_t deflating nominal series by the GDP deflator. Figure 1.1 plots the de-seasonalised data in log real terms.[3] Table 1.1 reports unit root tests on the series' log-levels and first differences. Both series are integrated of order one and show a similar upward trend. However, there appears to be significant divergence during the 1970s, 1980s and 1990s, indicating increasing deficits over those periods and structural breaks in any cointegrating relationship that may link the two series. Both

[3] The original data series include strong seasonal effects which I account for using a constant and three seasonal dummies.

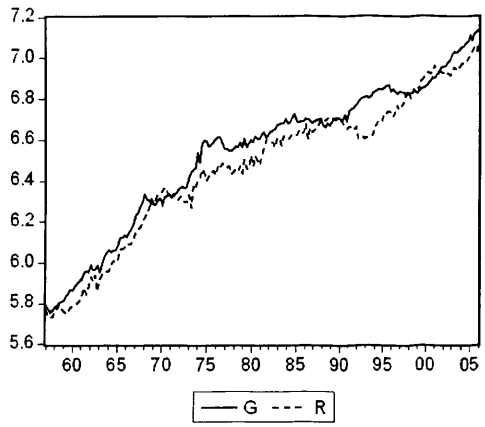

Fig. 1.1. Real government revenue and expenditure (both in log terms)

the Augmented Dickey Fuller (ADF) (1979, 1981) and Phillips-Perron (PP) (1988) tests confirm that government revenue and expenditure are first difference stationary - Table 1.1.

Unit root tests	TG_t		R_t	
	Levels	1st. Diff.	Levels	1st. Diff.
ADF test statistic	-1.744	-5.643	-1.086	-6.755
	(0.408)	(0.000)**	(0.722)	(0.000)**
PP test statistic	-1.274	-16.490	-0.848	-18.261
	(0.642)	(0.000)**	(0.803)	(0.000)**

Table 1.1. Tests for non-stationarity of deseasonalised real government expenditure TG and revenue R

*Notes on Table: MacKinnon's critical values (MacKinnon, 1996) for rejection of hypothesis of a unit root: values in parentheses are p-values, while ** indicates significance at the 1% level. The number of lags in the ADF tests is set using AIC criterion; for the PP tests using Newey-West bandwidth (see Newey and West, 1987).*

1.4.1 The issue of levels versus logs

According to Quintos (1995), in a regression of revenue on expenditure expressed in levels, if $\widehat{\alpha} = 0$ and $\widehat{\beta} = 1$ and ε_t is stationary then I have strong form sustainability; if $\widehat{\alpha} = 0$ and $\widehat{\beta} = 1$ and ε_t is non-stationary then I have weak form sustainability.If the equation is tested using the logs of the variables, these conditions continue to hold. However, some of the other conditions describing weak-form sustainability in a regression estimated using levels change. More specifically:

Suppose that the regression in levels is

$R_t = \alpha + \beta TG_t + \varepsilon_t$

The condition for strong-form sustainability is that α = 0, β =1 and ε_t is stationary. If this is the case, then the equilibrium relationship between R_t and TG_t is given by an (1, -1) cointegrating vector. In other words $R_t = \beta TG_t + \varepsilon_t$,where ε_t is a random error term with zero mean. In that case, I can claim that the equilibrium relationship in levels is $R_t = TG_t$.If this is the true long-run (cointegrating) relationship, then I can claim that in equilibrium $\log R_t = \log TG_t$, in which case I can also claim that a regression of the form $\log R_t = \alpha + \beta \log TG_t + \varepsilon_t$ is a test of strong form sustainability:

- If α = 0 and β = 1, and ε_t is stationary, then $\log R_t = \log TG_t$, i.e. $R_t = TG_t$ and I have strong-form sustainability

- If α = 0 and β=1, but ε_t is not non-stationary, then I am in the case of $R_t = TG_t$ but without cointegration, in which cases I have weak-form sustainability. In short, the conditions for strong form sustainability are the same, irrespective of whether the cointegrating relationship is estimated using levels or logs.

However, when the cointegrating regression is estimated in terms of logs, the conditions for weak form sustainability defined in terms of the value of β when the regression is estimated using levels do not hold any more. Suppose that I have weak form sustainability with $\alpha = 0$ and β less than 1, say β=0.90 In that case, the true relationship in levels is given by:

$R_t = 0.90TG_t$

Now, taking logs, this will give the following equation

$\log R_t = \log(0.90) + \log TG_t$

In other words, if I transform the level relation into logs, once again I will get a unity coefficient on $\log TG_t$ and, this time, a non-zero constant.

To conclude, when the cointegrating equation is estimated using logs, the necessary and sufficient conditions for weak form sustainability are:

$\alpha = 0$ and $\beta = 1$, and ε_t is stationary.

By contrast, I have weak form sustainability if $\alpha = 0$, $\beta = 1$ and ε_t is not stationary; or if $\beta = 1$ but α is not zero, irrespective of whether ε_t is stationary or not.

1.5 Linear tests on fiscal policy sustainability

I start the econometric investigation on the sustainability of UK fiscal policy using the linear cointegration framework. I first test for sustainability without accounting for structural breaks in the cointegrating equation given by (1.13). Dynamic OLS (DOLS) methodology is used; it is a cointegration method that is asymptotically equivalent to the Engle and Granger (1987) and Johansen (1988) cointegration methodologies with the extra advan-

tages of performing better in small samples and controlling for endogeneity among the regression's variables through the inclusion of lead and lag differences of the regressors (see Stock and Watson, 1993). Given that both series include one unit root, the DOLS regression is given by equation (1.19) below

$$R_t = \alpha + \beta TG_t + \sum_{t=-k}^{k} \gamma_k \Delta TG_{t-k} + \varepsilon_t \tag{1.19}$$

where Δ denotes the first difference operator and ε_t is a random error term. If the residual series ε_t is serially correlated, then equation (1.13) is estimated using the Dynamic Generalised Least Squares (DGLS) estimator. This augments equation (1.19) with autoregressive error terms under the Feasible Generalised Least Squares. Under both DOLS and DGLS the cointegrating vector is given by $CV = R_t - \widehat{\alpha} - \widehat{\beta} TG_t$. The results of estimating equation (1.13) using DGLS are reported in Table 1.2 below. Although the restriction $\beta = 1$ is not rejected, strong-from sustainability is rejected as the reported ADF test is not significant at the 5% level.

Cointegration analysis without breaks	
	DGLS
	1955Q1-2006Q1
Estimated equation	
$R_t = \alpha + \beta TG_t + \varepsilon_t$	
$\widehat{\alpha}$	0.044(0.128)
$\widehat{\beta}$	0.984(0.020)**
F-Wald test, $H_0 : \widehat{\beta} = 0$ [p-value]	2443.31[0.000]**
F-Wald test, $H_0 : \widehat{\beta} = 1$ [p-value]	0.609[0.435]
t-ADF test on $\widehat{\varepsilon}_t$	-2.693
5% critical value	[-4.250]
S.E.of regression	0.055

Table 1.2. Cointegrating test for real government revenue and expenditure without breaks

*Notes on Table: standard errors in parentheses. ** indicates significance at the 1% level. All DGLS estimates are corrected for heteroskedasticity and autocorrelation of unknown form in the residuals (DOLS-HAC, see Newey and West, 1987).*

Rejection of cointegration, however, may be due to structural breaks in the cointegrating relationship given by (1.13). To identify such breaks endogenously I use the sequential cointegration stability test proposed by Quintos (1995) described by equations (1.20) to (1.22) below:

$$R_t = \alpha + \beta TG_t + \delta(D_t TG_t) + \varepsilon_t \tag{1.20}$$

where,

$$D_t = 1 \text{ if } t \in T_1 = \{1, ..., m\} \tag{1.21}$$

$$D_t = 0 \text{ if } t \in T_2 = \{m+1, ..., T\} \tag{1.22}$$

In equations (1.21) and (1.22) D_t is a dummy variable taking the value of unity before period m and zero thereafter, where m represents the date of the tested breakpoint. The null hypothesis of stability assumes $\delta = 0$ and is tested using a Wald F-test. Equation (1.20) is estimated sequentially. Following Andrews (1993) I have trimmed 15 per cent from the beginning and the end of the sample. Equation (1.20) is estimated using DOLS and, for robustness, OLS. Figure 1.2 plots the sequential Wald test statistics testing the restriction $\delta = 0$ over the period of 1963Q1-1998Q4. It suggests that the cointegrating relationship between R_t and TG_t has been subject to multiple structural breaks. More specifically, it suggests a number of statistically significant values for the depicted Wald statistics in the early 1970s, the early 1980s and the second half of the 1990s. As structural

breaks cannot fall too close together, these three groupings of statistically significant values are very likely reflecting three distinct structural breaks. I define the exact timing of each of the three breaks on the basis of highest F-score in each grouping. Using this criterion, both estimators suggest breaks of almost identical timing, with DOLS suggesting the break points to be 1972Q3, 1981Q3 and 1997Q4, while OLS suggests 1972Q1, 1981Q4 and 1997Q3. These dates can be related to important exogenous shifts in UK macroeconomic policy. The break in 1972 is close to the introduction of UK expansionary fiscal policies targeting the unemployment rate through wage and income controls. The break in 1981 coincides with the introduction of the fiscal consolidation effort pursued by the Medium Term Financial Strategy (MTFS), a monetary and fiscal policy programme announced by the Conservative Government in early 1980. Finally, the break date in 1997 is close to the endorsement of the then newly-elected Labour government of its predecessor's relatively restrictive fiscal policies and the granting of operational independence to the Bank of England, establishing further the 'monetary-dominance' rather than 'fiscal-dominance' nature of the UK macroeconomic outlook.

The next step is to test for UK fiscal policy sustainability accounting for the effect of the structural breaks identified above. I do so by estimating equation (1.23) below

$$R_t = \alpha + \beta TG_t + \sum_{i=1}^{j} \phi_i (D_{it} TG_t) + \varepsilon_t \tag{1.23}$$

Equation (1.23) modifies the cointegrating regression given by equation (1.13) by including slope dummy variables corresponding to each of the three breaks identified above. Each of the three dummies D_{it} ($j = 1, 2, 3$) takes a zero value before the date of the corresponding break and the value of unity thereafter (see Table 1.3). A positive (nega-

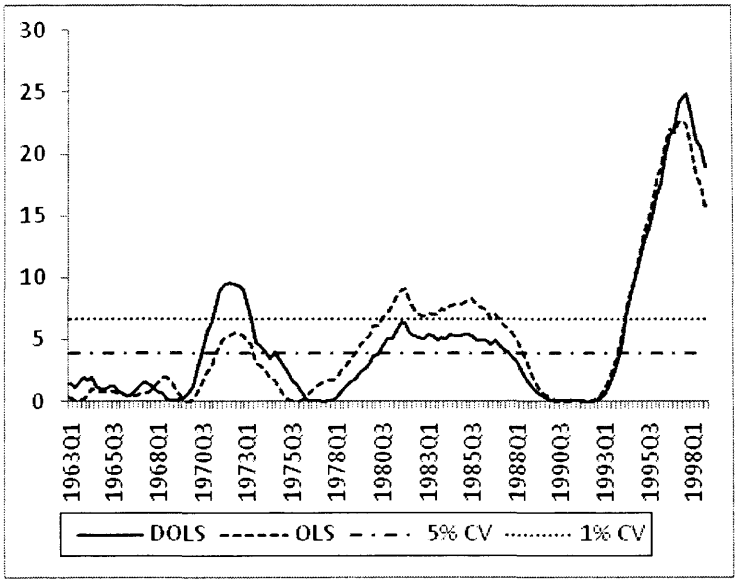

Fig. 1.2. Sequential Wald tests for structural breaks

tive) coefficient represents a movement towards (away from) sustainability. The augmented cointegrating vector obtained by equation (1.23) is then given by

$$CV = R_t - \alpha - \beta TG_t - \sum_{i=1}^{3} \phi_i (D_{it} TG_t) \tag{1.24}$$

Equation (1.23) is estimated using three alternative methodologies, namely DGLS, DOLS and simple OLS. The break dates for the DGLS/DOLS and OLS estimates of equation (1.23) are respectively defined on the basis of the highest score obtained from the DOLS and OLS estimator for each grouping of statistically significant F-statistics in Figure 1.2. The only exception is the break of the early 1970s when equation (1.23) is estimated using the DGLS methodology. By defining D_{1t} to take the value of unity after 1972Q3 I could not obtain DGLS estimates free of heteroskedasticity problems and obtained a marginally insignificant, at the 5% level, dummy coefficient. Experimenting with alternative

definitions of D_{1t} in the neighbourhood of 1972Q3 I obtained the best data representation (in terms of a minimum score for the Akaike information criterion and regression standard error) when D_{1t} took the value of unity from 1973Q3 onwards.

The results of the estimations are reported in Table 1.3. The coefficients of all break dummies turn out to be statistically significant at the 5% level with the expected signs. More specifically, the coefficient of the dummy capturing the break of the early 1970s is in all cases negatively suggesting a deteriorating fiscal outlook during the implementation of the fiscal expansion of that period. The positive and significant coefficients of the dummies capturing the break of the early 1980s confirm the partial reversal of the expansionary dynamics established in the early 1970s. Finally, the dummy variables capturing the break of 1997 have a positive and significant coefficient, suggesting further improvement of the UK's public finances over the period 1998-2006.

Finally, I use the findings reported in Table 1.3 to test for weak and strong-form sustainability. Similar to the findings reported in Table 1.2, the DGLS, DOLS and OLS results reported in Table 1.3 suggest there is no cointegration between government revenue and expenditure at the 5% level. As the DGLS model produces a significantly lower regression standard error, it seems to provide the best data representation. I then test the null hypothesis of a unity total multiplier for the coefficient of public expenditure, given by $H_0 : \beta + \sum_{j=1}^{k} \phi_j = 1$,for $j = 1, 2, 3$. For the DGLS and DOLS estimates the null of a unity total multiplier is maintained. However, given the finding of no cointegration in my prefered DGLS estimation of equation (1.23) the findings reported in Table 1.3 suggests that

following the structural breaks that occurred in the early 1970s, 1980s and late 1990s, over the period 1955-2006 UK fiscal policy was sustainable only in the weak-form sense.

Cointegration analysis with breaks			
	DGLS	DOLS	OLS
Estimated equation			
$R_t = \alpha + \beta TG_t + \phi_1 D_1 TG_t + \phi_2 D_2 TG_t + \phi_3 D_3 TG_t + \varepsilon_t$			
$\widehat{\alpha}$	0.014(0.253)	0.161(0.165)	0.323(0.154)*
$\widehat{\beta}$	0.997(0.042)**	0.968(0.027)**	0.939(0.025)**
$\widehat{\phi}_1(D_1 = 1$ in 1972Q1-2006Q1, 0 Otherwise)	-	-	-0.006(0.003)*
$\widehat{\phi}_1(D_1 = 1$ in 1972Q3-2006Q1 0 Otherwise)	-	-0.008(0.003)**	-
$\widehat{\phi}_1(D_1 = 1$ in 1973Q3-2006Q1, 0 Otherwise)	-0.015(0.004)**	-	-
$\widehat{\phi}_2(D_2 = 1$ in 1981Q3-2006Q1, 0 Otherwise)	0.007(0.003)*	0.005(0.002)**	-
$\widehat{\phi}_2(D_2 = 1$ in 1981Q4-2006Q1, 0 Otherwise)			0.007(0.002)**
$\widehat{\phi}_3(D_3 = 1$ in 1997Q3-2006Q1, 0 Otherwise)	-	-	0.009(0.002)**
$\widehat{\phi}_3(D_3 = 1$ in 1997Q4-2006Q1, 0 Otherwise)	0.013(0.005)**	0.010(0.002)**	
F-Wald test, $H_0 : \widehat{\beta} + \widehat{\phi}_1 + \widehat{\phi}_2 + \widehat{\phi}_3 = 1$ [p-value]	0.001[0.981]	1.040[0.309]	5.193[0.024]*
F-Wald test, $H_1 : \widehat{\beta} + \widehat{\phi}_1 + \widehat{\phi}_2 + \widehat{\phi}_3 = 0$ [p-value]	741.71[0.000]**	1688.07[0.000]**	1844.40[0.000]**
t-ADF test on $\widehat{\varepsilon}_t$	-2.880	-2.837	-2.851
5% critical value	[-4.250]	[-4.250]	[-4.250]
S.E.of regression	0.021	0.047	0.050

Table 1.3. Cointegration test for real government revenue and expenditure with endogenous structural breaks

*Notes on Table: standard errors in parentheses. *, ** indicate significance at the 5% and 1% level, respectively. All DGLS estimates are corrected for heteroskedasticity and autocorrelation of unknown form in the residuals (DOLS-HAC, see Newey and West, 1987).*

1.6 Linear error correction models

In the previous section I concluded that the post-war UK fiscal policy has been subject to three structural breaks, giving rise to four fiscal regimes over the sample period respectively covering the periods 1955-1972; 1973-1981; 1982-1997 and 1998-2006. In this section I estimate linear error correction models (ECM) for each of these periods with a dual objective. First, to establish whether fiscal policy reacts to fiscal disequilibrium as the latter

is captured by the cointegrating vector accounting for structural breaks. If for a particular period the disequilibrium term enters the ECM with an insignificant coefficient or a significant coefficient of positive sign, then the fiscal regime of that period is identified as 'non-Ricardian', characterised by non-sustainable fiscal policy. Second, if fiscal policy reacts to deviations from the long-run equilibrium path, estimates of ECMs will provide an information as to whether the adjustment comes through the revenue or expenditure side, or both. Then government revenue and expenditure may be considered to be generated by error-correction models of the form

$$\Delta R_t = \alpha + \sum_{i=1}^{k} \beta_i \Delta R_{t-k} + \sum_{i=1}^{k} \gamma_i \Delta TG_{t-k} + \delta \widehat{\varepsilon}_{t-1} + v_t \tag{1.25}$$

$$\Delta TG_t = \alpha + \sum_{i=1}^{k} \beta_i \Delta R_{t-k} + \sum_{i=1}^{k} \gamma_i \Delta TG_{t-k} + \delta \widehat{\varepsilon}_{t-1} + v_t \tag{1.26}$$

where, $\widehat{\varepsilon}_{t-1}$ is the estimated cointegrating vector, obtained from the DGLS estimation of equation (1.23) accounting for structural break and v_t is a random error. There are two single error correction models. Both δs are expected to capture the adjustments of ΔR_t and ΔTG_t towards long-run equilibrium, while ΔR_{t-k} and ΔTG_{t-k} are expected to capture the short-run dynamics of the model.

The results of the ECM estimations are reported in Table 1.4. The Table presents ECM models estimated for the whole of the sample period as well as for each of the four sub-periods defined by structural breaks identified in section 1.5 above. For each sample period I present two ECMs, ECM1 and ECM2, respectively defining the dependent variable to be ΔR_t and ΔTG_t. I report parsimonious estimates (i.e. excluding insignificant terms) obtained from initial models including four lags (i.e. $k = 4$) of ΔR_t and ΔTG_t.

The cointegrating vector which is calculated using $R_t - \widehat{\alpha} - \widehat{\beta} TG_t$ is defined as government surplus as it is normalized on government revenue. Hence, in ECM1, which models the adjustment of government revenue, a reduction in the value of the error correction term below zero giving rise to a deficit must be accompanied by an increase in government revenue to create a surplus and eliminate the disequilibrium caused by the deficit. In a similar fashion, if the error correction term takes positive values, giving rise to a surplus, government revenue may be allowed to decline until the disequilibrium term becomes zero and a balanced budget is restored. In other words, δ should take negative in the ECM1. On the other hand, ECM2 models the adjustment of government expenditure. If the disequilibrium term increases to take positive values, so that the public budget is in surplus, the government can afford to increase expenditure until the disequilibrium term becomes zero and a balanced budget is restored. By contrast, if the error correction term declines below zero, so that the public budget is in deficit, government expenditure must also decline to restore a balanced budget. In other words, in the case of ECM2 δ should take a positive sign. For the full-sample period and three out of four sub-periods, the coefficient of the error correction term δ is statistically significant with a negative sign in the equation modeling ΔR_t and however being insignificant in the equation modeling ΔTG_t. These findings suggest a Ricardian regime, consistent with fiscal policy sustainability and adjustment to any fiscal disequilibrium coming from the revenue rather than expenditure side. This is an indication of UK authorities relying more on tax increases rather than expenditure reductions to correct fiscal imbalances. On the other hand, the period 1973Q3-1981Q2 seems exceptional. For that period, the δ coefficient is insignificant in both ECM equations, suggesting lack

of policy reaction to the increasing at the time fiscal disequilibrium term. This is consistent with the findings in the previous section, suggesting a structural shift away from fiscal sustainability in the early 1970s and the presence of a 'non-Ricardian' fiscal regime.

ECM1	1955Q1-2006Q1	1955Q1-1973Q2	1973Q3-1981Q2	1981Q3-1997Q3	1997Q4-2006Q1
$\widehat{\delta}$	-0.131(0.031)**	-0.258(0.050)**	-0.249(0.170)	-0.095(0.042)*	-0.110(0.049)*
$\widehat{\alpha}$	0.005(0.002)**	-	-	0.006(0.003)*	-
$\widehat{\beta}_1$	-0.291(0.065)**	-	-0.488(0.141)**	-0.290(0.109)**	-
$\widehat{\beta}_2$	-	-	-	-	-
$\widehat{\beta}_3$	0.164(0.062)**	-	-	-	-
$\widehat{\beta}_4$	-	-	-	-	-
$\widehat{\gamma}_1$	-0.191(0.091)*	-0.329(0.137)*	-	-	-
$\widehat{\gamma}_2$	-	-	-	-	-
$\widehat{\gamma}_3$	-	-	0.414(0.183)*	-0.349(0.173)*	-
$\widehat{\gamma}_4$	-	-		-0.341(0.173)*	-
S.E.of regression	0.022	0.021	0.027	0.021	0.020
Misspe. tests (p-values)					
F-AR	0.91	0.68	1.00	0.29	0.18
F-$ARCH$	0.12	0.88	0.27	0.77	0.80
Norm	0.40	0.23	0.84	0.30	0.91
F-Het	0.44	0.16	0.07	0.72	0.21
ECM2	1955Q1-2006Q1	1955Q1-1973Q2	1973Q3-1981Q2	1981Q3-1997Q3	1997Q4-2006Q1
$\widehat{\delta}$	-0.040(0.023)	-0.035(0.051)	0.141(0.121)	-0.003(0.030)	-0.033(0.037)
$\widehat{\alpha}$	0.007(0.001)**	0.010(0.003)**	-	0.004(0.002)*	0.014(0.003)**
$\widehat{\beta}_1$	-	-	-	-	-
$\widehat{\beta}_2$	-	-0.215(0.088)*	-	-	-0.258(0.109)*
$\widehat{\beta}_3$	-	-	-0.291(0.132)*	-	-
$\widehat{\beta}_4$	-	-	-	-	-
$\widehat{\gamma}_1$	-0.219(0.066)**	-	-	-	-0.486(0.153)**
$\widehat{\gamma}_2$	-	-	0.376(0.171)*	-	-
$\widehat{\gamma}_3$	-	-	0.376(0.179)*	-	-
$\widehat{\gamma}_4$	-	-	-	-	-
S.E.of regression	0.017	0.018	0.026	0.015	0.011
Misspe. tests (p-values)		-			
F-AR	0.57	0.76	0.10	0.45	0.99
F-$ARCH$	0.10	0.81	0.38	0.44	0.28
Norm	0.32	0.75	0.61	0.20	0.46
F-Het	0.77	0.40	0.06	0.49	0.54

Table 1.4. Error correction models for short run dynamic behaviours

*Notes on Table: standard errors in parentheses. *, ** indicate significance at the 5% and 1% level, respectively. Some estimations include impulse dummy variables for outlier observations. These are: for period 1955Q1-2006Q1, 1962Q4 in ECM(1) and 1974Q1 and 1974Q3 in ECM(2); for period 1955Q1-1973Q2, 1962Q4 in ECM(1). The estimates for period 1973Q3-1981Q2 are corrected for heteroskedasticity and autocorrelation of un-*

known form in the residuals (OLS-HAC estimates). F-AR is the Lagrange Multiplier F-test for residual serial correlation up to forth order. F-$ARCH$ is an F-test for autoregressive conditional heteroskedasticity. Norm is the normality chi-Square Bera-Jarque test for residuals' non-normality. F-Het is F-test for residuals heteroskedasticity.

1.7 Non-linear fiscal adjustment

The empirical findings reported above suggest that over the last five decades UK fiscal policy has been subject to multiple structural shifts, giving rise to multiple fiscal regimes and one of which (in the 1970s) was non-Ricardian. Overall, however, I find that UK government finances over the period of 1955-2006 satisfy the IBC. This implies that that the UK's authorities ultimately correct fiscal disequilibrium, even after protracted negative fiscal shocks. An important question is whether the process of reversion to fiscal equilibrium involves non-linearities, i.e. whether fiscal adjustment occurs only when fiscal imbalances become sufficiently large. To test this hypothesis, I need to estimate econometric models able to capture such non-linearities.

The smooth transition autoregressive (STAR) model for a univariate time series, y_t is given by (van Dijk, 2002)

$$\begin{aligned} y_t &= \left(\phi_{1,0} + \phi_{1,1} y_{t-1}, + ... + \phi_{1,p} y_{t-p}\right)\left(1 - G\left(s_t; \gamma, c\right)\right) \\ &\quad + \left(\phi_{2,0} + \phi_{2,1} y_{t-1}, + ... + \phi_{2,p} y_{t-p}\right) G\left(s_t; \gamma, c\right) + \varepsilon_t \end{aligned}$$

for $t = 1, ..., T$,

or

$$y_t = \phi_1' x_t \left(1 - G\left(s_t; \gamma, c\right)\right) + \phi_2' x_t G\left(s_t; \gamma, c\right) + \varepsilon_t$$

where $x_t = \left(1, \widetilde{x}_t'\right)'$ with $\widetilde{x}_t' = \left(y_{t-1}, ...y_{t-p}\right)'$ and $\phi_i = \left(\phi_{i,0}, \phi_{i,1}, +... + \phi_{i,p}\right)'$, $i = 1, 2$.

The residual term ε_t is assumed to be a martingale difference sequence with respect to the history of the time series up to time $t-1$, which is denoted as $\Omega_{t-1} = \left\{y_{t-1}, y_{t-2}, \cdots y_{1-(p-1)}, y_{1-p}\right\}$, that is, $E\left[\varepsilon_t | \Omega_{t-1}\right] = \sigma^2$.

The transition function $G\left(s_t; \gamma, c\right)$ is a continuous function that is bounded between 0 and 1. The transition variable s_t can be either a lagged endogenous variable, or an endogenous variable or a (possible nonlinear) function of lagged endogenous variables (see Lin and Teräsvirta, 1994). There are two interpretations of the STAR model. On the one hand, the STAR model can be considered as a regime-switching model that allows for two regimes, associated with the extreme values of the transition function, $G\left(s_t; \gamma, c\right) = 0$ and $G\left(s_t; \gamma, c\right) = 1$, where the transition from one regime to the other is smooth. On the other hand, the STAR model allows for a 'continuum' of regimes, where the transition function $G\left(s_t; \gamma, c\right)$ each time has a different value between 0 and 1.

The adjustment towards this equilibrium is nonlinear and can be characterized as regime-switching, with the regimes determined by the size and/or sign of the deviation from equilibrium. In linear time series, this type of behaviour is captured by cointegration and error-correction models (ECMs). For the nonlinear behaviour I concentrate on incorporating the smooth transition mechanism in an ECM to allow for nonlinear or asymmetric adjustment. The regime that occurs at time t is determined by the observable variable s_t and the associated value of $G\left(s_t; \gamma, c\right)$. Different choices for the transition function $G\left(s_t; \gamma, c\right)$

give rise to different types of regime-switching behaviour. One of the choice for $G(s_t;\gamma,c)$ is the first-order logistic function

$$G(s_t;\gamma,c) = (1+\exp\{-\gamma(s_t-c)\})^{-1} \qquad \gamma > 0 \tag{1.27}$$

with the resultant model is called the Logistic STAR (LSTAR) model. The parameter c in (1.27) can be interpreted as the critical threshold distinguishing between the two regimes (i.e. the inner and the outer regimes), in the sense that the logistic function changes monotonically from 0 to 1 as s_t increases and $G(s_t;\gamma,c) = 0.5$. The parameter γ determines the smoothness of the change in the value of the logistic function and, thus, the smoothness of the transition from one regime to the other. In the LSTAR model, the two regimes are associated with small and large values of the transition variable s_t (relative to c).

The other possible choice for $G(s_t;\gamma,c)$ can be the second-order logistic function

$$G(s_t;\gamma,c) = (1+\exp\{-\gamma(s_t-c_1)(s_t-c_2)\})^{-1} \qquad \gamma > 0 \tag{1.28}$$

where $c = (c_1, c_2)'$. In this case, the two regimes are associated with small and large values of the transition variable s_t. The resultant model is called the Quadratic-Logistic STAR (QL-STAR) model. The specification is identical to that of the L-STAR model except the outer regimes are defined by two threshold values, the upper (c_2) and the lower (c_1) which define a band, where the speed of adjustment to equilibrium within the band is different from the one prevailing outside the band.

In order to choose the correct transition function the type of non-linear adjustment needs to be identified. The hypothesis of linear fiscal policy can be tested using the testing

procedure established by Granger and Teräsvirta (1993) and Teräsvirta (1994). This is based on the auxiliary regression given by equation (1.29) below

$$\widehat{\varepsilon}_t = \gamma_{00} + \sum_{j=1}^{\phi} \left(\gamma_{0j}\widehat{\varepsilon}_{t-j} + \gamma_{1j}\widehat{\varepsilon}_{t-j}\widehat{\varepsilon}_{t-d} + \gamma_{2j}\widehat{\varepsilon}_{t-j}\widehat{\varepsilon}^2_{t-d} + \gamma_{3j}\widehat{\varepsilon}_{t-j}\widehat{\varepsilon}^3_{t-d}\right) + \gamma_4\widehat{\varepsilon}^2_{t-d} + \gamma_5\widehat{\varepsilon}^3_{t-d} + \omega_t \tag{1.29}$$

In (1.29) $\widehat{\varepsilon}_t$ denotes the estimated fiscal disequilibrium term accounting for structural breaks given by the estimated residuals obtained from the DGLS estimation of equation (1.23); ϕ is the order of the autoregressive parameter determined by the partial autocorrelation function of ε_t (see Granger and Teräsvirta(1993)); d is the delay parameter of the transition function; and ω_t is an the error term with Gaussian distribution. The null hypothesis of linearity is described by $H_0 : \gamma_{1j} = \gamma_{2j} = \gamma_{3j} = \gamma_4 = \gamma_5 = 0$, for all $j \in (1, 2, ...\phi)$. This is tested using a general LM-type test, denoted by LM^G, estimated for all plausible values of d. If any of the LM^G statistics is statistically significant the linearity hypothesis is rejected. If more than one LM^G statistics are significant the value of d is determined by the highest F-score. If linearity is rejected I determine the specific form of non-linearity following the approach by Teräsvirta and Anderson (1992). In terms of equation (1.29) this involves three steps. First, conditional upon LM^G being significant, I test the null described by $H_0 : \gamma_{3j} = \gamma_5 = 0$, for all $j \in (1, 2, ...\phi)$. This test is denoted as LM^{L1}. If LM^{L1} is significant I conclude that non-linearity is of the logistic form. If LM^{L1} is not significant I test the null of $H_0 : \gamma_{1j} = \gamma_{2j} = \gamma_4 = 0 | \gamma_{3j} = \gamma_5 = 0$, for all $j \in (1, 2, ...\phi)$. I denote this test as LM^Q. If LM^Q is significant I conclude that non-linearity is quadratic. If both LM^{L1} and LM^Q are insignificant I perform a third test, LM^{L2}, where the null is

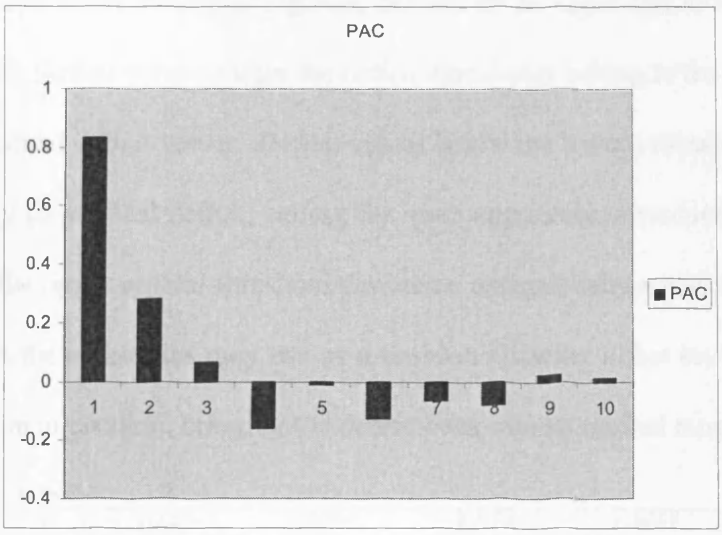

Fig. 1.3. Partial autocorrelation function (CV accounts for structural breaks, 1955Q1-2006Q1)

given by $H_0 : \gamma_{1j} = 0 | \gamma_{2j} = \gamma_{3j} = \gamma_4 = \gamma_5 = 0$, for all $j \in (1, 2, ...\phi)$. A statistically significant LM^{L2} indicates linearity of the logistic type.

Given the relatively small number of observations in each of the fiscal regimes identified in the previous section I test for non-linear fiscal adjustment using the whole of the available sample period. Figure 1.3 presents the partial autocorrelation function of the series obtained from estimating equation (1.23) with DGLS, i.e. the DGLS estimates of the cointegrating vector accounting for structural breaks. This is statistically significant up to the second lag, therefore equation (1.29) is estimated by setting $\phi = 2$.

The results of the non-linearity tests are reported in Table 1.5. I report findings for $d =$ 1, 2, 3 and 4. The significance of LM^G score for d=1 suggesting a rejection of the hypothesis of linear fiscal adjustment. For d=1 the LM^{L1} and LM^Q are insignificant and significant respectively. Therefore, I conclude the existence of non-linearity of quadratic type. This

implies the existence of two fiscal regimes, defined by an upper and lower critical deficit threshold value. Deficit values within the critical thresholds belong to the inner regime, interpreted as normal deficit values. Deficit values below the lower critical threshold denote an exceedingly large fiscal deficit, calling for more aggressive correction. Finally, deficit values above the upper critical threshold denote an exceptionally small deficit value, or a surplus, which the authorities may use as a cushion allowing a fast increase in spending and/or reduction in taxation, bringing the deficit back into its normal range.

$\phi = 2$

d	LM^G	LM^{L1}	LM^Q	LM^{L2}
1	7.523[0.000]**	1.717[0.182]	13.775[0.000]**	N/A
2	0.989[0.434]	1.321[0.269]	1.073[0.362]	0.064[0.801]
3	1.151[0.335]	2.115[0.124]	1.057[0.350]	0.265[0.767]
4	0.522[0.791]	1.130[0.325]	0.277[0.759]	0.161[0.852]

Table 1.5. Test for the type of non-linear fiscal adjustment

*Note on Table: p-value are in square brackets and ** represents significance at 1% level. Recall that LM^G is a general test of non-linearity where the null hypothesis is denoted as linear adjustment. LM^{L1} tests the linear adjustment against the alternative of non-linear logistic adjustment; LM^Q tests the null of linear adjustment against the alternative of non-linear quadratic adjustment; LM^{L2} tests the quadratic adjustment against the alternative of logistic adjustment.*

The quadratic non-linearity is modeled using the Quadratic-Logistic Smooth Threshold Error Correction Model (QL-STECM). This is given by equations (1.30) to (1.33)below

$$\Delta R_t = \theta_t S_{1t} + (1-\theta_t) S_{2t} + v_t \tag{1.30}$$

$$S_{1t} = \alpha_1 + \sum_{i=1}^{n} \beta_{1i} \Delta R_{t-i} + \sum_{i=0}^{p} \gamma_{1i} \Delta TG_{t-i} + \delta_1 \widehat{\varepsilon}_{t-1} \tag{1.31}$$

$$S_{2t} = \alpha_2 + \sum_{i=1}^{n} \beta_{2i} \Delta R_{t-i} + \sum_{i=0}^{p} \gamma_{2i} \Delta TG_{t-i} + \delta_2 \widehat{\varepsilon}_{t-1} \tag{1.32}$$

$$\theta_t = pr\left\{\tau^L \leqslant \widehat{\varepsilon}_{t-d} \leqslant \tau^U\right\} = 1 - \frac{1}{1+e^{-\sigma(\widehat{\varepsilon}_{t-d}-\tau^L)(\widehat{\varepsilon}_{t-d}-\tau^U)}} \tag{1.33}$$

Equations (1.31) and (1.32) are standard linear error-correction models, capturing the two fiscal regimes, the inner (S_1) and the outer (S_2). Within the inner regime adjustment towards equilibrium takes place at a speed described by δ_1. At the outer regime, adjustment takes place at a rate equal to δ_2. My expectation is that $|\delta_2| > |\delta_1|$ denoting faster adjustment in the outer rather than the inner regime. Equation (1.30) models period-to-period fiscal adjustment as a weighted average of S_1 and S_2. The regime weight θ_t is defined in (1.33) as the probability that the transition variable ε_{t-d} takes values within the inner regime boundaries, with σ denoting the speed of transition between these two regimes. Note that only ΔR_t is being considered in the nonlinear setup as the linear ECM results suggest that the coefficient of the error correction term δ is statistically insignificant in the equation modeling ΔTG_t. As argued by the nonlinear literature, fiscal adjustment takes place more rapidly when budget deficits or the stock of outstanding debt exceed certain critical thresholds beyond which they are considered exceedingly large. As a result, if there is no adjustment in the linear fashion (i.e. δ being insignificant in ECM2) I would expect the error correction term to be insignificant in the nonlinear framework.

The estimates of the parsimonious QL-STECM model are reported in Table 1.6. The estimated coefficient of the error correction term in the inner regime (S_1) is insignificant, suggesting no correction of deficits within the inner regime. By contrast, the coefficient of the error correction term in the outer regime (S_2) is significant, with both critical thresholds τ^U and τ^L being negative and significant. These suggest correction of excessive large deficits. They also suggest that UK governments correct (push back into the inner regime) any temporary small deficits and surpluses.[4] The QL-STECM has good econometric properties, as it passes all misspecification tests. It also fits the data better than its linear counterpart reported in the first column of Table 1.4, as suggested by its lower regression standard error.

[4] These findings are consistent with those reported by Considine and Gallagher (2008), who base their analysis on non-linearities indentified for the UK debt to GDP ratio series.

	S_1	S_2
Constant	0.028(0.012)*	-0.001(0.002)
ΔTG_t	-	0.440(0.097)**
ΔR_{t-1}	-0.336(0.139)*	
ΔR_{t-5}	-	0.204(0.077)**
$\widehat{\varepsilon}_{t-1}$	0.564(0.361)	-0.122(0.029)**
σ	768.64(929.9)	
τ^U	-0.012(0.003)**	
τ^L	-0.053(0.003)**	
S.E.of regression	0.021	
Misspecification tests (p-values)		
F-AR	0.42	
F-$ARCH$	0.21	
Norm	0.55	
F-Het	0.73	

Table 1.6. Non-linear fiscal adjustment model (QL-STECM)

*Notes on Table: standard errors in parentheses. *,** denote significance at 5% and 1% level, respectively. Misspecification tests are carried out. F-AR is the Lagrange Multiplier F-test for residual serial correlation up to forth order. F-$ARCH$ is an F-test for autoregressive conditional heteroskedasticity. Norm is normality chi-Square Bera-Jarque test for residuals' non-normality. F-Het is F-test for residuals heteroskedasticity. The model includes dummy variables for outlier observations (1962Q4 and 1973Q2).*

Figure 1.4 plots the transition function $\theta(\varepsilon_t; \sigma, \tau)$, i.e. the probability of a regime change in the current period against the transition variable ε_{t-1}, the value of fiscal disequilibrium in the previous period. I would intuitively expect $\theta(\varepsilon_t; \sigma, \tau)$ to increase as the fiscal outlook deteriorates beyond the lower deficit threshold, calling for a fast correction of deficits; or increases above the upper deficit threshold, providing the government the opportunity to introduce higher expenditure or reduce taxation. In both cases I would expect a high value of $\theta(\varepsilon_t; \sigma, \tau)$ capturing a high probability of a transition from the outer regime

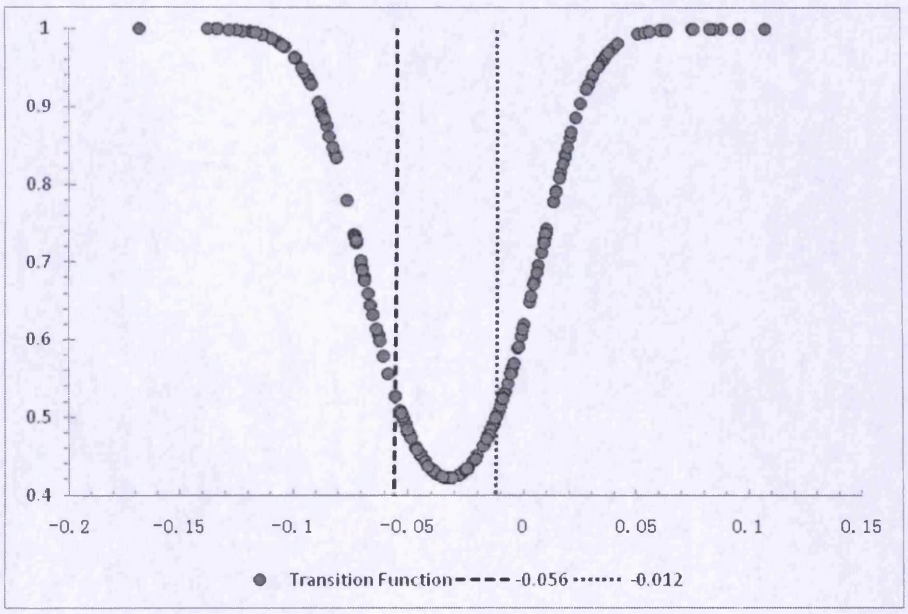

Fig. 1.4. Estimated transition function

to the inner. By contrast, when the fiscal disequilibrium term takes values within the inner regime, I would expect a low value for $\theta(\varepsilon_t; \sigma, \tau)$, denoting a low probability of switching from the inner regime to the outer. Figure 1.4 provides evidence consistent with my expectations. As expected, the probability of regime change is lowest when the transition variable takes the value of -0.026 which lies comfortably within the inner regime defined by [-0.053, -0.012]. On the other hand, the probability of a switch from the outer to the inner regime convergences to unity fast as the lagged disequilibrium term moves away from the model's estimated critical thresholds.

Finally, Figure 1.5 depicts the estimated θ_t parameter over the sample period and its smoothed two-year moving average value. The value of θ_t denotes the probability of being in the inner regime, i.e. expectations of being in the regime of 'normal', and by

implication sustainable, deficit values. From that point of view it can be seen as a rough measure of credibility of the current fiscal policy stance providing an indication regarding expectations of its sustainability. It is observed that that this probability is declining rapidly since the early 1970s, when the UK fiscal deficit had entered a period of non-sustainable fiscal dynamics as discussed in section 1.6 above. By contrast, the second half of the 1980s saw a significant increase in the value of θ_t , suggesting increasing confidence in the sustainability of the improved fiscal outlook achieved by the UK authorities initiated over that decade. Expectations of being in the inner regime record another marked reduction during the recession of the early 1990s, recovering however within a short period of time coinciding with the high growth rates the UK economy registered following its exit from the Exchange Rate Mechanism in 1992. Finally, there is another sharp decline in the value of θ_t in the late 1990s. Most likely, however, this is not the result of a substantial deterioration of the UK fiscal outlook but a substantial improvement, leading to expectations that the surpluses the UK economy had been recording over those years (see Figure 1.1) would not last for long. Indeed, and as Figure 1.1 suggests, in the subsequent decade of the 2000s public expenditure increased much faster than revenue expectations, eliminating the temporary surpluses achieved in the late 1990s pushing back the deficit within its 'normal' range and, as Figure 1.5 suggests, increasing the probability that the latter will stay there.

1.8 Conclusion

In this chapter I test for fiscal policy sustainability in the UK over the period 1955-2006. Using quarterly data and a unified framework of analysis I address four interrelated ques-

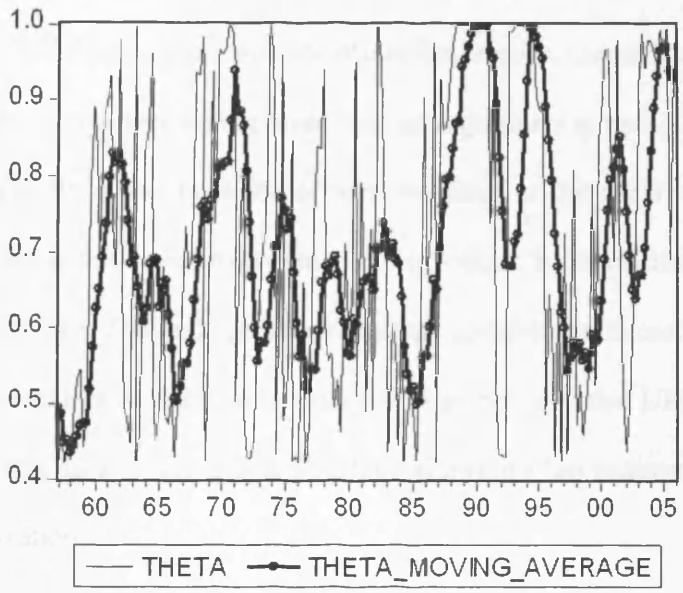

Fig. 1.5. Transition Function 1955Q1-2006Q1

tions. First, I test for fiscal policy sustainability accounting for exogenous shifts in UK fiscal policy, which I identify using tests for endogenous structural breaks. Second, I assess the nature of fiscal policy (Ricardian versus non-Ricardian) in each of the fiscal regimes identified by the structural stability analysis. Third, I test whether deviations from fiscal sustainability are corrected through adjustments in government revenue or expenditure. Finally, I test for non-linearities in UK fiscal policy.

The main findings can be summarized as follows: First, UK fiscal policy has been sustainable over the period under examination. Second, it has been subject to three structural breaks, respectively located in the early 1970s, early 1980s and late 1990s. These coincide with important shifts in UK fiscal policy, with the first break moving government finances away from sustainability and the remaining two towards it. Third, fiscal policy was

sustainable during all sub-periods identified by the analysis, with the exception of 1973-1981 when the UK fiscal regime was non-Ricardian. Fourth, correction of deviations from fiscal sustainability has been taking place through adjustment of public revenue rather than expenditure. Finally, I find evidence of non-linearities in UK fiscal policy, with the UK government not reacting to relatively small deficit values; but correcting exceedingly large deficits relatively fast. The findings also imply fast correction of exceedingly small deficits or temporary surpluses, which can be interpreted as evidence that UK authorities use unusually favourable fiscal conditions as a cushion allowing a fast increase in spending and/or reduction in taxation.

Overall, the results show weak-form sustainability, a sufficient condition in itself to establish that the government finances satisfy the IBC. Since UK authorities continue to carry out the deficit reduction plans and the economy is characterised by a low real rate of interest, it is rather unlikely that the rejection of strong-form sustainability will lead to infinite per-capita debt in the long-run.

Chapter 2
Can the Fiscal Theory of the Price Level explain UK inflation in the 1970s?

2.1 Introduction

In previous chapter I find a non-Ricardian fiscal policy regime for UK 1970s, where fiscal disequilibrium was restored by neither government expenditure nor taxes. Now I turn to this episode, the 1970s, in some detail where sustainability have been questioned.

In 1972 the UK government floated the pound while pursuing highly expansionary fiscal policies whose aim was to reduce rising unemployment. To control inflation the government introduced statutory wage and price controls. Monetary policy was given no targets for either the money supply or inflation; interest rates were held at rates that would accommodate growth and falling unemployment. Since wage and price controls would inevitably break down faced with the inflationary effects of such policies, this period appears to fit rather well with the policy requirements of the Fiscal Theory of the Price Level: fiscal policy appears to have been non-Ricardian (not limited by concerns with solvency) and monetary policy accommodative to inflation - in the language of Leeper (1991) fiscal policy was 'active' and monetary policy was 'passive'. Furthermore, there was no reason to believe that this policy regime would come to an end: both Conservative and Labour parties won elections in the 1970s and both pursued essentially the same policies. While Margaret Thatcher won the Conservative leadership in 1975 and also the election in 1979, during

the period it was not assumed that the monetarist policies she advocated would ever occur, since they were opposed by the two other parties, by a powerful group in her own party, as well as by the senior civil service. Only after her election and her actual implementation of them was this a reasonable assumption. So it appears that in the period 1972-79 there was a prevailing policy regime which was expected to continue. These are key assumptions about the policy environment; besides this narrative background I also check them empirically below.

Under FTPL the price level or inflation is determined by the need to impose fiscal solvency; thus it is set at the value necessary for the government's intertemporal budget constraint to hold at the market value of outstanding debt. Given this determinate price level, money supply growth, interest rates and output are determined recursively as the values required by the rest of the model to permit this price level. The theory implies a relationship between the trend in the level of inflation and trends in fiscal variables. This relationship can be tested by cointegration analysis in a familiar way. Indirect inference, less familiarly, can be used to evaluate the model's dynamics by checking whether its simulated dynamic behaviour is consistent with the data. The data under indirect inference is described by some time-series equation. The model's simulated behaviour implies a range of time-series behaviour depending on the shocks hitting it; this range can be described by the parameters of the same time-series equation that fits the data. I can derive the implied statistical joint distribution for the parameters of this equation and test whether the parameters of the time-series equation from the data lie jointly within this distribution at some confidence level.

The FTPL has been set out and developed in Leeper (1991), Sims (1994, 1997), Woodford (1996, 1998a, 1998b and 2001), and Cochrane (2000, 2001) - see also comments by McCallum (2001,2003), Buiter (1999, 2002), and for surveys Kocherlakota and Phelan (1999), Carlstrom and Fuerst (2000) and Christiano and Fitzgerald (2000). Empirical tests have been proposed by Canzoneri, Cumby and Diba (2001), Bohn (1998), Cochrane (1999) and Woodford (1999), Davig et al (2007). In particular Loyo (2000) argues that Brazilian policy in the late 1970s and early 1980s was non-Ricardian and that the FTPL provides a persuasive explanation for Brazil's high inflation during that time. The work of Tanner and Ramos (2003) also finds evidence of fiscal dominance for the case of Brazil for some important periods. Cochrane (1999, 2000) argues that the FTPL with a statistically exogenous surplus process explains the dynamics of U.S. inflation in the 1970s. This appears to be similar to what I see in the UK during the 1970s.

With fiscal policy of this type, the financial markets - forced to price the resulting supplies of government bonds - will take a view about future inflation and set interest rates and bond prices accordingly. It will set bond prices so that the government's solvency is assured ex post (i.e. in equilibrium); thus it will be ensuring that buyers of the bonds are paying a fair price. Future inflation is expected because if the bonds were priced at excessive value then consumers would have wealth to spend, in that their bonds would be worth more than their future tax liabilities; this would generate excess demand which would drive up inflation. However this mechanism would only come into play out of equilibrium. It is unobservable because markets anticipate it and so drive interest rates and expected inflation up in advance; inflation follows because of the standard Phillips Curve mechanism

by which workers and firms raise inflation in line with expected inflation. Thus the FTPL can be regarded as a particular policy regime within a sequence of different policy regimes.

The aim in this chapter is to test the Fiscal Theory of the Price Level (FTPL) as applied to the UK in the 1970s episode that described above. Buiter (1999, 2002) argue that the government's IBC is a constraint on the government's instruments that must be satisfied for all values of the economy-wide endogenous variables. It is an identity not just an equilibrium condition. The economy would not be correctly specified if the government IBC failed to hold in all actual price sequences. It is an ill-posed equilibrium model because of the unwarranted change in the assumption about when the government constraint applied. As argued by Buiter (1999), FTPL no longer requires that the government solvency constraint hold for all sequences of price levels and interest rates. Instead it requires only the solvency constraint hold in equilibrium, that is, for equilibrium sequences of prices and other endogenous variables. In addition, there is an arbitrary restriction on the permissible configuration of the exogenous public spending and revenue sequences and the predetermined initial stock of non-monetary nominal debt. This relaxation violates the normal rules for constructing a well-posed general equilibrium model. However, as argued by Cochrane (1999, 2001), this does not imply that FTPL does not hold; I observe an equilibrium sequence of prices where IBC holds but this can come from an FTPL economy or one with the traditionally assumed active monetary and passive fiscal policy.

McCallum (2001, 2003) establish a simple closed economy model and argue that under FTPL the solution is not learnable because the price level solution involves a bubble component. The traditional or monetarist representation can by contrast be learnt - see

also McCallum and Nelson (2005). Yet they fail to notice that FTPL creates a world that is observationally equivalent, as noted by Cochrane (see also Christiano and Fitzgerald, 2000); the price solution is determined by the fiscal policy and in this case the bubble components are eliminated. Specifically, there is no bubble in my model solution. As for 'learnability' this appears to be an irrelevant consideration since fiscal and monetary policies are often announced via numerous policy statements; certainly they can be so announced, thus bypassing learning. For further discussions of the criticism of the FTPL please refer to section 2.10.1

Cochrane (1998, 2000, and 2001) has noted that there is a basic identification problem affecting the FTPL: in the FTPL fiscal policy is exogenous and forces inflation to close the government constraint while monetary policy is endogenous and responds to that given inflation; but the same economic behaviour can be consistent with an exogenous monetary policy determining inflation in the normal way, with Ricardian fiscal policy endogenously responding to the government budget constraint to ensure solvency given that inflation path. Hence testing either or both models is not straightforward. My procedure is in two stages. In the first stage I identify which model set-up is operating with tests of exogeneity for the two policy regimes: I test for and do not reject, both the endogeneity of monetary policy and the exogeneity of fiscal policy. In the second stage I go on to test whether the FTPL can account for the behaviour of inflation in terms of the behaviour of fiscal variables alone; I do so first by testing for the trend cointegrating relationship and secondly carrying out a test for the dynamic relationship. The trend relationship is tested using Engle and Granger (1987) cointegration analysis. Indirect inference, less familiarly, is used

to evaluate the model's dynamic by checking whether its simulated dynamic behaviour is consistent with the data. In general the Indirect Inference test has substantial test power against a DSGE model. This would apply to the last chapter when I use a DSGE model to explain the processes of inflation, output gap and interest rates (see Le et al, 2011 for Monte Carlo experiments on the power of indirect inference in testing DSGE models).

In the context of testing a single variable, i.e. ARMA as here, I find in Monte Carlo experiments that the power of the Indirect Inference test is low when the model is just a single equation as here. However, the ordinary single parameter test on the autoregressive coefficient of the error term of this equation has its usual power. This turns out to be sufficient to reject the model if that parameter is nonzero. The basic point, which is discussed in the last part of Chapter 2, is that the error term in the model inflation equation is autocorrelated, whereas it should be white noise. Thus while the model does not reject a role for government spending and taxation in the creation of inflation, in that inflation is cointegrated with them and is also a nonstationary process with them as determinants, it does reject the notion that this is the only determinant of inflation: there is something else going on, that makes inflation a more persistent unit root process.

This chapter is organised as follows. I review the history of UK policy during the 1970s in section 2.2 - in this section I establish a narrative that in my view plainly supports the exogeneity features required for the FTPL to be identified. A comprehensive literature review is appended as an appendix to the chapter (section 2.10) ; In section 2.3 I set up the model of FTPL for this UK episode. In section 2.4 I discuss the data and test the two key policy exogeneity/endogeneity assumptions of the theory econometrically. In section

2.5 I carry out the cointegration test for the inflation trend. In section 2.6 I explain the methodology of indirect inference. In section 2.7 I carry out the tests for the dynamics of inflation. In section 2.8 I discuss what evidence the model throws on other dynamic factors affecting inflation that are included via the model's error term. Section 2.9 concludes.

2.2 UK policy background

From WWII until its breakdown in 1970 the Bretton Woods system governed the UK exchange rate and hence its monetary policy. While exchange controls gave some moderate freedom to manage interest rates away from foreign rates without the policy being overwhelmed by capital movements, such freedom was mainly only for the short term; the setting of interest rates was dominated in the longer term by the need to control the balance of payments sufficiently to hold the sterling exchange rate. Pegging the exchange rate implied that the price level was also pegged to the foreign price level. Through this mechanism monetary policy ensured price level determinacy. Fiscal policy was therefore disciplined by the inability to shift the price level from this trajectory and also by the consequent fixing of the home interest rate to the foreign level. While this discipline could in principle be overthrown by fiscal policy forcing a series of devaluations, the evidence suggests that this did not happen; there were just two devaluations during the whole post-war period up to 1970, in 1949 and 1967. On both occasions a Labour government viewed the devaluation as a one-off change permitting a brief period of monetary and fiscal ease, to be followed by a return to the previous regime.

However, after the collapse of Bretton Woods, the UK moved in a series of steps to a floating exchange rate. Initially sterling was fixed to continental currencies through a European exchange rate system known as 'the snake in the tunnel', designed to hold rates within a general range (the tunnel) and if possible even closer (the snake). Sterling proved difficult to keep within these ranges, and was in practice kept within a range against the dollar and an 'effective' (currency basket) rate. Finally it was formally floated in June 1972.

UK monetary policy was not given a new nominal target to replace the exchange rate. Instead the Conservative government of Edward Heath assigned the determination of inflation to wage and price controls. A statutory 'incomes policy' was introduced in late 1972. After the 1974 election the incoming Labour government set up a 'voluntary incomes policy', buttressed by food subsidies and cuts in indirect tax rates. Fiscal policy was expansionary until 1975 and monetary policy was accommodative, with interest rates kept low to encourage falling unemployment. In 1976 the Labour government invited the IMF to stabilise the falling sterling exchange rate; the IMF terms included the setting of targets for Domestic Credit Expansion. These were largely met by a form of control on deposits (the 'corset') which forced banks to reduce deposits in favour of other forms of liability. But by 1978 these restraints had effectively been abandoned and prices and incomes controls reinstated in the context of a pre-election fiscal and monetary expansion - see Minford (1993), Nelson (2003), Nelson and Nikolov (2003), DiCecio and Nelson (2007), and Meenagh et al (2009b) for further discussions of the UK policy environment for this and other post-war UK periods.

The description of policy suggests that the role of nominal anchor for inflation will have been played during the 1970s by fiscal policy, if only because monetary policy was not given this task and was purely accommodative. Thus the FTPL can be regarded as a particular policy regime within a sequence of different policy regimes. One could therefore think of money supply as endogenously determined by whatever demand for money would emerge at acceptably low unemployment and at whatever the rate of inflation was at the time. Hence plainly monetary policy would not define an inflation equilibrium and fiscal policy plays a key role. This is now I turn to.

2.3 The FTPL model for the UK in the 1970s

I assume that the UK finances its deficit by issuing nominal perpetuities, each paying one pound per period and whose present value is therefore $\frac{1}{R_t}$, where R_t is the long-term rate of interest. I use perpetuities here rather than the usual one-period bond because of the preponderance of long-term bonds in the UK debt issue: the average maturity of UK debt at this time was approximately ten years. All bonds at this time were nominal (indexed bonds were not issued until 1981).

The government budget constraint can then be written as

$$\frac{B_{t+1}}{R_t} + M_{t+1} = G_t - T_t + B_t + \frac{B_t}{R_t} + M_t \tag{2.34}$$

where G_t is government spending in money terms, T_t is government taxation in money terms, B_t is the number of perpetuities issued. Note that when perpetuities are assumed the debt interest in period t is B_t while the stock of debt at the start of period t has

the value during the period of $\frac{B_t}{R_t}$; end-period debt therefore has the value $\frac{B_{t+1}}{R_t}.M_t$ is the beginning of period nominal money holdings and M_{t+1} is the end of period money holdings. Note too the perpetuity interest rate is by construction expected to remain constant into the future.

In order to avoid discussion about the assumption about the money demand function, much of the fiscalist literature opts for eliminating money from the model- see Sims (1997), Cochrane (1999) and Woodford (2001). The reason is that money creation responds endogenously to inflation and other determinants of money demand. The resulting seigniorage revenue from the monetary base as a fraction of GDP is therefore $\frac{M_t}{GDP_t}\frac{\Delta M_{t+1}}{M_t}$ where in the UK $\frac{M_t}{GDP_t}[= \mu]$ is very small, of the order of 0.03-0.05; hence it will contribute a negligible fraction of movements in the UK government's overall revenue. If I write the $\log_e$ of money demand as $\ln P_t + \zeta \ln y_t - \xi R_t$ then this seigniorage term becomes $\mu[\pi_t + \zeta\Delta \ln y_t - \xi\Delta R_t]$. If I include this in the government budget constraint its permanent value, $\mu[\pi_t + \zeta\gamma]$ would enter (2.36) as an extra element in t_t; thus in (2.43) I would have an extra term on the right hand side $\frac{\Delta\mu\pi_t}{\bar{t}-\bar{g}}$ (γ being fixed). The solution equation for inflation is unaltered, viz $\Delta\pi_t = \kappa(\Delta g_t - \Delta t_t) + \eta_t$. In principle the value of κ and λ should allow for this extra term but as μ is so small I ignore it and so omit this seigniorage from the budget constraint in line with the majority of the earlier literature. Therefore, I exclude money from the government budget constraint and rewrite equation(2.34) as follows

$$\frac{B_{t+1}}{R_t} = G_t - T_t + B_t + \frac{B_t}{R_t} \tag{2.35}$$

The implied value of current bonds outstanding can be obtained by substituting forwards for future bonds outstanding yields

$$\frac{B_t}{R_t} = \sum_{i=0}^{\infty} (T_{t+i} - G_{t+i}) \frac{1}{(1+R_t)^{i+1}} \tag{2.36}$$

I represent equation (2.36) in terms of each period's expected 'permanent' tax and spending share, t_t and g_t, and assume that $T_{t+i} = t_t P_{t+i} y_{t+i}$ and $G_{t+i} = g_t P_{t+i} y_{t+i}$.

Simplifying equation (2.36) (see Appendix 2.10.3) yields

$$\frac{B_t}{R_t P_t y_t} = \frac{t_t - g_t}{(1+\pi_t+\gamma)(r_t^* - \gamma)} \tag{2.37}$$

where $R_t = r_t^* + \pi_t$ (respectively the perpetuity real interest rate and perpetuity inflation rate, both 'permanent' variables), γ is the 'permanent' growth rate of real GDP. Permanent growth in this period it is assumed to be constant so that output (which is an I(1) variable during this period) is assumed to be a random walk with constant drift equal to γ. All these expected permanent variables are by construction expected to be constant in the future at today's level.

In the case of inflation I impose on the model the simplifying assumption that it is a random walk, so that future expected inflation is equal to current inflation and is also therefore permanent inflation. Notice that in the rest of the model I have equations for output and real interest rates, in the IS and Phillips Curves (details are discussed in chapter 3); but these cannot determine inflation as well. Hence if inflation had some dynamic time-path other than the random walk I would have to determine it exogenously; I choose the random walk for simplicity, on the basis that the off-equilibrium wealth effect would operate so powerfully on excess demand that it would drive inflation at once to its permanent value.

The pricing condition on bonds in equation (2.37) thus sets their value consistently with expected future primary surpluses. Suppose now the government reduces the present value of future primary surpluses. At an unchanged real value of the debt this would be a 'non-Ricardian' fiscal policy move. According to the FTPL prices will adjust to reduce the real value of the debt to ensure the government budget constraint holds and thus the solvency condition is met. This is to be compared with the normal Ricardian situation, in which fiscal surpluses are endogenous so that fiscal shocks today lead to adjustments in future surpluses, the price level remaining unaffected.

Since the pricing equation sets the ratio of debt value to GDP equal to a function of permanent variables, it follows that this ratio b_t follows a random walk such that [5]

$$b_t = \frac{B_t}{R_t P_t y_t} = E_t b_{t+1} \tag{2.38}$$

and

$$\Delta b_t = \eta_t \tag{2.39}$$

where η_t is an $i.i.d.$ process.

This in turn allows me to solve for the inflation shock as a function of other shocks (especially shocks to government tax and spending). With the number of government bonds issued, B_t, being pre-determined (issued last period) and therefore known at t-1, equation (2.37) could be written as follows (taking logs and letting $\log x_t^{ue} = \log x_t - E_{t-1} \log x_t$, the unexpected change in $\log x_t$)

[5] A 'permanent' variable $\overline{x}_t$ is by definition a variable expected not to change in the future so that $E_t\, \overline{x}_{t+1} = \overline{x}_t$
Thus $\overline{x}_{t+1} = \overline{x}_t + \epsilon_{t+1}$
where ϵ_{t+1} is an $i.i.d.$error making the process a random walk.

LHS of equation (2.37)

$$\log b_t^{ue} = -\log R_t^{ue} - \log P_t^{ue} - \log y_t^{ue} \tag{2.40}$$

RHS of equation (2.37)

$$= \log\left(t_t - g_t\right)^{ue} - \log\left(1 + \pi_t + \gamma\right)^{ue} - \log(r_t^* - \gamma)^{ue} \tag{2.41}$$

With all the variables in the equation defined to follow a random walk, I can rewrite the above expression as

$$-\Delta \log\left(\pi_t + r_t^*\right) - \Delta\pi_t - \Delta\gamma = \Delta \log(t_t - g_t) - \Delta\pi_t - \Delta\gamma - \Delta \log(r_t^* - \gamma) \tag{2.42}$$

Simplifying

$$-\Delta \log\left(\pi_t + r_t^*\right) = \Delta \log(t_t - g_t) - \Delta \log(r_t^* - \gamma) \tag{2.43}$$

Using a first-order Taylor Series expansion around the sample means

$$-\frac{\Delta(\pi_t + r_t^*)}{\overline{\pi} + \overline{r^*}} = \frac{\Delta(t_t - g_t)}{\overline{t} - \overline{g}} - \frac{\Delta(r_t^* - \gamma)}{\overline{r^*} - \gamma} \tag{2.44}$$

Simplifying equation (2.44) I can obtain a solution for $\Delta\pi_t$ as a function of change in government expenditure and tax rates

$$\Delta\pi_t = \kappa(\Delta g_t - \Delta t_t) + \eta_t \tag{2.45}$$

where $\eta_t = \lambda\Delta r_t^*$; $\kappa = \frac{\overline{\pi}+\overline{r^*}}{\overline{t}-\overline{g}}$, $\lambda = \frac{\overline{\pi}+\gamma}{\overline{r^*}-\gamma}$; $\overline{\pi}, \overline{r^*}, \overline{t}$ and $\overline{g}$ are mean values of the corresponding variables. The term $\eta_t = \lambda\Delta r_t^*$ is treated as an error term.

2.4 Data, estimation and testing

2.4.1 Time series properties of the data

I begin with some notes on the time-series behaviour of inflation and the other macro variables I am dealing with for this period (1970Q4-1978Q4). Table 2.7 shows that inflation, output, interest rates and money supply (M4) growth are all I(1).

Unit root tests	π_t		y_t		R_t		M_t	
	Levels	1st Diff.	Levels	1st Diff.	Levels	1st Diff.	Levels	1st Diff.
ADF test statistic	-2.107	-5.218	-0.906	-2.082	-1.952	-4.782	-3.177	-6.058
	(0.243)	(0.000)**	(0.772)	(0.038)*	(0.605)	(0.000)**	(0.108)	(0.000)**
PP test statistic	-2.127	-7.561	-1.104	-7.201	-2.129	-4.666	-3.010	-8.591
	(0.236)	(0.000)**	(0.702)	(0.000)**	(0.511)	(0.000)**	(0.146)	(0.000)**

Table 2.7. Tests for non-stationarity of inflation, output, nominal interest rates and money supply (M4)

*Notes on Table: MacKinnon's critical values (MacKinnon, 1996) for rejection of hypothesis of a unit root: values in parentheses are p-values, while *, ** indicate significance at the 5% and 1% level, respectively. The number of lags in the ADF tests is set using AIC criterion; for the PP tests using Newey-West bandwidth (Newey and West, 1987).*

I now go on to estimate the best fitting $ARMA$ for the inflation first difference. Starting with $ARMA$ (0,0), I raise the order of the AR and MA each by one, and apply an F-test to test the validity of the lower order restriction. I find that any $ARMA$ coefficients added to a random walk are insignificant, suggesting that UK inflation first difference, $\Delta\pi_t$, may well simply be $ARMA$ (0,0), a pure random walk. However, of course it is also possible the dynamics are more complex, even if I cannot reject the simple random walk at the 5% level. I show below how the AIC varies as one raises the order - Table 2.8. The approach I take to the dynamics of $\Delta\pi_t$ is to examine all these $ARMA$ equations (except

order 3,3 whose MA roots lie outside the unit circle), in order to achieve robustness in the face of possibly more complex dynamics.

	$AR(1)$	$AR(2)$	$AR(3)$	$MA(1)$	$MA(2)$	$MA(3)$	AIC	$\overline{R}^2$
$ARMA(0,0)$	-	-	-	-	-	-	-6.261	0.000
$ARMA(1,1)$	-0.371	-	-	0.512	-	-	-6.138	-0.009
$ARMA(1,2)$	0.270	-	-	-0.296	-0.310	-	-6.185	0.065
$ARMA(1,3)$	-0.403	-	-	0.664	-0.351	-0.791	-6.280	0.174
$ARMA(2,0)$	0.056	-0.360	-	-	-	-	-6.213	0.097
$ARMA(2,1)$	0.382	-0.367	-	-0.393	-	-	-6.173	0.088
$ARMA(2,2)$	0.421	-0.463	-	-0.441	0.115	-	-6.110	0.097
$ARMA(2,3)$	-0.666	-0.312	-	0.859	-0.020	-0.597	-6.251	0.203
$ARMA(3,0)$	-0.003	-0.374	-0.160	-	-	-	-6.179	0.103
$ARMA(3,1)$	-0.835	-0.333	-0.458	0.952	-	-	-6.186	0.135
$ARMA(3,2)$	-0.955	-0.432	-0.449	1.094	0.131	-	-6.119	0.101
$ARMA(3,3)$*	-0.599	-0.482	0.377	1.565	0.738	-1.814	-7.356	0.746

Table 2.8. ARMA regressions:*=AR or MA roots outside unit circle

The fiscal variables, G/GDP and T/GDP, are shown in Figure 2.6. G/GDP is non-stationary: both the ADF and PP test suggest that it follows a pure random walk (Table 2.9), which implies that its current value is also its trend or permanent value, g_t. T/GDP is stationary around its mean with no significant deterministic trend; hence its trend or permanent value, t_t, is simply a constant. I conclude that government expenditure is the only driving force for inflation that can be observed in the data.[6]

*Notes on Table: MacKinnon's critical values (MacKinnon, 1996) for rejection of hypothesis of a unit root: values in parentheses are p-values, while *, ** indicate significance*

[6] For model convergence, the amount of government expenditure is required be less than taxation for government bonds to have a positive value. Note that since government expenditure of a capital variety is expected to produce future returns in line with real interest rates, I should deduct the trend in such spending from the trend in g (derived from the data shown in the Figure 2.6). To implement this, it is assumed that the average share of expenditure in the period devoted to fixed capital, health and education can be regarded as the (constant) trend in such capital spending; of course the 'capital' element in total government spending is essentially unobservable and hence the assumption is intended merely to adjust the level of the g trend in an approximate way but not its movement over time which would be regarded as accurately capturing changes in current spending. The adjustment for these is of the order of 10% of GDP.

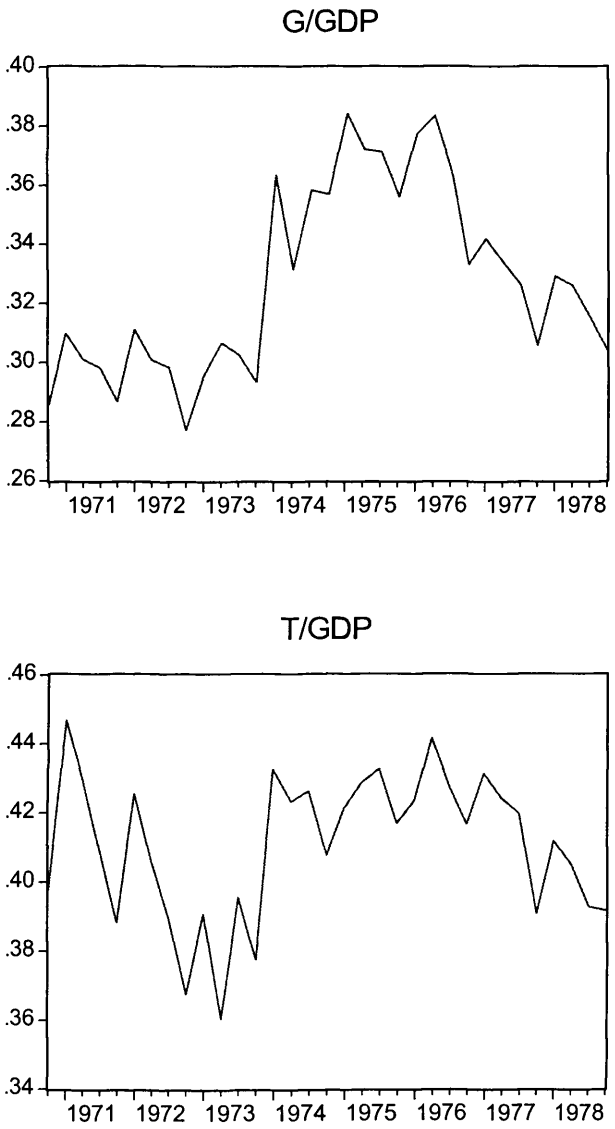

Fig. 2.6. The patterns of government expenditure rate G/GDP and tax rate T/GDP.

Unit root tests	G/GDP		T/GDP	
	Levels	1st Diff.	Levels	1st Diff.
ADF test statistic	-0.814	-8.393	-2.180	-
	(0.953)	(0.000)**	(0.030)*	-
PP test statistic	-1.576	-8.410	-3.379	-
	(0.780)	(0.000)**	(0.001)**	-

Table 2.9. Tests for non-stationarity of deseasonalised government expenditure and tax-both to GDP ratio

at the 5% and 1% level, respectively. Number of lags in the ADF test is set upon AIC criterion and PP test upon Newey-West bandwidth (Newey and West, 1987).

2.4.2 Testing the assumptions about policy exogeneity

The FTPL makes particular assumptions about the exogeneity of monetary and fiscal variables. First, it assumes that money is entirely accommodating; thus it is produced as needed to equal the demand for money resulting from the behaviour of output, inflation and interest rates. Second, it assumes that government spending and taxation are set in a non-Ricardian way, that is they do not respond to the state of the public finances in a way that would restore fiscal balance. They can respond to other factors, such as unemployment or special interests but are independent of the finances. I now set out the tests of these two assumptions.

Endogeneity of money supply

The focus is on M4, as M0 is generally agreed to have been supplied on demand during this period and indeed generally since WWII. The question therefore I ask here is whether M4 responds to the lagged behaviour of inflation, output and interest rates, all of which should enter the demand for money; I did not attempt to estimate a stable demand for

money function as this has proved elusive (see for instance Fisher and Vega (1993), Astley and Haldane (1995), Fiess and MacDonald (2001)). However my aim is narrower: to check on whether M4 responds to these minimum determinants. I found that the growth of M4 was I(1); other I(1) variables were inflation, the log of output, and the level of interest rates. Thus I checked an equation in the first differences of these variables, relating the change in M4 growth to the lagged changes in inflation, in output and in interest rates - Table 2.10. One can see that this equation finds highly significant feedback of money growth to these determinants.[7] I also find below - section 2.8 - that there is no effect of lagged money growth on the error in the model. Thus there is evidence here that money growth is endogenous.

[7] It might be suggested that a test should be made for whether there is an interest rate setting, Taylor rule in this period. However, as noted by Minford et al (2002), a Taylor rule equation is not identified on its own, since a variety of full models imply an equation indistinguishable from it. The long-run cointegrating relationship here gives a unit coefficient on inflation plus a relationship with the natural real rate; dynamics add further relationships with inflation both directly and via the two errors' correlation with inflation. Hence after detrending one will obtain some relationship between interest rates and inflation; this cannot be distinguished from the Taylor rule family of relationships. One can achieve identification by specifying a full alternative DSGE model with a Taylor rule. This could be tested by indirect inference in the manner of this chapter (Minford and Ou, 2009, do this for the US data post-1984); however, there are difficulties in specifying a Taylor rule for this period as it would need to permit very high annual rates of inflation (up to 50%) in the mid-1970s. Some, eg Nelson (2003), has argued that the correct rule would imply indeterminacy, hence a sunspot solution. Since the sunspot can be any number and has infinite variance, a model that includes it is simply untestable. One could take the approach of Minford and Srinivasan (2009) and impose a terminal condition to create determinacy in which case the Taylor rule could have a time-varying inflation target to accommodate the large swings in inflation. However, resolving these issues and specifying an alternative Taylor rule model for testing against the FTPL model here lie well outside the scope of this chapter.

Dependent variable	$\Delta_2 M_t$
Variable	coefficient
constant	-0.000(0.001)
$\Delta\pi_{t-1}$	-0.117(0.157)
Δy_{t-1}	0.133(0.048)**
ΔR_{t-1}	-0.484(0.109)**
F-test on joint significance (p-value)	0.000**
S.E.of regression	0.01
Misspecification tests (p-values)	
F-AR	0.13
F-$ARCH$	0.12
Norm	0.40
F-Het	0.41

Table 2.10. Money endogeneity test

*Notes on Table: standard errors in parenthesis. ** denotes significance at 1% level. Misspecification tests are carried out. F-AR is the Lagrange Multiplier F-test for residual serial correlation up to forth order. F-$ARCH$ is an F-test for autoregressive conditional heteroskedasticity. Norm is normality chi-Square Bera-Jarque test for residuals' non-normality. F-Het is F-test for residuals heteroskedasticity.*

Fiscal policy: non-Ricardian?

I test the fiscal policy assumption with the following equation

$$\Delta s_t = \alpha + \sum_{i=0}^{T} \gamma_i \Delta d_{t-i} + u_t \tag{2.46}$$

where s_t is government primary surplus as a percentage to GDP and d_t is the debt to GDP ratio. Both variables are I(1)-confirmed by both ADF and PP tests - Table 2.11. Thus I test whether the budget surplus responds to the public debt, both in first differences - Table 2.12 . There is evidently no feedback from changes in the debt/GDP ratio onto the primary

surplus to GDP ratio, as a Ricardian regime would require: this is clear evidence therefore of a non-Ricardian fiscal regime.

Unit root tests	s_t		d_t	
	Levels	1st Diff.	Levels	1st Diff.
ADF test statistic	-2.679	-11.251	-2.920	-1.810
	(0.251)	(0.000)**	(0.172)	(0.067)
PP test statistic	-0.963	-12.422	-3.051	-7.580
	(0.293)	(0.0000)**	(0.135)	(0.000)**

Table 2.11. Tests for non-stationarity of deseasonalised government surplus and debt- both to GDP ratio

*Notes on Table: MacKinnon's critical values (MacKinnon, 1996) for rejection of hypothesis of a unit root: values in parentheses are p-values, while ** indicate significance at the 1% level. The number of lags in the ADF tests is set using AIC criterion; for the PP tests using Newey-West bandwidth (Newey and West, 1987).*

	$T=1$	$T=2$	$T=3$	$T=4$
$\widehat{\alpha}$	-0.001(0.003)	0.001(0.003)	0.000(0.003)	0.000(0.003)
$\widehat{\gamma}_0$	0.407(0.215)	**0.625(0.255)***	0.544(0.278)	0.497(0.316)
$\widehat{\gamma}_1$	-0.020(0.217)	-0.046(0.215)	-0.221(0.268)	-0.368(0.306)
$\widehat{\gamma}_2$	-	0.401(0.251)	0.396(0.267)	0.448(0.319)
$\widehat{\gamma}_3$	-	-	-0.165(0.262)	-0.258(0.292)
$\widehat{\gamma}_4$	-	-		0.167(0.297)
F-test on joint significance (p-value)	0.19	0.13	0.43	0.54
S.E.of regression	0.02	0.02	0.02	0.02
Misspecification tests (p-values)				
F-AR	0.25	0.30	0.44	0.01
F-$ARCH$	0.66	0.16	0.23	0.56
Norm	0.93	0.03	0.08	0.24
F-Het	0.64	0.83	0.13	0.13

Table 2.12. Fiscal policy exogeneity test

*Notes on Table: standard errors in parenthesis. * denotes significance at 5% level. Misspecification tests are carried out. F-AR is the Lagrange Multiplier F-test for resid-*

ual serial correlation up to forth order. F-$ARCH$ is an F-test for autoregressive conditional heteroskedasticity. Norm is normality chi-Square Bera-Jarque test for residuals' non-normality. F-Het is F-test for residuals heteroskedasticity.

2.5 Can FTPL account for the trend in inflation?

The theory above implies that there is a cointegrating relation between inflation and the other arguments of (2.45): if one integrates (2.45) one obtains

$$\pi_t = \kappa g_t + \lambda r_t^* - \kappa t_t + c \tag{2.47}$$

While c is a constant, both t_t and r_t^* are in principle random walks, I found empirically that t_t was constant during this period. As for r_t^* again it is entirely possible that permanent real interest rates moved little. Thus it is likely that the only non-stationary variable on the right hand side that moves substantially is g_t. If so one would expect government spending and inflation to be cointegrated. Figure 2.7 compares the pattern of inflation (π_t) and public spending (g_t): both are I(1) variables and plainly share some similarities in behaviour.

I examine the following relationship with an Engle and Granger (1987) cointegration test

$$\pi_t = \alpha + \beta g_t + \varepsilon_t \tag{2.48}$$

Table 2.13 shows the estimating results. The stationarity of the estimated cointegrating vector $\widehat{cv} = \pi_t - \widehat{\alpha} - \widehat{\beta} g_t$ is established on ADF and PP tests, both of which reject the null hypothesis of non-stationarity. The result suggests there is a strong positive asso-

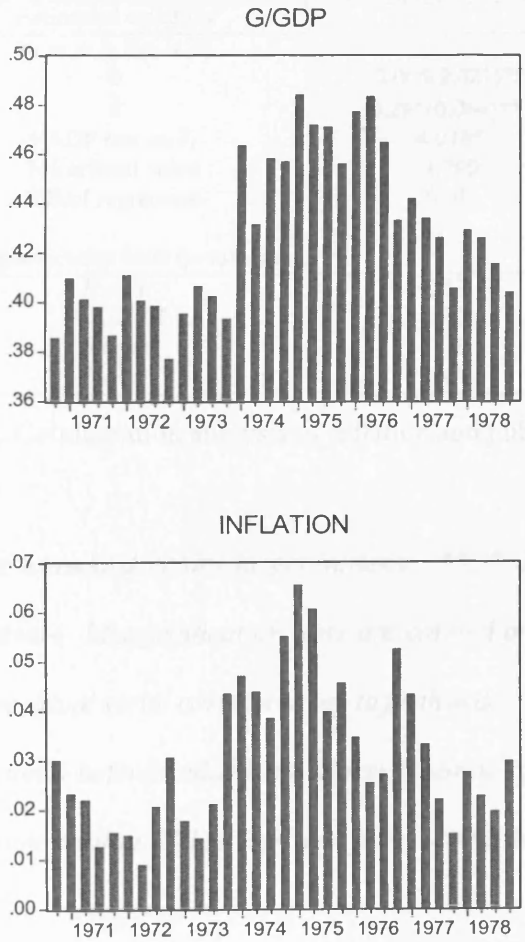

Fig. 2.7. Patterns of inflation (π_t) and public spending (g_t)

ciation between these two as suggested by the theory. Thus the FTPL's implication that fiscal trends drive inflation is quite consistent with the data. Now I turn to the test I use for evaluating the inflation dynamics.

	Engle-Granger(1987) approach
Estimated equation	
$\pi_t = \alpha + \beta g_t + \varepsilon_t$	
$\widehat{\alpha}$	-0.065(0.021)**
$\widehat{\beta}$	0.295(0.064)**
t-ADF test on $\widehat{\varepsilon}_t$	-4.018*
5% critical value	-3.390
S.E.of regression	0.01
Misspecification tests (p-values)	
F-AR	0.12
F-$ARCH$	0.37
Norm	0.49
F-Het	0.04

Table 2.13. Cointegration analysis of inflation and public expenditure

*Notes on table: standard errors in parenthesis. **, * denote significance at 5% and 1% level respectively. Misspecification tests are carried out. F-AR is the Lagrange Multiplier F-test for residual serial correlation up to forth order. F-$ARCH$ is an F-test for autoregressive conditional heteroskedasticity. Norm is normality chi-Square Bera-Jarque test for residuals' non-normality. F-Het is F-test for residuals heteroskedasticity.*

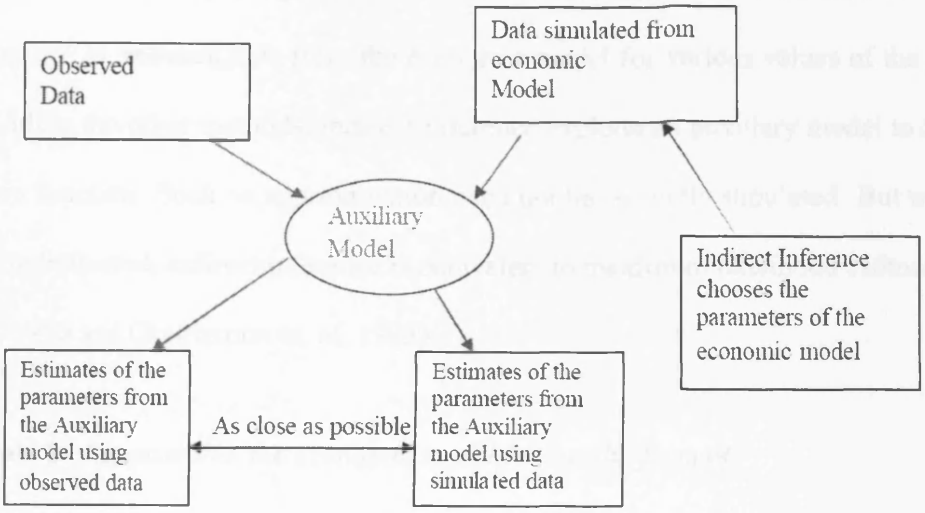

Fig. 2.8. The flow chart of indirect inference

2.6 The testing procedure

General speaking, indirect inference is a simulation-based method for estimating the parameters of economic models and it can be easily understood from Figure 2.8.

This diagram shows the way that indirect inference operates in the estimation process. This method chooses the parameters of the economic model and simulates the model with chosen parameters. By carefully selecting an auxiliary model both simulated and observed data go through the auxiliary model and one could obtain a set of estimates based on these two types of data. One could compare these two sets of estimates and indirect inference is to pick the value of the parameters to minimise the differences between these two estimates. The diagram indicates the importance of using auxiliary as a bridge for observed data and simulated data. Its usefulness lies on its ability of estimating models whose like-

lihood function is analytically difficult to evaluate. It is similar to other simulation-based techniques - to simulate data from the economic model for various values of the parameters. Unlike the other methods, indirect inference exploits an auxiliary model to form the criterion function. Such an approximation need not be correctly stipulated. But when it is correctly indicated, indirect inference is equivalent to maximum likelihood estimation (see Smith, 1990 and Gourieroux et. al, 1993).

Example 1 *Suppose that the economic model M has the form of*

$$y_t = g\left(y_t, (L)y_t, x_t, (L)x_t, \varepsilon_t, \beta\right) \qquad t = 1, 2, \ldots, T, \tag{2.49}$$

where y_t denotes a series of observed endogenous variables, x_t is a sequence of observed exogenous variables, L is the lag series $L, L^2, L^3, \ldots L^k$ (where k is the maximum lag length) and ε_t is the vector of structural equation errors. β is an n-dimensional parameter vector.

The auxiliary model, Γ, on the other hand, is defined by a conditional probability density function $f\left(y_t|(L)y_t, x_t, (L)x_t, \theta\right)$ that depends on an m-dimensional parameter vector θ. It is required that in the application of indirect inference method - the number of parameters in the auxiliary model must be greater or at least equal to the number of parameters in the economic model.

The curial point is to choose the most appropriate value of β so that the estimates of $\widetilde{\theta}(\beta)$ and $\widehat{\theta}$ are as close as possible. In general, if the distance between those two is zero, one could conclude that the estimated parameters exactly the same in the economic and auxiliary models. The task of indirect inference is to pick the value to minimise the

distance. There is a large literature on indirect inference, firstly introduced by Smith (1993) and later extended in important ways by Gregory and Smith (1991, 1993) and Gourieroux et al (1993). Gourieroux and Monfort (1995) provide a useful survey of indirect inference. There have been many interesting applications of indirect inference to the estimation of economic models - see also Dridi et al (2007) and Hall et al (2010). But here I make a different use of indirect inference by taking value of estimated parameters as given (i.e. the calibrated values). The method constitutes a test of the model's ability to 'fit the key facts'. The facts involved are suggested to be the key ones that a model must fit to be of 'use' or 'interest' in the context. Such key facts imply that model's ability to replicate the data. Hence I represent the facts in the data by some distributions, say at the 95% confidence level, if the data facts lie inside these distributions, it is suggested that the model fits.

The reason behind this method is idea of 'the ability for the model to generate the data'. It is argued that eventhough the models fit well it may fail to behave dynamically like the data. While fitting data conditional on lagged endogenous and current exogeneous variable values is not the same as replicating data's behaviour over time unconditionally (Le et al, 2009a). Also, the model's shocks should be estimated from the data, not imposed, and that all the shocks should be used; but more importantly that model's properties should be assessed against the data using their joint distributions which generally pose more stringent requirements than their single distributions viewed collectively (Le et al, 2009b).

The general idea is to test the model's dynamic behaviour against that of the economy. It assumes that the structural model, M, in the case of the true model is the null hypothesis. The structural errors based on the data are used to generate the model's simu-

lated performance. Under the null, the random parts of the residuals are bootstrapped and the model simulated with these to generate a large number of sample replications. One then can derive the moments, impulse responses and the time series properties of the data, and test whether the model's prediction can fit those of the data.

Indirect inference uses 'auxiliary' representation such as a VAR (for a group of variables) or an $ARMA$ (for a single variable) as the vehicle to estimate and test the structural model. Meenagh et al (2009c) explain how the test can be conducted. There are three steps in the test. First, the structural models are solved and the errors which implied by the models conditional on actual data are computed. The models under the null hypothesis are the true ones hence the errors calculated based on the models are also considered to be the true errors. Second, the errors under the null are omitted variables, modelled by autoregressive processes of identically and independently distributed shocks, where the shocks which are extracted as the residual from their autoregressive processes. The empirical distribution is assumed to be given by the actual sample of the residuals, and its variance and covariance is therefore the actual one. Given the true sample shocks, one could repeatedly resample them to obtain new samples from the distribution. Third, the parameters of the structural model are chosen to ensure the estimates simulated from the model are similar to those obtained from actual data (which is measured by the distance).

I now replicate the stochastic environment for the FTPL model to see whether the estimated dynamic equations for $\Delta\pi_t$ could have been generated by this model. This I do by bootstrapping the model above with their error processes. The aim is to compare the performance of the auxiliary model based on observed data with its performance based

on simulations of the macroeconomic model derived from the given distributions of the parameters. This is relevant as here, when I am interested in the behaviour of a structural model whose structure is rather precisely specified by the theory.

I create pseudo data samples - here 1000 - for $\Delta\pi_t$. I randomly draw $i.i.d.$ shocks in the error processes with replacement; I then input them into their error processes and these in turn into the model to solve for the implied path of $\Delta\pi_t$ over the sample period. I then run $ARMA$ regressions of the inflation first difference on all the pseudo-samples to derive the implied 95% confidence intervals for all the coefficient values found. Finally I compare the $ARMA$ coefficients estimated from the actual data to see whether they lie within these 95% confidence intervals: under the null hypothesis these values represent the sampling variation for the $ARMA$ coefficients which are generated by the model. The portmanteau Wald statistic - the 95% confidence limit for the joint distribution of the $ARMA$ parameters- is also computed based on the distributions of these functions of the parameters of the auxiliary model. The Wald-statistic is derived from the bootstrap distribution of the $ARMA$ parameters under the null hypothesis of the model. The Wald-statistic (Meenagh 2009a, 2009c) is calculated using following formula

$$(\widehat{\gamma} - \overline{\gamma})' \sum_{\gamma}^{-1} (\widehat{\gamma} - \overline{\gamma})' \tag{2.50}$$

where, $\sum_{\gamma}^{-1}$ is the inverse of the variance-covariance matrix of $\widehat{\gamma}$, the $ARMA$ parameter vector here generated by the bootstrap ($\overline{\gamma}$ is the mean of the bootstrap distribution). I arrange the values in ascending order and get the 5% critical percentile value for the model to be accepted as a whole. For the particular case of an $ARMA(0,0)$ I use the joint distrib-

ution of the coefficients of an $ARMA(1,1)$ to check whether it encompasses the two zero coefficients found in the data.

Figure 2.9 below illustrates the method for two parameters in the auxiliary equation such as in an $ARMA$(1,1). The bootstrap distributions of these two parameters under the null are shown for two cases: one where the two parameter estimates are uncorrelated, the other where they are highly correlated (with a coefficient of 0.9). One can think of estimation via indirect inference as changing the parameters of the structural model, thus changing the implied distribution, so as to push the observed data point as far into the centre of the distribution as possible. The test however takes the structural parameters (and hence the bivariate distribution) as given and merely notes the position of the observed data point (here given as 0.1 and 0.9) in the distribution. The Wald statistic is computed as this position expressed as a percentile; thus for example 96 indicates that the observed parameter estimates lie on the 96% 'contour', i.e. in the 95% rejection region.

2.7 The indirect inference results

I now use equation (2.45) for $\Delta\pi_t$ and bootstrap the random components of these Δg and η processes (since t is stationary and its trend value is a constant, it drops out on first-differencing). I obtain 1000 pseudo-samples of $\Delta\pi_t$ then run an $ARMA$ on each of these samples to generate the distribution of the $ARMA$ parameters. The Wald statistic then tests the model at the 95% level of confidence on the basis of the complete set of $ARMA$ parameters. I use the bootstrapped samples to compare the model with the data

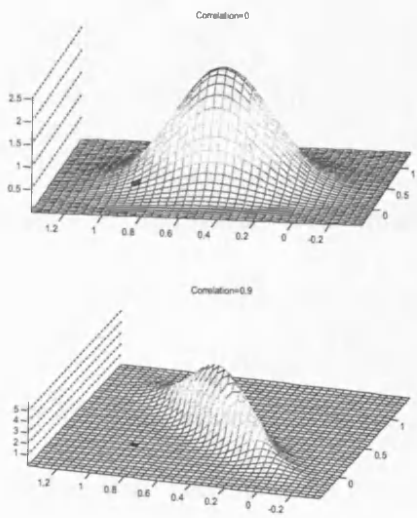

Fig. 2.9. Bivariate normal distributions (0.1, 0.9 shaded) with correlation of 0 and 0.9.

on its dynamic aspects - here the coefficients of the $ARMA$ for $\Delta\pi_t$. Of course I have already established that g is a pure random walk and that inflation is close to that too, which suggests that the model will generate similar dynamics. I first show the $ARMA(1,1)$ case for illustration. I run 1000 $ARMA$ regressions on the pseudo-samples to derive the implied 95% confidence intervals for both AR and MA coefficients. Then I compare the $ARMA$ coefficients estimated from the observed data and I conclude that both AR and MA coefficients lie within these 95% confidence intervals. I also look at the bootstrap sample variance: the variance of $\Delta\pi_t$ in levels in the data sample is 0.000116 lies inside the 95% model bounds; the 95% bounds for the bootstrap samples are 0.000088 (lower) and 0.000223 (upper) respectively. Thus the model replicates the data variance. The Full Wald statistic includes all $ARMA$ parameters and variance of $\Delta\pi_t$ and the result suggests the model as a whole is not rejected by the data.

	Model	Estimated	95% Confidence interval		IN/OUT
			Lower	Upper	
	$AR(1)$	-0.371	-0.936	0.913	IN
	$MA(1)$	0.512	-1.052	1.379	IN
	$Var(\Delta\pi_t)$	0.00011551	0.00008759	0.00022325	IN
Wald statistic	16.9%				
Full wald Statistic	18.3%				

Table 2.14. Confidence limits of first-differenced inflation process for theoretical ARMA(1,1)

If I disregard the $ARMA(3,3)$ as unstable, Table 2.15 reports that the Wald statistics for all the $ARMA$s as well as Full Wald statistics for $ARMA$s plus variance lie inside the 95% bounds. The more elaborate the dynamics that are estimated, the closer the model gets to being rejected; but this is a normal occurrence with indirect inference. The DSGE models impose stringent theoretical assumptions on behaviour, so that the more complex the representation of the data's behaviour the less well does the model replicate that behaviour. Here I find the model very easily encompasses the random walk but encompasses less well the more complicated dynamic schemes that can be found in the data.

	Wald statistic ($ARMA$ only)	Full wald statistic ($ARMA$ and variances)
$ARMA(0,0)$	0.01%	20.2%
$ARMA(1,1)$	16.9%	18.3%
$ARMA(1,2)$	21.9%	24.1%
$ARMA(1,3)$	76.8%	70.2%
$ARMA(2,0)$	85.7%	79.9%
$ARMA(2,1)$	67.5%	61.0%
$ARMA(2,2)$	45.2%	38.0%
$ARMA(2,3)$	58.7%	56.9%
$ARMA(3,0)$	80.2%	75.2%
$ARMA(3,1)$	94.3%	93.2%
$ARMA(3,2)$	84.3%	82.0%

Table 2.15. Wald statistics and Full wald statistics for variety of ARMA representations

2.8 What other dynamic factors could be affecting inflation?

The results reported in previous section suggest that the theoretical model does replicate the dynamic behaviour of inflation. It is worth noting however, that it does so when the model's structural error, η_t, in equation (2.45) is included as implied by the model and the data. This error, which has the interpretation of omitted variables, is found to be serially correlated which implies that the theory above only works approximately because if it were exact then this error would be serially uncorrelated. In the rest of this section I explore what these omitted influences on inflation may have been.

I begin by carefully identifying the time-series properties of this error. Note that since ε_t, the error in the cointegrating equation (2.48), is stationary, the error in the dynamic equation (2.45), $\eta_t = \Delta\varepsilon_t$, is also stationary but will in general not be $i.i.d.$, rather an $ARMA$ process. Estimating it I find indeed that this is the case - Table 2.16. If I ignore $ARMA$s with roots outside the unit circle (asterisked) I find that the best relationship is $ARMA$(1,3).

	$AR(1)$	$AR(2)$	$AR(3)$	$MA(1)$	$MA(2)$	$MA(3)$	AIC	$\overline{R}^2$
$ARMA$(1,1)	0.115	-	-	0.262	-	-	-6.166	0.101
$ARMA$(1,2)	-0.695	-	-	1.422	0.774	-	-6.352	0.274
$ARMA$(1,3)	-0.479	-	-	1.139	0.313	-0.394	-6.363	0.302
$ARMA$(2,0)	0.408	-0.181	-	-	-	-	-6.152	0.118
$ARMA$(2,1)	1.264	-0.417	-	-0.955	-	-	-6.133	0.126
$ARMA$(2,2)	1.107	-0.272	-	-0.779	-0.190	-	-6.082	0.107
$ARMA$(2,3)	0.144	0.507	-	0.439	-0.611	-0.819	-6.295	0.297
$ARMA$(3,0)	0.392	-0.141	-0.114	-	-	-	-6.062	0.096
$ARMA$(3,1)	-0.364	0.190	-0.411	0.979	-	-	-6.279	0.293
$ARMA$(3,2)*	0.098	0.665	-0.460	0.858	-0.990	-	-6.665	0.533
$ARMA$(3,3)*	0.315	0.794	-0.440	1.527	-1.215	-2.120	-7.555	0.813

Table 2.16. ARMA regressions for change in error of cointegrating vector. *= AR or MA roots outside unit circle

What now interests me is what lagged factors are influencing this error; current factors I know include all the innovations in the shocks to the economy. My method of investigation is to regress this error as an Error Correction Mechanism on a variety of candidate variables that could influence the dynamics of inflation via the usual channels of aggregate demand and supply that are omitted from the model's inflation determination. Significant factors could suggest ways the model could be enriched dynamically in future versions. The significance of both the ϕ_i and δ - Table 2.17 - suggest the presence of dynamic effects on inflation that the model does not capture; there is rather rapid error correction and positive reaction to lagged interest rate rises. Thus the model's dynamics could be enriched in ways that further work could investigate. Notice that money growth is insignificant, consistently with my earlier finding that it is entirely endogenous.

Estimated equation	
$\eta_t = \Delta\varepsilon_t = \alpha + \sum_{i=1}^{T} \beta_i \Delta_2 M_{t-i} + \sum_{i=1}^{T} \gamma_i \Delta y_{t-i} + \sum_{i=1}^{T} \phi_i \Delta R_{t-i} + \delta\varepsilon_{t-1} + u_t$	
$\widehat{\alpha}$	-0.000(0.002)
$\widehat{\beta}_1$	0.256(0.188)
$\widehat{\gamma}_1$	-0.022(0.050)
$\widehat{\phi}_1$	0.306(0.147)*
$\widehat{\phi}_2$	0.271(0.131)*
$\widehat{\delta}$	-0.587(0.186)**
S.E.of regression	0.01
Misspecification tests (p-values)	
F-AR	0.17
F-$ARCH$	0.35
Norm	0.57
F-Het	0.83

Table 2.17. Omitted variable test

*Notes to table: standard errors in parenthesis. *, ** indicate significance at the 5% and 1% level, respectively. Misspecification tests are carried out. F-AR is the Lagrange Multiplier F-test for residual serial correlation up to forth order. F-ARCH is an F-test for autoregressive conditional heteroskedasticity. Norm is normality chi-Square Bera-Jarque test for residuals' non-normality. F-Het is F-test for residuals heteroskedasticity.*

2.9 Conclusion

The chapter investigates whether the Fiscal Theory of the Price Level can explain UK inflation in the 1970s, a period in which the government greatly increased public spending without raising taxes and monetary policy was accommodative; I find evidence that fiscal policy behaved exogenously with respect to the state of the public finances and that money growth behaved entirely endogenously, thus identifying the policy assumptions of the Fiscal Theory. Its implied model of inflation is tested in two ways: for its trend using cointegration analysis and for its dynamics using the method of indirect inference. I find that it is not rejected. But I also find that the model's errors indicate omitted dynamics. Taken with the low power of the indirect inference procedure for this single equation structural model of inflation, this suggests that the inflation equation's dynamics merit further research.

2.10 Appendices

2.10.1 FTPL literature review

The theoretical literature

Traditionally, monetary policy sets the money supply (or interest rates) in order to meet an inflation or price level target while fiscal policy is assumed to adjust to ensure balance on the budget. This is the situation in which monetary policy is described as 'active' and fiscal policy as 'passive' or 'Ricardian' (Leeper, 1991). The change in the quantity of money would lead to an equal proportional change in the price level hence an unchanged level of real money. Another case is where fiscal policy is 'active' (i.e. sets government spending and taxes without regard to ultimate budget balance - 'non-Ricardian') and monetary policy is 'passive', that is it sets money supply endogenously in response to money demand at whatever market interest rates prevail. Here the government intertemporal budget constraint (IBC) is not satisfied at any point in time for any arbitrary sequence of price levels; and in order for there to be equilibrium in the economy with the IBC satisfied, a particular sequence of price levels/inflation must be chosen for which the IBC is satisfied. This is the case of FTPL - see also Sims (1994, 1997) and Woodford (1995, 1996, 1998a, 1998b, 2001). For FTPL to be an interesting theory, it is not necessary to have it always hold, but only in some contexts.[8]

[8] A related but subtly different case is where fiscal policy is active in the above sense but monetary policy ensures by suitable choice of money supply or interest rates to generate an inflation path and so seigniorage that exactly ensures the satisfaction of government IBC. This sets inflation so as to ensure budget balance, so that if the present value of future government surpluses is reduced, seigniorage must rise to ensure the government IBC is satisfied. Future inflation would be higher if the monetary authority is trying to reduce

The Quantity Theory of Money argues that the equilibrium price level is set by equating the real purchasing power of money stock to the real demand for money. Hence, the change of price level can be controlled by changing the money stock in that sense. Sargent and Wallace (1981) highlight the conditions under which the monetary authority might lose control of the price level. They argue that under the open-market operations if the government debt is positive at some level, controlling inflation by continuing to sell the stationary stock of interest-bearing bonds can be unsuccessful. Sargent (1987) states that "monetary and fiscal policies must be coordinated in the sense that, given a process for government expenditure, processes for inflation and the money stock cannot be chosen independently if they are to satisfy the government's present-value budget constraint". Even though their framework still supports monetary price determination, inflation is not a purely monetary phenomenon. The traditional view of price determination has thus been challenged by the new theory. The main framework of the Fiscal Theory of the Price Level concerns the government's IBC, where $\frac{\text{nominal debt}}{\text{general price level}}$ = present value of real surpluses. This equation works in the same way as the quantity theory of money determination, where the price level is determined by the ratio of debt and surplus.

I review the FTPL using a simple model in order to provide a better understanding of the theory. This theory begins with the idea that the price level can be determined even if the interest rate is fixed. It differs to the conventional view because it provides an additional constraint to pin down the price level. Such an idea comes from that the government sol-

inflation and seigniorage today with a fixed government surplus trajectory. This is the case of 'unpleasant monetarist arithmetic' (Sargent and Wallace, 1981). Here monetary policy is 'active' like fiscal policy but subordinates itself to the needs of the IBC, and thus can be described as 'following' fiscal policy.

vency condition is being satisfied only in equilibrium. The price level, which determined by the solvency condition also ensures the government IBC holds in equilibrium. Unlike the process developed by Sargent and Wallace (1981), FTPL does not consider the active and passive aspect of the story, it introduces a theory of the "jumping general price level without increasing money" which replaces the quantity theory of money with a quantity theory of the public debt (Woodford, 1995).

Consider the following money-in-utility (MIU) model where the representative agent has preferences over consumption and real money balances

$\sum_{t=0}^{\infty} \beta^t u\left(C_t, \frac{M_{t+1}}{P_t}\right)$ (A.1)

where, β is the discount factor, C_t denotes the real consumption and $\frac{M_{t+1}}{P_t}$ is the end of period real money holdings. The use of the end of money stock instead of beginning of period money stock is crucial here. As argued by the Buiter (1998) the cash-in-arrears framework ensures price level determinacy under a fixed nominal interest rate.[9] I follow Woodford (1995) cash-in-arrears approach.

Household

The representative household maximises the expected present discounted value of utility of the form (i.e. Buiter, 2002)

$\sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} + \frac{\gamma}{1-b}\left(\frac{M_{t+1}}{P_t}\right)^{1-b}\right]$ (A.2)

The representative agent budget constraint can be written as

$C_t + \frac{M_{t+1}}{P_t} + \frac{B_{t+1}}{(1+R_{t+1})P_t} + T_t = \frac{B_t}{P_t} + \frac{M_t}{P_t} + A_t$ (A.3)

[9] Buiter (1998) argues that the price level is overdetermined if transactions technology has 'cash-in-advance' features.

where, A_t is the real endowment, R_t is the nominal interest rate, that is, the government is issuing a nominal bond this period costing £1 and promises to pay interest rate of R_t£ next period. T_t is lump-sum tax that the household must pay. The representative agent maximizes the discounted stream of utility by choosing C_t, M_{t+1} and B_{t+1} subject to the budget constraint. The first order conditions of the household's optimization problems are

$C_t^{-\sigma} = \lambda_t$ (A.4)

$\frac{\gamma}{P_t}\left(\frac{M_{t+1}}{P_t}\right)^{-b} = \frac{1}{P_t}\lambda_t - \frac{1}{P_{t+1}}\lambda_{t+1}$ (A.5)

$\frac{\lambda_t}{(1+R_{t+1})P_t} = \beta\frac{\lambda_{t+1}}{P_{t+1}}$ (A.6)

where, λ_t is the Lagrange multiplier to the representative agent's budget constraint.

Substituting equation (A.6) in (A.4), as well as using the Fisher equation $(1+r_t)\left(\frac{P_{t+1}}{P_t}\right) = (1+R_t)$ yields the consumption Euler equation

$\left(\frac{C_t}{C_{t+1}}\right)^{-\sigma} = \beta\left(1+r_t\right)$ (A.7)[10]

The money demand function is similar obtained by combining equations (A.4), (A.5) and (A.6)

$\frac{1}{\gamma}\left(\frac{M_{t+1}}{P_t}\right)^{-b} C_t^{-\sigma} = \left(\frac{R_{t+1}}{1+R_{t+1}}\right)$ (A.8)

or

$\left(\frac{M_{t+1}}{P_t}\right)^{b} = \gamma\left(\frac{1+R_{t+1}}{R_{t+1}}\right) C_t^{\sigma}$ (A.9)[11]

Equation (A.9) shows the relationship between the demand for real balances and the consumption. The higher the consumption the higher the real money required. While the

[10] The Euler equation has the conventional meaning.

[11] I first divide equation (A.5) by λ_t yields:
$\frac{\gamma}{\lambda_t P_t}\left(\frac{M_{t+1}}{P_t}\right)^{-b} = \frac{1}{P_t} - \beta\frac{1}{P_{t+1}}\frac{\lambda_{t+1}}{\lambda_t}$.Combining with equations (A.4) and (A.6) one would obtain the expression for money demand function.

high the nominal interest rate leads to lower money balances. Equations (A.7) and (A.9) are the equilibrium conditions for an identical household.

Government

The government IBC

$$\frac{M_t}{P_t} + \frac{B_t}{P_t} = T_t - G_t + \frac{M_{t+1}}{P_t} + \frac{B_{t+1}}{(1+R_{t+1})P_t} \quad \text{(A.10)}$$

where T_t includes all tax revenues except the seigniorage revenues, G_t denotes the government expenditure. The nominal value of government bond B_t is discounted by $\frac{1}{1+R_{t+1}}$. I denote the government's financial liabilities by comprise both outstanding bonds and money issued to the public. Substituting $L_t = M_t + B_t$ into equation (A.10) the constraint can be expressed as

$$L_t = (T_t - G_t)\, P_t + \frac{1}{1+R_{t+1}} L_{t+1} + \left(\frac{R_{t+1}}{1+R_{t+1}}\right) M_{t+1}$$

Substituting forwards for future liabilities

$$L_t = \sum_{j=1}^{\infty} \left\{ \prod_{i=1}^{j} \left(\frac{1}{1+R_{t+i}}\right) \left[(T_{t+j} - G_{t+j})\, P_{t+j} + \left(\frac{R_{t+j+1}}{1+R_{t+j+1}}\right) M_{t+j+1} \right] \right\} \text{(A.11)}$$

To ensure government not to involve in the Ponzi game, it is required that the government transversality condition must be satisfied at all time

$$\lim_{T\to\infty} \prod_{i=1}^{T} \left(\frac{1}{1+R_{t+i}}\right) L_{t+T} = 0 \text{ (A.12)}$$

In equilibrium, the entire model can be summarised as follows

$$1 = \beta\,(1 + r_t) \text{ (A.13)}$$

$$\left(\frac{M_{t+1}}{P_t}\right)^b = \gamma \left(\frac{1+R_{t+1}}{R_{t+1}}\right) C_t^{\sigma} \text{ (A.14)}$$

$$L_t = \sum_{j=1}^{\infty} \left\{ \prod_{i=1}^{j} \left(\frac{1}{1+R_{t+i}}\right) \left[(T_{t+j} - G_{t+j})\, P_{t+j} + \left(\frac{R_{t+j+1}}{1+R_{t+j+1}}\right) M_{t+j+1} \right] \right\} \text{ (A.15)}$$

Woodford (1995) argues that price level can be determinable under a fixed interest rate rule. Price level is adjusted automatically to ensure the government solvency condition

is satisfied. The problem of price indeterminacy rises because the nominal interest rate is fixed in equation (A.14). Although the real balances are determined from this equation, there are no unique solutions for price level and money supply. In short, any combination of price level and money supply can lead to a same level of real money balances. This is what called 'nominal indeterminacy' in dynamic models. But this problem can be solved under the case of FTPL. Rewrite condition (A.15) by dividing both side of the equation by the general price level

$$\frac{L_t}{P_t} = \sum_{j=1}^{\infty} \left\{ \prod_{i=1}^{j} \left(\frac{1}{1+r_{t+i}} \right) \left[(T_{t+j} - G_{t+j}) + \left(\frac{R_{t+j+1}}{1+R_{t+j+1}} \right) \frac{M_{t+j+1}}{P_{t+j}} \right] \right\} \text{ (A.16)}$$

With an additional constraint (A.16), the price level can be identified as both real interest rate and real money balances are determined in equations A.13 and A.14 respectively.

In Woodford's 'Really Unpleasant Arithmetic' he argues that instability in fiscal policy have an impact on the price level no matter how much the monetary authority commits to its policy. This implies that fiscal policy alone can influence price level. Thus, to ensure price stability, monetary authorities must work well with fiscal authorities when adopting a policy. Christiano and Fitzgerald (2000) use a simple example to illustrate the how the monetary authorities may fail to control price stability in non-Ricardian framework. I can also easily to show in the example under exogenous government's fiscal rule (specific policy rule) the monetary authorities would lose control over the price stability. For simplicity, it is assumed the government nominal taxes follow an exogenous process where the government can adjust its tax revenues to fully compensate the loss of seigniorage revenues

$$T_t P_t = \widetilde{T} P_t + \triangle M_{t+1} \text{ (A.17)}$$

Thus deficits are fully backed by bonds and taxes. Hence the government budget constraint can be rewritten as

$B_t = \frac{B_{t+1}}{1+R_t} + (T_t - G_t) P_t$ (A.18)

and the corresponding government solvency condition becomes

$\frac{B_t}{P_t} = \sum_{j=0}^{\infty} \left[\left\{ \prod_{i=0}^{j} \left(\frac{1}{1+r_{t+i}} \right) \right\} (T_t - G_t) \right]$ (A.19)

Equation (A.19) is the fiscal equation for the price determination. Note that both money and nominal interest rate are absent from the equation. Monetary authorities have no control of price using monetary instruments. Price level is purely determined by the value of government bonds and its primary surplus (deficit if negative).

In addition, Woodford (1995) argues that price level can be determined even if the demand and supply for government fiat money is nonexistent. This has important policy implication as what should be included in the government IBC is questioned. The substitution between government fiat money and other financial equivalent is more possible if price is determined by government budget debt. As one can see from equation (A.19), there is no government fiat money enters the equation because government can adjust its tax revenues to fully compensate the loss of seigniorage revenues. Hence price level is completely independent to money. With money redundancy, the price level still can be formally pinned down.

In particular, Woodford (1996 and 1998a) argue that in non-Ricardian environments there are possibilities that aggregate demand would be affected by fiscal shocks. This is because, as he states, part of households' net wealth is government debt hence their future consumption is affected by the exogeneity of government deficits. Woodford (2001) em-

phasizes that price stability requires an appropriate commitment on fiscal rules, even when monetary policy is active. He shows that inflation targeting cannot be successful if fiscal policy is not consistent with such an equilibrium determined by monetary policy. Even if these two policies are consistent with an equilibrium stable price level, there is a possibility for the government budget choice to influence people's expectations. In such a case a policy rule to commit the government budget deficit is necessary for price stabilization.

Sims (1997) argues that fiscal policy should play an equal role with monetary policy in the process of price determination. For a monetary union, so long as there is no central fiscal authority to control fiscal instruments, individual countries with positive debt should commit themselves to increased primary surpluses in the future. This is because an individual government can increase the welfare of its citizens in equilibrium by committing to a lower or even a negative level of primary surpluses, hence leading to an upward jump of that country's price level and a permanent increase in the consumption of its citizens, transferred from the rest. This implies that there should be institutional controls and penalties for preventing individual countries from over-issuing debt at the expense of other countries' wealth. The only successful monetary union would be the one with limits on borrowing.

Thus theoretically there is the possibility for fiscal policy to influence the price level, even if the relevance of the FTPL is questioned in contributions such as Buiter (1999) and McCallum (2001). Cochrane (1999) argues that the government IBC holds in equilibrium for both Ricardian and non-Ricardian regimes. Hence whether the equilibrium is restored by either budget surplus (Ricardian) or by prices (non-Ricardian) is an empirical matter

that requires testing a full model of the economy - see also Cochrane (2000, 2001). He also argues that "the FTPL per se has no testable implications for the time series of debt, surpluses and price level". This is due to the fact that the government budget constraint written in nominal terms holds in both Ricardian and non-Ricardian regimes - a problem of 'observational equivalence'. Whether this equilibrium is restored by price or surplus adjustments remains unclear. The variables determined by the equilibrium conditions (A.13) to (A.15) are valid under both regimes. Hence, all one observes are equilibrium points, but not the fundamentals behind them. The FTPL requires that the government is pursuing a 'non-Ricardian' fiscal policy. But simply testing government sustainability is not evident to identify the regimes. This is because present value of government budget constrain is rather an equilibrium condition. By observing the equation (A.19) does not permit to distinguish between Ricardian and non-Ricardian regimes.

Buiter (1999, 2002) supports this view and states that "the government's IBC is a constraint on the government's instruments that must be satisfied for all values of the economy-wide endogenous variables". In particular, Buiter (1999) argues that economy would not be correctly specified if the government IBC failed to hold in all actual price sequences. However, as I can see from Cochrane's argument, this does not imply that FTPL does not hold; I observe an equilibrium sequence of prices where IBC holds but this can come from an FTPL economy or one with the traditionally assumed active monetary and passive fiscal policy.

McCallum (2001, 2003) establish a simple closed economy model and argue that under FTPL the solution is not learnable because the price level solution involves a bubble

component. The traditional or monetarist representation can by contrast be learnt - see also McCallum and Nelson (2005). Yet they fail to notice that FTPL creates a world that is observationally equivalent, as noted by Cochrane (see also Christiano and Fitzgerald, 2000); the price solution is determined by the fiscal policy and in this case the bubble components are eliminated. As for 'learnability' this appears to be an irrelevant consideration since fiscal and monetary policies are often announced via numerous policy statements; certainly they can be so announced, thus bypassing learning.

Gordon and Leeper (2006) generalise the FTPL, arguing that the price level is jointly determined by both current and expected future monetary and fiscal policies. The quantity theory of money and FTPL are considered as two restricted special cases. It is argued that with expenditures fully backed by tax revenues the quantity theory should account for price determination. On the other hand, when the central bank engages in the bond transaction (i.e. supports the bond prices during war time), the FTPL is more plausible. They conclude that "both theories require special assumptions on policy behaviour which are unlikely to hold in general. They are two special cases of price determination".

The empirical literature

In recent years FTPL have been received a lot of attention empirically. Bohn (1998) establishes a backward-looking econometric model to analyse the relationship between primary surplus-to-GDP and the (lagged) debt-to-GDP for U.S. He finds a positive response of surpluses to debt and hence that government budget is sustainable over the sample period. Although his work does not directly link to the FTPL, his findings support the view

of monetary dominance. Canzoneri et al (2001) investigate U.S. data for the period 1951-1995. Unlike Bohn (1998) they use a forward-looking approach for testing the theory of fiscal dominance and focus on a set of impulse response functions involving the primary surplus and total government liabilities (both as ratios to GDP). They find that a positive innovation in the surplus decreases liabilities for several periods and increases future surpluses. A positive innovation in surplus in period t pays off some of the debt in period t, since the response of surplus in period t+1 is also positive, then more debt are paid off in period t+1. The positive autocorrelations in the surplus process is also plausible under the Ricardian regime: the disturbances are likely to be persistent as election cycles and business cycles take years to complete. They argue that this is most plausibly interpreted as a Ricardian regime. Loyo (1999) addresses the inflationary episodes in Brazil using the FTPL and shows that a tight monetary policy along with lose fiscal policy would lead to hyperinflation. The work of Tanner and Ramos (2003) also finds evidence of fiscal dominance for the case of Brazil for some important periods.

Working within the context of a full DSGE model, Davig and Leeper (2007), Davig et al (2007) examine regime switches between fiscal and monetary policy for U.S. They define the Ricardian policy regime as an 'active' monetary policy coupled with a 'passive' fiscal policy- the policy mix implicit in the literature on the Taylor principle. In contrast, the non-Ricardian regime is where fiscal policy dominates (is 'active') and monetary policy is accommodative or 'passive' - this is the combination associated with the FTPL (Davig et al 2007). They model regime change as an on-going process and show that as long as agents are allowed to place probability on both kinds of regimes happening, and if active

fiscal policy is expected to occur next period, then tax changes would have wealth effects and lead to non-Ricardian outcomes. Another attempt to locate regime switching is due to Favero and Monacelli (2005). They investigate U.S. data for the period of 1960-2002 and conclude that U.S. monetary policy shifted between active and passive. Other work on regime changes includes Leeper and Zha (2003), Lubik and Schorfheide (2004), Davig and Leeper (2009a, 2009b). Thus the FTPL can be regarded as a particular policy regime- one of 'active' or 'dominant' fiscal policy - within a sequence of different policy regimes.

For European economies, Afonso (2002) applies a panel data approach for the EU-15 countries during the period 1970-2001 and the result does not support the view of FTPL. He concludes that countries tend to react positively to the increases in the government liabilities - a Ricardian regime. Janssen et al (2002) analyse the change of the UK inflation path under the impact of both monetary and fiscal policy. By examining 300 years data, they find that there is little inter-relationship between fiscal policy and the general price level. Creel et al (2005) test the FTPL by investigating the interactions between government surplus, debt accumulation and price dynamics on French data. Although some findings agree with FTPL their main results support the Ricardian fiscal policy rule. Semmler and Zhang (2004) consider empirical evidence on monetary and fiscal policy interactions in the Euro area. They explore fiscal regimes with a VAR model and find empirical evidence that a non-Ricardian fiscal policy has been pursued in both France (1970-1998) and Germany (1967-1998).

Bihan and Creel (2006) use the Canzoneri et al (2001) VAR methodology on French, German, Italian, UK and US data. In this approach they assume a FTPL regime and try to

reject the assumption of FTPL through the results acquired from impulse response functions (IRFs). They conclude that the FTPL is rejected in the case of all five countries. However, when structural balance data is used FTPL seems plausible for the UK. Sabate et al (2006) argue that Spanish government does not have barriers to monetize budget deficits for some important periods. They estimate a stationary VAR model and to examine the dynamic link between budget and money for Spain for the period 1874 to 1935. Although their work does not concentrate on the theory of the price determination their findings support the dominance of fiscal policy.

Thams (2007) uses a Bayesian VAR to test for the FTPL on German and Spanish data over the period 1980 to 2000 based on the Canzoneri et al (2001) framework. They find evidence of non-Ricardian equilibria for Spain. Bajo-Rubio et al (2009) argue that fiscal sustainability can be achieved in two ways: one is through the endogenous adjustment of the primary budget surplus (Ricardian), the other is through the endogenous adjustment of the price level (FTPL). In their empirical approach, they estimate a cointegration relationship between primary surplus-to-GDP and debt-to-GDP over the period 1970-2005 for eleven EU countries on the country-to-country basis. Their findings support the fiscal policy sustainability in all the EMU countries except Finland where there is no significant response of primary surplus to the debt-GDP ratio, pointing to the possibility of a non-Ricardian or FD regime. However, the solvency condition holds under both Ricardian and Non-Ricardian regimes. In order to distinguish regimes, they perform Granger-causality tests between these two variables for those countries. But the results from the tests did not give a clear conclusion about the prevalence of either regime.

2.10.2 Data and Sources

- Inflation: defined as the Consumer Price Level (CPI) deflator, $\frac{\text{Nominal Total Consumption (NTC)}}{\text{Real Total Consumption (RTC)}}$

- Government Expenditure: Total Managed Expenditure excludes debt interest payment. (TME= Total current expenditure + Net Investment +Depreciation)

- Government Revenue: Total Current Receipts

- GDP: Gross Domestic Product: chained volume measures: Seasonally adjusted

- Nominal Interest Rate: Sterling certificates of deposit: 3 months: bid rate: end period observation

- Money Supply M4: Money Stock M4 (end period), Level, Seasonally adjusted

- Primary Surplus: Difference between Government Expenditure and Revenue - Figure 2.10

- Debt: Public Sector Finances Net Debt (data is unavailable in quarterly frequency, author converts) - Figure 2.10

Source: *UK Office for National Statistics (ONS) databank*

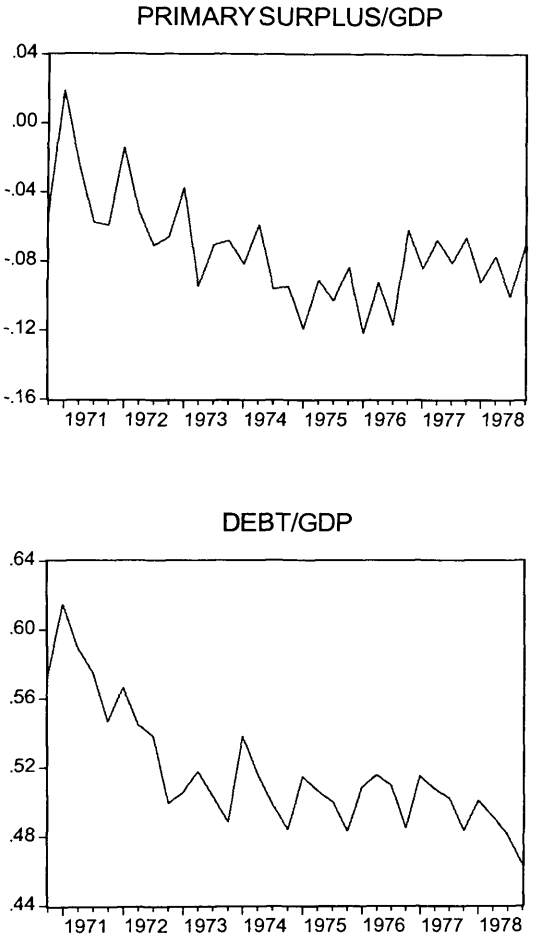

Fig. 2.10. Patterns of government primary surplus-to-GDP ratio and debt-to-GDP ratio.

2.10.3 Derivation of government budget constraint

The government budget constraint can be written as

$\frac{B_{t+1}}{R_t} = G_t - T_t + B_t + \frac{B_t}{R_t}$

where,

G_t is the government spending in money terms,

T_t is the government taxation in money terms,

R_t is the amount of nominal interest the government must pay. The value of the bonds outstanding is $B \times \frac{1}{R}$.

I can derive an expression for government budget constraint in the forward direction by substituting forwards for future bonds outstanding, yields

$\frac{B_t}{R_t} = \sum_{i=0}^{\infty} (T_{t+i} - G_{t+i}) \frac{1}{(1+R_t)^{i+1}}$

If T_{t+i} and G_{t+i} are growing with money GDP,

i.e. $T_{t+i} = t_t P_{t+i} y_{t+i}$

$G_{t+i} = g_t P_{t+i} y_{t+i}$

$\frac{B_t}{R_t} = \sum_{i=0}^{\infty} \frac{(t_t - g_t) P_{t+i} y_{t+i}}{(1+R_t)^{1+i}}$

$= \sum_{i=0}^{\infty} \frac{(t_t - g_t) P_t y_t (1+\gamma+\pi_t)^i}{(1+R_t)^{1+i}}$ (note that real output grows at rate γ)

$= (t_t - g_t) P_t y_t \sum_{i=0}^{\infty} \frac{(1+\gamma+\pi_t)^i}{(1+R_t)^{1+i}}$

$= (t_t - g_t) P_t y_t \sum_{i=0}^{\infty} \frac{(1+\gamma+\pi_t)^{1+i}}{(1+R_t)^{1+i}(1+\gamma+\pi_t)}$

If γ and π_t are both small enough,

$\sum_{i=0}^{\infty} \frac{(1+\gamma+\pi_t)^{1+i}}{(1+R_t)^{1+i}} = \sum_{i=0}^{\infty} \left(\frac{1}{1+R_t-\gamma-\pi_t}\right)^{1+i} = \left(\frac{1}{1-\frac{1}{1+r_t^*-\gamma}} - 1\right) = \left(\frac{1}{r_t^*-\gamma}\right)$

Hence, $\frac{B_t}{R_t P_t y_t} = \frac{(t_t - g_t)}{(1+\gamma+\pi_t)(r_t^*-\gamma)}$

2.10.4 Details of indirect inference tests

2.10.1 $ARMA$(1,2)

Model	Estimated	95% Confidence interval		IN/OUT
		Lower	Upper	
$AR(1)$	0.270	-0.929	0.902	IN
$MA(1)$	-0.296	-1.215	1.233	IN
$MA(2)$	-0.310	-0.586	0.917	IN
Wald statistic	21.9%			
Full wald statistic	24.1%			

Table 2.18. Confidence limits of change in inflation process for theoretical ARMA(1,2)

2.10.2 $ARMA$(1,3)

Model	Estimated	95% Confidence interval		IN/OUT
		Lower	Upper	
$AR(1)$	-0.403	-0.947	0.948	IN
$MA(1)$	0.664	-1.343	1.329	IN
$MA(2)$	-0.351	-0.718	0.739	IN
$MA(3)$	-0.791	-0.872	0.865	IN
Wald statistic	76.8%			
Full wald statistic	70.2%			

Table 2.19. Confidence limits of change in inflation process for theoretical ARMA(1,3)

2.10.3 $ARMA$(2,0)

Model	Estimated	95% Confidence Interval		IN/OUT
		Lower	Upper	
$AR(1)$	0.056	-0.361	0.360	IN
$AR(2)$	-0.360	-0.371	0.299	IN
Wald statistic	85.7%			
Full Wald Statistic	79.9%			

Table 2.20. Confidence limits of change in inflation process for theoretical ARMA(2,0)

2.10.4 $ARMA$(2,1)

Model	Estimated	95% Confidence interval		IN/OUT
		Lower	Upper	
$AR(1)$	0.382	-1.033	1.040	IN
$AR(2)$	-0.367	-0.423	0.344	IN
$MA(1)$	-0.393	-1.060	1.407	IN
Wald statistic	67.5%			
Full wald statistic	61.0%			

Table 2.21. Confidence limits of change in inflation process for theoretical ARMA(2,1)

2.10.5 $ARMA$(2,2)

Model	Estimated	95% Confidence interval		IN/OUT
		Lower	Upper	
$AR(1)$	0.421	-1.296	1.219	IN
$AR(2)$	-0.463	-0.953	0.713	IN
$MA(1)$	-0.441	-1.529	1.502	IN
$MA(2)$	0.115	-0.983	2.101	IN
Wald statistic	45.2%			
Full Wald Statistic	38.0%			

Table 2.22. Confidence limits of change in inflation process for theoretical ARMA(2,2)

2.10.6 $ARMA$(2,3)

Model	Estimated	95% Confidence interval		IN/OUT
		Lower	Upper	
$AR(1)$	-0.666	-1.274	1.323	IN
$AR(2)$	-0.312	-0.911	0.745	IN
$MA(1)$	0.859	-1.584	1.511	IN
$MA(2)$	-0.020	-1.084	1.831	IN
$MA(3)$	-0.597	-0.903	0.938	IN
Wald statistic	58.7%			
Full wald statistic	56.9%			

Table 2.23. Confidence limits of change in inflation process for theoretical ARMA(2,3)

2.10.7 $ARMA$(3,0)

Model	Estimated	95% Confidence Interval		IN/OUT
		Lower	Upper	
$AR(1)$	-0.003	-0.383	0.364	IN
$AR(2)$	-0.374	-0.382	0.319	IN
$AR(3)$	-0.160	-0.358	0.353	IN
Wald statistic	80.2%			
Full Wald Statistic	75.2%			

Table 2.24. Confidence limits of change in inflation process for theoretical ARMA(3,0)

2.10.8 $ARMA$(3,1)

Model	Estimated	95% Confidence interval		IN/OUT
		Lower	Upper	
$AR(1)$	-0.835	-0.977	0.995	IN
$AR(2)$	-0.333	-0.466	0.383	IN
$AR(3)$	-0.458	-0.398	0.394	OUT
$MA(1)$	0.952	-1.468	1.426	IN
Wald statistic	94.3%			
Full wald statistic	93.2%			

Table 2.25. Confidence limits of change in inflation process for theoretical ARMA(3,1)

2.10.9 $ARMA$(3,2)

Model	Estimated	95% Confidence interval		IN/OUT
		Lower	Upper	
$AR(1)$	-0.955	-1.193	1.239	IN
$AR(2)$	-0.432	-0.984	0.743	IN
$AR(3)$	-0.449	-0.449	0.446	OUT
$MA(1)$	1.094	-1.624	1.568	IN
$MA(2)$	0.131	-1.099	2.129	IN
Wald statistic	84.3%			
Full wald statistic	82.0%			

Table 2.26. Confidence limits of change in inflation process for theoretical ARMA(3,2)

Chapter 3
The open economy DSGE model

I now introduce a completed strand of the model by adding a forward-looking IS curve, derived in the usual way from the household Euler equations and the goods market-clearing condition and a New Classical Phillips Curve. New Classical model is more appropriate here as it allows that price/inflation to jump to whatever level is needed to satisfy the government budget constraint. I assume that inflation moves because of excess demand hence the implicit price/inflation rigidity will not be considered in this setup. I would like to see under the Fiscal Theory of the Price Level framework how the other variables (interest rates and output gap) can be explained within the model.

In this chapter, first I describe the model and this follows Meenagh et al (2009b); this model was developed in order to test theories of the persistence of UK inflation in different policy regimes over the post-war period. It was only applied to ARMA models of inflation for each of those regimes, effectively excluding the period I am dealing with here. Here I am using the general framework to test a full model of the economy. In contrast to Meenagh et al (2009b), I express all variables in term of per capita for simplicity. In section 3.1, I derive the IS curve from the market clearing condition and Phillips curve from equations of production function, demand for labour and supply of labour, that are used to determine the path of output gap and real interest rate. In the previous chapter inflation is determined by the fiscal equation. Output and interest rates are then determined by inflation interacting with the IS curve and the Phillips curve. In section 3.2, I present the data

and model calibration and briefly discuss the method used in analysis and testing. I test the model by creating my own Matlab code to bootstrap the model's errors and generate 95% confidence limits for a VAR representation of the data. To preserve the contemporaneous correlation between the innovations I draw the bootstraps as a time vector and draw each period with replacement. I then input them into their error processes and these in turn into the model to solve for their implied path over the sample period. I then run $VAR(1)$ regressions of x_t and $\widetilde{r}_t$ on all the pseudo-samples to derive the implied 95% confidence intervals for all the coefficient values found. Finally I compare the VAR coefficients estimated from the actual data to see whether they lie within these 95% confidence intervals: under the null hypothesis these values represent the sampling variation for the VAR coefficients which are generated by the model. The portmanteau Wald statistic - the 95% confidence limit for the joint distribution of the VAR parameters - is also computed. The Wald statistic is derived from the bootstrap joint distribution of the VAR parameters under the null hypothesis that the structural model holds. In the section 3.3 I report the results. The last section concludes.

3.1 The model

It is a medium-sized open economy and assumed that the real interest in UK economy has no real effect on the world interest rates (world interest rates are assumed to be exogenous) but allowing home interest rates to deviate from the world interest rates. To keep the representation simple, I assume identical agent who produces a single good as output and use it for consumption and investment. There are no market imperfections such as transactions

costs. There is no population growth and all variables are expressed as per capita (In this case all variables used in this chapter are expressed as per capita).

Each period the representative agent chooses the commodity bundle for consumption and leisure. During the period the necessary total amounts of factor inputs are also determined for production. In addition, goods do not enter in the production process hence they are only for trading purpose. The consumption comprises those consumption of domestic goods C_t^d and consumption of foreign goods C_t^f, or imports. I express the composite consumption utility function of the form[12]

$$C_t = \left[\omega \left(C_t^d\right)^{-\rho} + (1-\omega)\left(C_t^f\right)^{-\rho}\right]^{\left(\frac{-1}{\rho}\right)} \tag{3.51}$$

where ω is the weight of home goods in the consumption function, σ, the elasticity of substitution is equal to $\frac{1}{1+\rho}$.

The general price level P_t also called the consumption-based price index is the minimum expenditure that agent needs to acquire one unit of composite good C_t, provided the prices of home good and foreign good are as given. The resulting expression for the index is

$$P_t = \omega^{\frac{1}{1+\rho}}\left(P_t^d\right)^{\frac{1}{1+\rho}} + (1-\omega)^{\frac{1}{1+\rho}}\left(P_t^F\right)^{\frac{\rho}{1+\rho}} \tag{3.52}$$

where, P_t^d denotes the domestic price level, P_t^F denotes the foreign price level in domestic currency, P_t^f denotes the foreign price level in foreign currency and $P_t^F = S_t P_t^f$ and S_t is the nominal exchange rate.

[12] Armington (1969) CES aggregator.

The corresponding expression for home goods demand in term of the composite consumption is

$$\frac{C_t^d}{C_t} = \left(\frac{P_t^d}{\omega P_t}\right)^{-\frac{1}{1+\rho}}$$

And the corresponding expression for foreign goods demand in term of the composite consumption is

$$\frac{C_t^f}{C_t} = \left(\frac{P_t^F}{(1-\omega)\,P_t}\right)^{-\frac{1}{1+\rho}} \tag{3.53}$$

The consumer maximises this composite utility index, given that an amount C_t has been chosen for total expenditure/consumption, with respect to its components, C_t^d and C_t^f subject to $C_t = p_t^d C_t^d + Q_t C_t^f$ [13]

Note that

$$1 = \omega^{\frac{1}{1+\rho}} \left(p_t^d\right)^{\frac{\rho}{1+\rho}} + (1-\omega)^{\frac{1}{1+\rho}} Q_t^{\frac{\rho}{1+\rho}} \tag{3.54}$$

where Q_t is defined as the foreign price level in domestic currency relative to the general price level (the real exchange rate and also the terms of trade).

Hence the resultant logarithmic approximation can be written as

$$\log p_t^d = -\left(\frac{1-\omega}{\omega}\right)^{\frac{1}{1+\rho}} \log\left(Q_t\right) + \text{constant} \tag{3.55}$$

---

[13] The Lagrangian problem can be formed as $L = \left[\omega\left(C_t^d\right)^{-\rho} + (1-\omega)\left(C_t^f\right)^{-\rho}\right]^{\left(\frac{-1}{\rho}\right)} + \mu(C_t - \frac{P_t^d}{P_t}C_t^d - \frac{P_t^F}{P_t}C_t^f)$. Thus $\frac{\partial L}{\partial C_t} = \mu$; also at its maximum with the constraint binding $L = C_t$ so that $\frac{\partial L}{\partial C_t} = 1$. Thus $\mu = 1$ - the change in the utility index from a one unit rise in consumption is unity. Substituting this into the first order condition $0 = \frac{\partial L}{\partial C_t^f}$ yields equation (3.53) . $0 = \frac{\partial L}{\partial C_t^d}$ gives the equivalent equation: $\frac{C_t^d}{C_t} = \omega^{\frac{1}{1+\rho}}(p_t^d)^{-\frac{1}{1+\rho}}$ where $p_t^d = \frac{P_t^d}{P_t}$ Divide (3.51) through by C_t to obtain $1 = \left[\omega\left(\frac{C_t^d}{C_t}\right)^{-\rho} + (1-\omega)\left(\frac{C_t^f}{C_t}\right)^{-\rho}\right]^{\left(\frac{-1}{\rho}\right)}$;substituting into this for $\frac{C_t^f}{C_t}$ and $\frac{C_t^d}{C_t}$ from the previous two equations gives equation (3.54).

The representative agent maximises his/her expected utility subject to the given budget constraint. Each agent's preferences are given as

$$U = MaxE_0 \left[\sum_{t=0}^{\infty} \beta^t \left(C_t, 1 - N_t \right) \right] \tag{3.56}$$

where β is the discount factor, C_t is real consumption in period t, N_t is the labour supply, 1-N_t is the amount of leisure time consumed in period t and E_0 is the mathematical expectations operator.

The agent's tastes are assumed to be constant over time and would not be influenced by any exogenous shocks in the economy. The utility function has the normal properties

1) it is increasing in space of consumption and leisure

$$u^{'} \left(C, 1 - N \right) > 0 \tag{3.57}$$

2) it is concave

$$u^{''} \left(C, 1 - N \right) < 0 \tag{3.58}$$

3) it satisfies Inada conditions

For consumption

$$u^{'} \left(C, 1 - N \right) \longrightarrow 0 \text{ as } c \rightarrow \infty \tag{3.59}$$

and

$$u^{'} \left(C, 1 - N \right) \longrightarrow \infty \text{ as } c \rightarrow 0 \tag{3.60}$$

For leisure

$$u^{'} \left(C, 1 - N \right) \longrightarrow 0 \text{ as } l \rightarrow \infty \tag{3.61}$$

and

$$u^{'} \left(C, 1 - N \right) \longrightarrow \infty \text{ as } l \rightarrow 0 \tag{3.62}$$

The objective of this chapter is to specify a fully articulated model of an open economy which is proposed to be calibrated/estimated using data for the UK. This model is used to explain the behaviour of interest rates, output gap and inflation under FTPL environment.

3.1.1 The representative household

The model economy is assumed to have a large number of identical households who are making decisions on consumption, labour supply and investment interact in competitive market. Each household chooses sequence of consumption and hours of leisure that maximise its expected discounted level of utility. The utility function is time-separable and has the following form

$$u\left(C_t, 1-N_t\right) = \theta \log C_t + (1-\theta)\left(1-\rho_1\right)^{-1}\left(1-N_t\right)^{(1-\rho_1)} \tag{3.63}$$

where $0 < \theta < 1$, and $\rho_1 > 0$ is the leisure substitution parameter.

Each individual agent involve in a dynamic stochastic game where changes in expectations about future events would generally affecting current decisions. Each household endowed with a fixed amount of time which is spent between leisure and work. The total endowment of time is normalised to unity implies that

$$N_t + L_t = 1 \text{ or } L_t = 1 - N_t \tag{3.64}$$

Furthermore for convenience in the logarithmic transformations I assume that approximately $L = N$ on average.

The representative agent's budget constraint is

$$C_t + \frac{b_{t+1}}{1+r_t} + \frac{Q_t b_{t+1}^f}{\left(1+r_t^f\right)} + p_t S_t^p = \left(1-\tau_t\right) v_t N_t + b_t + Q_t b_t^f + \left(p_t + d_t\right) S_{t-1}^p \tag{3.65}$$

where,

p_t denotes the real present value of shares,

τ_t denotes labour income tax rate, which includes all taxes on households and is assumed to be a stochastic process,

b_t^f denotes foreign bonds,

b_t denotes the domestic bonds,

S_t^p demand for domestic shares,

d_t denotes dividends,

Q_t is the real exchange rate,

$$Q_t = \frac{P_t^F}{P_t} \tag{3.66}$$

v_t is the real consumer wage,

$$v_t = \frac{W_t}{P_t} \tag{3.67}$$

or

$$v_t = w_t p_t^d \tag{3.68}$$

where w_t denotes the producer real wage. It is the wage relative to the domestic goods price level.

Note that only consumer real wages are linked with both domestic and foreign prices. Producers do not take into account the foreign prices as they do not use imported intermediate goods.

In a stochastic environment the representative agent maximizes the expected discounted stream of utility subject to the budget constraint. The Lagrangian associated with

this problem can be written as

$$L = \max E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\theta \log C_t + (1-\theta)(1-\rho_1)^{-1}(1-N_t)^{(1-\rho_1)} \right] \right\}$$
$$+\lambda_t \left[(1-\tau_t) v_t N_t + b_t + Q_t b_t^f + (p_t + d_t) S_{t-1}^p - C_t - \frac{b_{t+1}}{1+r_t} - \frac{Q_t b_{t+1}^f}{\left(1+r_t^f\right)} - p_t S_t^p \right]$$

where λ is the Lagrangian multiplier. The conditions of expected behaviour of the economy and the time path of the values of consumption, labour and investment both domestic and foreign financial assets are listed below.

The first order conditions with respect to C_t, the marginal utility of consumption equals to the shadow price of output

$$E_0 \beta^t \theta C_t^{-1} = E_0 \lambda_t \tag{3.69}$$

With respect to N_t, the marginal utility of leisure equals to marginal utility of labour which is the real wage net of income tax

$$E_0 (1-\theta)(1-N_t)^{-\rho_1} = E_0 \lambda_t (1-\tau_t) v_t \tag{3.70}$$

With respect to b_{t+1}, the intratemporal consumption can be obtained by

$$E_0 \frac{\lambda_t}{1+r_t} = E_0 \lambda_{t+1} \tag{3.71}$$

With respect to b_{t+1}^f, the expression for Uncovered Real Interest Rate Parity (URIP) is

$$E_0 \frac{\lambda_t Q_t}{(1+r_t^f)} = E_0 \lambda_{t+1} Q_{t+1} \tag{3.72}$$

With respect to S_t^p, the expression for the real present value of shares can be obtained by

$$E_0 \lambda_t p_t = E_0 \lambda_{t+1} (p_{t+1} + d_{t+1}) \tag{3.73}$$

Substituting equation (3.71) in (3.69) and letting t=0 yields

$$(1+r_t) = \left(\frac{1}{\beta}\right) E_t \left(\frac{C_t}{C_{t+1}}\right)^{-1} \tag{3.74}$$

Now substituting (3.69) and (3.71) in (3.70) yields

$$(1-N_t) = E_t \left\{ \frac{\theta C_t^{-1} (1-\tau_t) v_t}{(1-\theta)} \right\}^{\frac{-1}{\rho_1}} \tag{3.75}$$

C_t is the composite consumption level- it is the weighted average of domestic goods consumption and foreign goods consumption (imported goods).

Substituting out for $v_t = w_t p_t^d$ (the consumer real wage is the product of real producer wage and the domestic goods price), and noting that $E_t \log v_t = \log v_t + \log p_t^{ue} = \log w_t + \log p_t^d + \log p_t^{ue}$, where the ue superscript means 'unexpected'.

Using (3.55) equation (3.75) becomes

$$(1-N_t) = \left\{ \frac{\theta C_t^{-1} \{(1-\tau_t) \exp[\log w_t - (\frac{1-\omega}{\omega})^{\frac{1}{1+\rho}} (\log Q_t) + \log p_t^{ue}]\}}{(1-\theta)} \right\}^{\frac{-1}{\rho_1}} \tag{3.76}$$

Substituting (3.71) in (3.73) yields

$$p_t = E_t \left(\frac{p_{t+1} + d_{t+1}}{(1+r_t)} \right) \tag{3.77}$$

Using the arbitrage condition and forward substitution the expression for real present value per share equation (3.77) can be written as

$$p_t = E_t \sum_{i=1}^{\infty} \frac{d_{t+i}}{\prod_{j=1}^{i} (1+r_{t+j})} \tag{3.78}$$

This equation shows that the present value of shares is equal to the expected income stream from the dividend payment.

It is argued that UK should be considered as a medium sized economy for which the domestic economy is small enough to continue with the assumption that world interest

rates are exogenous (in small open economy models the domestic real interest rate is taken as given which is equal to the world interest rate) but large enough for the domestic rate to deviate from the world rate (Meenagh et al, 2010).

To derive the uncovered interest parity condition by substituting the representative agent's demand for home goods (equation (3.71)) into the household's Lagrangian (equation (3.72)) yields

$$\left(\frac{1+r_t}{1+r_t^f}\right) = E_t \frac{Q_{t+1}}{Q_t} \tag{3.79}$$

In logs this yields

$$r_t = r_t^f + \log E_t \frac{Q_{t+1}}{Q_t} \tag{3.80}$$

3.1.2 The government

The government must finance its exogenously amount of expenditure G_t (expressed in per capita) by levying a labour income tax τ_t and by issuing bonds b_t, each period which pays a return next period. It is assumed that government spends less than the national income, where the output is always at its 'desired' level in equilibrium.

The government budget constraint is

$$G_t + b_t = \tau_t v_t N_t + \frac{b_{t+1}}{1+r_t} \tag{3.81}$$

where b_t is real bonds.

3.1.3 The representative firm with fixed capital

Firms rent labour from households, who own their shares, and transform them into output according to a production technology and sell consumption goods to households and gov-

ernment. The technology available to the economy is described by following production function where capital is fixed

$$Y_t = Z_t {N_t}^{\alpha} \tag{3.82}$$

where,

$0 < \alpha < 1$,

Y_t is aggregate output per capita,

Z_t reflects the state of technology.

In a stochastic environment the firm maximises present discounted stream, V, of cash flows, subject to the production technology

$$MaxV = E_t \sum_{i=0}^{T} d^i (Y_{t+i} - w_{t+i} N_{t+i}) \tag{3.83}$$

Here w_t is the producer real wage. The firm optimally chooses labour so that

$$N_t = \frac{\alpha Y_t}{w_t} \tag{3.84}$$

3.1.4 The foreign sector

Recall that the household's consumption bundle is

$$C_t = p_t^d C_t^d + Q_t C_t^f \tag{3.85}$$

where

$$C_t = \left[\omega \left(C_t^d \right)^{-\rho} + (1 - \omega) \left(C_t^f \right)^{-\rho} \right]^{\left(\frac{-1}{\rho} \right)} \tag{3.86}$$

The Lagrangian problem can be formalised as

$$\begin{aligned} L &= \max E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\theta \log C_t + (1-\theta)(1-\rho_1)^{-1}(1-N_t)^{(1-\rho_1)} \right] \right\} \\ &+\lambda_t \left[(1-\tau_t) v_t N_t + b_t + Q_t b_t^f + (p_t + d_t) S_{t-1}^p - C_t - \frac{b_{t+1}}{1+r_t} - \frac{Q_t b_{t+1}^f}{\left(1+r_t^f\right)} - p_t S_t^p \right] \\ &+\mu_t \left(p_t^d C_t^d + Q_t C_t^f - C_t \right) \end{aligned}$$

The first order condition with respect to C_t^d is

$$-\rho\omega \left(C_t^d\right)^{-\rho-1} \left[\omega \left(C_t^d\right)^{-\rho} + (1-\omega)\left(C_t^f\right)^{-\rho} \right]^{\left(\frac{-1}{\rho}-1\right)} = \mu_t p_t^d \tag{3.87}$$

With respect to C_t^f is

$$-\rho(1-\omega)\left(C_t^f\right)^{-\rho-1} \left[\omega \left(C_t^d\right)^{-\rho} + (1-\omega)\left(C_t^f\right)^{-\rho} \right]^{\left(\frac{-1}{\rho}-1\right)} = \mu_t Q_t \tag{3.88}$$

Combine equations (3.87) and (3.88) yields

$$\left(\frac{1-\omega}{\omega}\right)\left(\frac{C_t^d}{C_t^f}\right)^{1+\rho} = \frac{Q_t}{p_t^d} \tag{3.89}$$

or

$$Q_t = \left(\frac{1-\omega}{\omega}\right)\left(\frac{C_t^d}{C_t^f}\right)^{1+\rho} p_t^d \tag{3.90}$$

The elasticity between domestic and foreign goods is

$$\sigma = \left[\frac{\partial\left(\frac{C_t^d}{C_t^f}\right)}{\partial\left(\frac{P_t^F}{P_t^d}\right)} \right] \left[\frac{\left(\frac{P_t^F}{P_t^d}\right)}{\left(\frac{C_t^d}{C_t^f}\right)} \right] = \frac{1}{1+\rho} \tag{3.91}$$

where σ is assumed to be a non-negative number and will take the finite value.

The amount of imports will be the consumption of foreign good C_t^f. Hence the import equation for UK can be written as

$$\log C_t^f = \sigma \log\left(\frac{1-\omega}{\omega}\right) + \log C_t^d - \sigma \log Q_t + \sigma \log p_t^d \tag{3.92}$$

Recall that the demand for home goods is a function of total composite consumption

$$\frac{C_t^d}{C_t} = \omega^{\frac{1}{1+\rho}} \left(p_t^d\right)^{-\frac{1}{1+\rho}} \tag{3.93}$$

Taking logs of the domestic good consumption equation yields

$$\log C_t^d = \sigma \log \omega + \log C_t - \sigma \log p_t^d \tag{3.94}$$

Substituting (3.94) into the equation (3.92) to give the import function

$$\log C_t^f = \log IM_t = \sigma \log (1-\omega) + \log C_t - \sigma \log Q_t \tag{3.95}$$

Similarly for the foreign country, the corresponding Armington aggregator consumption function can be written as

$$C_t^F = \left[\omega^F \left(C_t^{fd}\right)^{-\rho^f} + \left(1-\omega^f\right)\left(C_t^{ff}\right)^{-\rho^f}\right]^{\left(\frac{-1}{\rho^f}\right)} \tag{3.96}$$

and

$$P_t^* = (\omega^f)^{\frac{1}{1+\rho^f}} \left(P_t^{fd}\right)^{\frac{1}{1+\rho^f}} + (1-\omega^f)^{\frac{1}{1+\rho^f}} \left(P_t^D\right)^{\frac{\rho^f}{1+\rho^f}} \tag{3.97}$$

and

$$Q_t^f = \left(\frac{1-\omega^f}{\omega^f}\right)\left(\frac{C_t^{ff}}{C_t^{fd}}\right)^{1+\rho^f} p_t^{df} \tag{3.98}$$

where C_t^F is the composite consumption of the foreign country, C_t^{fd} is the consumption of its own goods in foreign country and C_t^{ff} is the consumption of domestic goods in foreign country. ω^f is the weight of foreign goods in the consumption function and Q_t^f is the real exchange rate in term of foreign aggregate price and $Q_t^f = \frac{1}{Q_t}$. P_t^{fd} is the foreign price in foreign currency (i.e. the foreign country's own price level) and P_t^D corresponds to the domestic price level in foreign currency, i.e. $P_t^D = \frac{P_t^d}{S_t}$. $\frac{1}{1+\rho^f}$denotes the elasticity of

substitution between its own goods (foreign country's production goods) and home goods (home exports).

The corresponding expression for foreign country's home goods demand in term of the composite consumption is

$$\frac{C_t^{fd}}{C_t^F} = (\omega^f)^{\frac{1}{1+\rho^f}} \left(p_t^{fd}\right)^{-\frac{1}{1+\rho^f}} \tag{3.99}$$

Taking logs of the foreign country's own goods consumption equation (3.99)

$$\log C_t^{fd} = \sigma^f \log\left(\omega^f\right) + \log C_t^F - \sigma^f \log p_t^{fd} \tag{3.100}$$

Taking logs of equation (3.98)

$$\log C_t^{ff} = \sigma^f \log\left(\frac{1-\omega^f}{\omega^f}\right) + \log C_t^{fd} + \sigma^f \log Q_t + \sigma^f \log p_t^{fd} \tag{3.101}$$

where $Q_t^f = \frac{1}{Q_t}$.

Substituting equation (3.100) into equation (3.101) yields

$$\log C_t^{ff} = \log EX_t = \sigma^f \log\left(1-\omega^f\right) + \log C_t^F + \sigma^f \log Q_t \tag{3.102}$$

Finally, the foreign bonds evolve over time according to the foreign bond holding and the difference between the imports and exports

$$\frac{Q_t b_{t+1}^f}{(1+r_t^f)} = Q_t b_t^f + p_t^d EX_t - Q_t IM_t \tag{3.103}$$

3.1.5 Behavioural equations

The complete listings of model's behavioural equations are

(1) The composite Consumption C_t which is the weighted average of UK domestic goods and foreign goods consumption

$$C_t = \left[\omega \left(C_t^d\right)^{-\rho} + (1-\omega)\left(C_t^f\right)^{-\rho}\right]^{\left(\frac{-1}{\rho}\right)} \tag{3.104}$$

Given the consumption C_t solves for r_t

$$(1+r_t) = \frac{1}{\beta} E_t \left(\frac{C_t}{C_{t+1}}\right)^{-1} \tag{3.105}$$

or

$$C_t = \xi C_{t+1} - \vartheta r_t + \varepsilon_{0t} \tag{3.106}$$

Using a first order Taylor series expansion around (average) $\overline{C_t}, \overline{E_t C_{t+1}}$, where $\xi = \overline{\left(\frac{C_t}{E_t C_{t+1}}\right)}$ and $\vartheta = \xi\beta\overline{C_t}$ and it would be typically assumed to be less than unity on the grounds of growth. By dividing both sides by $\overline{C_t}$, I can approximate this linear expression as

$$\log C_t = \xi E_t \log C_{t+1} - \xi\beta r_t + \varepsilon_{1t} \tag{3.107}$$

(2) UIP condition

$$r_t = r_t^f + E_t \log Q_{t+1} - \log Q_t + \varepsilon_{2t} \tag{3.108}$$

where r^f denotes the foreign real interest rate

(3) Production function Y_t

$$Y_t = Z_t (N_t)^{\alpha} \tag{3.109}$$

or

$$\log Y_t = \alpha \log N_t + \log Z_t \tag{3.110}$$

(4) Demand for labour

$$N_t = \left(\frac{\alpha Y_t}{w_t}\right) \tag{3.111}$$

or

$$\log N_t = \log \alpha + \log Y_t - \log w_t \tag{3.112}$$

(5) The producer wage is derived by equating demand for labour, N_t, to the supply of labour given by the consumer's first order conditions

$$(1 - N_t) = \left\{ \frac{\theta C_t^{-1} \left[(1-\tau_t) \ \exp\left(\log w_t - (\frac{1-\omega}{\omega})^{\sigma}(\log Q_t) + \log p_t^{ue}\right)\right]}{(1-\theta)} \right\}^{\frac{-1}{\rho_1}} \tag{3.113}$$

or

$$\log(1-N_t) = -\log N_t = \frac{1}{\rho_1}\left[\log C_t - \log(1-\tau_t) - \log w_t - \log p_t^{ue} + (\frac{1-\omega}{\omega})^{\sigma} \log Q_t\right] + \epsilon_{3_t}$$

where Q_t is the domestic real exchange rate, $(1-\omega)^{\sigma}$ is the weight of domestic prices in the CPI index.

(6) Imports IM_t

$$\log IM_t = \sigma \log(1-\omega) + \log C_t - \sigma \log Q_t \tag{3.114}$$

(7) Exports EX_t

$$\log EX_t = \sigma^f \log\left(1-\omega^f\right) + \log C_t^F + \sigma^f \log Q_t \tag{3.115}$$

(8) Market-clearing condition for goods

$$Y_t = C_t + G_t + EX_t - IM_t \tag{3.116}$$

(9) Balance of payment (give the accumulation of foreign bonds)

$$\frac{Q_t b_{t+1}^f}{(1+r_t^f)} = Q_t b_t^f + p_t^d EX_t - Q_t IM_t \tag{3.117}$$

(10) The present value of shares

$$p_t = E_t \sum_{i=1}^{\infty} \frac{d_{t+i}}{\prod_{j=1}^{i} (1 + r_{t+j})} \tag{3.118}$$

which is given by the expected income stream from dividend payments.

(11) The value of dividends

$$d_t S_t^P = Y_t - w_t N_t \tag{3.119}$$

(12) Nominal exchange rate

$$S_t = \frac{Q_t P_t^d}{P_t^f} \tag{3.120}$$

and the government expenditure share is assumed to be an exogenous process.

3.1.6 The resulting small models

The IS curve:

To derive the IS curve note that loglinearising the market-clearing condition directly yields

$$\log Y_t = c \log C_t + g_t + x(\log C_t^F - \log C_t) + x\sigma^* \log Q_t \tag{3.121}$$

where c is the share of consumption in GDP, x is the share of trade in GDP and $\sigma^* = \sigma + \sigma^f$. Hence substituting for $\log C_t$ from above yields

$$\log Y_t = \frac{-(c-x)\xi\beta}{1 - \xi B^{-1}} r_t + g_t + x C_t^F + x\sigma^* \log Q_t \tag{3.122}$$

and multiplying through by $1 - \xi B^{-1}$ gives

$$\log Y_t = -\xi\beta r_t + \xi E_t \log Y_{t+1} + x N X_t + v_t \tag{3.123}$$

This is the IS curve where $NX_t = (1 - \xi B^{-1}) \log C_t^F + \sigma^*(1 - \xi B^{-1}) \log Q_t$ (the external factors driving net exports) and $v_t = (1 - \xi B^{-1}) g_t$. Denote $w_t = xNX_t + v_t$ the composite error term in this IS curve so that

$$\log Y_t = -\xi \beta r_t + \xi E_t \log Y_{t+1} + w_t \tag{3.124}$$

The New Classical Phillips Curve:

The New Classical Phillips Curve can be obtained as follows. I solve the 3 equations production function- equation (3.110), demand for labour - equation (3.112), and supply of labour - equation (3.113), first for expected (equilibrium) values, assuming $\log C_t$ is also at its expected value. Then I solve the same three equations for the effect of unexpected prices, $\log p_t^{ue}$ (consumption is assumed to be smoothed to stay at its expected value). The production function implies that $\log Y_t^{ue} = \alpha \log N_t^{ue}$ hence the demand for labour can be written as $\log N_t^{ue} = -\frac{1}{1-\alpha} \log w_t^{ue}$. Finally the labour supply equation implies that $\log N_t^{ue} = \frac{1}{\rho_1}(\log w_t^{ue} + \log p_t^{ue})$ so it follows that $\log w_t^{ue} = \frac{\alpha-1}{\rho_1+1-\alpha} \log p_t^{ue}$. The labour supply equation implies that $\log w_t^{ue} = \frac{\rho_1+1-\alpha}{1-\alpha}(\log p_t^{ue})$ so that $\log Y_t^{ue} = \frac{\alpha}{\rho_1+1-\alpha}(\log p_t^{ue})$. This is of course the 'surprise' Phillips Curve

$$\log Y_t = \log Y^* + \delta(\log p_t - E_{t-1} \log p_t) \tag{3.125}$$

where $\delta = \frac{\alpha}{\rho_1+1-\alpha}$ and it has been assumed that households do not have contemporaneous knowledge of the general price level.

3.1.7 The completed model

Now the complete the DSGE model can be written as

Rewrite the IS curve

$$(y_t - y_t^*) = -\phi(r_t - r_t^*) + \xi E_t(y_{t+1} - y_{t+1}^*) + w_t \tag{3.126}$$

where, $\phi = \xi\beta(c - x)$

the New Classical Phillips Curve (NCPC)

$$x_t = \delta(\pi_t - E_{t-1}\pi_t) + u_t \tag{3.127}$$

Since inflation follows a random walk by assumption (so that expected inflation is simply lagged inflation), I can now establish from these two equations that

IS curve

$$\widetilde{r}_t = -\frac{1}{\phi}(\delta\Delta\pi_t - u_t) + \frac{1}{\phi}w_t - \frac{\xi}{\phi}E_t u_{t+1} \tag{3.128}$$

and New Classical Phillips Curve (NCPC)[14]

$$x_t = \delta\Delta\pi_t + u_t \tag{3.129}$$

Thus output and real interest rates are stationary processes around their natural rates. Both u_t and w_t may be serially correlated.

Inflation follows a random walk from the FTPL solution (so that expected inflation is simply lagged inflation)

$$\Delta\pi_t = \kappa \times (\Delta g_t - \Delta t_t) + \eta_t \tag{3.130}$$

where $\kappa = \frac{\overline{\pi}+\overline{r^*}}{\overline{t}-\overline{g}}$ is $\overline{\pi}, \overline{r^*}, \overline{t}$ and $\overline{g}$ are mean values of the corresponding variables. And η_t is the structural error which captures the other effects on the process of $\Delta\pi_t$.

[14] I only consider the New Classical Phillips Curve (NCPC) as price/inflation is required to ensure the balance of government intertemporal budget constraint. Inflation moves because of consumer spend bond income (the excess demand). The case of implicit price/inflation rigidity is not modeled here.

3.2 Data, calibration and testing

I begin with some notes on the time-series behaviour of the inflation and the other macro variables I am dealing with for this period (1970q4-1978q4) - Figure 3.11. All data is extracted from UK Office of National Statistic (ONS) data bank. Recall that the data for inflation is defined as the Consumer Price Level (CPI) deflator, $\frac{\text{Nominal Total Consumption (NTC)}}{\text{Real Total Consumption (RTC)}}$. The mean and standard deviation for the sample period (quarterly rates of change, in fraction per quarter) are 3% and 1% respectively. The real interest rates are the difference between the nominal interest rates and inflation. Output gap (x_t) and net real interest rate $(\widetilde{r}_t)$ are detrended from their long run equilibrium level using H-P filter (Hodrick and Prescott, 1997). Inflation is I(1), both x_t and $\widetilde{r}_t$ are stationary processes - confirmed by ADF and PP tests - Table 3.27.

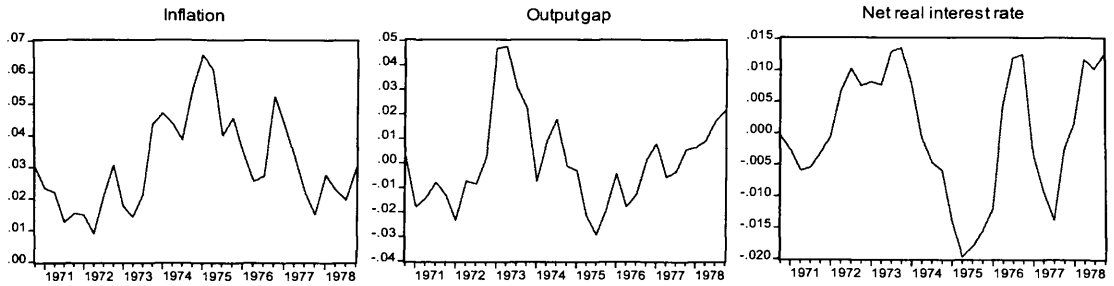

Fig. 3.11. Patterns of macro variables (1970Q4-1978Q4)

Unit Root Tests	π_t		x_t		$\widetilde{r}_t$	
	Levels	1st Diff.	Levels	1st Diff.	Levels	1st Diff.
ADF Test Statistic	-2.107	-5.218	-2.165	-	-3.049	-
	(0.243)	(0.000)**	(0.031)*	-	(0.004)**	-
PP Test Statistic	-2.127	-7.561	-2.322	-	-2.173	-
	(0.236)	(0.000)**	(0.022)*	-	(0.031)*	-

Table 3.27. Tests for non-stationarity of inflation, output gap and net real interest rates

*Notes on Table: MacKinnon's critical values (MacKinnon, 1996) for rejection of hypothesis of a unit root: values in parentheses are p-values, while *, ** indicate significance at the 5% and 1% level, respectively. Number of lags in the ADF test is set upon AIC criterion and PP test upon Newey-West bandwidth (see Newey and West, 1987).*

The calibration is taken from Meenagh et al (2010) and the completed lists of the model parameter are presented in Table 3.28.

Parameter	Calibrated Value
β	0.99
ξ	1.0
ρ_1	1.0
α	0.7
c	0.7
x	0.3
$\phi = \xi\beta(c-x)$	0.4
$\delta = \frac{\alpha}{\rho_1+1-\alpha}$	0.5

Table 3.28. Calibrated parameters

Time discount rate β is set to 0.99 implying an approximate 1% quarterly rate of interest. ξ which is defined as $\overline{(\frac{C_t}{E_t C_{t+1}})}$ is expected equals to one in the New Classical framework. The substitution elasticity between consumption and leisure ρ_1 is unity. $\alpha = 0.70$ impiles production is labour intensive. c and x are the consumption and trade share in GDP respectively.

3.2.1 Bootstrapping and the method of indirect inference

Note that in this model inflation is recursively determined by $\Delta g, \Delta t_t, \eta_t$; and I have already tested the model for its ability to replicate inflation behaviour in the previous chapter. Even though η_t could well be correlated with other model errors, these other errors cannot determine inflation conditional on η_t being chosen. Thus when I draw the vector of error innovations for the model the other ones can be discarded as they are of no impact on inflation. It follows that I have already tested the inflation model above and what remains is to test the other part of the model for x_t and $\widetilde{r}_t$.

I replicate the stochastic environment to see whether the estimated dynamic equations could have been generated by this model. This I do by bootstrapping the model above with their error processes and focus on its solution for these two variables alone. The detailed testing procedure is explained in previous chapter (section 2.6). This method (indirect inference) uses an 'auxiliary model' - in this case is a VAR - to describe the data and to evaluate the fit of a given structural model (rather than for estimation). The auxiliary model used is a $VAR(1)$. By raising the lag order of the VAR and increasing the number of variables, the stringency of the overall test of the model is increased. If the structural model is already rejected by a $VAR(1)$ then I do not proceed to a more stringent test based on a higher order VAR. Le et al (2001) illustrate a case for higher order VARs. By raising the lag order of the VAR in their model setup it worsens the fit to the data because greater complexity in the behaviour being captured. Thus having rejected the model VAR(1), I would reject any case for VAR(i) on i>1, whatever order was needed to get the 'best' VAR fit. The issue of misspecification of a VAR model is not relevant to the test I use here.

Output gap and net real interest rates are determined by inflation interacting with the IS curve and the Phillips curve. To preserve the contemporaneous correlation between the innovations I draw the bootstraps as a time vector and each time with a replacement. I then input them into their error processes and these in turn into the model to solve for their implied path over the sample period. I then run $VAR(1)$ regressions of x_t and $\widetilde{r}_t$ on all the pseudo-samples to derive the implied 95% confidence intervals for all the coefficient values found. Finally I compare the VAR coefficients estimated from the actual data to see whether they lie within these 95% confidence intervals: under the null hypothesis these

values represent the sampling variation for the VAR coefficients which are generated by the model. The portmanteau Wald statistic - the 95% confidence limit for the joint distribution of the VAR parameters - is also computed. The Wald statistic is derived from the bootstrap joint distribution of the VAR parameters under the null hypothesis that the structural model holds.

In addition to the Wald percentile, the Mahalanobis Distance is generally calculated to measure the extent to which the model fits poorly. It is computed based on the same joint distribution, normalised as a t-statistic, as an overall measure of closeness between the model and the data. In effect, this covey the same information as in the Wald test but is in the form of a t-value. The Mahalanobis Distance is the square root of the Wald value, defined as

$$\frac{\sqrt{2M_a} - \sqrt{2n}}{\sqrt{2M_c} - \sqrt{2n}} \times 1.645 \tag{3.131}$$

where, M_a is the Mahalanobis distance calculated using the actual data, M_c is the critical Mahalanobis distance based on the simulated data at 95% confidence interval, n is the degree of freedom (i.e. the number of parameters considered within the model). One could convert the chi-squared distribution into a standard t-statistic by adjusting the mean and the size. I normalise this here by ensuring that the resulting t-statistic is 1.645 at the 95% point of the distribution.

3.3 Testing results

I now use the equation for FTPL, the IS curve and the Philips curve and bootstrap the random components of these w_t and u_t processes and use the pre-determined samples of

inflation. I obtain 1000 pseudo-samples of x_t and $\widetilde{r}_t$ then run an $VAR(1)$ on each of these samples to generate the distribution of the $VAR(1)$ parameters. The Wald statistic then tests the model at the 95% level of confidence on the basis of the complete set of $VAR(1)$ parameters. The auxiliary VAR model is assumed to have the following form

$$\begin{bmatrix} x_t \\ \widetilde{r}_t \end{bmatrix} = \begin{bmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{bmatrix} \begin{bmatrix} x_{t-1} \\ \widetilde{r}_{t-1} \end{bmatrix} + \begin{bmatrix} \varrho_{1t} \\ \varrho_{2t} \end{bmatrix} \tag{3.132}$$

where, all βs are the coefficients to be tested. Table 3.29 lists the results of this exercise. The model as a whole captured by the full wald statistic of 100% implies that the model is rejected and cannot replicate the full property of the actual data. Dynamically, two out of four $VAR(1)$ estimates lie outside the 95% bounds implied by the model, including the output gap response to its own lag ($\widehat{\beta}_{11}$) and the $\widetilde{r}_t$ response to the lag of output gap ($\widehat{\beta}_{21}$). Both estimates are far from the bounds, the model generates an even larger dynamic for $\widetilde{r}_t$ response to the lag of output gap with a lower bound of 1.08 and upper bound of 3.10 .This makes it highly probable that the model will be rejected overall, since I would only expect 5% of the parameters to be rejected under overall acceptance. The Mahalanobis Distance is calculated to give an idea how bad the model can be where t-statistic of 1.645 corresponds to the 95% confidence interval. The large number of the transformed distance (i.e. 5.11) indicates the model overall performance is quite bad. In addition, the model cannot capture the joint distribution of the actual data volatility at 95% confidence interval as well as 99%.

$VAR(1)$ coefficient	Estimated	95% Confidence interval		IN/OUT
		Lower	Upper	
$\widehat{\beta}_{11}$	0.800	-0.671	0.3699	OUT
$\widehat{\beta}_{12}$	0.019	-0.285	0.185	IN
$\widehat{\beta}_{21}$	0.586	1.085	3.105	OUT
$\widehat{\beta}_{22}$	0.560	0.364	1.224	IN
$Var(x_t)$	0.000355	0.000340	0.001067	IN
$Var(\widetilde{r}_t)$	0.000103	0.001897	0.006035	OUT
		Transformed m-dis		
Wald statistic (Dynamics)	99.9%	4.333		
Wald statistic (Volatility)	99.2%	3.308		
Full wald statistic	100%	5.119		

Table 3.29. Confidence limits for theoretical VAR(1)

3.4 Conclusion

In this chapter I have extended the empirical tests of the model for the 1970s episode to the data for the output gap and interest rates. What I find is that while the model was accepted for inflation, which is recursively prior determined, it is rejected for these other two variables. With a normalised Mahalanobis Distance of 5.1 overall and 3.3 for the data variances, this rejection is not so catastrophic that some re-specification could not possibly repair the model. But, perhaps not surprisingly, it indicates that the rather simple set-up of the model, while well able to capture the wide fluctuations of inflation in this unusual policy environment, cannot capture the behaviour of output and interest rates.

Overall Conclusions

This thesis aims to make a contribution to the study of the UK economy in two main ways. First, I test the hypothesis of sustainability for the UK's public finances. A distinct feature of my analysis compared to previous studies is that I test for fiscal policy sustainability accounting simultaneously for endogenously identified structural breaks and non-linearities in UK fiscal policy. The main findings are as follows. First, UK fiscal policy has been sustainable over the period under examination, 1955-2006. Second UK fiscal policy has been subject to three multiple breaks, respectively occurring in the early 1970s, early 1980s and late 1990s. The first break moved the UK away from fiscal sustainability. Indeed, I find that during the period 1973-1980 UK fiscal policy was non-Ricardian. The remaining two breaks contributed towards the restoration of sustainable fiscal dynamics. Finally, I find that UK fiscal policy adjusts non-linearly towards long-run equilibrium. More specifically, the nonlinear analysis of fiscal adjustment shows that UK authorities react more vigorously when the fiscal deficit crosses certain thresholds.

The finding that the 1970s were characterized by non-Ricardian fiscal regime motivates the second part of the analysis. In such a regime, characterized by 'active' fiscal policy and 'passive' monetary policy (Leeper, 1991), fiscal equilibrium is assumed to be restored by price changes as suggested by the Fiscal Theory of the Price Level (Woodford, 1995). First, it assumes that money is entirely accommodating; thus it is produced as needed to equal the demand for money resulting from the behaviour of output, inflation and interest rates. For the money supply, I note that the demand for money function will

hold at all times with an error term. I then can check on the causal direction of lagged effects thus given a disequilibrium in the demand for money, I can ask whether it affects the money growth rate today (which is thus endogenously adapting to money demand). I find money is endogenous.

Second, it assumes that government spending and taxation are set in a non-Ricardian way, that is they do not respond to the state of the public finances in a way that would restore fiscal balance. They can respond to other factors, such as unemployment or special interests but are independent of the finances. For the fiscal variables, I look for a lagged response to indicators of fiscal imbalance. I find a non-Ricardian fiscal policy regime as no feedback from changes in the debt onto the primary surplus. I then use a bootstrap method, developed by Meenagh et al (2009a, 2009b, 2009c) to test the model according to their dynamic performance. While for its trend I use cointegration analysis (Engle and Granger, 1987). I find that FTPL provides a reasonable explanation for inflation in the UK 1970s. Nevertheless, when the model is extended to the data output gap and interest rates the model is 100% rejected.

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