Recruitment to Organised Crime

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Abstract

Organised crime is unique within the underground economy. Unlike individual criminals, criminal organisations can substitute between a variety of inputs; chiefly labour and effort. This paper considers the effect of several popular anti-crime policies in such an environment. Using a profit maximisation framework, I find that certain policies may cause the organisation to reduce its membership in favour of more intensive activity. Others may lead to increases in membership. Consequently, policies designed to reduce the social loss suffered as a result of criminal activities may actually increase it. Results prove robust to differences in hiring practices on the part of the criminal organisation.

Keywords: Organised crime; Crime policy; Occupational choice.

JEL: J24, J28, K42

1 Introduction

Over recent years, a myriad of policies have been suggested to reduce involvement in crime (e.g. Ballester et al. 2010, Fryer 2011). The rationale behind this is simple: crime is costly. Recent estimates (Cohen and Piquero 2009) place the external cost of one high-risk eighteen year-old’s potential life of crime between $2.6 million and $5.3 million. Over the last decade, estimates of these costs have increased substantially. In an earlier study employing a similar approach (Cohen 1998), the headline cost was between $1.7 million and $2.3 million.\(^1\) Society is forced to invest in security, pay for public prosecution and incarceration, and suffer from victimisation and the fear of crime. A recent survey (Egley and Howell 2012) estimated that there are 29,400 gangs active in the US, employing some 756,000 individuals. Combined with Cohen and Piquero’s estimates, the annual cost of organised crime may be as high as $480 billion in the US, or 3% of US GDP (Bureau of Economic Analysis 2013)\(^2\). The recent

\(^1\)Admittedly, a large proportion of the increase is the result of improvements in measurement techniques. Nevertheless, crime is much more costly than hitherto imagined.

\(^2\)Cohen and Piquero use a constant 2% discount rate. Author’s estimates are based upon youths being active from age 18 to 26 and use 2010 US nominal GDP.
This paper develops a simple framework in which to study a criminal organisation’s likely reaction
to the implementation of policy. A profit-maximising gang has two available inputs, labour (the gang’s
size) and gang members’ effort, which it uses generate revenue through illegal activity. Heterogeneous
youths grow up in the gang’s neighbourhood. During their early years, these youths have, to a varying
degree, the opportunity to acquire criminal skills by engaging in juvenile crime. Criminal skill lowers
the cost of future criminal effort. They then decide where to work. If they join the formal economy, they
are paid a flat wage (the expected wage when they are making career choices, incorporating possible
unemployment). If instead they opt for a criminal career, they join the gang. The gang requires that
they exert an observable amount of effort in exchange for a given wage. The exact amount of effort, and
the form of compensation provided, depends upon the gang’s ability to discriminate between youths
with different levels of skill. At its most simple, the gang will offer a single wage, and require all
members to apply identical effort. A separating gang, on the other hand, may offer a range of different
wages, each associated with a different level of effort. The model is one of adverse selection.

When a policy is introduced that affects the youths’ incentives, the gang’s reaction depends upon
how the varying of inputs affects profit. Consider a simple gang. It will recruit all youths who are
willing to apply the specified effort for the given wage. Those who have acquired the most criminal
skill, and hence have the lowest cost of effort, are the first to sign up. When a policy reduces the
incentive to join the gang, it is precisely these individuals who remain members after those with higher
costs of criminal effort have moved to the formal labour market. A smaller gang is likely to be less able
to generate revenue through increased effort (the marginal revenue product of effort may fall). However,
the amount the gang must compensate its members for effort has also declined; those who remain find
effort less costly. The marginal cost of effort has also declined. Whether the gang should optimally
increase or decrease the level of effort they require as a result of this policy depends upon which effect
dominates: size and effort can be profit complements or profit substitutes.

If effort and size are profit complements, any policy that aims to reduce the incentive to join the
gang also reduces the amount of effort the gang members employ, unequivocally lowering the social
loss from crime. If, on the other hand, effort and size are profit substitutes, a variety of outcomes may
arise. Falling size may cause the gang to substitute towards effort or vice versa. Perversely, this could
increase the social cost of crime. Policies are therefore most effective when they not only reduce gang
membership, but also hamper the gang’s ability to incite their members to work harder. For example,
prevention of juvenile crime may prove particularly effective. It not only increases the opportunity cost
of crime, but also reduces youths’ ability to learn criminal skills. This makes them more sensitive to

intensification of research into these policies is therefore unsurprising.


criminal activity. Conversely, improving labour market conditions, conditional on a youth still choosing to become a criminal, have no impact upon their incentives to learn criminal skills. Whilst membership is reduced, such policies unambiguously increases the marginal profitability of effort, causing the gang to intensify its activities.

This intuition is similar to two recent contributions by Poutvaara and Priks (2009 and 2011). In their papers, a gang leader gains utility from being in charge of a large gang whose members commit relatively severe crimes or violent acts. The leader must trade off attracting members with the severity of the actions he can expect of them. They find that, following changes in police tactics (2009) or unemployment (2011), the gang may indeed substitute away from members towards more severe activity. This paper extends their analysis in several important dimensions. Firstly, in considering a profit-maximising gang, my results allow for the possibility that a policy causes membership to increase. Secondly, I allow for a continuum of effort levels. This enables me to compare a situation in which all gang members are offered the same contract with a situation in which they fully separate. Results are shown to be broadly robust, although the intuition changes slightly. Finally, I incorporate criminal skill acquisition into the model, allowing me to assess the link between juvenile crime and more serious crime. The structure I develop provides a very intuitive way to analyse a wide range of policies in a relatively straightforward manner.

The remainder of the paper proceeds as follows. Section 2 reviews the related literature. I then introduce a model of recruitment to organised crime, and formally define the two types of gang in Section 3. Section 4 evaluates how the neighbourhood’s youth respond to the opportunities on offer. In Section 5, I discuss the behaviour of the first gang type: one which can only offer a single wage and effort contract. I then evaluate policy in this setting. Section 6 extends the analysis, discussing a situation in which the gang can offer a range of jobs to its members, before considering the effects of policy in this more complicated setting. Finally, Section 7 concludes.

2 Related Literature

2.1 Organised Crime as a Firm

This paper follows a long literature in modelling a criminal organisation as a profit-maximising monopolist (for example Garoupa 2000, Chang et al. 2005 or Kugler et al. 2005). In keeping with this approach, the principal input into production for the gang discussed herein is its members. In addition to fulfilling the traditional role of labour (applying costly effort to generate revenue), several network
externalities have been identified that a larger gang can take advantage of. For example, operating in an illegal market is a risky business. By vouching for its members, and inflicting severe punishments on those who cheat, a gang can reduce informational asymmetries. It thereby enables more trade to take place (Cook et al. 2007). Larger criminal organisations can also stretch police resources (Sah 1991). As the number of members increases, the probability than any one individual will be arrested diminishes. Consequently, larger organisations suffer proportionally less from police disruption.

In a market plagued with informational asymmetries, effort provides the other key input. Whilst it also has a traditional role in the production function, effort also represents a means of signalling ability. For example, firms operating a protection racket must occasionally demonstrate that they are capable of protecting their customers’ businesses (Gambetta 1996, Konrad and Skaperdas 1997, Dixit 2007). Violent effort may also be used to ensure that threats against those who cooperate with the police are credible (Baccara and Bar-Isaac 2008), and that competitors are deterred from entry into an illegal market (Silverman 2004).

The informational asymmetries extend to within an organisation. Jankowski 1991 suggests that a reasonably large proportion of violent activity attributed to gangs is actually committed by gang members for more selfish reasons. He suggests that, by engaging in these violent acts, the individual signals to their gang that they are capable, and hence worthy of advancement or respect. The widespread use of initiation by organisations (Morgan 1960, Iwai 1986, Decker 1996, Skaperdas 2001, Paoli 2003) as a means to test recruits’ ability provides further evidence that a model of adverse selection is indeed appropriate.

Criminal organisations’ costs are primarily the wages paid to their members. Whilst the organisation’s leaders dictate the effort employed, it is the foot soldiers that face the cost of implementing it during the commission of their crimes. Levitt and Venkatesh 2000 show that, over a four-year period, members of drug-selling gang in Chicago had a 25% chance of dying (versus a 0.4% chance nationwide in the same demographic). Over the same period, they also suffered an average of two non-fatal injuries (ranging from gunshots and knife wounds to beatings). In order to attract and retain members, wages must incorporate compensation for the costs associated with engaging in illegal activities. This provides an incentive for criminal organisations to substitute between size and effort. By increasing the amount of effort it requires its members to employ, the organisation’s wage bill grows substantially, as each member must be compensated. Levitt and Venkatesh find clear evidence of this during gang wars. When the cost of effort is increased (due to the greater risk of violence), foot soldiers’ wages increase significantly. In this sense, the marginal cost of size is increasing in effort, and vice versa.
2.2 Tackling Organised Crime

Policies designed to tackle crime, and organised crime in particular, are motivated by the fact that crime is associated with several negative externalities. Chief amongst these (Cohen and Piquero 2009) is the fear of crime and victimisation. The extent of this external cost is likely to be related to an organisation’s transient features - its size and the effort its members apply in the commission of their crimes. A larger gang generates more victims. Moreover, if each criminal applies more effort, the average victim cost per crime is likely to increase.

Secondly, society must expend resources protecting itself from the gang. Once more, we would expect these costs to be increasing in both size and effort (Ehrlich 1981). With more members, crime is more prevalent. The chance of becoming a victim of crime is higher. More crimes will need investigation, and will lead to more prosecutions. If effort is higher, a victim suffers a greater loss. The return to investing in protection is higher and more investment will occur.

Since the inception of the economics of crime, economists have been suggesting policies to reduce criminal activity. In Becker’s seminal paper of 1968, he proposed that a relatively cheap way to reduce crime was to simply increase the severity of punishment (fine) incurred when caught. By increasing the size of fine, the expected payoff to committing crime is reduced, given a constant arrest rate. This reduces the incentive to become involved in crime, relative to staying honest. Since then, an abundance of policies have been put forward, each aiming to manipulate the incentives of would-be criminals.

Another Beckerian policy involves increasing arrest and conviction rates. Increasing arrest rates reduces the number of members available for the organisation to utilise (what Levitt 1996 calls the incapacitation effect). This, in turn, will impact upon the wages they are willing to offer, and even their optimal levels of effort. Whilst increasing the length of prison terms may have similar effects, other increases in severity may not.

Primary labour market policies cover an extremely broad range of suggestions, all aiming to increase the wage paid in the formal economy. A by-product of this is a fall in crime. Two cases stand out, however, for their direct targeting of high-risk youths. The now famous Perry Preschool Programme (see Parks 2000), focused on poor black preschool children with low IQs in the 1960s. Participants attended intensive preschool classes, and their parents met regularly with teachers. They were then tracked over forty years, creating a reasonably comprehensive data set on their educational, employment and criminal outcomes. Recent analysis suggests that the project yielded an internal rate of return of 7%-10% (Heckman et al. 2010). More recently, a range of case studies have been conducted by EdLab. For example, the Paper Project in Chicago targets ninth and tenth grade students. The organisers pay
the students for passing their classes. They can earn up to $2,000 per year, with 50% payable upon graduation from high school (Fryer 2011).

Prevention of juvenile crime increases the cost of acquiring criminal skills, and is at the heart of the arguments put forward by Ballester et al. 2010. They suggest that there are positive learning externalities within juvenile criminal networks. By removing those most central to the network (so-called, “Key players”), youths cannot take advantage of these externalities and are forced to acquire criminal skills in isolation. They are thus less likely to join criminal organisations, as they will find applying effort relatively costly.

2.3 Are Size and Effort Substitutes?

The introduction highlighted the main insight of this paper: anti-crime policies can backfire in the case of organised crime if gang size and effort are profit substitutes. Endogenous effects counteract the impact of the policy, causing the gang to substitute away from one input towards another. Is there any evidence of this?

One widely cited example of anti-crime policy backfiring relates to the United States’ ‘War on Drugs’ (summarised in Lee 1993). Over the course of the 1980s, arrests for heroin and cocaine trafficking rose dramatically (from 4% of all drug arrests in 1980 to almost 20% of arrests in 1989). Moreover, a far higher proportion of traffickers were incarcerated (up from 85% in 1985 to 92% in 1989) and for much longer periods of time (up from 61 months to 76 months). However, over the same decade, the availability of both drugs increased substantially. In 1985, 21% of high school seniors reported that heroin was, “Fairly easy of very easy to get hold of”. By 1989, that figure had increased to 31%. For cocaine, the percentage of positive responses rose from 49% to 59%. Over the period, the price of both drugs were relatively stable. Several attempts to explain this paradox have been put forward based upon changes in consumer behaviour (Lee 1993), market structure (Poret 2002), demand elasticity (Becker et al. 2006) or competition (Mansour et al. 2006). This paper provides an arguably simpler explanation. As drug gang members were incarcerated, the drugs gangs found it more profitable to increase the amount of effort those members who were still at liberty applied in the sale of drugs. Thus, whilst the number of drugs dealers may have declined, the overall supply of drugs remained relatively unaltered, or may have increased.

Another more recent example relates to the London riots on 2011. As a result of the rioting, the Metropolitan Police developed a strategy of arresting known gang members in London. A recent report (The Centre for Social Justice 2012) suggests that, as a result of this policy, gang violence in the city
has increased. Whilst there may be other explanations, it is again suggestive of direct substitution away from membership towards increased violent activity on the part of remaining gang members.

3 A Model of Recruitment to Organised Crime

The economic environment, hereafter referred to as the *neighbourhood*, consists of two sectors: the *primary labour market* and the *gang*. A mass $N$ of heterogeneous youths grows up in the neighbourhood. After investing in appropriate skills, they decide where to seek employment. Whilst the primary labour market is passive, the gang acts as a monopsonistic employer of criminals.

The gang offers a *contract schedule* comprising of wage and effort pairs $(g(s), e(s))$. Recruits to the gang are able to choose any contract they wish from the available menu, $s \in S$. I assume that contracts are binding on both sides, so by choosing a contract the recruit commits to a given level of effort in exchange for the associated wage.

The revenue each gang member, $i$, generates depends primarily upon the size of the gang (total membership, denoted by $M \in [0, N]$) and the effort they apply, $e(s_i)$. Each individual’s revenue stream is given by $r(M, e(s_i))$. I assume that $r(M, e)$ is subject to positive but diminishing marginal returns with regard to effort, that it is homogeneous of degree zero, and that $r(M, 0) = r(0, e) = 0$ for all $M, e \geq 0$. The homogeneity assumption is not necessary, but makes for a simpler characterisation of the equilibrium and policy effects.

As each gang member receives a wage, $g(s_i)$, the profit they generate for the gang is given by:

$$\pi(M, s_i) \equiv r(M, e(s_i)) - g(s_i)$$

(1)

The gang chooses its contract schedule to maximise total profits:

$$\Pi(M, (g(s), e(s))_{s \in S}) \equiv ME[\pi(M, s_i) \mid i \text{ joins}]$$

(2)

The contract schedule is announced prior to any decisions by youths and becomes common knowledge.

A parameter that proves critical in the analysis to follow is the cross elasticity of the marginal

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3 This terminology follows Huang et al. 2004, who develop a similar model of predation.

4 Whilst the terminology used refers to a street gang, the model presented is equally relevant to alternative forms of organised crime.

5 The black box nature of revenue (as opposed to production) is purely for notational ease. One can think about it as an indirect revenue function: the one resulting from the optimal allocation of inputs across the wide range of activities the gang engages in. Kugler et al 2005 consider a more structured approach, decomposing revenue into the number of crimes committed, and the booty collected from each crime.
revenue product of effort with respect to gang size:

\[ \eta (M, e (s_i)) \equiv \frac{M \times r_{Me} (M, e (s_i))}{r_e (M, e (s_i))} > -1 \]

where the subscripts after functions denote derivatives. \( \eta \) provides a measure of the degree of revenue complementarity between effort and size for each contract, \( s_i \). When \( \eta \) is relatively large, a small decrease in one input results in a relatively large decline in the other’s marginal revenue product. Size and effort are strong revenue complements in this case. The strength of revenue complementarity (as measured by \( \eta \)) is clearly important in determining whether size and effort are profit complements or substitutes.

Youths vary in their intrinsic criminal ability, denoted by \( \sigma_i \) and distributed exponentially with parameter \( \lambda > 0 \). Youths simultaneously make two decisions. Firstly, they choose how much criminal skill to acquire. Acquiring criminal skill is a costly process. However, those with a higher criminal ability find it easier than those with a lower ability. Denoting the amount of criminal skill acquired by youth \( i \) by \( c_i \), the cost of acquiring criminal skill is given by \( kC \left( \frac{c_i}{\sigma_i} \right) \). \( C (\cdot) \) is a strictly increasing and convex function. \( k > 0 \) reflects the fact that policy (such as key player policies) can influence how easy it is to acquire criminal skills. An important characteristic of the criminal skill cost function proves to be the following elasticity:

\[ \varepsilon \left( \frac{c}{\sigma} \right) \equiv \frac{\frac{c''(\varepsilon)}{c'('\varepsilon')}}{C'(\frac{c}{\sigma})} \]

where \( C'(\frac{c}{\sigma}) \) denotes the derivative of \( C \) with respect to \( \frac{c}{\sigma} \). \( \varepsilon \) plays a role in determining how youths respond to changes in the structure of the contracts offered by the gang. If it is low, they find it relatively easy to adjust their level of skill, allowing the gang additional flexibility in responding to policy. Accordingly, this will affect how the marginal cost of labour or effort adjust to changes in the policy environment.

Youths also decide where to work. For simplicity, the primary labour market pays an exogenously given flat wage rate, \( w \geq 0 \). This can be thought of as the wage youths expect to receive when they make career choices, net of any costs of education and incorporating the probability of unemployment.

Should the youth join the gang instead, they choose a contract, \( s_i \), from the available menu. However being a gang member is a dangerous affair. There is a possibility of arrest and conviction. Following

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6In the models of delinquent networks discussed earlier, this parameter can be thought of as a measure of a youth’s centrality within the network.

7Continuing the analogy regarding networks, those who are more central have more opportunities to learn from others, rather than by relatively costly trial and error.
Becker 1968, arrest occurs with probability $p$, resulting in a fine of size $f^8$ and wages being confiscated.\footnote{\textit{8}Whilst $f$ is a constant in this model, the results that follow would apply equally to situations in which different crimes receive different punishments. In that case, an increase in $f$ would be equivalent to a situation in which all punishments became more severe, but the gradient of the punishment schedule remained unchanged. The qualitative results do not change if we allow $p$ and $f$ to depend upon the individual level of effort (at least when we assume that $p'(e) > 0$ and $f'(e) > 0$). The expressions do, however, become more complicated. To keep the exposition simple, I will abstract from this, although any differences in the results will be highlighted.} Moreover, effort is costly. Whilst the gang leaders choose the level(s) of effort the gang applies, it is the members who must bear the cost of applying it. However, by learning to be a criminal, youths lower the cost of criminal effort. In particular, each youth suffer disutility $-\frac{e(s)}{c_i}$. Arrests are always made after a crime has been committed. Since youths apply effort during their crimes, they suffer this disutility irrespective of whether they are subsequently arrested. The payoff from joining the gang is therefore:

$$G(c_i, s_i; \sigma_i) \equiv (1 - p)g(s_i) - pf - \frac{e(s_i)}{c_i} - kC\left(\frac{c_i}{\sigma_i}\right)$$

(3)

In the remainder of the paper, I will distinguish between two extreme types of gang. A gang is \textit{simple} if it can only offer a single contract, $(g, e)$, to all members. In this sense, it is a single-price monopsonist. At the other end of the spectrum, a gang is \textit{separating} if it can offer a full range of contracts, $(g(s), e(s))_{s \geq 0}$, and, in particular, if these contracts cause recruits fully separate according to their abilities; equivalent to second degree price discrimination. Whilst I am agnostic regarding which type of gang better represents real criminal organisations, this enables me to demonstrate the robustness of my policy results.

Each of the policies outlined in the introduction is associated with a parameter in the model. Specifically:

1. Increasing the severity of punishment increases $f$.
2. Primary labour market policies increase $w$.
3. Increasing the arrest or conviction rate increases $p$.
4. Prevention of juvenile crime disrupts the ability of youths to learn criminal skills, increasing $k$.

When discussing the impact of new policies, I will first derive results for a generic parameter, $\phi \in \{f, w, p, f\}$, before turning attention to each specific policy in turn.

\footnote{\textit{9}This is a simplification. In reality, there is some evidence to suggest that criminal organisations pay members’ families whilst they are incarcerated. However, as they are unable to take advantage of other membership benefits, their gang wage does go down.}
4 Youth Decisions

We begin the discussion of the equilibria of these models by evaluating the decisions made by the neighbourhood’s youth. Taking contract schedule, $S$, as given, a youth with criminal ability $\sigma_i$ faces the following utility maximisation problem:

$$\max_{j \in \{0,1\}, c \geq 0, s \in S} \left\{ (1 - j) w + j \left[ (1 - p) g(s) - p f - \frac{e(s)}{c} \right] - k C \left( \frac{c}{\sigma_i} \right) \right\}$$  \hspace{1cm} (4)

$j \in \{0,1\}$ takes value one when the youth chooses to join the gang and zero otherwise.

Consider first the choice of criminal skill, conditional upon career choice. If the youth chooses to join the primary labour market, criminal skill is of no use to them. They consequently select $c_i^* = 0$. Conversely, if they decide to join the gang, they choose $c^* (s^*_i; \sigma_i)$ satisfying:

$$\frac{e (s_i^*)}{C^* (s^*_i; \sigma_i)} \equiv \frac{k}{\sigma_i} C' \left( \frac{c^* (s_i^*; \sigma_i)}{\sigma_i} \right)$$  \hspace{1cm} (5)

The resulting $c^* (s_i^*; \sigma_i)$ is strictly positive, and increasing in both the level of effort specified by the gang and the criminal ability of the youth. More effort increases the marginal benefit of acquiring criminal skills, whereas increasing criminal ability reduces the marginal cost.

A youth will join the gang if and only if:

$$G(c_i^*, s_i^*; \sigma_i) \geq w$$  \hspace{1cm} (6)

If a youth of ability $\sigma_i$ joins the gang, it is straightforward to show that all youths with ability $\sigma > \sigma_i$ also join the gang. Suppose that the youth with ability $\sigma_i$ chooses contract $s_i^*$. If they join, it must be the case that $G(c_i^*, s_i^*; \sigma_i) \geq w$. Now, any youth choosing $s_i^*$ is offered $(g(s_i^*), e(s_i^*))$. Consider a youth with ability $\sigma_j > \sigma_i$. If this youth joins the gang, chooses the same contract and acquires the same amount of criminal skill, then they will enjoy the same wage and suffer the same disutility from effort. However, since they have higher criminal ability, the cost of acquiring $c_i^*$ is lower. They can therefore guarantee themselves a strictly higher payoff than the youth with criminal ability $\sigma_i$. Conversely, the payoff from joining the primary labour market, $w$, is independent of a youth’s criminal ability. We can therefore conclude that there exists some $\sigma_M$ defined by:

$$G(c_M^*, s_M^*; \sigma_M) \equiv w$$  \hspace{1cm} (7)
such that a youth will join the gang if and only if $\sigma_i \geq \sigma_M$. We call the youth with ability $\sigma_M$ the *marginal youth*. By definition, the gang’s size is given by $M = N (1 - p) \exp \{-\lambda \sigma_M\}$.

Finally, in the case of a separating gang, each youth who decides to join the gang chooses a contract to maximise the payoff they receive from their membership:

$$
(1 - p) \frac{\partial g}{\partial s} (s^* (\sigma_i)) \equiv \frac{1}{c^* (\sigma_i, s^*_i)} \frac{\partial e}{\partial s} (s^* (\sigma_i))
$$

(8)

The marginal change in wage must compensate the youth for the associated change in effort. A youth will therefore truthfully reveal their ability if and only if $\sigma_i$ satisfies the above equation, i.e. $s^* (\sigma_i) = \sigma_i$.

5 The Simple Gang

The simple gang’s the profit maximisation problem can be viewed as the gang choosing an effort level, and then compensating gang members sufficiently to induce a chosen gang size. The extent of the compensation is derived as follows. In order to acquire gang size of $M$, it is necessary that the marginal youth have criminal ability:

$$
\sigma_M = \ln \frac{N + \ln (1 - p) - \ln M}{\lambda}
$$

(9)

This youth must therefore be indifferent between the gang and the primary labour market. In order to ensure this with effort level $e$, the gang must offer a wage:

$$
g (M, e) \equiv \frac{w + pf}{1 - p} + \frac{e}{c^* M (1 - p)} + \frac{k}{1 - p} C \left( \frac{\lambda c^*_M}{\ln N + \ln (1 - p) - \ln M} \right)
$$

(10)

The gang must compensate the youth for the opportunity cost of joining, the possibility of being punished (which also entails a loss of wages), the effort the gang requires them to apply and the cost of acquiring appropriate criminal skills. The gang leadership’s profit maximisation problem thus becomes:

$$
(M^*, e^*) = \arg \max_{M \in [0, N], e \geq 0} \{M (r (M, e) - g (M, e))\}
$$

(11)

Before outlining the solution to (11), it is expedient to discuss an issue alluded to in the introduction. When effort increases, the marginal revenue product of size increases. Gang members face less disruption, a stronger monopoly, and may even be able to extort higher prices (a *revenue effect*). Concurrently, however, the marginal cost of labour also increases. Each member is being forced to apply more effort, increasing the disutility they suffer as a result. The gang must offer additional compensation according
to (10), in order to prevent those with relatively low criminal ability from opting to join the primary labour market instead (a cost effect). These two effects counteract one another, and consequently, the net effect on the marginal profit generate by size is unclear. Determining which effect dominates is not only helpful when describing the equilibrium, but proves to have important implications for the impact of policy. The cross-derivative of gang profits with respect to both inputs is:

\[ \Pi_{Me} = \frac{1}{M C^*_M (1 - p)} \left[ \eta - \frac{1 + \varepsilon_M}{\lambda} \right] \]

where \( \eta \equiv \eta(M, e) \) and \( \varepsilon_M \equiv \varepsilon \left( \frac{e}{\sigma_M} \right) \). If \( \Pi_{Me} \) is positive, size and effort are profit complements. Otherwise size and effort are profit substitutes. An increase in size reduces the profitability of effort and vice-versa. The term in parentheses determines which case we are in. \( \eta \) represents the revenue effect - the degree of revenue complementarity between size and effort. The second term in parenthesis reflects the cost effect. Which one dominates depends upon the functional forms of \( r(\cdot, \cdot) \) and \( C(\cdot) \) respectively. It is therefore convenient to make one of two assumptions:

**Assumption 1 (Simple Complements)** \( \eta \) is sufficiently large to ensure that the revenue effect always dominates the cost effect.

**Assumption 2 (Simple Substitutes)** \( \eta \) is sufficiently small to ensure that revenue effect is always dominated by the cost effect.

![Figure 1: The range of values over which size and effort are complements or substitutes.](image)

These assumptions are illustrated in Figure 1, and are clearly exhaustive and mutually exclusive. Starting from any degree of revenue complementarity, moving to the right increases the cost effect. Near the vertical axis, the gang is very small. All of its members are highly skilled, and have a very
low cost of applying effort. The cost effect is small, and we have complements. As size increases, gang members with less skill are recruited. These individuals suffer a greater disutility from effort, increasing the marginal cost to the gang of increasing $e$. The cost effect increases. Eventually, we reach a threshold gang size such that the cost effect dominates the revenue effect. Size and effort become substitutes. Larger criminal organisations (in the sense that they must resort to recruiting relatively low skill criminals) are therefore more likely to view size and effort as substitutes. Smaller ones are more likely to view them as complements. This is broadly consistent with the evidence. The two examples of policies backfiring cited earlier focused on relatively large scale, well established criminal organisations.

We are now in a position to outline the gang leaders’ choices.

**Proposition 1 (Profit Maximisation)** Suppose that $\eta > 0$, and that either Assumption 1 or Assumption 2 holds. Then the gang leadership’s profit maximisation problem given by (11) has a unique solution, with $e^* > 0$ and $M^* \in [0, N]$.

**Proof.** See Appendix A. ■

Requiring that $\eta > 0$ serves two purposes. The marginal revenue product of effort declines as effort increases. However, as the gang requires greater effort from its members, each youth optimally invests more heavily in acquiring criminal skills. As a result, each marginal increase in effort, $de$, has a smaller impact upon the cost members bear from applying effort, $dc^*$, since $c^*$ is larger. Consequently, the gang must increase its compensation for applying effort by smaller amounts as effort increases: the marginal cost of effort is also decreasing. In order for a maximum to exist, we require that marginal revenue decline faster than the marginal cost. A sufficient condition for ensuring this is that $\eta > -\frac{1}{2}$.

Under Assumption 1, $\eta > -\frac{1}{2}$ guarantees that a unique maximum exists. Unfortunately, if effort and size are substitutes, the incentive to substitute may be strong enough to move the gang towards one of the extremes (high effort, tiny membership or vice versa). To ensure an interior solution, we require that a decline in effort reduces the marginal revenue product of size sufficiently. This, in turn, requires the slightly stronger condition that $\eta > 0$.

Assumptions 1 and 2 are not necessary, but are sufficient to ensure uniqueness of the equilibrium. To see this, consider the (restricted) factor demand functions, $\tilde{e}(M)$ and $\tilde{M}(e)$. These are derived directly from the first order conditions (below), and give the gang’s optimal choice of effort and size respectively, holding the other input constant:

$$r_e(M, \tilde{e}) - \frac{1}{c^*_M (1 - p)} \equiv 0 \quad (13)$$

$$r(M, \tilde{e}) + Mr_M(M, \tilde{e}) - g(e, \tilde{M}) - \frac{\lambda \sigma_M e^*_M (1 - p)}{\lambda \sigma_M e^*_M (1 - p)} \equiv 0 \quad (14)$$
(13) gives the optimal choice of effort, given gang size. The gang trades off the increase in revenue associated with greater effort against the increase in wages it must offer to maintain the indifference of the marginal youth. (14) considers the optimal choice of size. Again, the gang trades off higher revenue against higher costs. By increasing its size, the gang has more youths to pay. Moreover, it involves attracting lower ability youths, necessitating an increase in the wage it offers all its members.

![Diagram showing factor demand with complements and substitutes.](image)

Figure 2: Factor demand with complements and substitutes.

Maintaining the assumption that \( \eta > 0 \), Figure 2 displays the restricted demand functions for complements and substitutes. In both cases, the gang leaders’ equilibrium choices are described by the intersection of the two curves, where their choice of effort is optimal given their size, and their choice of membership size is optimal given their level of effort. It is clear from Figure 2, however, that the comparative statics are different. When membership size and effort are complementary, both demand functions slope upwards.\(^{10}\) In this case, an exogenous increase in, say, the gang’s restricted demand for effort (\( \tilde{e}(M) \) shifts upwards) makes increasing size more profitable (the revenue effect dominates the cost effect). Consequently, both increase concurrently. In contrast, when size and effort are substitutes, both curves slope downwards. An exogenous increase in the gang’s restricted demand for effort makes size less profitable (the cost effect dominates). In this case, the gang optimally reduces size as effort increases.

\(^{10}\)The slopes of the restricted demand curves are derived by a simple application of the Implicit Function Theorem. \( \tilde{e}(M) \) is defined by \( \Pi_e (M, \tilde{e}) \equiv 0 \). Differentiating with respect to \( M \) yields:

\[
\tilde{e}_M = -\frac{\Pi_{Me}}{\Pi_{ee}}
\]

Since \( \eta > -\frac{1}{2} \), \( \Pi_{ee} < 0 \). Under Assumption 1, \( \Pi_{Me} > 0 \), so \( \tilde{e}_M > 0 \). Under Assumption 2, \( \Pi_{Me} < 0 \), so \( \tilde{e}_M < 0 \). An equivalent argument holds for \( \tilde{M}_e \), noting that the slope of the curve in Figure 2 is \( \frac{1}{\tilde{M}_e} \).
5.1 Results for a Generic Policy with a Simple Gang

We can view the impact of a generic policy by considering its effects on the restricted demand functions. When a policy is implemented, it can change youth’s incentives in two ways. Firstly, it may reduce the net benefit they gain from joining the gang. In order to retain members, this may necessitate the gang raising the wage they offer, or reducing the cost of effort they inflict upon their membership. Higher wages increase the marginal cost of size for the gang, as each new youth must be paid more, as shown in (14). Faced with the increased marginal cost, and no equivalent increase in marginal revenue, a profit-maximising gang will reduce its restricted demand for members, $\tilde{M}$ as marginal profit derived from size, $\Pi_M$, becomes negative.

Secondly, a policy may affect how youths respond to changes in effort or gang wages. Some policies increase youths’ sensitivity to criminal activity. Any increases in the level of effort the gang enforces will now require a larger amount of compensation. The marginal cost of effort will increase. As there is no equivalent increase in marginal revenue, the marginal profit derived from effort, $\Pi_e$, will also become negative. In this case, the gang will optimally reduce its restricted demand for effort, $\tilde{e}$.

This intuition, combined with the first panel of Figure 2, leads us very quickly to our first result:

**Proposition 2 (Policy with Simple Complements)** Suppose that $\eta > 0$ and that size and effort are complements. Then any policy which reduces either $\Pi_M$ or $\Pi_e$ and does not increase the other, reduces both the amount of effort gang members employ, and the number of members that the gang chooses to recruit.

![Figure 3: Policy effects with complements.](image)

This is illustrated in Figure 3. If $\Pi_M$ declines, then the restricted demand for size shifts inwards. Similarly, if $\Pi_e$ declines, then the restricted demand for effort shifts down. Since both curves are upward
sloping, any inward shift leads, unambiguously, to a fall in both size and effort. Any fall in, say, size reduces the marginal revenue product of effort (the revenue effect). However, since fewer, more capable, gang members need to be compensated for changes in effort, the marginal cost of effort also falls (the cost effect). If effort and size are complements, the fall in marginal revenue exceeds the fall in marginal cost, and the gang reduces its optimal effort level. In turn, this causes a further reduction in size. These endogenous effects reinforce the decline in both size and effort, leading to the result. We can therefore conclude that, if membership size and effort are sufficiently strong revenue complements, any policy will be effective in reducing the loss society suffers at the hands of the gang. This results in a smaller social loss.

Unfortunately, the case with substitutes is not so clear cut, as shown in Figure 4. In contrast to complements, reductions in size cause the gang to substitute towards effort, and vice versa. As before, a fall in say, size, reduces both the marginal revenue product and the marginal cost of effort. However, the decline in marginal cost exceeds the decline in marginal revenue, causing an endogenous increase in the restricted demand for effort. Similarly, a decline in effort makes size more profitable, causing an endogenous increase in the restricted demand for size. Consequently, if one of these effects were to offset the initial impact of the policy, we could have a situation in which either membership size or effort increases (but, fortunately, not both). It may even be possible that a policy designed to reduce the social loss from organised crime may actually increase it.

**Proposition 3 (Policy with Simple Substitutes)** Suppose that \( \eta > 0 \) and that size and effort are substitutes. Consider any policy which reduces either \( \Pi_M \) or \( \Pi_e \) and does not increase the other. There exist thresholds, \( \eta^e \) and \( \eta^M \), such that:

1. If \( \eta^M < \eta^e \) and \( \eta < \eta^e \), the policy reduces effort, but increases size.
2. If \( \eta > \max\{\eta^e, \eta^M\} \) then the policy reduces both size and effort.
3. If \( \eta^e < \eta^M \) and \( \eta < \eta^M \), then the policy reduces size, but increases effort.

In order for a policy to inadvertently increase either size or effort, we need that the substitution effects are sufficiently large. These are determined by the relative size of the revenue and cost effects. Proposition 3 states that, for each given policy, there exists an upper bound on the magnitude of the revenue effect (equivalently, on the degree of revenue complementarity) such that the endogenous increase in an input following the introduction of a policy exceeds its exogenous reduction.

In case one, introducing policy \( \phi \) has a much greater direct effect on size than effort, as shown in the first panel of Figure 4 (case three is analogous, but with a greater direct effect on effort). Since
demand for size declines so much, the marginal cost of effort also declines substantially. For any given revenue effect, this creates a greater incentive to increase effort. Conversely, since demand for effort declines relatively little as a direct result of the policy, the marginal cost of size is relatively unaffected. For any given revenue effect, there is little incentive to increase size. We have that $\eta_M^e < \eta_e^e$. If $\eta$ is below $\eta_M^e$, the reduction in the marginal cost of effort is sufficient to completely reverse the direct effect of the policy on effort. As effort subsequently increases, the gang begins to substitute away from size, further increasing the marginal profitability of effort.

Conversely, in case two, the direct effects of the policy are relatively balanced (second panel of Figure 4). The resulting reductions in the marginal cost of either input are not sufficient to cause an overall increase in either size or effort. Despite having substitutes, the revenue effect is still sufficiently strong to ensure that the endogenous effects are small.

5.2 Results for Specific Policies with a Simple Gang

The direct effect of each of class of policy is to reduce the marginal profit of either effort, size or both inputs. As such, under Assumption 1, Proposition 2 holds, and both effort and size are unequivocally diminished. The loss society suffers as a result of the gang will always decline. For each policy, we will therefore focus on what happens under Assumption 2. With substitutes, a fall in demand for one input causes an increase in demand for the other, counteracting the immediate effects of the policy. The social loss the gang inflicts may therefore increase.

5.2.1 Severity of Punishment ($f$)

When severity increases, the marginal cost of size increases (from (14)). Recruits require a greater degree of compensation for the possibility of being punished, increasing the wage the gang offers for any given gang size. As a result, the gang’s restricted demand for members falls ($\Pi_M f < 0$). However, if they
still decide to join the gang, greater severity of punishment has no impact upon members’ willingness to acquire criminal skills (given by (5)). For each given gang size, the criminal skills acquired by the marginal youth remain unchanged. Consequently, there is no exogenous change in the gang’s marginal cost of effort. By (13), the gang’s restricted demand for effort remains unchanged ($\Pi_{ef} = 0$).\footnote{If we allowed $f$ to depend positively on $e$, this would no longer be the case. Instead, the restricted demand curve for effort would shift in.} We are firmly in case three:

**Corollary 1** Suppose that $\eta > 0$ and that size and effort are substitutes. Then any increase in the severity of punishment will result in fewer gang members, each of whom applies more effort.

If society suffers sufficiently from increases in the intensity of effort that the gang chooses to inflict, then this policy could lead to an increase in the social loss the gang creates.

### 5.2.2 Primary Labour Market Policies ($w$)

The effect of an increase in the market wage is similar to that of an increase in the severity of punishment. The opportunity cost of joining the gang is increased. The gang must offer higher wages at every gang size, increasing the marginal cost of size. Consequently, by (14), the restricted demand for membership size declines ($\Pi_{Mw} < 0$). Upon deciding to join the gang, youths’ incentives to acquire criminal skill are unaffected by the increase in $w$. As a result, by (13), the restricted demand for effort once again remains the same:

**Corollary 2** Suppose that $\eta > 0$ and that size and effort are substitutes. Then any improvement in the primary labour market will result in fewer gang members, each of whom applies more effort.

Again, if society is particularly sensitive to changes in the intensity of effort, relative to changes in size, labour market policies could actually increase the social cost of the gang.

### 5.2.3 Arrest and Conviction Rate ($p$)

The arrest and conviction rate has the most complex effect of any of the policies considered. An increase in the probability of conviction reduces the restricted demand for membership size for three reasons. Not only does it become more likely that gang members will be punished (increasing $pf$), but the probability that they will be deprived of their wages also rises. Youths discount for this in (10), and consequently require even more pay in order to join. Moreover, maintaining the same size of gang involves recruiting lower ability members, as more members are locked away (and are thus unproductive). Since lower
ability individuals are more sensitive to effort, a third increase in the wage the gang offers is required. This is Levitt’s incapacitation effect. Combined, these three wage increases raise the marginal cost of size in (14). The restricted demand for size declines ($\Pi_{Mp} < 0$).

In contrast to the previous two policies, increasing the probability of conviction also reduces the restricted demand for effort. For given gang size, the ability of the marginal youth is lower. The marginal youth is thus relatively sensitive to changes in effort ($c_M^*$ falls). Any increase in effort thus requires a greater increase in the gang wage. In addition, members incur the cost of effort irrespective of whether they are arrested or not. Since there is a greater chance that they will not receive their wages, they require proportionally more compensation should the amount of effort they apply increase. Both these effects increase the marginal cost of effort in (13), reducing the restricted demand for effort ($\Pi_{ep} < 0$).\(^{12}\)

In sum, both the restricted demand curves shift inward. Depending upon the size of the shifts, any of the cases outlined in Proposition 3 appear feasible. This turns out not to be the case, as we have the following result:

**Corollary 3** Suppose that $\eta > 0$ and that size and effort are substitutes. Then any improvement in the arrest and conviction rate may result in:

1. Fewer gang members, each of whom applies more effort; or
2. Fewer gang members, each of whom applies less effort.

It can never be the case that more gang members, each of whom applies less effort result.

The result hinges upon the revenue effect. It can be shown that $\eta_{e} > 0 > \eta_{M}$ so that we can immediately rule out the possibility that the gang increases its membership. The increase in the marginal cost of size always dominates the increase in marginal revenue, even taking endogenous effects into account.

Relative to the previous two policies, increasing the arrest and conviction rate is relatively successful at reducing the loss society suffers. Whilst it may be the case that effort increases, this only happens when size and effort are extremely strong profit substitutes. Otherwise, even under Assumption 2, both size and effort decline, leading to an unambiguous fall in the social loss.

5.2.4 Prevention of Juvenile Crime ($k$)

As with the conviction rate, preventing juvenile crime impacts upon both the restricted demand for size and the restricted demand for effort. Youths find it more difficult to acquire criminal skills (from (5)).

\(^{12}\)If we allowed $p$ to depend positively on $e$, this would also contribute to the decline in the restricted demand for effort.
Members thus suffer more from the effort the gang requires them to apply. They require a larger wage for each level of effort in order to retain their membership. Any attempt by the gang to increase its membership therefore require a larger increase in the wage than before the introduction of the policy. Both the higher wage and the larger wage increase required to recruit more members increase the marginal cost of size for the gang (in (14)), reducing its restricted demand ($\Pi_{Mk} < 0$).

Since all youths acquire fewer criminal skills, the marginal cost of effort increases as well. In particular, for each gang size, increasing effort whilst retaining the marginal youth is more expensive. As the marginal revenue product of effort is unaffected by the policy, the increased marginal cost induces the gang to reduce its restricted demand for effort as well ($\Pi_{ek} < 0$).

Once more, it appears that all three cases in Proposition 3 are feasible. Both restricted demand curves have shifted inwards. Again, this turns out to be incorrect:

**Corollary 4** Suppose that $\eta > 0$ and that size and effort are substitutes. Then any improvement in the prevention of juvenile crime may result in:

1. *Fewer gang members, each of whom applies more effort; or*

2. *fewer gang members, each of whom applies less effort.*

*It can never be the case that more gang members, each of whom applies less effort result.*

Once again, it is straightforward to show that $\eta_{ek} > 0 > \eta_{Mk}$, so the gang never increases its size. It is possible that its members will increase effort. Tackling juvenile crime always has a greater effect on the demand for size than for effort.

Prevention of juvenile crime also proves to be relatively effective at reducing the loss society suffers. Once again, size and effort need to be extremely weak revenue complements to cause an increase in gang effort. It is quite possible that both will decline as a result of the policy, reducing the social cost of the gang.

### 6 The Separating Gang

The analysis performed in the previous section was done under the assumption that the gang was unable to discriminate between individuals. I now relax that assumption, and instead ask how the various policies perform when the gang is free to implement any incentive compatible wage scheme and associated effort schedule focusing, of course, on direct mechanisms.
The gang leaders’ decisions are significantly more complicated. They must now choose a profit-maximising contract schedule, subject to its being implementable. The form of an implementable contract schedule is given by the following:

**Proposition 4 (Implementable Contracts)** A contract schedule, \( \{(g(\sigma), e(\sigma))\}_{\sigma \geq 0} \), is implementable if and only if it is of the form:

\[
g(\sigma) = \frac{w + pf}{1 - p} + \frac{e(\sigma)}{c^* (1 - p)} + k \frac{C}{1 - p} \left( \frac{c^*}{\sigma} \right) + \frac{1}{1 - p} \int_{s=\sigma_M}^{\sigma} \frac{e(s)}{sc^*} ds
\]  

(15)

**Proof.** See Appendix B. 

This relationship between wages and effort bears a tremendous similarity to that of the marginal youth in a simple gang, given by (10). The first three terms simply state that each youth must be compensated for the expected costs incurred by joining the gang. The difference arises in the final term of (15), which constitutes informational rent.

![Figure 5: An example of an implementable contract.](image)

An example of an implementable contract schedule is shown in Figure 5, displaying indifference curves for two youths. Higher ability youths are less sensitive to effort. As such, less compensation is required when effort is increased: their indifference curves are steeper. The contract schedule is designed so that each youth weakly prefers the contract designed for their ability to all others. For example, the youth with ability \( \sigma_1 > \sigma_0 \) prefers \((g_1, e_1)\) to \((g_0, e_0)\).

An important feature of an implementable contract is made clear by Figure 5: wages and effort must both be increasing in ability. For a youth with ability \( \sigma_1 \) to prefer a bundle \((g_1, e_1)\) to \((g_0, e_0)\) it must lie to the south-east of the \( \sigma_1 \)-indifference curve passing through \((g_0, e_0)\). Similarly, for a youth with ability \( \sigma_0 \) to prefer \((g_0, e_0)\) over \((g_1, e_1)\), \((g_1, e_1)\) must lie to the north-west of the \( \sigma_0 \)-indifference curve.
curve passing through \((g_0, e_0)\). Only contracts in the shaded region satisfy both properties, so \(g\) and \(e\) must both be increasing in ability.

Restricting attention to implementable contracts, we now turn our attention to the gang’s profit maximisation decision. Since the gang’s choice of effort uniquely determines the wage it must pay to its members, it is sufficient once again to think of the gang as maximising profits with respect to \(\{e(s)\}_{s \geq 0}\) and \(M\). Its optimal choice of \(g(s)\) will then be given by (15). In other words, the gang solves:

\[
\left( (e^* (s))_{s \geq 0}, M^* \right) = \arg \max_{(e(s))_{s \geq 0}, M \geq 0} \left\{ N (1 - p) \int_{s = \sigma_M}^{\infty} \pi (s, M) \lambda \exp \{ - \lambda s \} ds \right\} \tag{16}
\]

subject to : (15)

The solution is described in two stages. Firstly, for each ability and each gang size, I describe the optimal choice of effort. This provides a restricted effort schedule, dependent upon the gang’s membership size \((\tilde{e}(s, M))_{s \geq 0}\). Then, incorporating this restricted effort schedule into (16), the gang chooses membership size to maximise its profits. Without further ado:

**Proposition 5 (Restricted Effort Schedule)** Suppose that, for each \(\sigma\) and each \(M\), \(\eta(M, \cdot) > -\frac{1}{2}\) and \(\varepsilon'(\cdot) > 0\). Then there exists a unique effort schedule, \((\tilde{e}(s, M))_{s \geq 0}\) that maximises profits.

**Proof.** See Appendix C. \(\blacksquare\)

For each gang size and each \(\sigma \geq \sigma_M\), the gang selects \(\tilde{e}(\sigma)\) to satisfy:

\[
r_e (M, \tilde{e}(\sigma)) - \frac{1}{c^* (1 - p)} - \frac{1}{c^* (1 - p) \lambda \sigma (2 + \varepsilon (c^*, \sigma))} \equiv 0 \tag{17}
\]

where \(c^* = c^*(\sigma; \sigma)\). This expression is very similar to (13). The marginal benefit to the gang of increasing a member’s effort comes in the form of additional revenue they will be able to generate. \(\eta > -\frac{1}{2}\) ensures the marginal revenue product is declining in effort. The marginal cost comprises two elements. Firstly, it is necessary to compensate the individual for the disutility they suffer from greater activity. The second element of the marginal cost represents the need to increase the informational rent paid to members. Increasing effort for a youth with ability \(\sigma\) increases the informational rent to all members with ability greater than \(\sigma\). The rent paid to those with lower ability is unaffected (see (15)). \(\varepsilon' > 0\) ensures that the marginal cost is increasing in effort.

Substituting the restricted effort schedule into (16), we are now in a position to calculate the gang’s optimal size.
Proposition 6 (Profit Maximisation with Separation) Suppose that Proposition 5 is satisfied, and that \(r_{MM} > 0, r_{MMe} < 0, r_{Me} r_{Mee} < 0\) and \(\pi_{ee} < 0\). Then there exists a unique gang size, \(0 < M^* < N\), that maximises profits.

**Proof.** See Appendix D.

Given an effort schedule, the gang choose a membership size to satisfy:

\[
r(M, e(M)) - g(M) + N (1-p) \int_{s=M}^{\infty} r_M(M, e(s)) \lambda \exp{-\lambda s} ds - \frac{e(M)}{\lambda M c^*(M)} (1-p) \equiv 0
\]

(18)

where \(c^*_M = c^*(\sigma_M, \sigma_M)\). When the gang increases its size, its new members generate revenue equal to \(r(M, e(M))\). The marginal cost has several components. First, each new member must be paid. Their wage is given by (15), but does not need to incorporate any informational rent since a youth would never choose a contract designed for someone with a higher ability. Secondly, since the aggregate revenue function has diminishing marginal returns, it must be the case that an increase in size reduces the revenue each inframarginal member is able to generate. Finally, the gang must increase the informational rent paid to all other members.

### 6.1 Generic Policy with a Separating Gang

In the previous section, the extent to which effort and membership size were revenue complements was critical in determining the effects of policy. A similar intuition holds when the gang is capable of separating out recruits. When gang size increases, there are two opposing effects on the marginal revenue product of effort. Firstly, as there are more members applying effort, aggregate revenue from effort increases. The increased membership will also impact upon the personal marginal revenue products of effort of those already in the gang. Gang members may be able to take advantage of network externalities and returns to scale, increasing their marginal revenue products of effort (network effects). Conversely, congestion effects may reduce the marginal revenue product, as more individuals are attempting to extract rents from the neighbourhood. So long as the aggregate effect dominates the individual ones, we are consistent with the model of the previous section (\(\eta(M, e(s_i)) > -1\)).\(^{13}\) However, the degree of revenue complementarity will be strongly affected by whether the network or congestion effects dominate at the individual level. This leads us to make one of two assumptions:

**Assumption 3 (Separating Complements)** Network effects dominate congestion effects, so that each gang member’s marginal revenue product of effort is strictly increasing in the gang’s size: \(\eta(M, e(s_i)) > -1\).\(^{13}\)

\(^{13}\)In fact, equilibrium with a simple gang required that the network effects dominated the congestion effect.
Assumption 4 (Separating Substitutes) Network effects are dominated by congestion effects, so that each gang member’s marginal revenue product of effort is strictly decreasing in the gang’s size: 

\[-1 < \eta(M, e(s_i)) < 0.\]

The consequences of these assumptions are shown in Figure 6. If Assumption 3 holds, then size and effort are very strong revenue complements. When the gang’s membership increases, additional members enable existing members to take advantage of network externalities. This makes effort more profitable for each member. Their individual restricted demand for effort is increasing in gang size. Similarly, the marginal revenue product of size is increasing in the restricted demand for effort of each individual. If Assumption 4 holds, on the other hand, size and effort are weak revenue complements. As size increases, the neighbourhood becomes satiated and existing gang members see their personal returns to effort fall. The individual restricted demand for effort is decreasing in gang size. Moreover, an increase in the level of effort each member applies reduces the marginal revenue product of size.

Note that Assumption 4 combined with the conditions in Proposition 6 imply that the degree of revenue complementarity, \(\eta\), is decreasing in size for each effort level. Larger criminal organisations (in the sense that they recruit low skilled criminals) are more likely to view size and effort as substitutes, even in this more complicated environment.

The impact of a generic policy, \(\phi\), is similar to that discussed in Section 5. Each policy still acts to increase the gang’s marginal cost of membership size and (in certain cases) effort. For a given effort schedule, policies reduce the surplus members receive from the gang. Any increase in membership necessitates paying a higher wage than before. This is compounded by the need to maintain truthful revelation. If the gang offers higher wages to new (low ability) members, and do not ask them to apply more effort, then there will exist a higher ability member who will strictly prefer to accept the contract offered to the new members. The gang must therefore increase the informational rent paid to all its members. Given this increase in marginal cost, the gang will optimally choose to reduce its size.

Policy may also affect members’ response to changes in effort. In contrast to the previous model, however, each member applies different levels of effort. Assuming that a policy affects every member in a similar way, each youth will require an increase in their wages to compensate them for increases in effort. For given gang size, the marginal revenue product of effort is unaffected. So, once again, the gang will optimally reduce effort levels.

The overall effect also depends upon the strength of revenue complementarity between membership size and effort. If they are strong complements, we have the following result, equivalent to Proposition
Proposition 7 (Policy with Separating Complements) Suppose the conditions given in Proposition 6 hold and size and effort are complements. Then any policy which reduces either $\pi_e$ or $\Pi_M$ and does not increase the other reduces both the amount of effort each member of the gang applies and the number of members the gang chooses to recruit.

The proposition is illustrated in Figure 7. Consider a policy that reduces the gang’s size by increasing the marginal cost of size. Under Assumption 3, gang members are no longer able to take advantage of economies that were previously available to them. As a result, the marginal revenue product of effort declines for each gang member. The marginal cost of effort is unaffected by changes in size. Consequently, $\pi_e < 0$ for every gang member, and the gang chooses to reduce effort levels.

Now consider a policy that reduces the amount of effort the gang’s members apply for each $M$. Each individual’s restricted demand for effort shifts down. By the Envelope Theorem, the only effect on the marginal profitability of size manifests itself in a decline of the individual marginal revenue products of
size of the inframarginal recruits. The \( MRP \) curve also shifts down. As a result, \( \Pi_M < 0 \) and the gang chooses to reduce its size.

These two endogenous effects reinforce declines in both gang size and effort, giving rise to the result in Proposition 7. If Assumption 3 is satisfied, just as before, any policy will be effective at reducing the loss society suffers at the hands of the gang.

![Figure 7: Policy effects with separation and complements.](image)

If increases in membership cause congestion, effort and membership are relatively weak revenue complements. As with the simple gang environment, this makes the policy effects much more difficult to predict, as shown in Figure 8. Consider a policy that increases the marginal cost of size. The gang optimally reduces the number of youths it recruits, increasing the marginal revenue product of effort for each gang member. With fewer members, each individual is able to extract greater rents from the neighbourhood by employing effort. Thus reducing size causes an endogenous increase in effort.

Similarly, any policy which reduces individuals’ restricted demand for effort increases the marginal revenue product of size. With the neighbourhood less satiated with criminal activity, there are greater opportunities for new members to generate revenue. The gang will opt to endogenously increase its membership.

When a policy is implemented that directly reduces both size and effort, these endogenous effects counteract the initial fall in both inputs. The final outcome is not immediately clear, as formalised in the following proposition, equivalent to Proposition 3 in Section 5:

**Proposition 8 (Policy with Separation and Substitutes)** Suppose the conditions given in Proposition 6 hold and size and effort are substitutes. Consider any policy which reduces either \( \pi_e \) or \( \Pi_M \) and does not increase the other:

1. If \( \Pi_{M}\phi < M \int_{s=\sigma_M}^{\infty} \pi_{e\phi} |e_M| \lambda \exp\{-\lambda (s - \sigma_M)\} ds \) then the policy reduces the amount of effort each member of the gang applies, but increases size.
2. If $\Pi_{M\phi} > M \int_{s=\sigma_M}^{\infty} \pi_{e\phi} e_M \lambda \exp\{-\lambda(s - \sigma_M)\} ds$ then the policy reduces the size of the gang, and:

(a) Individual gang members for whom $\frac{\pi_{e\phi}}{r_{M\phi}} > M\phi$ apply less effort.
(b) Individual gang members for whom $\frac{\pi_{e\phi}}{r_{M\phi}} < M\phi$ apply more effort.

Size is defined by $\Pi_M = 0$. The marginal profitability of size increases as a result of $\phi$ if and only if:

$$\frac{d\Pi_M}{d\phi} = \Pi_{M\phi} + M \int_{s=\sigma_M}^{\infty} \pi_{e\phi} e_M \lambda \exp\{-\lambda(s - \sigma_M)\} ds > 0 \quad (19)$$

The first term is negative, since $\phi$ reduces profitability directly. The second term is positive since $\phi$ reduces the marginal profitability of effort, and effort and size are substitutes. This yields the first condition in the proposition.

As different members engage in different levels of effort, the effect of policy on effort is more complex. In particular, there is no such thing as $\tilde{M}_e$. Instead, consider the effect on the marginal profitability of effort of policy for a given level of effort:

$$\frac{d\pi_e}{d\phi} = \pi_{e\phi} + r_{M\phi} M\phi \quad (20)$$

When a policy is implemented, its direct effect may incorporate an immediate decline in the profitability of effort. However, if size also declines, the neighbourhood becomes less congested. This enables each member to generate more revenue through effort. The marginal cost of each member increasing effort is unaffected by size, as changes in effort only affect the informational rents of those members with higher ability. If the decline in gang size is sufficiently large, the increase in the marginal revenue product of effort may dominate the fall in profitability caused by the policy. The gang optimally increases the member’s effort. The conditions given in Proposition 8 are thus precisely those that dictate whether the marginal profitability of effort increases or decline.

Each case is illustrated in Figure 8. The direct effects of the policy are both a decline in the restricted demand for effort for each individual, and an increase in the marginal cost of size. The decline in effort stimulates an increase in the marginal revenue product of effort, and it is this (relative to the change in marginal cost) that leads to the different outcomes. In case 1, the marginal revenue product endogenously increases by more than the increase in marginal cost. The gang optimally increases its size. However, this creates additional congestion. The marginal revenue product of effort declines for each individual, further reducing the amount of effort they employ.
Case 1: Larger gang with less effort

Case 2: Smaller gang with: (a) less effort; or (b) more effort

Figure 8: Policy effects with separation and substitutes.

In case two, the increase in the marginal revenue product of size is insufficient to cause the gang to recruit more members. Congestion is reduced, increasing the marginal revenue product of effort for each individual. Individuals whose marginal cost of effort was strongly affected by the policy, or who have are relatively unaffected by congestion optimally reduce the extent of their effort (Case (a)). Otherwise, they increase it (Case (b)).

6.2 Results for Specific Policies

When the gang can discriminate between its members, and Assumption 3 holds, the effects of any of the policies I consider are unambiguous. Since all policies’ direct effects include reducing the marginal profit generated by membership size, size declines. This causes a reduction in the marginal revenue product of effort for all members, in turn reducing effort. Proposition 7 holds. Under Assumption 4, the results are less clear, and are outlined below.
6.2.1 Severity of Punishment ($f$)

Increasing the severity of punishment only affects the restricted demand for size. It increases the expected cost of joining the gang. However, once a youth has decided to join, it leaves their incentive to acquire criminal skills (and hence their sensitivity to effort) unaltered.\footnote{Again, if we allowed $f(e)$ with $f'(e) > 0$, an increase in $f'(e)$ would increase youths' sensitivity to effort.} In order to maintain the size of its membership, the gang must increase the wages it pays to all of its members for a given effort schedule (see (15)). The marginal cost of size has increased, as shown in (18). The gang chooses to reduce its membership. As effort is unaffected, we are firmly in case 2(b):

**Corollary 5** Suppose the conditions given in Proposition 6 hold and size and effort are substitutes. Then any increase in the severity of punishment will result in fewer gang members, but each member will increase effort.

As in Section 5, if society is relatively sensitive to increases in criminal effort, introducing more severe punishment across the board could lead to an increase in the damage the gang inflicts on society. Whilst membership does decline, reducing the loss, each remaining member may become increasingly active. This could more than offsets these gains.

6.2.2 Primary Labour Market Policies ($w$)

The effect of improvement in the primary labour market is, once again, identical to an increase in the severity of punishment. As the opportunity cost of joining the gang increases, the gang must offer the marginal youth a higher wage, given the effort schedule. This increases the marginal cost of size in (18), whilst having no effect upon its marginal revenue product. The gang optimally reduces its size:

**Corollary 6** Suppose the conditions given in Proposition 6 hold and size and effort are substitutes. Then any improvement in the primary labour market will result in fewer gang members, but each member will increase effort.

Unsurprisingly, even under this more complicated wage setting, improvements in the primary labour market can also yield greater social losses from the gang.

6.2.3 Arrest and Conviction Rate ($p$)

When there is an increase in the arrest and conviction rate, the marginal cost of size increases. In a similar manner to the previous two policies, the expected cost of joining the gang has increased, as it is more likely that an individual will be punished ($pf$ is higher in (15)). Furthermore, if members are...
caught, their wages are confiscated. When deciding upon whether to join the gang, they discount their wages for this possibility, and consequently require higher wages to encourage them into a criminal career. The gang chooses to reduce its size.

The marginal cost of effort also increases for every youth. Gang members are only arrested after committing crime. However, it is during the commission of crime that they apply effort. Irrespective of whether they are caught, they therefore incur the cost of effort. The compensation they receive for doing so, on the other hand, is conditional on their evading arrest. When the arrest rate increases, and gang members discount their wage further, any increase in effort requires a more substantial increase in pay. The gang to reduce levels of effort.\textsuperscript{15}

Corollary 7 Suppose the conditions given in Proposition 6 hold and size and effort are substitutes. Then any improvement in the arrest and conviction rate may result in:

1. More, less active gang members;

2. Fewer gang members, some of whom may become more active, others of whom may become less active.

Unlike in the case of the simple gang, we are unable to rule out the possibility that the gang increases its size. Sufficed to say that, in comparison to increasing the severity of punishment or the primary labour market wage, improvements in the arrest and conviction rate are more likely to result in a reduction in the social loss caused by the gang’s activities.

6.2.4 Prevention of Juvenile Crime ($k$)

Increases in the prevention of juvenile crime reduces the restricted demand for size. The cost of acquiring criminal skills increases. For a given effort schedule, every prospective member invests less, and consequently suffers a greater disutility from the effort they are forced to apply. The marginal youth is particularly affected. As the youth with the lowest intrinsic ability, they are more sensitive to changes in the cost of acquiring criminal skills, and consequently reduce their skills dramatically. The cost of retaining their membership increases. They require higher wages. This leads to increases in the informational rent paid to all inframarginal members. The marginal cost of size increases. The gang optimally reduces the number of members it recruits.

Concurrently, the policy also reduces the restricted demand for effort. Each member suffers a greater disutility from effort. Moreover, they also become more sensitive to changes in effort. Each

\textsuperscript{15}This effect would be enhanced if $p$ depended upon effort.
youth therefore requires a greater level of compensation for any changes in effort levels. The gang reduces every element of its effort schedule:

**Corollary 8** Suppose the conditions given in Proposition 6 hold and size and effort are substitutes. Then any improvement in the prevention of juvenile crime may result in:

1. More, less active gang members;

2. Fewer gang members, some of whom may become more active, others of whom may become less active.

Again, it is not possible to rule out the gang increasing its membership. In spite of this, increasing efforts to prevent juvenile crime could yield situations in which both the gang’s size and the effort levels decline.

### 7 Conclusions

Over recent years, numerous policies have been put forward to combat the social loss associated with crime. These policies aim to decrease individuals’ incentive to engage in crime and, in doing so, reduce the amount of crime that occurs. However, when applied to neighbourhoods where organised crime is prevalent, this argument breaks down. Instead, the policy may increase the loss society suffers at the hands of organised crime.

This paper has shown the effects of several popular policies in such an environment, and developed an intuitive way to assess other policies. As criminal organisations tend to operate within a well-defined geographical territory, they act as a monopsonist employer for all criminals within that territory. Irrespective of whether the organisation operated a single wage or more complicated recruitment strategy, results were shown to be robust.

The effects of policy depend upon the degree of complementarity between inputs in the criminal organisation’s revenue function. If they are strong complements, policies that one input reduce the marginal profitability of the other. Both size and effort decline. Conversely, if they are weak revenue complements, the organisation may choose to substitute between size and effort, possibly undoing some of the effects of the policy. In this case the loss society suffers may increase.

When there is an incentive to substitute, policies which simply increase the opportunity cost of joining a criminal organisation, such as improved labour market wages fail badly. As they do not affect youths’ incentive to acquire criminal skill, they actually reduce the marginal cost of effort. Those who
chose to remain in the organisation after the policy is implemented are highly skilled. They do not require as much compensation when effort is intensified. As such, criminal organisations will always choose to increase effort, at the expense of membership.

Other policies prove more effective. Prevention of juvenile crime and improved arrest or conviction rates may cause an intensification of effort, but only in relatively extreme circumstances. Otherwise, these policies diminish both the organisation’s size and effort. Preventing juvenile crime not only increases the opportunity cost of joining a criminal organisation, but also reduces the incentive to acquire criminal skill. By doing so, it increases not only the marginal cost of acquiring members, but also the marginal cost of effort. Improving arrest rates have a similar effect. As youths may be prevented from receiving their wages, they require more compensation \textit{ex ante} for effort. As such, the marginal cost of effort once again increases. If the degree of substitutability between size and effort is particularly large, then the criminal organisation may still choose to substitute away from size towards effort. Otherwise, it will reduce both its size and effort.

In summary, anti-crime policies are most effective against organised crime when they not only reduce the incentive of youths to join the organisation, but also hamper its ability to induce effort.

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\textbf{A Proof of Proposition 1}

Firstly, note that $M = N$ is never profit maximising, as it involves the gang paying infinitely large wages. Also, if $e^* = 0$ or $M^* = 0$, equilibrium profit for the gang, $\Pi^*$, is non-positive. So, to prove that the gang will operate with positive $e$ and $M$, it will be necessary to show that positive profits will result.
Now, the first order conditions for profit maximisation are:

\[ \Pi_e = Mr_e (M^*, e^*) - \frac{M^*}{c^*_M (1-p)} = 0 \]

\[ \Pi_M = r (M^*, e^*) + Mr_M (M^*, e^*) - g (M^*, e^*) - e^* \frac{\lambda \sigma_M e^*_M (1-p)}{\lambda \sigma_M (1-p)} = 0 \]

Given that the revenue function has constant returns to scale, \( r + Mr_M = ce^* (2r_M + Mr_M) \).

Substituting appropriately, this yields second-order conditions:

\[ \Pi_{MM} = - \frac{e^*}{M^*} \frac{1}{M^* c^*_M (1-p)} \left[ \eta + 1 + \frac{1}{\lambda \sigma_M} \left( 1 + \frac{1}{\lambda \sigma_M} + \frac{1}{\lambda \sigma_M 2 + e_M} \right) \right] \]

\[ \Pi_{Me} = \frac{1}{M^* c^*_M (1-p)} \left[ \eta - \frac{1}{\lambda \sigma_M} \frac{1 + e_M}{2 + e_M} \right] \]

\[ \Pi_{ee} = - \frac{M^*}{e^*} \frac{1}{M^* c^*_M (1-p)} \left[ \eta + 1 - \frac{1}{2 + e_M} \right] \]

Necessary and sufficient conditions for a maximum are that \( \Pi_{MM} < 0, \Pi_{ee} < 0, \) and \( \Pi_{ee} \Pi_{MM} - \Pi^2_{Me} > 0 \). The first of these conditions is satisfied unambiguously upon inspection, since \( r + Mr_M > 0 \).

The second condition is satisfied if and only if \( \eta > -\frac{1}{2} \).

Thirdly, we require that \( \Pi_{ee} \Pi_{MM} > \Pi^2_{Me} \). With relatively little work, it can be shown that this is satisfied if and only if:

\[ \eta > \frac{1 + \frac{1}{\lambda \sigma_M} (1 + \frac{e_M}{2 + e_M}) + \frac{1}{\lambda \sigma_M} \left( 1 + \frac{e_M}{2 + e_M} \right)}{2 - \frac{1}{\lambda \sigma_M} (1 + \frac{e_M}{2 + e_M}) + \frac{1}{\lambda \sigma_M} \left( 1 + \frac{e_M}{2 + e_M} \right)} - 1 \]

A sufficient condition is that \( \eta > 0 \). So any point where both first order conditions are satisfied constitutes a local maximum. Rearranging these conditions yields:

\[ \left| \frac{\Pi_{Me}}{\Pi_{ee}} \right| < \left| \frac{\Pi_{MM}}{\Pi_{Me}} \right| \]

Note that these inequalities do not simply hold at a point of profit maximisation - they hold everywhere. Therefore, assuming that the sign of \( \Pi_{Me} \) never changes, any profit maximising point will be unique.

Finally, it remains to show that the profit derived by the gang in any such equilibrium is positive.

From the first-order conditions, we have that \( e^* M^* r_e (M^*, e^*) = \frac{M^* e^*}{c^*_M (1-p)} \) and \( M^* (r (M^*, e^*) + Mr_M (M^*, e^*) - g (M^*, e^*) - e^* \frac{\lambda \sigma_M e^*_M (1-p)}{\lambda \sigma_M (1-p)} = 0 \). Noting that the gang level revenue function is homogeneous of degree one,
it is clear that:

$$II^* = M^* e^* (1 + \ln N + \ln (1 - p) - \ln M^*) > 0$$

This completes the proof.

**B Proof of Proposition 4**

A contract is implementable if it is incentive compatible and individually rational. Considering first the issue of incentive compatibility, a youth has a strict incentive to truthfully reveal their type if:

$$\sigma = \arg \max_{s \geq 0} \left\{ (1 - p) g(s) - p f - \frac{e(s)}{c^*} - k C \left( \frac{c^*}{\sigma} \right) \right\}$$

where $c^*$ is a function of both $\sigma$ and $e$. Taking first-order conditions, this is equivalent to:

$$(1 - p) \frac{\partial g}{\partial s} (\sigma) = \frac{1}{c^*} \frac{\partial e}{\partial s} (\sigma)$$

Integrating both sides over the range $[\sigma M, \sigma]$ yields:

$$g(\sigma) = g(\sigma M) + \frac{e(\sigma)}{c^*_M (1 - p)} - \frac{e(\sigma_M)}{c^*_M (1 - p)} + \frac{1}{1 - p} \int_{t=\sigma_M}^{\sigma} \frac{V(t)}{c^*} \frac{\partial c^*}{\partial t} dt$$

Now, for $\sigma_M$ to be the marginal youth, it must be the case that $G(\sigma_M, \sigma_M) = w$. Otherwise, if $G(\sigma_M, \sigma_M) > w$, a lower ability youth will be able to gain a larger payoff by joining the gang and sending signal $s_i = \sigma_M$, contradicting the fact that $\hat{\sigma}$ is the marginal youth. On the other hand, if $G(\sigma_M, \sigma_M) < w$, then the marginal youth would strictly prefer to join the primary labour market, again providing a contradiction. So:

$$g(\sigma_M) = \frac{w + p f}{1 - p} + \frac{e(\sigma_M)}{c^*_M (1 - p)} + \frac{k}{1 - p} C \left( \frac{c^*_M}{\sigma_M} \right)$$

Also, we have that:

$$\frac{\partial}{\partial \sigma} \left( k C \left( \frac{c^*}{\sigma} \right) \right) = \frac{k}{\sigma} C' \left( \frac{c^*}{\sigma} \right) \frac{\partial c^*}{\partial \sigma} - \frac{kc^*}{\sigma^2} C'' \left( \frac{c^*}{\sigma} \right)$$

$$= \frac{e(\sigma)}{c^*} \frac{\partial c^*}{\partial \sigma} - \frac{kc^*}{\sigma^2} C'' \left( \frac{c^*}{\sigma} \right)$$
So, by (5):
\[
\int_{t=\sigma_M}^{\sigma} \frac{e(t)}{\sigma^2} \frac{\partial c^*}{\partial t} \, dt = k \int_{t=\sigma_M}^{\sigma} \frac{\partial}{\partial t} \left( C \left( \frac{c^*}{t} \right) \right) \, dt + \int_{t=\sigma_M}^{\sigma} \frac{kc^*}{t^2} C' \left( \frac{c^*}{t} \right) \, dt
\]
\[
= kC' \left( \frac{c^*}{\sigma} \right) - kC \left( \frac{c^*_M}{\sigma_M} \right) + \int_{t=\sigma_M}^{\sigma} \frac{e(t)}{tc^*} \, dt
\]

Substituting, we have that:
\[
g(\sigma) = \frac{w + pf}{1 - p} + \frac{e(\sigma)}{c^*(1 - p)} + \frac{k}{1 - p} C \left( \frac{c^*}{\sigma} \right) + \frac{1}{1 - p} \int_{t=\sigma_M}^{\sigma} \frac{e(t)}{tc^*} \, dt
\]

Finally, we must show that this is individually rational. The implementable payoff from joining the gang is:
\[
w + \int_{t=\sigma_M}^{\sigma} \frac{e(t)}{tc^*} \, dt
\]

For any youth with \( \sigma > \sigma_M \), the payoff from joining the gang strictly exceeds the wage they would earn in the primary labour market. For the marginal youth, the two are equal. For any youth with ability less than the marginal youth, they strictly prefer the primary labour market. This completes the proof.

C Proof of Proposition 5

Before evaluating the profit maximisation problem, consider the expected cost of informational rent for the gang:
\[
I = \frac{1}{1 - p} \int_{s=\sigma_M}^{\infty} \int_{t=\sigma_M}^{s} \frac{e(t)}{tc^*} \, dt \lambda \exp\{-\lambda(s - \sigma_M)\} \, ds
\]

Performing a standard integration by parts yields:
\[
I = \frac{1}{1 - p} \int_{s=\sigma_M}^{\infty} \frac{e(s)}{\lambda sc^*} \lambda \exp\{-\lambda(s - \sigma_M)\} \, ds
\]

so the gang leadership’s objective function becomes:
\[
N(1 - p) \int_{s=\sigma_M}^{\infty} \left[ r(M, e(s)) - \frac{w + pf}{1 - p} - \frac{e(s)}{c^*(1 - p)} \left( 1 + \frac{1}{\lambda s} \right) - \frac{k}{1 - p} C \left( \frac{c^*}{s} \right) \right] \lambda \exp\{-\lambda s\} \, ds
\]

Now, given gang size, for each \( \sigma \geq \sigma_M \), the restricted demand for \( e \) must maximise \( \pi(M, e) \). It
must therefore satisfy:

\[ \pi_e = r_e (\bar{e}, M) - \frac{1}{e^* (1 - p)} \left[ 1 + \frac{1 + \varepsilon}{\lambda \sigma 2 + \varepsilon} \right] \equiv 0 \]

The associated second-order condition is:

\[ \pi_{ee} = r_{ee} (M, \bar{e}) + \frac{r_e (\bar{e}, M)}{\bar{e} (2 + \varepsilon)} - \frac{\varepsilon}{\bar{e} e^* (1 - p) (2 + \varepsilon)} \left[ 1 + \frac{\varepsilon C'' (\varepsilon^*)}{C'' (\varepsilon^*)} - \varepsilon \right] \]

It is straightforward to show that the final term is positive, since \( \varepsilon \) is increasing. So the second-order condition is unambiguously negative if:

\[ r_{ee} (M, \bar{e}) + \frac{r_e (M, \bar{e})}{\bar{e} (2 + \varepsilon)} < 0 \]

A sufficient condition is \( \eta (M, \bar{e} (\sigma)) > -\frac{1}{2} \). This completes the proof.

### D Proof of Proposition 6

Making liberal use of the envelope theorem, the gang’s optimal size, \( M^* \), satisfies the following first-order condition:

\[
\Pi_M = \hat{r} - \frac{w + pf}{1 - p} - \frac{\hat{e}}{\bar{e} (1 - p)} \left[ 1 + \frac{1}{\lambda \sigma M^*} \right] - \frac{k}{1 - p} C \left( \frac{e^*_M}{\sigma M^*} \right) + N (1 - p) \int_{s = \sigma M^*}^{\infty} r_M (M^*, \bar{e} (s, M^*)) \lambda \exp(-\lambda s) ds \equiv 0
\]

where \( \hat{r} \equiv r (M^*, \bar{e} (\sigma M^*, M^*)) \) and \( \bar{e} \equiv \bar{e} (\sigma M^*, M^*) \). The associated second-order condition is:

\[
\Pi_{MM} = 2 \hat{r}_M - \frac{\hat{e}}{\bar{e} \sigma M^* \sigma^* (1 - p)} \left[ 1 + \frac{1}{\lambda \sigma M^*} \right] \left[ 3 + 2 \varepsilon M^* \right] + N (1 - p) \int_{s = \sigma M^*}^{\infty} \left( r_{MM} - \frac{r_{Me}^2}{\pi_{ee}} \right) \lambda \exp(-\lambda s) ds
\]

Consider the final term:

\[
I = N (1 - p) \int_{s = \sigma M^*}^{\infty} \left( r_{MM} - \frac{r_{Me}^2}{\pi_{ee}} \right) \lambda \exp(-\lambda s) ds
\]

\[
= M \hat{r}_{MM} - M \frac{r_{Me}^2}{\pi_{ee}} + N (1 - p) \int_{s = \sigma M^*}^{\infty} r_{Me} \frac{\partial \pi}{\partial s} \exp(-\lambda s) ds + 2 N (1 - p) \int_{s = \sigma M^*}^{\infty} \frac{r_{Me} r_{Me} \partial \pi}{\pi_{ee} \partial s} \exp(-\lambda s) ds
\]
where $\hat{\pi}_{ee} = \frac{\partial \pi}{\partial M} (M^*)$. By assumption:

$$N (1 - p) \int_{s = \sigma_{M^*}}^{\infty} \frac{r_{MMe} \partial e}{\partial s} \exp \{ - \lambda s \} ds - 2N (1 - p) \int_{s = \sigma_{M^*}}^{\infty} \frac{r_{MMe} \partial e}{\partial s} \exp \{ - \lambda s \} ds + N (1 - p) \int_{s = \sigma_{M^*}}^{\infty} \frac{r_{M^2 Me} \partial^3 \pi}{\partial e^2 \partial s^2} \exp \{ - \lambda s \} ds < 0.$$  

So:

$$\Pi_{MM} < 2\hat{r}_M - \frac{\hat{e}}{\lambda \sigma_{M^*} \hat{M} \hat{r}_{ee} (1 - p)} \left[ 1 + \frac{1}{\lambda \sigma_{M^*}} \frac{3 + 2\hat{e}_{M^*}}{2 + \hat{e}_{M^*}} \right] + M^* \hat{r}_{MM} - M^* \frac{\hat{r}_{Me}^2}{\hat{r}_{ee}} > 0$$

The right hand side is negative if and only if:

$$2\hat{r}_M \hat{\pi}_{ee} - \frac{\hat{e}}{\lambda \sigma_{M^*} \hat{M} M^* \hat{r}_{ee} (1 - p)} \left[ \frac{\hat{e}}{\hat{r}_{ee} + \hat{r}_M \hat{e}} + M^* \hat{r}_{MM} \hat{\pi}_{ee} - M^* \frac{\hat{r}_{Me}^2}{\hat{r}_{ee}} > 0 \right]$$

where $\gamma = \frac{\hat{e}}{\hat{r}_{ee} + \hat{r}_M \hat{e}} - \frac{1}{\hat{r}_{ee} (1 - p) (2 + \hat{e})} \frac{1}{\lambda \sigma_{M^*} \hat{M} M^* \hat{r}_{ee} (1 - p)} \left[ 1 + \frac{\hat{e}}{\hat{r}_{ee} + \hat{r}_M \hat{e}} \right]$. Substituting for $\gamma$, and given the properties of $r (\cdot, \cdot)$, it is possible to show that this is unambiguously positive. Thus the second-order condition is invariably negative.

Now, in order to prove that this is profit-maximising, we need to show that equilibrium profits are positive. Otherwise, the gang will simply shut down. Note that, by the first-order condition, $\pi (\sigma_{M^*}, M^*) > 0$, since $r_M < 0$. Moreover, the profit each youth generates for the gang is increasing in ability for any given size:

$$\pi_{\sigma_i} (\sigma_i, M) = \frac{k c_i^* \sigma_i^2}{\sigma_i^2 (1 - p)} C^{\sigma_i} \left( \frac{c_i^*}{\sigma_i} \right) + \hat{\epsilon}_i \frac{3 + 2\hat{e}_{M^*}}{\lambda \sigma_i (2 + \hat{e}_{M^*})} > 0$$

So the gang must generate positive profits in equilibrium. This completes the proof.

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