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Migration and trade with external economies of scale

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Migration and trade with external economies of scale

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Abstract

The analysis of migration in Findlay (1982) is extended by adding external economies of scale to the Ricardian model as in Ethier (1982). With external economies, the larger country always gains from trade but the smaller country may lose from trade unless the external economies of scale are sufficiently strong. The smaller country will always gain from emigration but the larger country may lose from immigration unless the external economies of scale are sufficiently strong. Both countries gain from complete economic integration (free labour migration with free trade). Finally, the optimal migration policies of the two countries are derived.

Keywords: Immigration, emigration, international trade, factor mobility.

JEL classification: F22, F12, J61.

1. Introduction

Since the formation of the GATT in 1947 there has been substantial progress on the liberalisation of international trade but over this time, there has not been a corresponding liberalisation of international labour migration. The existence of substantial wage differences between countries shows that there are very large potential gains from the liberalisation of labour migration.¹ This has led to suggestions that there should be multilateral negotiations about labour migration and possibly a WTO for international migration. Hatton (2007) argues that because trade is driven by comparative advantage whereas labour migration is driven by absolute advantage then there is no basis for WTO-style multilateral negotiations. This paper presents a two-country model with external economies of scale where the full benefits of economic integration can only be achieved by free trade and free labour migration, but one country may lose from free trade and the other country may lose from free labour migration. However, by linking the issues of trade and migration in multilateral negotiations, it is possible to achieve the full benefits of economic integration.

The simplest analysis of the effects of labour migration used in many textbooks is based upon what Ruffin (1984) calls the MacDougall-Kemp model. There are two factors (capital and labour) and there is one good that is produced using constant returns to scale technology under perfect competition. Since there is only one good there will be no trade in the absence of factor mobility. Labour migration from the labour-abundant country to the capital-abundant country will reduce the wage and increase the return to capital in the capital-abundant country. The overall effect will be positive for the native residents in the capital-abundant country due to what Borjas (1995) calls the immigration surplus and Freeman (2006) calls the native gain. Overall the original residents of the labour-abundant country will gain from emigration. The migrant workers obviously gain as they receive higher wages, but

¹ Freeman (2006) argues that the worldwide variation in wages is much larger than variation in the prices of goods or the variation in the cost of capital.

the residents that remain in the labour-abundant country will lose unless they receive remittances from the migrants sufficient to offset their losses. Free labour migration results in a Pareto efficient allocation of capital and labour in the world.

A general analysis of factor mobility under perfect competition was undertaken by Grossman (1984), who assumed that factor payments were remitted to the source country.² He showed that although the equilibrium with free trade and free factor mobility is superior to the autarky equilibrium, a move from the free trade and no factor mobility equilibrium to the free trade and free factor mobility equilibrium would not always yield a welfare gain for the country. This results from the terms of trade effect of introducing free factor mobility starting from an initial equilibrium with free trade. If the terms of trade effect is negative then it may outweigh any allocative efficiencies from free factor mobility. Obviously, the terms of trade effect does not occur in the MacDougall-Kemp model as there is no trade in the absence of factor mobility.

In the Ricardian model, Findlay (1982) showed that the residents of the host country will lose as a result of a move from the free trade equilibrium to the free trade and free labour migration equilibrium due to the negative terms of trade effect from immigration.³ Immigration in the Ricardian model does not provide any allocative or productivity gain for the host country since the productivity of its native workers is unchanged, but there is a productivity gain for the migrant workers whose productivity increases to be the same as that in the host country. However, there is a negative terms of trade effect for the native residents in the host country as the migrants are employed in the export industry that expands thereby reducing the price of its exports.⁴ The residents that remain in the source country will gain

² The focus of the Grossman (1984) was not the migration of labour and the model had one consumer in each country.

³ One should note that Findlay (1982) was using distributive justice to argue the case for free migration.

⁴ This is similar to the negative terms of trade effect that causes immiserizing growth in Bhagwati (1958).

from a positive terms of trade effects as the export industry in the source country contracts thereby increasing the price of its exports. Both countries are better off in the free trade and free migration equilibrium, which is Pareto efficient, than in the autarky equilibrium. Recently, Davis and Weinstein (2002) have used the Findlay (1982) analysis as the basis for their estimates of the terms of trade loss from immigration for the USA and they claimed that the terms of trade loss amounted to \$72 billion or 0.8% of GDP for the USA.

This paper extends the analysis of Findlay (1982) by adding external economies of scale to the Ricardian model as in Ethier (1982). The implications of external economies of scale for the size of the immigration surplus were discussed by Borjas (1995), and Trefler (1998) presents a numerical example based upon the simple model in Helpman (1984). The main concern of Ethier (1982) was the pattern of trade and the welfare effects of free trade. He showed that the larger country will specialise in the production of the good with external economies of scale and will always gain from trade but, as suggested by Graham (1923), the smaller country may lose from trade if the larger country completely specialises in the production of the good with external economies of scale.⁵ This paper will extend the analysis of the gains from free trade in Ethier (1982) by deriving the critical value for the degree of external economies of scale required for gains from trade in the smaller country. However, the main contribution of this paper will be to analyse the welfare effects of free migration in the Ethier (1982) model when the larger country completely specialises in the production of the good with external economies of scale. It will be shown that the smaller (source) country will always gain from emigration while the larger (host) country may lose from immigration. The critical value for the degree of external economies of scale required for gains from migration in the larger country will be derived. It will be shown that both countries gain from complete economic integration (free trade and free migration). However, since the smaller

⁵ The Graham argument and the debate that ensued are discussed in detail by Irwin (1996, chapter nine).

country may lose from trade and the larger country may lose from migration, sequential liberalisation of international trade then labour migration may not yield the full gains from economic integration. This suggests that the issues of trade and migration should be linked in multilateral negotiations in order to achieve the full benefits of economic integration. Finally, the optimal migration policies of the two countries will be considered, where the host country maximises the welfare of its native residents and the source country maximises the welfare of its remaining residents. For the host country, it will be shown that the no immigration is optimal when the degree of external economies of scale is low and that unrestricted immigration is optimal when the degree of external economies of scale is high. For intermediate values of the degree of external economies of scale, restricted immigration is optimal.

2. The Model

The model is basically the same as that used by Ethier (1982). There are two almost identical countries (labelled A and B) with \bar{L}_A (\bar{L}_B) workers each endowed with one unit of labour in country A (B). The larger country is labelled as country A so it has more or the same number of workers as the smaller country labelled as country B , $\bar{L}_A \geq \bar{L}_B$, and the relative labour endowment is defined as the ratio $\lambda \equiv \bar{L}_A / \bar{L}_B$, which is greater than or equal to one, $\lambda \geq 1$. The technology available in the two countries is assumed to be identical. The labour can be used to produce two goods that are labelled X and Y . It is perfectly mobile between these two industries, but initially it is assumed to be internationally immobile. There are external economies of scale in the production of good X that are national in scope whereas there are constant returns to scale in the production of good Y . Although there are economies of scale, there is perfect competition in both industries, and in the labour market. Thus, firms

producing good X are small relative to the size of the industry and take the size of the X industry as given. The labour input requirement to produce one unit of good X in country A is $a_X = X_A^{-\theta}$, which is decreasing in the output of the X industry in country A and where $\theta \in (0,1)$ is a measure of the degree of external economies of scale. Similarly, since technology is identical, the labour input requirement in country B is $b_X = X_B^{-\theta}$. The marginal product of labour in country A is $1/a_X = X_A^\theta$ and country B is $1/b_X = X_B^\theta$ so the marginal product of labour is increasing in the output of the X industry in the country. In the Y industry, the labour input requirement is the same in both countries and normalised to one, so $a_Y = b_Y = 1$. If L_{AX} workers are employed in the X industry in country A then the output of the industry is $X_A = (L_{AX})^{\frac{1}{1-\theta}}$ and similarly in country B if L_{BX} workers are employed then the output is $X_B = (L_{BX})^{\frac{1}{1-\theta}}$. Consequently, full employment of labour implies that the production possibility frontier for country A is $Y_A = \bar{L}_A - X_A^{1-\theta}$ and for country B is $Y_B = \bar{L}_B - X_B^{1-\theta}$. Hence, the production possibility frontiers are convex to the origin as shown in figure one.

The preferences of the worker/consumers are identical in the two countries and can be represented by a Cobb-Douglas utility function: $U_A = U_B = x^\gamma y^{1-\gamma}$ where $0 < \gamma < 1$ is the proportion of income spent on good X . The wage in country A is w_A and in country B is w_B and each worker/consumer is endowed with one unit of labour. Therefore, the Marshallian demands of a worker/consumer in country A are $x_A = \gamma w_A / p_X$ and $y_A = (1-\gamma) w_A / p_Y$, and in country B are $x_B = \gamma w_B / p_X$ and $y_B = (1-\gamma) w_B / p_Y$. Since preferences are identical and homothetic, the aggregate demands will be functions of total income so the aggregate demand for good X in country A is $X_A^D = \gamma (w_A \bar{L}_A) / p_X$ and in country B is $X_B^D = \gamma (w_B \bar{L}_B) / p_X$. These demands have the Mill-Graham property, as assumed by Ethier (1982), that a constant

fraction of income γ is spent on good X and $1-\gamma$ is spent on good Y . The indirect utility function of worker/consumers will prove to be useful in the evaluation of welfare and these are obtained by substituting the individual demand functions into the utility functions, which yields:

$$V_A = w_A \left(\frac{\gamma}{p_X} \right)^\gamma \left(\frac{1-\gamma}{p_Y} \right)^{1-\gamma} \quad V_B = w_B \left(\frac{\gamma}{p_X} \right)^\gamma \left(\frac{1-\gamma}{p_Y} \right)^{1-\gamma} \quad (1)$$

In autarky, each country must supply whatever consumers demand so if it consumes both goods then it must produce both goods. If both goods are produced in positive quantities then both industries must pay the same wage so the wage in country A is $w_A = p_Y = p_X X_A^\theta$ and the relative price of X is $p_X/p_Y = X_A^{-\theta}$. Substituting this relative price and the wage into the demand function and solving for the output of the X industry yields: $X_A = (\gamma \bar{L}_A)^{\frac{1}{1-\theta}}$ so the autarky equilibrium relative price of X is $(p_X/p_Y)_A^N = (\gamma \bar{L}_A)^{\frac{\theta}{1-\theta}}$. Similarly, in country B , the autarky relative price of X is $(p_X/p_Y)_B^N = (\gamma \bar{L}_B)^{\frac{\theta}{1-\theta}}$, which is higher than in country A as the X industry is smaller in country B . If the countries are the same size then the autarky relative prices will be equal. Substituting the autarky relative prices into the indirect utility functions yields the autarky equilibrium utility of a worker/consumer in country A and country B :

$$V_A^N = (1-\gamma)^{1-\gamma} \gamma^{\frac{\gamma}{1-\theta}} (\bar{L}_A)^{\frac{\gamma\theta}{1-\theta}} \quad V_B^N = (1-\gamma)^{1-\gamma} \gamma^{\frac{\gamma}{1-\theta}} (\bar{L}_B)^{\frac{\gamma\theta}{1-\theta}} \quad (2)$$

The utility of a worker/consumer is higher in country A than in country B . The reason is that the output of the X industry is inefficiently low in both countries but country A is closer to its efficient output. The autarky equilibrium is shown in figure one where it can be seen that the output of the X industry is inefficiently low. Given the technology and the preferences, the autarky equilibrium in each country is unique and stable as was shown by

Ethier (1982). The utility levels under autarky will provide a basis for welfare comparisons with utility levels under free trade, free migration, and free migration with free trade.

3. Gains and Losses from Trade

Ethier (1982) analysed the welfare effects of free trade in this model and demonstrated the possibility of losses from trade as suggested by Graham (1923). The existence of external economies of scale means that there are multiple equilibria under free trade with the possibility of complete specialisation in production. Ethier (1982) analysed the possible equilibria under free trade using allocation curves and assuming a Marshallian (quantity) adjustment process. Before free trade is established between the two countries, the relative price of good X will be lower in country A as it is larger than country B so has more labour employed in the X industry and thereby better exploits the economies of scale. Therefore, when free trade is established, the X industry in country A will have a cost advantage and it will expand its production while the X industry in country B will contract. The result will be that country B would specialise in the production of good Y while good X would only be produced in country A . In the special case when the two countries are the same size ($\bar{L}_A = \bar{L}_B$), the autarky equilibrium would still be an equilibrium as the autarky prices would then be same in the two countries. However, this equilibrium would not be stable, as an increase in the output of the X industry in one country would give it a cost advantage that would lead it to expand while the X industry would contract in the other country. In this case, good X could be produced in either country but, without any loss of generality, it will be

assumed that it is produced in country A. As in Ethier (1982), there are two possible equilibria that will be considered.⁶

First, if the share of income spent on good X is sufficiently low then good Y will be produced in both countries while good X will only be produced in country A. Since both countries produce good Y and its price is the same in both countries under free trade, the wage will be the same in both countries, $w_A = w_B$. The price of good Y will be $p_Y = w_A$ and the price of good X is $p_X = X_A^{-\theta} w_A$. Substituting these prices into the aggregate demand function for good X, $X_A^D + X_B^D = \gamma(w_A \bar{L}_A + w_B \bar{L}_B) / p_X$, yields the output of the X industry:

$X_A^T = [\gamma(\bar{L}_A + \bar{L}_B)]^{\frac{1}{1-\theta}}$ and therefore the relative price of good X under free trade is:

$(p_X / p_Y)^T = [\gamma(\bar{L}_A + \bar{L}_B)]^{\frac{-\theta}{1-\theta}}$, which is lower than the autarky equilibrium relative price in

either country. Therefore, since worker/consumers in both countries receive the same wage and face the same prices, they will have the same utility. Substituting the prices and wages into the indirect utility functions yields the utility of a worker/consumer under free trade:

$V_A^T = V_B^T = \gamma^{\frac{\gamma}{1-\theta}} (1-\gamma)^{1-\gamma} (\bar{L}_A + \bar{L}_B)^{\frac{\gamma\theta}{1-\theta}}$, which is higher than autarky utility in either country.

Hence, both countries gain from trade as shown in figure two (assuming two identical countries) where country A produces at PA and consumes at CA while country B produces at PB and consumes at CB. This will be an equilibrium if the demand for labour from the X industry in country A at the wage $w_A = p_Y$ is less than or equal to the total number of workers in country A otherwise there will be an excess demand for labour from the X industry as this wage. The number of workers employed in the X industry is $\gamma(\bar{L}_A + \bar{L}_B)$ so this will be an equilibrium if $\gamma(\bar{L}_A + \bar{L}_B) \leq \bar{L}_A$ or if $\gamma < \lambda / (1 + \lambda)$. Since wages in the two countries are

⁶ When the degree of external economies of scale is low, there may be other equilibria where both countries produce good X. It is not possible to derive these equilibria using analytical techniques and they will be ignored.

equalised there is no incentive for migration in this equilibrium so it is of little interest for the later analysis of migration.

Secondly, if the share of income spent on good X is sufficiently high, $\gamma > \lambda/(1 + \lambda)$, then there would be an excess demand for labour from the X industry in country A at the wage $w_A = p_Y$. Therefore, the wage in country A must be greater than marginal product of labour in the Y industry, $w_A > p_Y$, so good Y will not be produced in country A as it is unprofitable, and country A will completely specialise in the production of good X . Country B will completely specialise in the production of good Y and wages will not be equalised in the two countries. Since country A produces only good X its output will be: $X_A = (\bar{L}_A)^{\frac{1}{1-\theta}}$, the wage in country A will be equal to the marginal product of labour: $w_A = p_X X_A^\theta = p_X (\bar{L}_A)^{\frac{\theta}{1-\theta}}$, and the wage in country B will be $w_B = p_Y$. Substituting these values into the aggregate demand function and solving yields the free trade equilibrium price:

$$\left(\frac{p_X}{p_Y} \right)^T = \frac{\gamma}{1-\gamma} (\bar{L}_A)^{\frac{-1}{1-\theta}} \bar{L}_B \quad (3)$$

Since wages are equal to the marginal product of labour, the wage in country B is $w_B = p_Y$ and the wage in country A is $w_A = p_X (\bar{L}_A)^{\frac{\theta}{1-\theta}}$. Hence, using (3), the wage in country A relative to the wage in country B is:

$$\frac{w_A}{w_B} = \frac{\gamma}{1-\gamma} \frac{\bar{L}_B}{\bar{L}_A} = \frac{\gamma}{1-\gamma} \frac{1}{\lambda} > 1 \quad (4)$$

The wage in country A will exceed the wage in country B since $\gamma > \lambda/(1 + \lambda)$ in this equilibrium with complete specialisation. Note that $w_A \bar{L}_A / w_B \bar{L}_B = \gamma/(1 - \gamma)$. The wage in

country A will be higher relative to the wage in country B , the larger is the share of income spent on good X and the lower is the relative labour endowment of country A .

Substituting the free trade equilibrium relative price and the wages into the indirect utility functions yields the utility of worker/consumers in the two countries under free trade:

$$V_A^T = \gamma (\bar{L}_A)^{\frac{\gamma+\theta-1}{1-\theta}} (\bar{L}_B)^{1-\gamma} \quad V_B^T = (1-\gamma) (\bar{L}_A)^{\frac{\gamma}{1-\theta}} (\bar{L}_B)^{-\gamma} \quad (5)$$

It is now possible to compare free trade and autarky to ascertain whether there are gains or losses from free trade. Dividing the utility of a worker/consumer in country A under free trade from (5) by their utility under autarky from (2) yields:

$$\frac{V_A^T}{V_A^N} = \left[\frac{\gamma}{(1-\gamma)\lambda} \right]^{1-\gamma} \gamma^{\frac{-\theta\gamma}{1-\theta}} > 1 \quad (6)$$

This is greater than unity since from (4) the term in brackets is greater than one and its exponent is positive while $\gamma < 1$ and its exponent is negative. Therefore, there are always gains from trade for country A , where there has been an increase in the output of the industry with external economies of scale, which is in keeping with the results of Kemp and Negishi (1970) and Helpman (1984). For country B , the production of the good with external economies of scale has decreased so there is the possibility of losses from trade. Dividing the utility of a worker/consumer in country B under free trade from (5) by their utility under autarky from (2) yields:

$$\frac{V_B^T}{V_B^N} = \left[\frac{(1-\gamma)\lambda}{\gamma} \right]^\gamma \gamma^{\frac{-\gamma\theta}{1-\theta}} \quad (7)$$

This may be less than or greater than unity since the term in brackets is less than one and its exponent is positive while $\gamma < 1$ and its exponent is negative. To find the critical value

for the degree of external economies of scale that ensures there are gains from trade, take logarithms and rearrange:

$$\theta > \theta_B^T \equiv \frac{G}{-\ln(1-\gamma)} \quad (8)$$

Where $G \equiv \ln \gamma - \ln(1-\gamma) - \ln \lambda > 0$, which is positive from (4), and since $\ln \gamma - \ln \lambda < 0$ then $-\ln(1-\gamma) > G > 0$ so therefore $0 < \theta_B^T < 1$. There will be gains from trade for country B if the degree of external economies of scale exceeds the critical value, $\theta > \theta_B^T$. The critical value for the degree of external economies of scale depends upon two parameters: the demand parameter γ and the relative labour endowment ratio $\lambda = \bar{L}_A / \bar{L}_B$. To see how these parameters affect the critical value, differentiate (8) with respect to the two parameters, which yields:

$$\begin{aligned} \frac{\partial \theta_B^T}{\partial \gamma} &= \frac{\gamma(\ln \lambda - \ln \gamma) - (1-\gamma)\ln(1-\gamma)}{\gamma(1-\gamma)[\ln(1-\gamma)]^2} > 0 \\ \frac{\partial \theta_B^T}{\partial \lambda} &= \frac{1}{\lambda \ln(1-\gamma)} < 0 \end{aligned} \quad (9)$$

The larger the relative labour endowment or the smaller the demand parameter then the more likely that there will be gains from trade for country B . This leads to the following proposition:

Proposition 1: *Country A will always gain from free trade and country B will gain (lose) from free trade if $\theta > \theta_B^T$ ($\theta < \theta_B^T$) where $\partial \theta_B^T / \partial \gamma > 0$ and $\partial \theta_B^T / \partial \lambda < 0$.*

This result extends propositions nine and ten in Ethier (1982) by deriving the critical value for the degree of external economies and showing how it depends upon the parameters of the model. The possibility of losses for country B was suggested by Graham (1923), and

shown in this setting by Ethier (1982).⁷ The free trade equilibrium (assuming two identical countries) is shown in figure three where there are gains for both countries and figure four where country B loses from trade. In both figures, country A produces at PA and consumes at CA while country B produces at PB and consumes at CB . It can be seen that if the relative price of X under free trade exceeds the relative price under autarky then the smaller country will lose from trade. This is more likely the larger is the demand for good X (the larger is γ) and the smaller is the number of workers in country A (the smaller is λ).

4. Gains and Losses from Migration

When there is complete specialisation in both countries under free trade, $\gamma > \lambda/(1 + \lambda)$, the real wage is higher in country A than in country B . Therefore, there is an incentive for workers to migrate from the high real wage country to the low real wage country. If free migration is allowed then the result will be that workers in country B will migrate to country A until the real wage is equalised. Since there is free trade, workers will face the same goods prices in the two countries and they will migrate to the country that pays the highest wage. Let the number of workers that choose to work in country A be L_A and the number in country B be L_B , where $L_A + L_B = \bar{L}_A + \bar{L}_B$ so the aggregate demands for good X in the two countries are now: $X_A^D = \gamma(w_A L_A)/p_X$ and $X_B^D = \gamma(w_B L_B)/p_X$. Since country A is completely specialised in the production of good X , the output of the X industry in country A will be $X_A = L_A^{\frac{1}{1-\theta}}$ and hence the wage in country A will be equal to the marginal product of labour: $w_A = p_X L_A^{\frac{\theta}{1-\theta}}$. Country B completely specialises in good Y so its production of good Y is $Y_B = L_B$ and hence the wage in country B is $w_B = p_Y$. Since free migration equalises the

⁷ The possibility of losses was also demonstrated by Panagariya (1981) in a model with increasing returns to scale in one industry and decreasing returns to scale in the other industry.

wages in the two countries, $w_A = w_B$, therefore the relative price of good X is: $p_X/p_Y = \bar{L}_A^{-\frac{\theta}{1-\theta}}$.

Substituting the relative price and the wages into the demand functions for X and equating with the production of X yields the number of workers in each country under free migration:

$L_A = \gamma(\bar{L}_A + \bar{L}_B)$ and $L_B = (1-\gamma)(\bar{L}_A + \bar{L}_B)$. With free migration, a fraction γ of the workers in the world is in country A producing good X and a fraction $1-\gamma$ is in country B producing good Y . Therefore, the equilibrium relative price under free labour migration is:

$$\left(\frac{p_X}{p_Y}\right)^M = \left[\gamma(\bar{L}_A + \bar{L}_B)\right]^{-\frac{\theta}{1-\theta}} \quad (10)$$

To compare the relative price of good X under free trade with that under free labour migration divide the free trade price (3) by the autarky price (10) yields:

$$\left(\frac{p_X}{p_Y}\right)^M \bigg/ \left(\frac{p_X}{p_Y}\right)^T = (1-\gamma)(1+\lambda) \left[\frac{\lambda}{\gamma(1+\lambda)}\right]^{\frac{1}{1-\theta}} < 1 \quad (11)$$

The term in square brackets is less than one and its exponent is positive while $(1-\gamma)(1+\lambda) < 1$ since $(1-\gamma)(1+\lambda) = 1 - [\gamma - \lambda(1-\gamma)] < 1$ and the term in square brackets is positive since $\lambda < \gamma/(1-\gamma)$. Therefore, since labour has migrated from country B to country A and country A specialises in the production of good X , the relative price of good X is lower under free labour migration than under free trade.

The welfare effects of the move from free trade to free migration will be analysed by looking at the utility of the native workers in country A , the workers remaining in country B , and the migrant workers.⁸ Substituting the price under free migration and the wages into the indirect utility function yields the utility of a worker/consumer in each country under free migration:

⁸ The possibility of remittances from the migrant workers in country A to the workers remaining in country B is ignored.

$$V_A^M = V_B^M = (\bar{L}_A + \bar{L}_B)^{\frac{\gamma\theta}{1-\theta}} (1-\gamma)^{1-\gamma} \gamma^{\frac{\gamma}{1-\theta}} \quad (12)$$

Since workers receive the same wage and face the same goods price, they have the same utility in both countries. Dividing the utility of a worker/consumer remaining in country *B* under free migration (12) by their utility under free trade (5) yields:

$$\frac{V_B^M}{V_B^T} = \left[\frac{\gamma}{(1-\gamma)\lambda} \right]^\gamma \left[\frac{(1+\lambda)\gamma}{\lambda} \right]^{\frac{\gamma\theta}{1-\theta}} > 1 \quad (13)$$

Both the expressions in square brackets are greater than unity and the exponents are positive so the ratio is greater than one. There are undoubtedly gains from free migration for worker/consumers remaining in country *B* due to the positive terms of trade effect. Migrant workers from country *B* obviously gain from migration to country *A* and their gain is the same as for a worker/consumer remaining in country *B*.

Dividing the utility of a native worker/consumer in country *A* under free migration (12) by their utility under free trade (5) yields:

$$\frac{V_A^M}{V_A^T} = \left[\frac{\gamma}{(1-\gamma)\lambda} \right]^{\gamma-1} \left[\frac{(1+\lambda)\gamma}{\lambda} \right]^{\frac{\gamma\theta}{1-\theta}} \quad (14)$$

This ratio is ambiguous since although both terms in square brackets are greater than unity, the exponent on the first term is negative and on the second term is positive. By taking logarithms, it can be shown that the ratio will be greater than one if the degree of external economies of scale is greater than the critical value:

$$\theta > \theta_A^M \equiv \frac{(1-\gamma)G}{\Delta} \quad (15)$$

where $G \equiv \ln \gamma - \ln(1-\gamma) - \ln \lambda > 0$ from (4), and $\Delta = (1-\gamma)G + \gamma H > 0$ since

$H \equiv \ln \gamma + \ln(1+\lambda) - \ln \lambda > 0$ from (4). Hence, the critical value for the degree of external

economies of scale is $\theta_A^M \in (0,1)$. This critical value is a function of two parameters and to see how it depends upon these two parameters differentiate (15) first with respect to γ :

$$\frac{\partial \theta_A^M}{\partial \gamma} = \frac{1}{\Delta^2} [G(\gamma - H) + H - G] < 0 \quad (16)$$

The sign of the derivative depends upon the term in square brackets and it seems to be ambiguous as $H - G = \ln(1 - \gamma) + \ln(1 + \lambda) < 0$ and $\gamma - H > 0$. However, the sign of the derivative can be determined by looking at the term in square brackets: $Z = G(\gamma - H) + H - G$. When $\gamma = \lambda/(1 + \lambda)$, $H = G = 0$ and hence $Z = 0$. Differentiating Z with respect to γ yields:

$$\frac{\partial Z}{\partial \gamma} = \frac{-1}{\gamma(1 - \gamma)} [H + (1 - \gamma)^2 G] < 0 \quad (17)$$

Therefore, $Z < 0$ for $\lambda/(1 + \lambda) < \gamma < 1$ and hence the derivative is negative, $\partial \theta_A^M / \partial \gamma < 0$, as claimed. An increase in the demand parameter reduces the critical value for the degree of external economies of scale for which country A will gain from free migration. To see how the critical value for the degree of external economies of scale depends on the relative labour endowment, differentiate (15) with respect to λ :

$$\frac{\partial \theta_A^M}{\partial \lambda} = \frac{\gamma(1 - \gamma)}{\lambda(1 + \lambda)\Delta^2} [\ln(1 + \lambda) - \ln(1 - \gamma) + \lambda H] > 0 \quad (18)$$

An increase in the relative labour endowment increases the critical value for the degree of external economies of scale for which country A will gain from free migration. These results lead to the following proposition:

Proposition 2: *Country B will always gain from free migration and country A will gain (lose) from free migration if $\theta > \theta_A^M$ ($\theta < \theta_A^M$) where $\partial \theta_A^M / \partial \gamma < 0$ and $\partial \theta_A^M / \partial \lambda > 0$.*

Migration of workers from country B to country A expands production of good X in country A , contracts the production of good Y in country B . This leads to a fall in the relative price of good X that has a positive effect on the terms of trade in country B and a negative effect on the terms of trade in country A . Workers that remain in country B undoubtedly gain from migration due to the positive terms of trade effect, but native workers in country A may lose if the productivity gain due to the external economies of scale does not outweigh the negative terms of trade effect. If the degree of external economies of scale is sufficiently large then everyone (native workers in country A , workers that remain in country B , and migrant workers) will gain from labour migration. Therefore, international migration may result in a Pareto improvement!

Since imports represent a fraction $1-\gamma$ of total expenditure in country A , an increase in the demand parameter, γ , will reduce the negative terms of trade effect for country A and decrease the critical value for the degree of external economies of scale, $\partial\theta_A^M/\partial\gamma < 0$. With migration, the increase in the number of workers in country A is the same as the decrease in the number in country B , but the proportionate change in country B will be larger, the smaller is country B relative to country A (or the larger is λ). Thus, the larger will be the terms of trade effect due to the contraction of the Y industry in country B relative to the productivity gains and the terms of trade effect due to the expansion of the X industry in country A . Therefore, for country A , an increase in the relative labour endowment, λ , will increase the negative terms of trade effect relative to the productivity gain from migration thereby increasing the critical value for the degree of external economies of scale, $\partial\theta_A^M/\partial\lambda > 0$.

5. Gains from Migration and Trade

The analysis in this paper so far has examined the welfare effects on the two countries of a move from autarky to free trade and then of a move from free trade to free migration with free trade. With complete specialisation, $\gamma > \lambda/(1 + \lambda)$ it has been shown that country A always gains from a move from autarky to free trade while country B may lose whereas country A may lose from a move from free trade to free migration with free trade while country B always gains. An obvious question is what are the welfare effects for the two countries of a move from autarky to complete economic integration (free migration with free trade). The welfare effects for workers in the two countries can be seen by dividing the utility of a worker/consumer under free migration with free trade (12) by utility under autarky (2), which yields:

$$\frac{V_A^M}{V_A^N} = \left(\frac{1 + \lambda}{\lambda} \right)^{\frac{\gamma\theta}{1-\theta}} > 1 \quad \frac{V_B^M}{V_B^N} = (1 + \lambda)^{\frac{\gamma\theta}{1-\theta}} > 1 \quad (19)$$

Since the terms in brackets are greater than one and the exponent is positive, then both expressions are greater than one. Therefore, both countries gain from a move from autarky to complete economic integration (free migration with free trade), but note that the gain is greatest for the smaller country B since $\lambda > 1$. The gain for the migrant workers from country B is the same as the gain for the workers that remain in country B . This leads to the following proposition:

Proposition 3: *Both countries gain from a move from autarky to complete economic integration (free labour migration with free trade).*

Hence, everyone (native workers in country A , workers that remain in country B and migrant workers) gain from complete economic integration (free migration with free trade). Although this equilibrium realises the full benefits of economic integration, it is not Pareto

efficient as the production of good X is still lower than that required for Pareto efficiency due to the presence of external economies of scale.⁹ Therefore, both countries would benefit from an economic liberalisation that involved the simultaneous introduction of free trade and free labour migration.

However, recent history suggests that economic liberalisation has not involved the simultaneous introduction of free trade and free migration of labour. Instead, international trade has been liberalised by multilateral negotiations at successive GATT/WTO rounds, and it is hoped that the liberalisation of labour migration will follow trade liberalisation. The implications of sequential economic liberalisation, where trade liberalisation is followed by the liberalisation of labour migration, in this model can be considered by looking at figures 5 and 6. Respectively, these show the critical values for the degree of external economies of scale, θ_A^M and θ_B^T , as functions of the demand parameter, γ , and the relative labour endowment, λ , with $\lambda = 3/2$ in figure 5 and $\gamma = 4/5$ in figure 6. In both figures, free trade will increase the welfare of country B if $\theta > \theta_B^T$ and will always increase the welfare of country A while free migration will increase the welfare of country A if $\theta > \theta_A^M$ and will always increase the welfare of country B . Therefore, there are four regions labelled I to IV where in region III , for example, {Gains, Losses} denotes that there are gains for country A from free migration and losses for country B from free trade.

With sequential liberalisation by multilateral negotiations, where each country has a veto at each stage, trade liberalisation will occur in regions I and II since both countries will gain from free trade but it will not occur in regions III and IV since country B will lose from free trade. If the liberalisation of labour migration can only occur after trade liberalisation then there will be liberalisation of labour migration in region I since both countries gain from

⁹ Pareto efficiency would require a subsidy to be given for the production of good X , see proposition 3 in Panagariya (1981).

free migration but not in region *II* since country *A* loses from free migration. Hence, only in region *I* will the full benefits of economic integration identified in Proposition 3 be achieved and, in the other regions, there are worldwide efficiency gains from further economic integration that cannot be achieved by sequential liberalisation.¹⁰ This suggests that trade liberalisation and the liberalisation of labour migration should be linked together in multilateral negotiations. In that case, since both countries gain from complete economic integration (free migration with free trade), as shown in Proposition 3, both countries would agree to the simultaneous liberalisation of trade and labour migration and the full gains from economic integration would be achieved in regions *I* to *IV*.

Finally, in this section, a technical point. To check that the parameter space (γ, θ) is always divided into four regions for any value of the relative labour endowment, λ , one can derive the intersection of the two critical values for the degree of external economies of scale θ_A^M and θ_B^T . Equating (8) and (15) yields:

$$\ln \gamma + \gamma \ln(1 + \lambda) = \ln \lambda \Rightarrow \gamma \ln(1 + \lambda) e^{\gamma \ln(1 + \lambda)} = \lambda \ln(1 + \lambda) \quad (20)$$

As the solution to equations of the form $We^W = z$ is given by the Lambert W (or Omega) function: $W(z)$; see Corless *et al* (1996) for details of its properties and its many applications. Hence, by letting $W = \gamma \ln(1 + \lambda)$ and $z = \lambda \ln(1 + \lambda)$ equation (20) can be solved to give the value of the demand parameter at the intersection:

$$\gamma^I = \frac{W[\lambda \ln(1 + \lambda)]}{\ln(1 + \lambda)} \quad (21)$$

¹⁰ It is assumed that there will no liberalisation of labour migration if there has been no trade liberalisation, but labour migration could still occur without international trade. Workers in country *A* would gain from migration but workers remaining in country *B* would lose. Free migration without trade would lead to an equilibrium where all the workers were in country *A*.

This is plotted in figure 7 where it can be seen that the intersection always occurs in the range of feasible values for the demand parameter, $\gamma^l \in [\lambda/(1+\lambda), 1]$. Hence, the parameter space (γ, θ) is always divided into four regions for any value of the relative labour endowment, λ .

6. Optimal Migration Policy

Having considered the gains from free trade and free labour migration one can now consider the optimal migration policy for the two countries under free trade. Suppose that country A can restrict the number of immigrants or that country B can restrict the number of emigrants.¹¹ For country A , there is a trade-off between the gains from the increase in productivity due to the external economies of scale and the terms of trade loss. The optimal policy restricts the number of immigrants to maximise the utility of a native worker/consumer in country A .¹² If the number of workers in country A is L_A and in country B is L_B then the equilibrium can be obtained simply by replacing \bar{L}_A and \bar{L}_B in equations (3), (4) and (5) from section three with L_A and L_B . Hence, the utility of a native worker in country A is:

$$V_A = \gamma (L_A)^{\frac{\gamma+\theta-1}{1-\theta}} (L_B)^{1-\gamma}.$$

To determine the optimal immigration policy for country A it is convenient to use the logarithm of the utility of a native worker in country A , which is:

$$v_A = \ln V_A = \ln \gamma + \frac{\gamma + \theta - 1}{1 - \theta} \ln L_A + (1 - \gamma) \ln L_B \quad (22)$$

¹¹ Freeman (2006) suggests some radical economic policies, such as auctioning immigration visas, that would transfer some of the benefits from migration to the native workers and thereby increase the optimal level of immigration.

¹² Trefler (1998) considers the optimal immigration policy in a numerical example based upon Helpman (1984) where the degree of external economies is one-half.

Migration from country B to country A involves an increase in the labour force in country A and a corresponding decrease in the labour force in country B so $dL_B/dL_A = -1$.

Differentiating (22) yields the welfare effect of immigration for country A :

$$\frac{dv_A}{dL_A} = \frac{\partial v_A}{\partial L_A} + \frac{\partial v_A}{\partial L_B} \frac{dL_B}{dL_A} = \frac{\partial v_A}{\partial L_A} - \frac{\partial v_A}{\partial L_B} = \frac{(\gamma + \theta - 1)L_B - (1 - \gamma)(1 - \theta)L_A}{(1 - \theta)L_A L_B} \quad (23)$$

Before deriving the interior solution, consider the two possible corner solutions where no immigration is optimal or where unrestricted immigration is optimal. No immigration is optimal if the derivative is negative when the labour force in the two countries is equal to the initial labour endowments so $L_A/L_B = \bar{L}_A/\bar{L}_B = \lambda$. Then, the derivative (23) is:

$$\frac{dv_A}{dL_A} = \frac{1}{(1 - \theta)L_A} [(\gamma + \theta - 1) - (1 - \gamma)(1 - \theta)\lambda] < 0 \quad (24)$$

This will be negative if the relative labour endowment is sufficiently large or if the degree of external economies of scale is sufficiently small. Solving for the critical value for the degree of external economies of scale yields:

$$\theta < \theta_A^0 \equiv \frac{(1 - \gamma)(1 + \lambda)}{1 + (1 - \gamma)\lambda} > 1 - \gamma \quad (25)$$

Therefore, no immigration is optimal for country A if the degree of external economies of scale $\theta < \theta_A^0$. Then, the terms of trade loss dominates any productivity gain due to external economies of scale. It can be shown that $\partial \theta_A^0 / \partial \gamma < 0$ as an increase in the demand parameter decreases the size of the negative terms of trade effect, and that $\partial \theta_A^0 / \partial \lambda > 0$ as an increase in the relative labour endowment increases the size of the negative terms of trade effect.

At the other extreme, unrestricted immigration will be optimal if the derivative is positive when wages are equalised in the two countries and this will happen when $L_A/L_B = \gamma/(1-\gamma)$. Then, the derivative is:

$$\frac{dv_A}{dL_A} = \frac{1}{(1-\theta)L_A} [(1+\gamma)\theta - 1] > 0 \quad (26)$$

This will be positive if the degree of external economies of scale is sufficiently large. Solving for the critical value yields:

$$\theta \geq \theta_A^U \equiv \frac{1}{1+\gamma} > \frac{1}{2} \quad (27)$$

Therefore, unrestricted immigration is optimal for country A if the degree of external economies of scale $\theta > \theta_A^U$. Then, the productivity gain due to external economies of scale dominates the terms of trade loss. It is obvious that $\partial\theta_A^U/\partial\gamma < 0$ as an increase in the demand parameter increases the size of the negative terms of trade effect, and that $\partial\theta_A^U/\partial\lambda = 0$ as the relative labour endowment does not affect the welfare effect of immigration when unrestricted immigration has equalised wages.

For the interior solution, the optimal immigration policy is obtained by setting the derivative (23) equal to zero and involves allowing immigration until the labour force in country A relative to the labour force in country B is:

$$\frac{L_A}{L_B} = \frac{\gamma + \theta - 1}{(1-\gamma)(1-\theta)} \quad (28)$$

Since the denominator is clearly positive, the numerator has to be positive so $\gamma + \theta - 1 > 0$ or $\theta > 1 - \gamma$ for any sensible interior solution. For any immigration to actually occur, $L_A/L_B > \lambda$, which implies that $\theta > \theta_A^0$. The corresponding relative wage is:

$$\frac{w_A}{w_B} = \frac{\gamma(1-\theta)}{\gamma+\theta-1} > 1 \quad (29)$$

If immigration is restricted then this must be greater than one, which will be the case if $\theta < \theta_A^U$. Therefore, there will be an interior solution if $\theta_A^0 < \theta < \theta_A^U$. The second-order conditions for a welfare maximum are:

$$\begin{aligned} \frac{d^2 v_A}{dL_A^2} &= \frac{\partial^2 v_A}{\partial L_A^2} + \frac{\partial^2 v_A}{\partial L_B \partial L_A} \frac{dL_B}{dL_A} - \frac{\partial^2 v_A}{\partial L_A \partial L_B} - \frac{\partial^2 v_A}{\partial L_B^2} \frac{dL_B}{dL_A} = \frac{\partial^2 v_A}{\partial L_A^2} - 2 \frac{\partial^2 v_A}{\partial L_B \partial L_A} + \frac{\partial^2 v_A}{\partial L_B^2} \\ &= \frac{-1}{(1-\theta)L_A^2 L_B^2} [(\gamma+\theta-1)L_B^2 + (1-\gamma)(1-\theta)L_A^2] < 0 \end{aligned} \quad (30)$$

Since $\gamma+\theta-1 > 0$ in any sensible interior solution, then the second order conditions are clearly satisfied. These results lead to the following proposition:

Proposition 4: *The optimal immigration policy for country A is unrestricted immigration if $\theta > \theta_A^U$ and no immigration if $\theta < \theta_A^0$. Otherwise, the optimal immigration policy is to allow immigration until the relative wage $w_A/w_B = \gamma(1-\theta)/(\gamma+\theta-1)$.*

Clearly, the optimal immigration policy depends upon the size of the positive productivity effect relative to the negative terms of trade effect. The critical values are shown in figure 8 as a function of the demand parameter and in figure 9 as a function of the relative labour endowment. Perhaps the most striking feature of the figures is the fairly narrow range of parameter values where there is an interior solution. In contrast to the numerical example in Trefler (1998), the optimal immigration policy might involve unrestricted immigration if the degree of external economies of scale is sufficiently high.¹³ Figure 10 shows the relative wage as a function of the degree of external economies of scale.

¹³ Since Trefler (1998) uses a value for the degree of external economies of scale equal to one-half, the possibility of unrestricted immigration is precluded by this restrictive assumption.

Similarly, for country B , it may restrict emigration to maximise the utility of workers that remain in country B . The utility of a worker that remains in country B is $V_B = (1-\gamma)L_A^{\frac{\gamma}{1-\theta}}L_B^{-\gamma}$ and the logarithm of utility is:

$$v_B = \ln V_B = \ln(1-\gamma) + \frac{\gamma}{1-\theta} \ln L_A - \gamma \ln L_B \quad (31)$$

Migration involves an increase in the labour force in country A and a decrease in the labour force in country B . Therefore, the effect of emigration from country B to country A is:

$$\frac{dv_B}{dL_A} = \frac{\partial v_B}{\partial L_A} + \frac{\partial v_B}{\partial L_B} \frac{dL_B}{dL_A} = \frac{\partial v_B}{\partial L_A} - \frac{\partial v_B}{\partial L_B} = \frac{\gamma}{(1-\theta)L_A L_B} [L_B + (1-\theta)L_A] > 0 \quad (32)$$

This is unambiguously positive due to the positive terms of trade effect and this leads to the following proposition:

Proposition 5: *Emigration always increases the utility of a worker/consumer in country B hence unrestricted emigration is the optimal policy for country B .*

The only effect of emigration on workers that remain in country B is the positive terms of trade effect that comes from the expansion of the X industry in country A and the contraction of the Y industry in country B . Therefore, country B would not want to restrict emigration.

7. Conclusions

Using the Ethier (1982) model, this paper has analysed the welfare effects of free trade, free labour migration and economic integration (free labour migration with free trade). Critical values for the degree of external economies have been derived as functions of the demand parameter and the relative labour endowment. When the larger country completely specialises in the production of the good with external economies of scale under free trade, it

was shown that the larger country gains from free trade and that the smaller country will only gain if the degree of external economies of scale is sufficiently high. Starting from free trade, it was shown that the larger country would only gain from free migration if the degree of external economies is sufficiently high and that the smaller country will always gain. If the larger country gains from free migration then free migration will be a Pareto-improving policy since both countries and the migrant workers gain. Since the smaller country may lose from free trade and the larger country may lose from free migration, multilateral negotiations about sequential liberalisation (free trade followed by free migration) will not always bring about complete economic integration (free trade and free migration) even though this would benefit both countries. This inefficient outcome could be avoided by the linkage of trade liberalisation and the liberalisation of labour migration in multilateral negotiations.

The optimal immigration policy for the larger country was derived and it was shown that it involves a trade off of the positive productivity gains due to the external economies of scale with the negative terms of trade effect due to the expansion of the export industry in the larger country and the contraction of the export industry in the smaller country. When the degree of external economies of scale is sufficiently low then the optimal policy is no immigration and when it is sufficiently high then the optimal policy is unrestricted immigration. For intermediate values of the degree of external economies of scale, the optimal policy is restricted immigration. The smaller country always benefits from emigration since it results in a positive terms of trade effect as it imports the good produced by the larger country.

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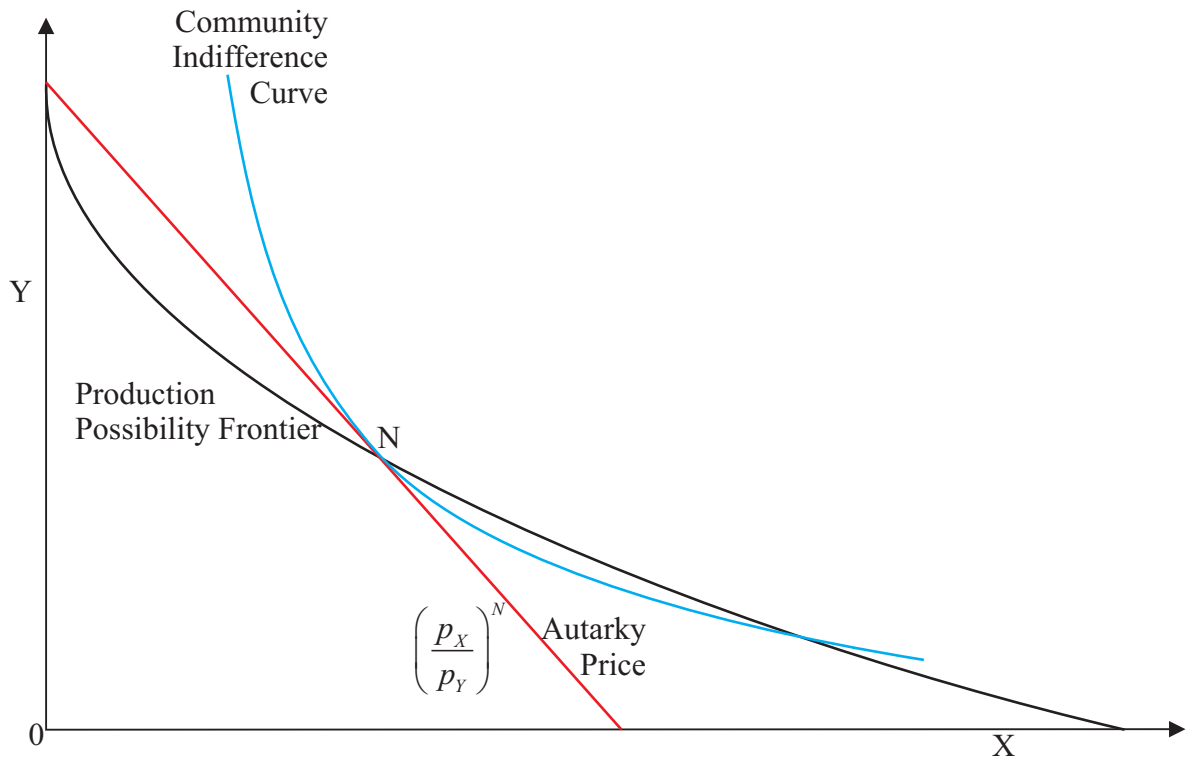
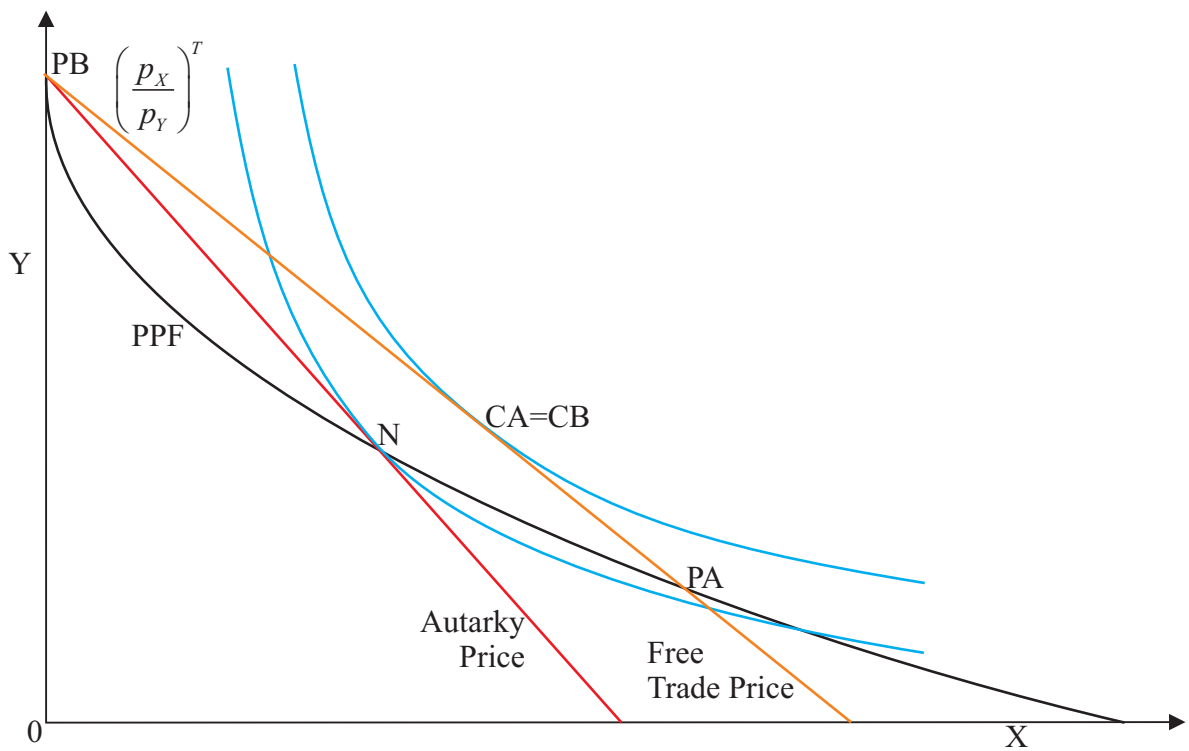


Figure 1: Autarky Equilibrium

Figure 2: Free Trade Equilibrium with Factor Price Equalisation



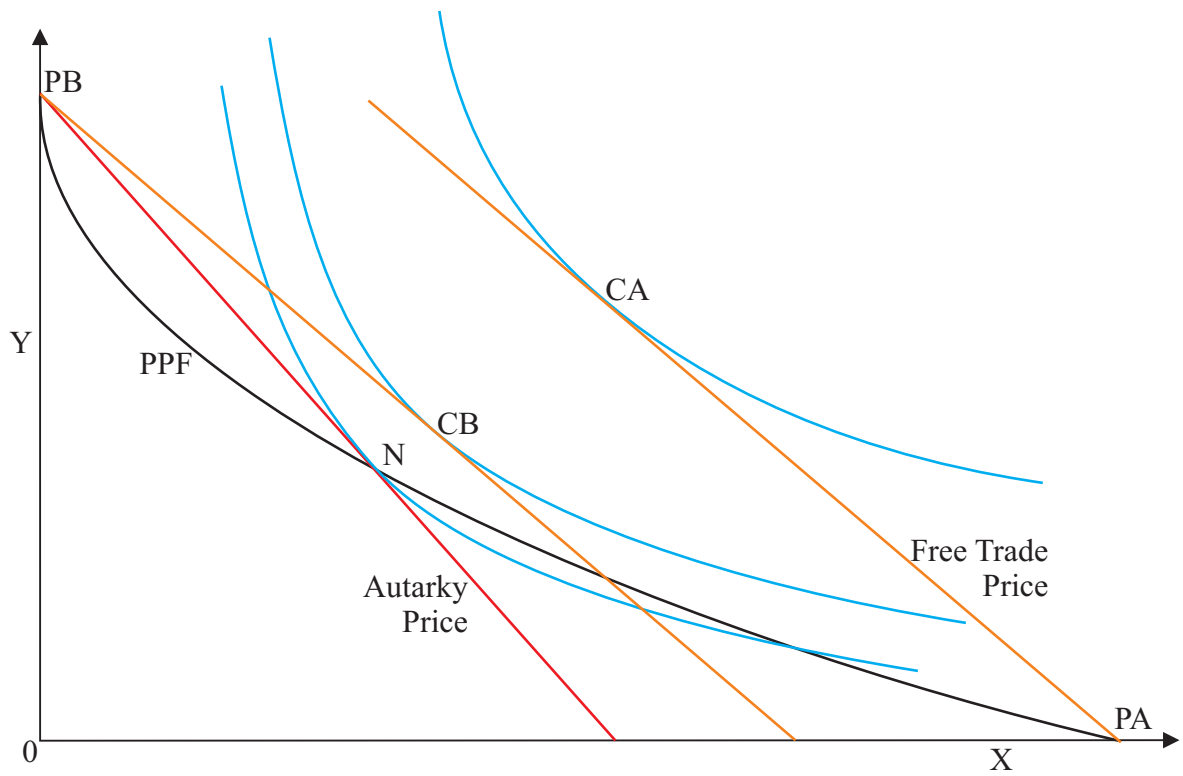
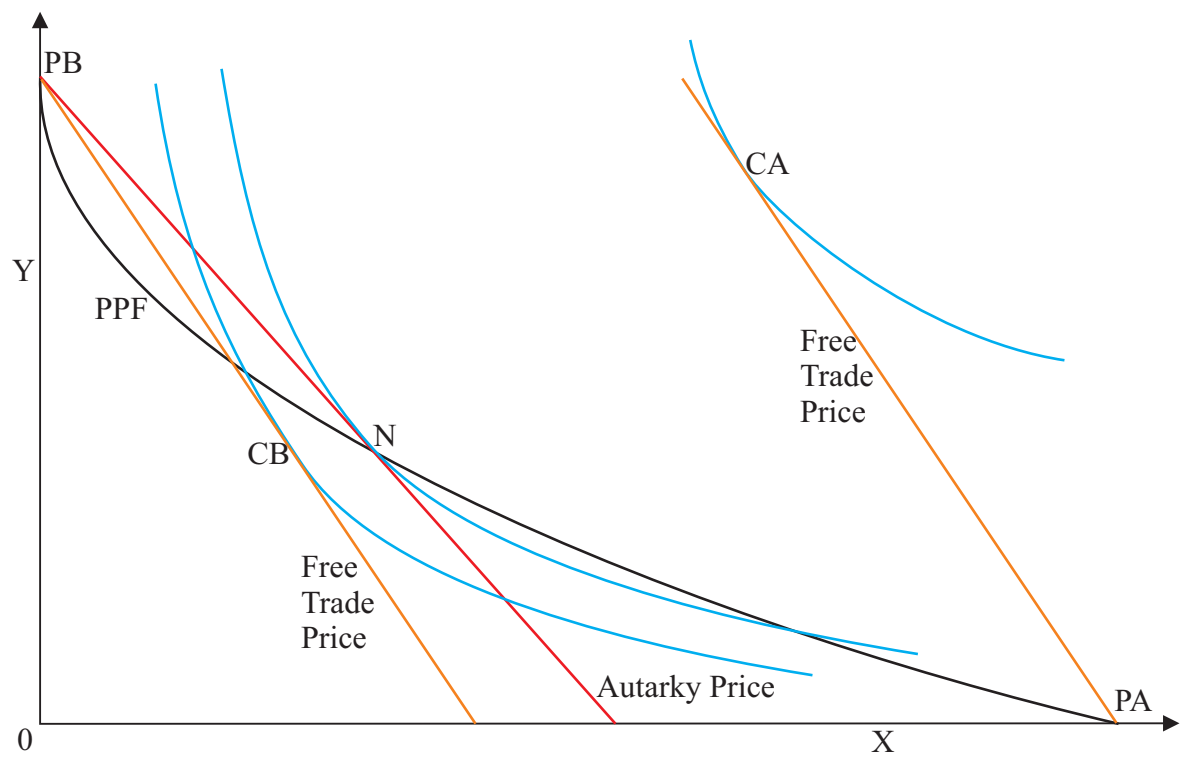


Figure 3: Gains from Trade with Complete Specialisation

Figure 4: Losses from Trade with Complete Specialisation



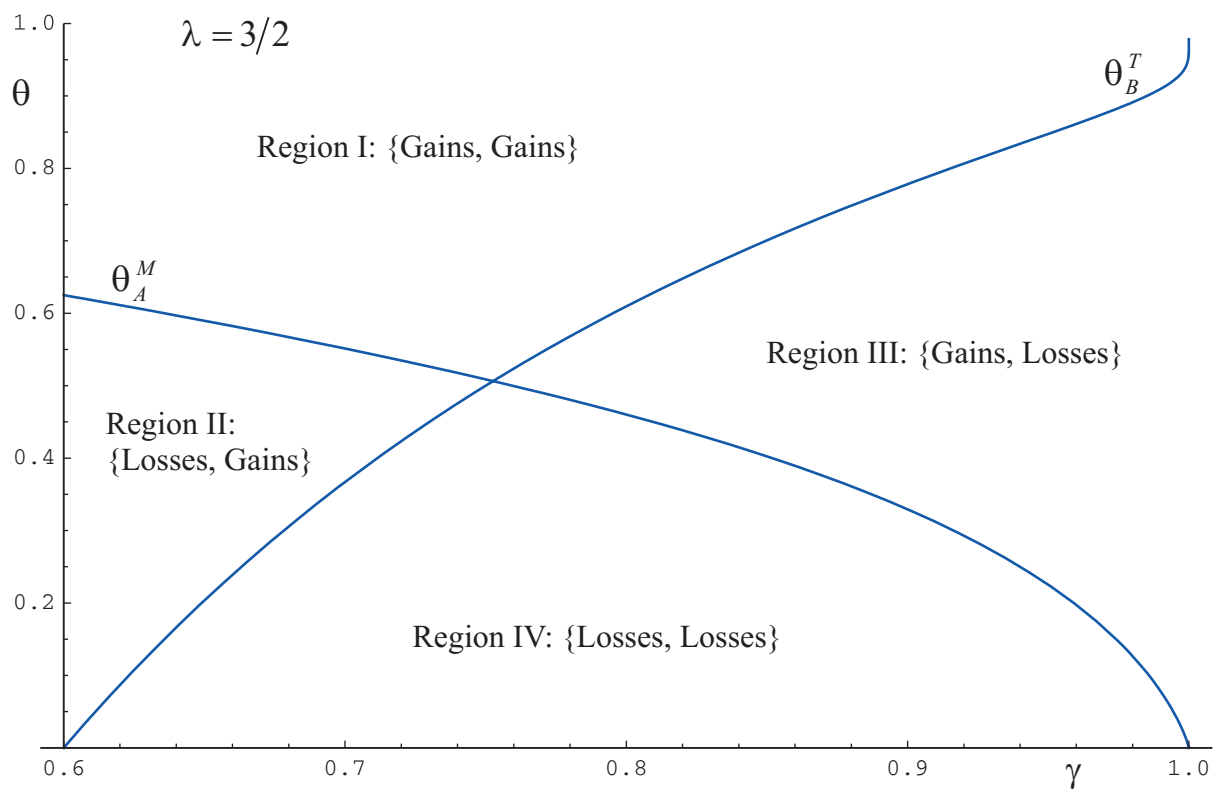
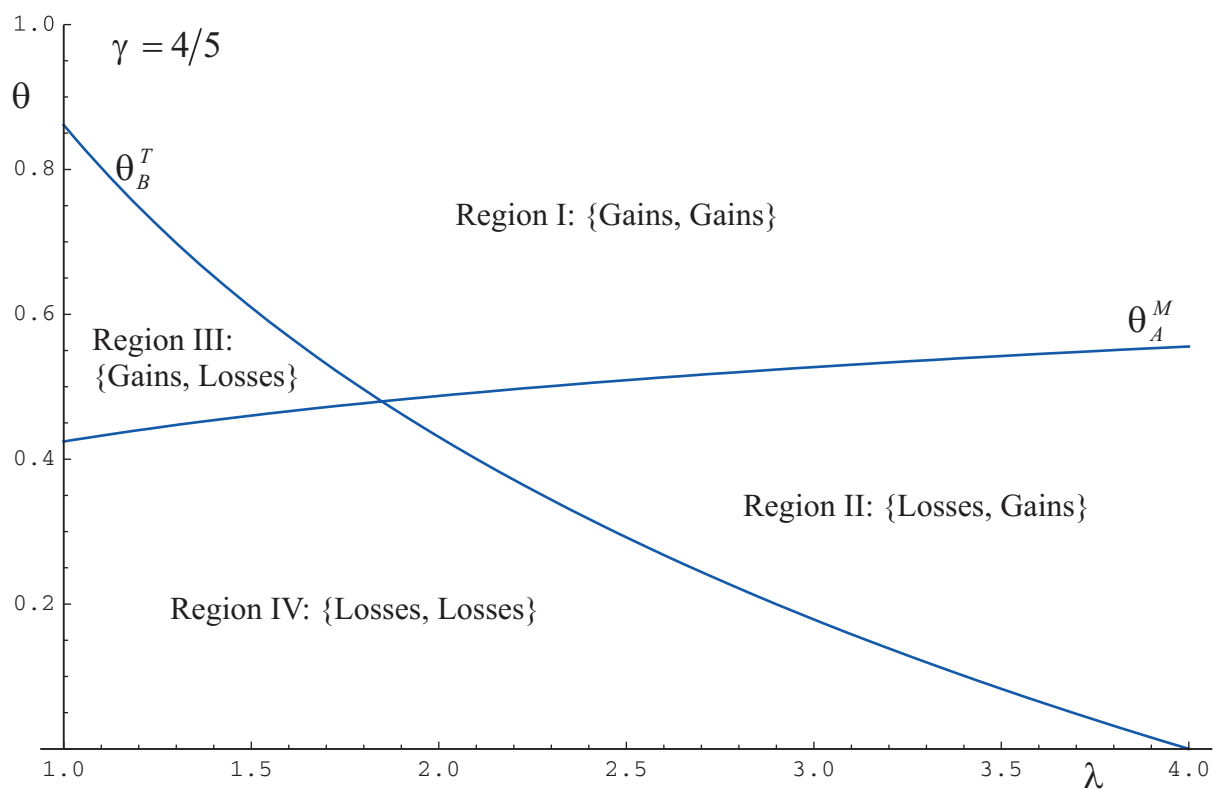


Figure 5: The Demand Parameter and the Gains and Losses from Trade and Migration

Figure 6: The Relative Labour Endowment and the Gains and Losses from Trade and Migration



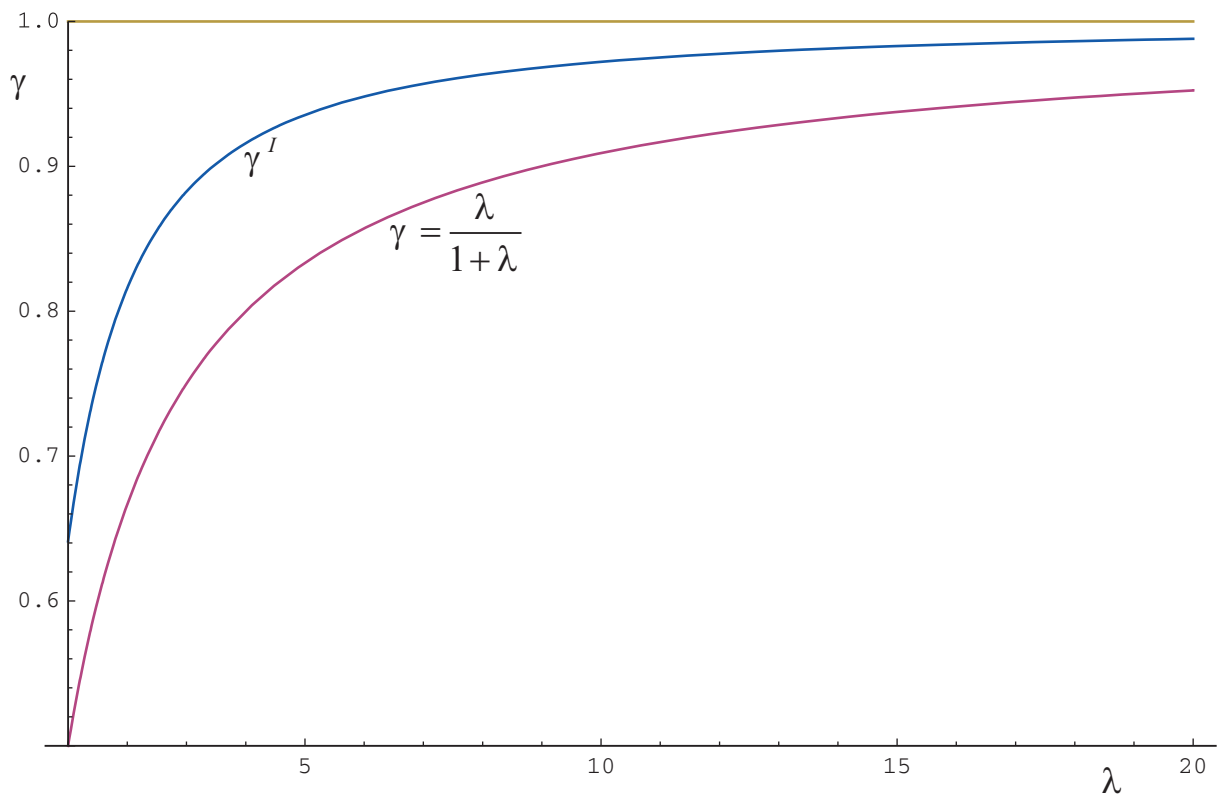
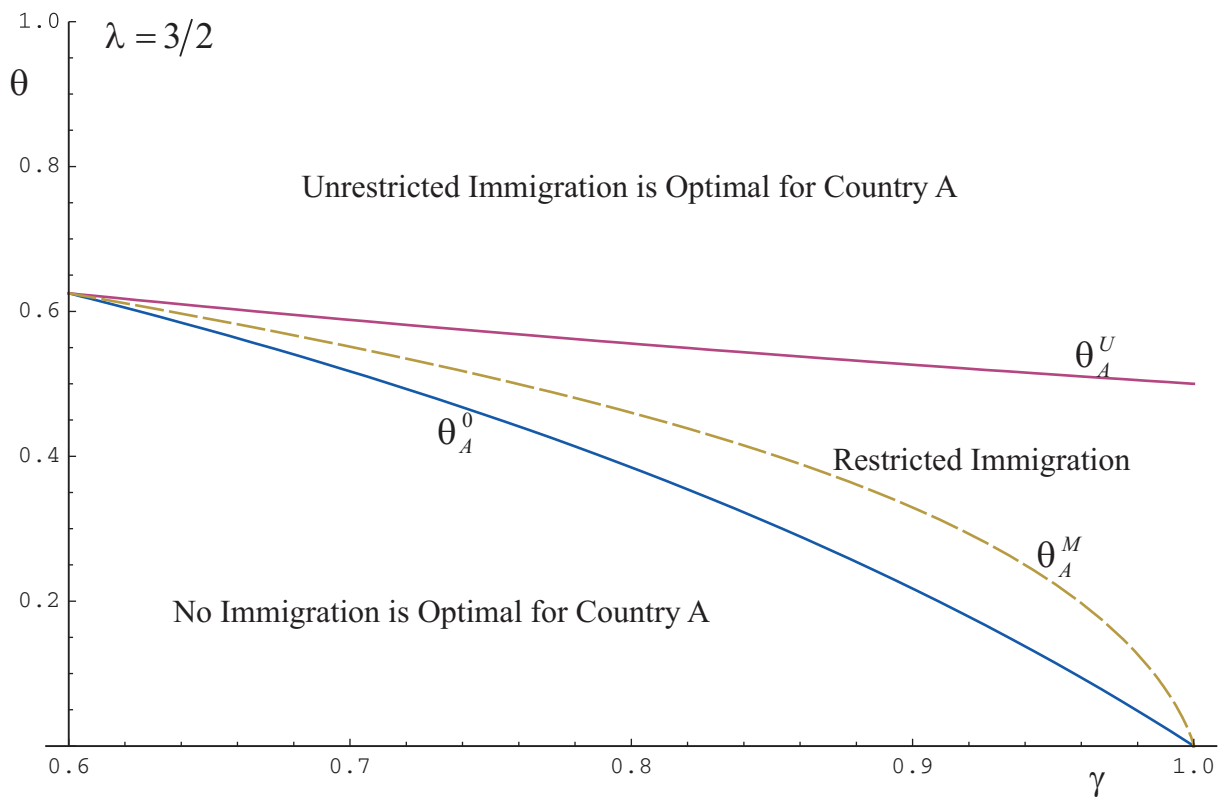


Figure 7: Demand Parameter at Intercept

Figure 8: Optimal Immigration Policy and the Demand Parameter



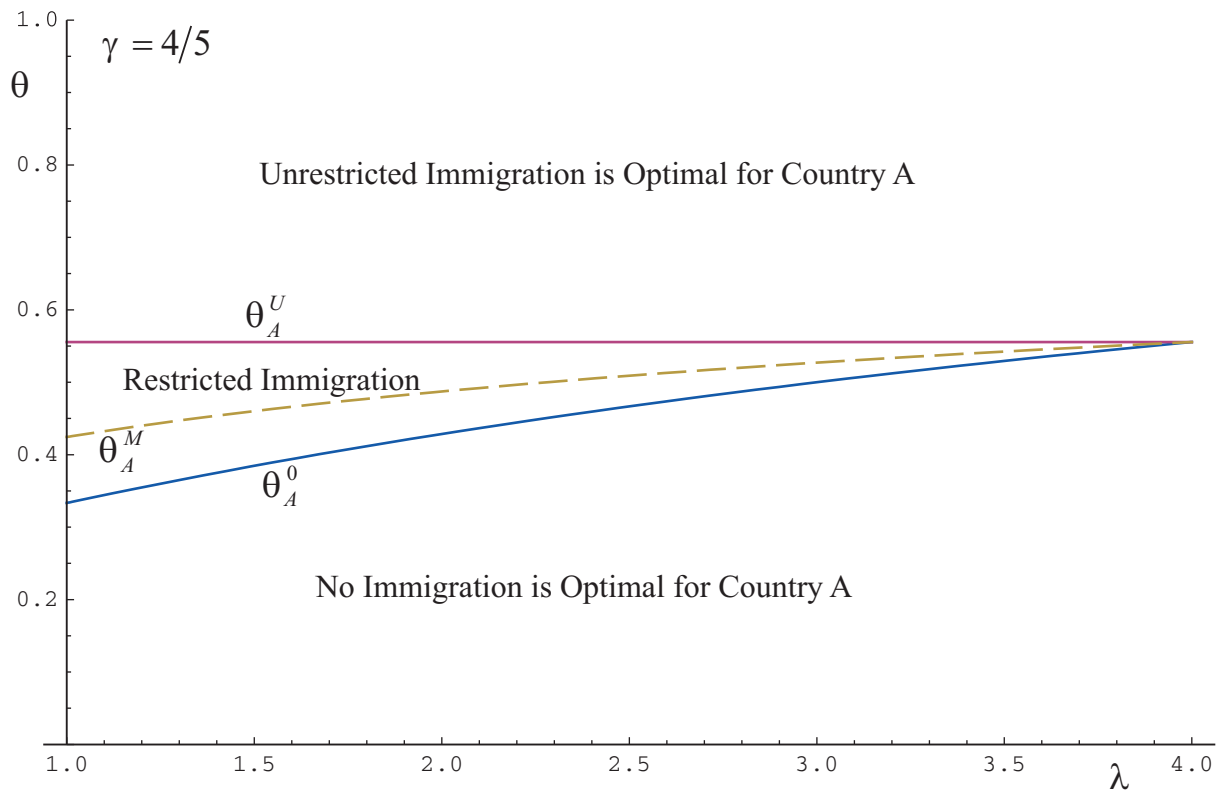


Figure 9: Optimal Immigration Policy and the Relative Labour Endowment

Figure 10: Economies of Scale and Relative Wages with the Optimal Immigration Policy

