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The impact of information enrichment on the Bullwhip Effect in supply chains: A control engineering perspective

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Abstract

This paper examines the beneficial impact of information sharing in multi-echelon supply chains. We compare a traditional supply chain, in which only the first stage in the chain observes end consumer demand and upstream stages have to base their forecasts on incoming orders, with an information enriched supply chain where customer demand data (e.g. EPOS data) is shared throughout the chain. Two types of replenishment rules are analysed: order-up-to policies and smoothing policies (policies used to reduce or dampen variability in the demand). For the class of order-up-to policies, we will show that information sharing helps to reduce the bullwhip effect (variance amplification of ordering quantities in supply chains) significantly, especially at higher levels in the chain. However, the bullwhip problem is not completely eliminated and it still increases as one moves up the chain. For the smoothing policies, we show that information sharing is necessary to reduce order variance at higher levels of the chain. The methodology is based on control systems engineering and allows us to gain valuable insights into the dynamic behaviour of supply chain replenishment rules. We also introduce a control engineering based measure to quantify the variance amplification (bullwhip) or variance reduction.

Keywords

Supply chain management, replenishment rule, bullwhip effect, production smoothing, control theory.

Nomenclature

$\mapsto$	Forecasting constant used in exponential smoothing forecast, $\mapsto$=1/(1+Ta)
$\bullet$	Signalling factor
AR	Amplitude Ratio
D	Demand
D_t	Demand at time, t
$\hat{D}_t$	Demand forecast at time, t
$\hat{D}_t^L$	Demand forecast over L time units at time, t
$\hat{D}_t^{Ta}$	Demand forecasted with the smoothing constant Ta at time, t
$DWIP_t$	Desired Work In Progress at time t
e	The base of the natural logarithm, 2.7182….
FR	Frequency response
i	The imaginary number $\sqrt{-1}$
i.i.d.	Independent and identically distribution normal distribution
n, j	Echelon of the supply chain
k	The normal variant used to determine the safety stock in the Order-Up-To model
L	The physical production/ distribution lead-time, plus a time unit for safety stock and/or a time unit to ensure the correct order of events
NS_t	Net Stock at time t
$\hat{\sigma}$	Estimated standard deviation
$\hat{\sigma}_n^L$	Estimated standard deviation at echelon n over L time units
OUT	Order Up To
O_n	Orders at echelon n
O_t	Orders at time t
O_t^n	Orders at echelon n at time t
S_t	Stock at time t
t	Time
Ta	Average age of exponential smoothing forecast
TF	Transfer Function of Orders
Tm	Number of periods used in the moving average
Tn	Time to adjust for errors in net stock
TNS_t	Target Net Stock at time t
Tp	The physical production/ distribution lead-time
Tw	Time to adjust for WIP errors
Var	Variance
VMI	Vendor Managed Inventory
w	Frequency (Radians per sampling period)
WIP	Work In Progress
WIP_t	Work In Progress at time t
W_N	Noise Bandwidth
z	z-transform operator

1. Introduction

In supply chains, the variability in the ordering patterns often increases as one moves up the chain, towards the factory and the suppliers. This variance or demand amplification was originally observed and studied by Forrester (1958, 1961). The Forrester paper inspired many authors to develop management games to demonstrate the variance amplification. The well-known Beer Game was developed at MIT at the end of the fifties and Sterman (1989) reports on the major findings. In the Beer Game, order decisions have to be made in a supply chain consisting of a retailer, a wholesaler, a distributor and a factory. Only the retailer observes the customer demand, so the other supply chain members have to base their decisions on the incoming orders. Players of the Beer Game will experience that very small variations in customer demand can lead to exaggerated order swings further up the supply chain. Demand amplification has been very popular in the research community in the last few years and the phenomenon is now mostly denoted by the term 'bullwhip effect' (Lee, Padmanabhan and Whang 1997a, 1997b). Lee et al. (1997a and 1997b) give five important causes for the bullwhip effect: the use of 'demand signal processing', non-zero lead times, order batching, supply shortages and price fluctuations. Under demand signal processing, we understand the practice of adjusting the demand forecasts and, as a result of this practice, adjusting the parameters of the inventory replenishment rules frequently. By doing this, short-run fluctuations maybe overreacted too, which induces variance amplification. In other words, the replenishment rule used by the members of the chain may be a contributory factor to the bullwhip effect. Chen, Drezner, Ryan and Simchi-Levi (2000a and 2000b) have quantified the bullwhip effect for order-up-to policies based on exponential smoothing forecasts as well as moving average forecasts. Dejonckheere, Disney, Lambrecht and Towill (2001a) have shown that order-up-to policies always result in variance amplification, irrespective of the forecasting method being used and state this without making any assumptions about the demand pattern.

Research has shown that such demand amplification can be constrained by proper design and re-engineering of the supply chain (see Berry, Naim and Towill, 1995). In particular the importance of designing the chain according to established Principles of Material Flow Control has been recognised (Towill and McCullen, 1999). In order to avoid the bullwhip effect, all causes should be eliminated at source. Van Ackere, Larsen, and Morecroft (1993) provide a useful framework to classify the counter measures that can be taken in any supply chain to reduce or avoid the bullwhip effect. They distinguish three different approaches: (1) redesigning the physical process (such as lead-time reduction and eliminating a channel in the supply chain), (2) redesigning the information channels (such as providing customer demand data throughout the chain), (3) and redesigning the decision process (using different replenishment rules). ***In this paper, we are especially dealing with the second approach: the benefits of information sharing in supply chains.***

Van Ackere, Larsen, and Moorcroft's third approach concerns the careful selection of the replenishment rule. Order-up-to policies are very popular both in research as in practice since they are known to minimise inventory holding and shortage costs. We are not concerned in this paper with the variance of the inventory levels. Here we are concerned with order variance as order-up-to policies always seem to result in order rate variance amplification. When production is inflexible and significant costs are incurred by switching up and down production quantities frequently, order-up-to policies may no longer be desirable or even achievable. These production (ordering) switching costs may be very large requiring a smooth production or ordering pattern. This problem is well known in the literature as the 'production smoothing problem' and a lot of decision rules exist that can avoid bullwhip, as they reduce variability in the demand pattern. Although smoothing the demand pattern might increase the inventory related costs, the decision rule may still

outperform order-up-to policies in terms of total costs (inventory holding and shortage costs plus production switching costs). This will of course depend on the cost structure of the supply chain under consideration. We do not investigate the link between bullwhip and inventory variance here although Disney and Towill (2002) and Dejonckheere, Disney, Farasyn, Janssen, Lambrecht and Towill (2002) have examined this issue in a single echelon of a supply chain.

In this paper we will show that in a multi-level supply chain, it is very beneficial to share customer demand information throughout the chain. Sharing customer demand information (e.g. EPOS data) is possible through new technologies available in many supply chains nowadays. All replenishment rules in this paper are analysed in two scenarios. The first scenario is the ***traditional supply chain*** in which only the retailer has access to customer demand, just as the original Beer Game is played. The second scenario is the supply chain with information sharing, which this paper denotes as the ***information enriched supply chain,*** after the seminal work of Mason-Jones (1998). The difference between the two scenarios is made clear in the Figure 1 below. The only difference to the traditional supply chain is that every chain member can now base its forecasts on the end consumer demand (instead of incoming orders); Note that we are not modelling a Vendor Managed Inventory supply chain since, in our information enriched supply chain, every stage still has to ship the goods ordered by the previous member of the chain, i.e. the supplier does not have the authority (or the necessary inventory information) to deliver goods to the customer at his discretion. For a detailed analysis of a Vendor Managed Inventory supply chain see Disney (2001). For an extended guide to several variations of the information enrichment strategy see Mason-Jones (1998).

We will look at two general classes of replenishment rules: the order-up-to policies, and the smoothing decision rules. For both policies, we will show the benefits of information sharing. When <u>order-up-to policies</u> are applied in a multi-level traditional supply chain, bullwhip can be dramatic at higher levels of the chain. In the next section, we give a real life case were the variance of the orders to the suppliers is over 100 times the variance of the customer demand. Chen et al. (2000b) quantified the bullwhip effect for order-up-to policies based on moving average forecasts. We will confirm those results and extend them to other forecasting policies. We show that when using order-up-to policies in a multi-level information enriched supply chain, the increase in variance will be much less (see also Chen et al. (2000b) and Lee et al. (2000)) than in a traditional supply chain. The last part of the paper examines a <u>smoothing replenishment rule</u> in a multi level supply chain. We show that information sharing is very crucial here as well. In the traditional supply chain, although bullwhip is reduced, the replenishment rule loses its smoothing characteristics at higher levels of the chain, whereas in the enriched chain, the replenishment rule is able to ensure order variability is not increased at higher levels of the supply chain. A summary of the bullwhip generated by all polices analysed in this paper is presented together with several closed form expressions for bullwhip that we have found.

Most of the available research on bullwhip and information sharing uses a statistical methodology (see Lee et al. (1997a and 2000) and Chen et al. (2000a and 2000b)). They quantify the magnitude of variance amplification in supply chains for a few types of demand processes. In this paper, we will advocate a control engineering approach and are able to confirm and extend the results obtained through statistical analysis. The control engineering approach goes back to the work of Simon (1952), Vassian (1955), Adelson (1966), Forrester (1961) and Towill (1970, 1982). We also refer to Deziel and Eilon (1967), Bertrand (1986), John, Naim and Towill (1994), Towill (1999), Dejonckheere, Disney, Lambrecht and Towill (2001) and Disney (2001) for other contributions on replenishment rules and inventory fluctuations. The control engineering methodology will enable us to gain important insights in the dynamic behaviour of the replenishment rules examined in this paper. The

methodology also allows us to quantify variance amplification (or variance reduction) at each level of the supply chain in response to a normally (i.i.d.) distributed demand from the end consumer. Note that we do not investigate the effect on performance of the types of forecasting method chosen (and it parameter(s) value) in relation to the demand process.

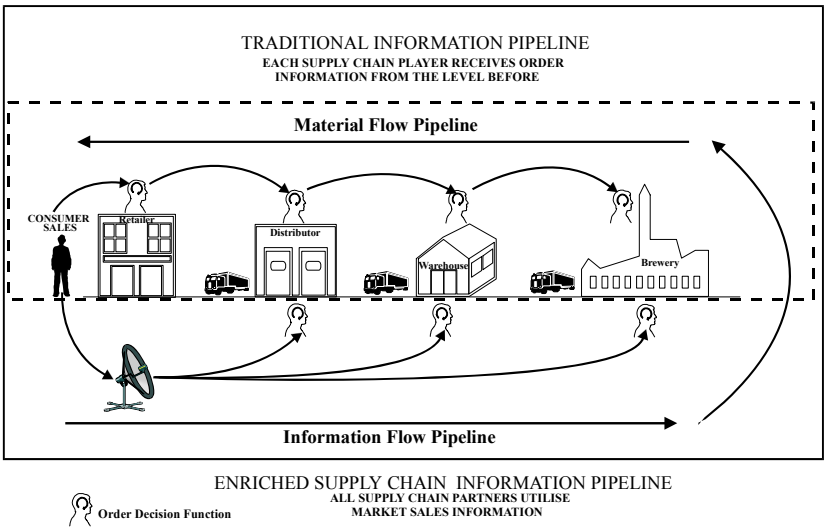

Figure 1. The traditional supply chain versus the information enriched supply chain (Taken from Mason-Jones and Towill (1997))

The remainder of the paper is organised as follows. In section 2, bullwhip is analysed in a real life supply chain. In section 3, the control engineering methodology used to analyse replenishment rules and quantify variance amplification is explained. Section 4 analyses the benefits of information sharing for order-up-to policies. Three forecasting systems are integrated in the order-up-to system: exponential smoothing (Holt, 1957), moving average, and demand signal processing (Lee, 1997a and 2000a). Section 5 examines a smoothing replenishment rule and the impact of information enrichment.

2. Bullwhip in a grocery supply chain

Our real-world bullwhip example is based on a European retail supply chain. Holmström (1997) analysed the orders flowing upstream from retail outlets. He studied in depth a traffic building (high volume), low margin product, and a low traffic (low volume), high margin product. Demand amplification was estimated via the bullwhip measure (standard deviation/average value) as discussed in detail by Fransoo and Wouters (2000). We have then calculated the Bullwhip Factor as the ratio of the standard deviation estimated as the orders are passed up successive echelons in the supply chain.

The results are shown in Table I. They confirm the fears of many academic researchers and management theorists by demonstrating that bullwhip exists in the real world. It is even larger than many theories predict, and promulgates wildly upstream exactly as Van Aken (1978) suggested based on his experiences at Philips plants (Eindhoven in Holland). Note that the Bullwhip Factors yield important insights into the behaviour of the various "players" in the chain. The downstream players (shops and wholesalers) are the biggest culprits in the sense of bullwhip generation. They exhibit little difference in attitudes towards ordering policies for either low margin or high margin products with Bullwhip Factors around 3 to 1.

Player	High volume/Low margin	Low volume/High margin
Retailer	2.60	3.14
Wholesaler	2.88	3.05
Factory	0.72	2.39
Supplier	1.67	1.25
Total SC bullwhip	9.0	28.61

Table I. Bullwhip effect in a grocery supply chain (Source; Holmström 1997)

Not so the factory scheduler. Not only does the scheduler treat the two products significantly differently, but adequately dampens down the volatility in the orders for the high volume product. This is most likely to have been achieved via some version of level scheduling (Suzaki, 1987). In contrast the scheduler is quite prepared to induce further substantial bullwhip into the system when considering the low volume product. Deliveries from the factories exhibit some bullwhip, but of a secondary effect composed to the downstream "players". The total Bullwhip Factor over the entire chain is 9 to 1 (high volume product), and 28.60 to 1 (low volume product).

Thus the highly volatile supply chain really does exist - it is not a figment of the imagination of academic researchers, and justifies the study of the many causes of bullwhip. Furthermore, the reduction of bullwhip via Business Process Re-Engineering Programmes to systematically root out the various causes is now a proven method of supply chain improvement. Indeed, there is much more to the bullwhip problem than algorithm design, it clearly needs to be linked with business strategy. Examples include this particular retail chain where bullwhip has been reduced by eliminating a major delay in the information flow path (Holmström, 1997). Furthermore, in a global precision products supply chain, bullwhip has been reduced by 2 to 1 whilst concurrently increasing stock turns and reducing stock variability by similar factors (Towill and McCullen, 1999). This is particularly important as Metters concludes that a 30% reduction in factory costs can follow from a bullwhip reduction program.

3. A control engineering method to investigate replenishment rules

The methodology used in the paper is control systems engineering, complemented with spreadsheet simulation. We briefly introduce the most important concepts and techniques.

a. Derive the transfer function

In control systems engineering, the <u>transfer function</u> of a system represents the relationship describing the dynamics of the system under consideration. It algebraically relates a system's output to its input and is defined as the ratio of the z-transform of the output variable to the z-transform of the input variable. The general form of the transfer function relating two signals within a system is then:

$$F(z) = K\left(\frac{1 + b_1 z + b_2 z^2 + \ldots + b_q z^q}{1 + a_1 z + a_2 z^2 + \ldots + a_p z^p}\right) \tag{1}$$

Since supply chains can be seen as a system, with complex interactions between different parts of the chain, we may use a transfer function approach to model these interactions. For every replenishment rule, a transfer function will be developed that completely represents the dynamics of a particular replenishment rule. Input to the system corresponds to the demand pattern and the output refers to the corresponding replenishment or production orders. For more details on control engineering and transfer functions, we refer the reader to the

appropriate literature, although Nise (1995) provides a good introduction. Dejonckheere et al. (2001) show how the transfer function may be obtained by constructing a 'causal loop diagram' and a 'block diagram' for the replenishment rule under consideration. For a historical overview of using the z-transform in production and inventory control applications we refer to Disney (2001).

b. The Frequency Response plot

To derive the 'Frequency Response' plot (FR) of a replenishment rule, we will present the rule with sinusoidal inputs of different frequencies; that means we want to know what orders (output) are generated when the demand (input) is sinusoidal. Since we are dealing with linear systems, we know that the output will also be a sine wave with the same frequency, but the amplitude and the phase angle may have changed. We will be particularly interested in the ratio of the amplitude of the generated orders (output) to the amplitude of the sinusoidal demand (input) i.e. the Amplitude Ratio (AR). We will present the replenishment rule with sine waves of frequencies ranging from 0 to π radians per sample interval. It is well known from Shannon's Sampling Theorem, (Shannon et al, 1948) that sampled data systems can only detect inputs of frequencies up to π radians per sampling interval unit, hence the plot is only required for the frequencies 0 to π. For all these frequencies, we can find the AR and in this way draw the FR plot. Technically, the FR plot is made by letting $z= e^{iw}$ in the transfer function and calculating the modulus of the vector in the complex plane. Because of the fact that any real life demand data can be seen as composed of different sinusoids, it makes sense to analyse responses to different sine waves. The FR plot immediately yields insight into the dynamic behaviour of the replenishment rule, without making any assumptions on the distribution of the demand pattern. It will be used to make predictions on whether or not the replenishment rule will lead to variance amplification.

c. The Noise Bandwidth

The Noise Bandwidth is a common metric in communications engineering. It is formally defined as:

$$W_N = \int_0^{\pi} \left| TF(e^{iw}) \right|^2 dw \tag{2}$$

where $TF(e^{iw})$ is the steady state response to the excitation frequency w. (Garnell and East, 1977). In words, W_N is defined as the area under the squared frequency response of the system. In order to calculate the noise bandwidth, the frequency response was calculated at set points of w and W_N estimated via numerical integration with strips of 0.0001 radians per sampling interval of frequency.

W_N has an important attribute that allows us to use it as a metric for bullwhip effect. This is that for an input of "white" random noise (constant power density at all frequencies) and zero mean, W_N is a direct measure of the variance at the output from the filter (see Towill, 1999). The noise bandwidth is thus directly related to the variance of the output when the system is subject to an input of "white random noise", better known as a random input, or else, normally distributed input with no correlation. More specifically, ***W_N / π is a very accurate metric to predict the magnitude of change in the variance when the replenishment rule under consideration is applied to independently and identically distributed (i.i.d) normal demands.*** When $(W_N / \pi) > 1$, the variance of the output (the orders) is larger than the variance of the input; hence we have variance amplification or bullwhip. This bullwhip measure has already been successfully used by Dejonckheere et al (2001) and later in this contribution we will verify the measure via a simulation based cross check. We will show that ***W_N / π is equivalent to the Variation Ratio measure (Variance Orders/ Variance Demand) used by many authors to quantify the Bullwhip Effect***. When $(W_N / \pi) < 1$

however, the variance of demand is reduced by applying the replenishment rule and we do not have any bullwhip effect but a smoothing of the demand pattern.

The control engineering based methodology used in the paper can be summarised by the following ***three-step procedure***:

1. Derive the transfer function of the replenishment rule under consideration. This is done by first drawing the block diagram of the replenishment rule.
2. Draw the Frequency Response plot. The FR plot gives a valuable insight in the dynamic behaviour of the replenishment rule under consideration (bullwhip or smoothing) without making any assumptions on the demand pattern.
3. Calculate W_N/π. This metric quantifies the magnitude of the Variance Ratio (Variance of output/ Variance of input) when the replenishment rule is applied to normally distributed demand patterns from the end consumer. If $(W_N / \pi) > 1$, the replenishment rule creates bullwhip, if $(W_N / \pi) < 1$, the replenishment rule smoothes order variance.

4. Information enrichment in order up-to-policies

Consider a simple supply chain consisting of a retailer, a distributor, a warehouse and a factory. The supply chain is denoted in this paper as a four level supply chain since ordering decisions have to be made by four members of the chain. We have two scenarios: the traditional supply chain scenario, and the information enriched supply chain. In the traditional scenario, we assume the following order of events: in each period t, the retailer first receives goods, then customer demand D_t is observed and satisfied; next, the retailer observes the new inventory level and places an order O_t^{ret} to the distributor. The distributor immediately receives the order in period t; but unlike the Beer Game, there is *no ordering delay.* The distributor also receives goods, fulfils the retailer order, and places an order O_t^{dist} to the wholesaler. The wholesaler immediately receives the order,…, and the process continues until the wholesaler and the factory have placed their orders. Any unfilled demand is backlogged in our model. There is a fixed lead time between the time an order is placed at a stage i and when it is received at that stage, such that an order placed at the end of period t is received at the start of period $t+L$, where $L = Tp+1$ and Tp is the physical production lead-time.

In the information-enriched scenario, everything remains the same, except the fact that every stage in the chain in any period not only receives an order from the previous chain member, but also he receives the end consumer demand for the current period. This end consumer demand can then be used to forecast from. Of course, the goods ordered by the previous member still have to be shipped downstream.

In this section, we integrate three different forecasting systems in the order-up-to policy: exponential smoothing, moving average, and demand signal processing.

4.1. Using exponential smoothing forecasting in order-up-to policies

In any order-up-to policy, ordering decisions are as follows:

$$O_t = S_t - \textit{inventory position}_t \tag{3}$$

where O_t is the ordering decision made at the end of period t, S_t is the order-up-to level used in period t and the inventory position equals net stock plus on order (or WIP), and net stock equals inventory on hand minus backlog. The order-up-to level is updated every period according to $S_t = \hat{D}_t^{\,L} + k\ \hat{\sigma}_t^{\,L}$ (4)

where $\hat{D}_t^L$ is an estimate of mean demand over L periods ($\hat{D}_t^L = L\hat{D}_t$), $\hat{\sigma}_t^L$ is an estimation of the standard deviation of the demand over L periods, and k is a chosen constant to meet a desired service level. To simplify the analysis, we set k equal to zero and increase the lead-time by one. Policies of this form are often used in practice: the value of L is inflated and the extra inventory represents the safety stock.

Additionally, L is increased by one in order to ensure the correct order of events. For example, we receive inventory and satisfy demand throughout the planning period and at the end of the planning period we observe inventory and place an order. Thus, even if the physical production / distribution lead-time is zero, it does not appear in the order decision until the end of the next planning period. Hence, L includes a nominal order of events delay. In other words L not only represents the physical lead-time, but also a safety lead-time and an order of events delay. Thus we have $L=Tp+2$, where Tp is the physical production / distribution lead-time.

In this section, we use simple exponential smoothing to forecast demand. The formula for simple exponential smoothing is well known to be:

$$\hat{D}_t = \hat{D}_{t-1} + \alpha * (D_t - \hat{D}_{t-1}) \text{ or } \hat{D}_t = \alpha * D_t + (1-\alpha) * \hat{D}_{t-1} \tag{5}$$

Note that since we make the ordering decision at the end of the period, the current demand D_t can be used in the forecast $\hat{D}_t$. For simple exponential smoothing, the average age of the data in the forecast is equal to $(1-\alpha)/\alpha$ (Makridakis, 1978). Let Ta be the average age of the data in the forecast, consequently $\alpha = 1/(1+Ta)$.

Dejonckheere et al. (2001) have shown that for a single level order up to system (only one chain member (e.g. the retailer) has to make ordering decisions) the transfer function equals:

$$TF_1 = \frac{z(Ta+Tp+3)-(Tp+2+Ta)}{-Ta+z+Taz} \tag{6}$$

They've also shown that the replenishment rule has to result in bullwhip for every possible demand pattern, and they have quantified the variance ratio for independently and identical distributed (i.i.d). normally distributed demands.

The goal of this section is to extend the analysis to multi-echelon supply chains for both the traditional supply chain scenario and the information enriched supply chain. In the analysis, we will follow the three-step procedure proposed in section 3.

<u>Step 1. Derive the transfer functions</u>

In order to draw the frequency response plots, the transfer functions first have to be derived for all levels in the supply chain. It is can be easily shown with control engineering that in the <u>traditional scenario</u>, the single level transfer function (7) can just be multiplied n times in order to find the nth level transfer function.

$$TF_n = \frac{O_n}{D} = \prod_{j=1}^{n}\left[\frac{z(Ta_j+Tp_j+3)-(Tp_j+2+Ta_j)}{-Ta_j+z+Ta_j z}\right] \tag{7}$$

In the case where all members use the same smoothing constant (Ta) and have the same lead-times (Tp), the multiplications can be translated into an exponent.

For the <u>information enriched scenario</u>, the transfer functions for the first level in the chain TF_1 is, of course, still equal to the single level transfer function; however, the higher level transfer functions TF_n have to be derived. The block diagram for this derivation is given in Appendix A. The transfer functions turn out to be very simple and elegant. If all supply chain members use different Ta-values and have different lead times Tp, then we have:

$$TF_n = \frac{O_n}{D} = TF_1 + \sum_{j=2}^{n}\left[\frac{(2+Tp_j)(z-1)}{-Ta_j+z+Ta_j z}\right] \tag{8}$$

If the lead times and the forecasting parameters however are the same throughout the chain, then the transfer function further simplifies to:

$$TF_n = \frac{O_n}{D} = \frac{z(2n+1+Ta+nTp)-(2n+Ta+Tp)}{-Ta+z+Ta\,z} \qquad (9)$$

Step 2. Plot the Frequency Response

Using these transfer functions (7) and (9), we now plot for both scenarios the FR's for an illustrative case where all four levels have *Tp=3* and *Ta*=9.

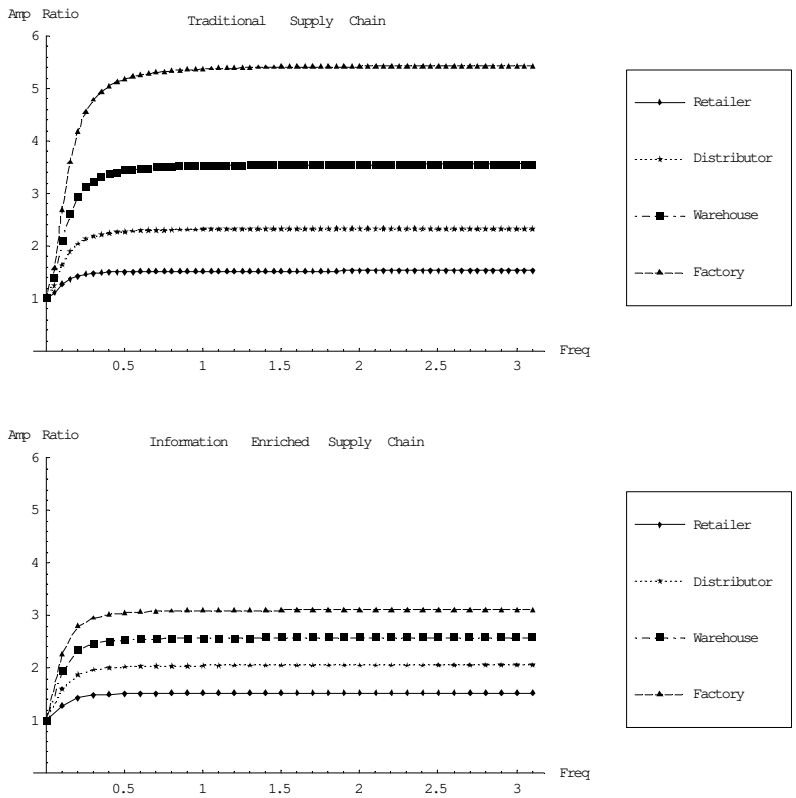

Figure 2. Frequency Responses for an order-up-to policy with exponential smoothing forecasts in the traditional supply chain scenario (upper graph) and in the information enriched scenario (lower graph)

From the Figure 2 above, we can see the impact of information sharing for the higher levels in the supply chain. The shape of the frequency responses remains the same, but the amplitude ratios are clearly lower in the second scenario.

Step 3. The Noise Bandwidth

By calculating the $W_N / \prec$ metric at all levels in both scenarios (still with *L=Tp+1=4* and *Ta=9*), we obtain the following graph:

From Figure 3, we can observe that the bullwhip factor (the ratio of the variance of the orders at an echelon in relation to the variance of the end consumer demand) gets very large (up to 25 for echelon 4) in higher levels of the supply chain. This is in line with the real world supply chain of Table 1, but here again the beneficial impact of information sharing in the supply chain is clearly illustrated. There is still bullwhip at all levels of the chain in the information enriched scenario, however the increase in variance amplification is much less than it is in the traditional supply chain. *The increase now* ***seems*** *to be linear with the level in*

the chain, whereas it seemed to be geometrical in the traditional case. The same conclusion can be made for other values of *Ta*.

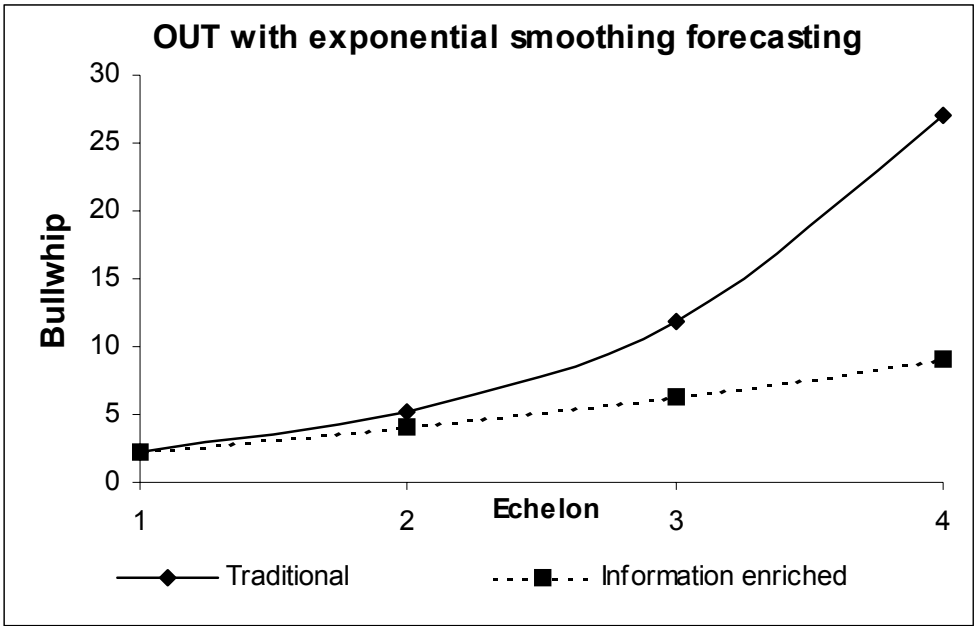

Figure 3. Variance ratio for order-up-to (OUT) policies with exponential smoothing for both scenarios

In order to test our control engineering based Variance Ratio predictions (W_N /$\prec$), we developed a spreadsheet, simulating a four level supply chain with order-up-to policies based on exponential smoothing. We inserted a random input signal[1], and then computed the variances of the ordering patterns of the four supply chain members. Next, we computed the variance ratios (variance of orders at level *n* / variance of end consumer demand) at all four levels. We then repeated the simulation for the information enriched supply chain. The results are presented in Table II below. Row a gives the control engineering based predictions of the variance ratios at the four levels in both scenarios and row c gives the simulation-based predictions. (Note that we still use Ta=9 and L=Tp+1=4).

	Traditional Supply Chain				**Supply chain with information enrichment**			
Echelon	**1**	**2**	**3**	**4**	**1**	**2**	**3**	**4**
a) Control engineering theory	2.263	5.164	11.839	27.223	2.263	4.053	6.368	9.211
b) Statistical lower bound	2.263	5.062	11.39	25.628	2.263	4.052	6.368	9.21
c) Simulation results	2.259	5.15	11.889	27.146	2.259	4.044	6.358	9.203
% GAP a-c	0.176	0.271	0.422	0.282	0.176	0.222	0.157	0.086
d) Average GAP a-c	**0.288 %**				**0.16%**			
% GAP b-c	0.177	1.708	4.197	5.591	0.177	0.197	0.157	0.076
e) Average GAP b-c	**2.91%**				**0.152%**			

Table II. Verifying the control engineering based predictions through simulation for the exponential smoothing forecasting procedure

[1] An independent and identically distributed normal demand pattern with a mean of 100 and a standard deviation of 10, 10000 time-periods long was used. The same demand signal is used throughout this paper.

From Table II we can conclude that our control engineering based predictions are accurate, (but remember that the demand signal may not be statistically perfect in the simulation cross-check). The percentage deviation is never more than 0.5%. On average the gap is 0.288% for the traditional mode and 0.16% for the information-enriched mode. Table II also includes a cross-check against the statistical lower bound given by Chen et al. (2000a) and extended[2] here to for the information-enriched supply chain.

4.2. Using moving average forecasting in order-up-to policies

We still use the order-up-to policy described in (1-2), but now with a moving average forecasting procedure used to update the order-up-to levels S_t. The demand forecast of period t, $\hat{D}_t$, is defined as;

$$\hat{D}_t = \frac{\sum_{m=0}^{Tm-1} D_{t-m}}{Tm} \tag{10}$$

This policy in a single level supply chain is analysed in Dejonckheere et al. (2001). We now extend the analysis to multi-level supply chains. We therefore have to repeat the three-step procedure, analogous to the previous section (4.1), in order to analyse the benefits of information sharing on the bullwhip effect. For the transfer functions (step 1 of the procedure), we refer to Appendix B. The block diagram needed to derive the multi-level transfer functions is given in Appendix A. The transfer functions are quite simple and elegant, even for higher levels in the supply chain. We will now immediately plot the FRs for both scenarios for an illustrative case were all four levels have $L=Tp+1=4$ and Tm=19 (step 2), and also the variance ratio predictions for both scenarios in our 4 level supply chain (step 3).

The conclusions are completely similar to the order-up-to policies based on exponential smoothing forecasts analysed in section (4.1). We still have bullwhip at all levels of the supply chain and the variance ratio increase with the level in the chain. However, it can be seen from the Figures 4 and 5 that information sharing can significantly reduce the increase in variance amplification at higher levels of the chain. Without information enrichment, the variance amplification increases in a geometric manner, whereas in the information-enriched scenario, the bullwhip effect increases linearly.

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[2] Chen et al 2000a gives the lower bound for a single level of the supply chain as $\frac{Var(O)}{Var(D)} \geq 1 + 2L\alpha + \frac{2L^2\alpha^2}{2-\alpha}$, which is easily extended for a multi-level traditional supply chain to $\frac{Var(O_n)}{Var(D)} \geq \prod_{j=1}^{n}\left(1 + 2L_j\alpha_j + \frac{2L_j{}^2\alpha_j{}^2}{2-\alpha_j}\right)$. The statistical lower bounds for the information enriched supply chain is given by $\frac{Var(O_n)}{VarD} = 1 + 2\left(\sum_{j}^{n} L_j\right)\alpha_i + \frac{2\left(\sum_{j=1}^{n} L_j\right)^2 \alpha_n}{2-\alpha_n}$. This was found by the authors via an analogy with the tight bounds for the moving average forecast presented by Chen et al 2000b.

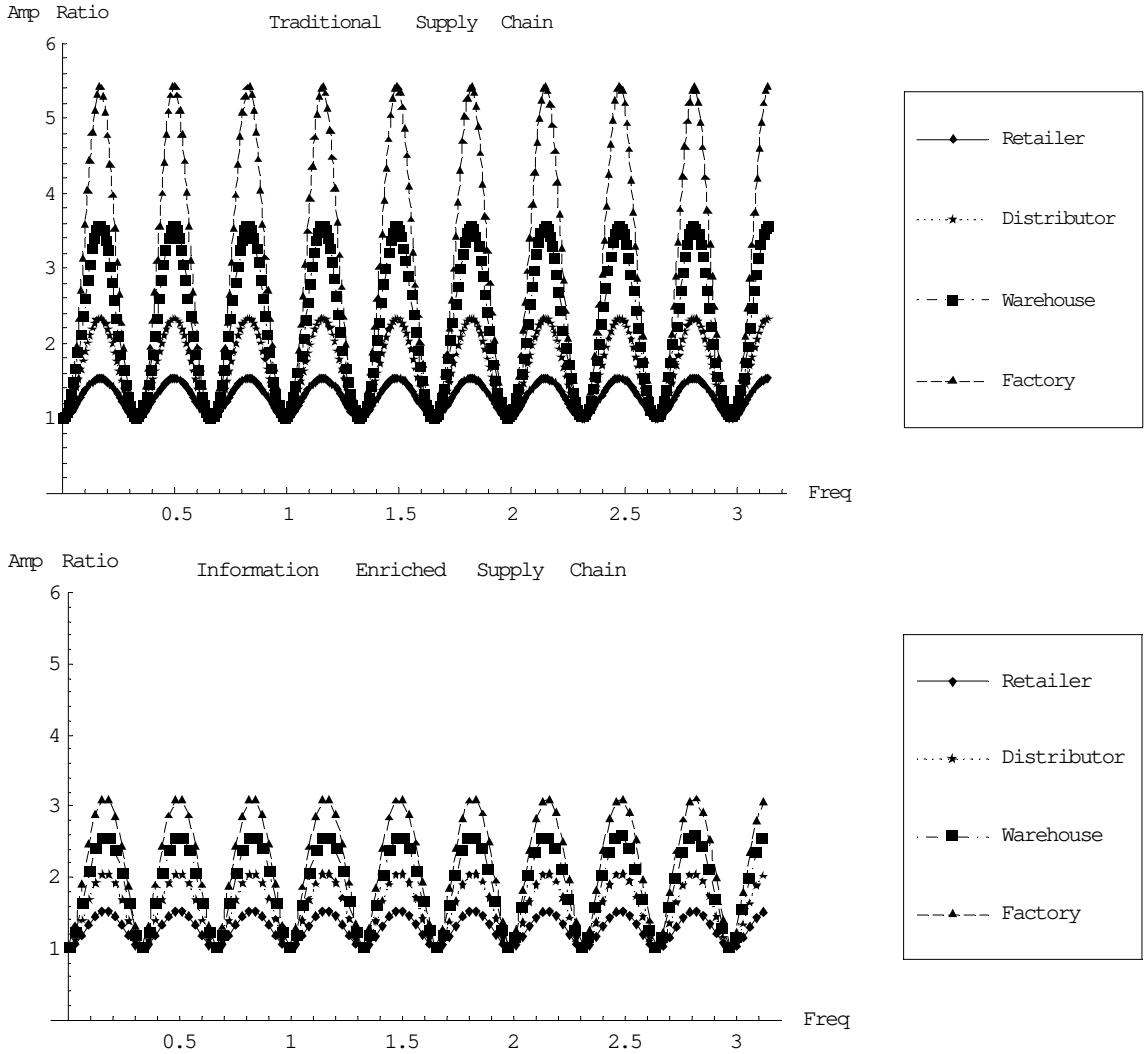


Figure 4. Frequency Responses for an order-up-to policy with moving average forecasts in the traditional supply chain scenario (upper graph) and in the information enriched scenario (lower graph).

The control engineering based variance ratios for both scenarios have been tested with our spreadsheet simulation. Moreover, for this order-up-to policy based on moving average forecasts, a statistical bound can be found in literature. Chen et al. (2000b) have statistically computed a lower bound on the variance ratio (variance amplification) for the traditional supply chain scenario (also often called a *decentralised* scenario):

$$\frac{\mathrm{Var}(O_n)}{\mathrm{Var}(D)} \geq \prod_{j=1}^{n}\left(1+\frac{2L_j}{Tm}+\frac{2L_j^2}{Tm^2}\right) \quad \forall i \tag{11}$$

with n being the level in the chain, Tm the same moving average constant at all levels, and L_i the lead time for level n. And for the information enriched (also called *centralised* scenario), they have found the following tight bound for variance amplification at level n in the chain:

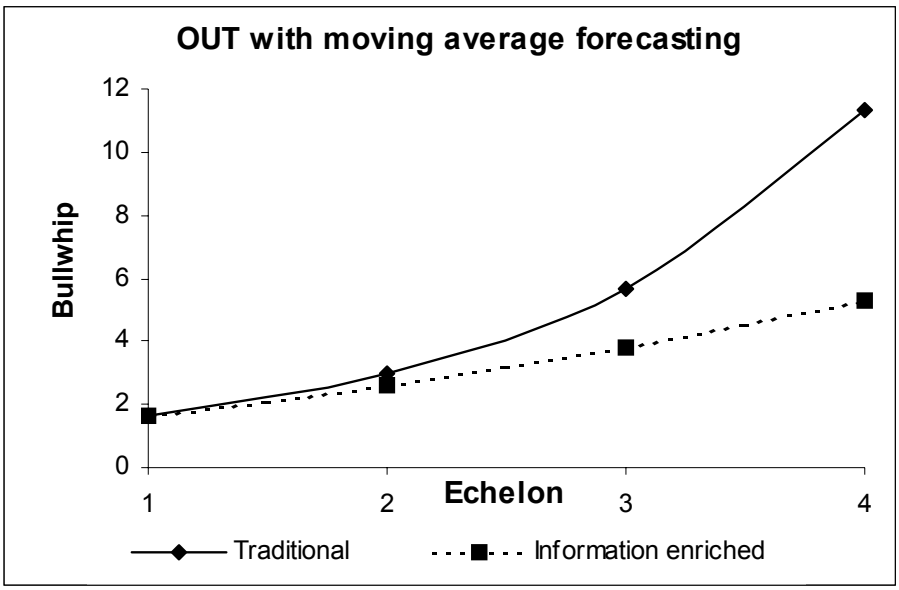


Figure 5. Variance ratios for order-up-to policies with moving average forecasts for both scenarios

$$\frac{Var(O^n)}{Var(D)} \geq 1 + \frac{2\left(\sum_{j=1}^{n} L_j\right)}{Tm} + \frac{2\left(\sum_{j=1}^{n} L_j\right)^2}{Tm^2} \tag{12}$$

In Table III below, we present the variance ratio predictions in both supply chain scenarios based on control engineering (row a), on the statistical bounds give in Equations 11 and 12 (row b) and based on simulation (row c). The parameters used for the comparison were Tm =19 and L=Tp+1=4 for all levels in the chain.

	Traditional supply chain				**Supply chain with information enrichment**			
Echelon	**1**	**2**	**3**	**4**	**1**	**2**	**3**	**4**
a) Control engineering theory	1.665	2.993	5.718	11.43	1.665	2.607	3.826	5.321
b) Statistical lower bound	1.665	2.772	4.614	7.682	1.665	2.607	3.826	5.321
c) Simulation results	1.653	2.974	5.821	12.604	1.653	2.587	3.82	5.387
% GAP a-c	0.72	0.634	1.801	10.271	0.721	0.767	0.157	1.24
d) Average GAP a-c	**3.357%**				**0.721%**			
% GAP b-c	0.72	7.287	26.159	64.07	0.721	0.767	0.157	1.24
e) Average GAP b-c	**24.56%**				**0.721%**			

Table III. Comparison of different variance ratio predictions for order-up-to policies based on moving average forecasts

From Table III it can be seen that for the information enriched scenario, the statistical tight bound are exactly equal to our control engineering predictions, even up to three decimal places. This confirms that our transfer function analysis is correct and $W_N/\prec$ is indeed a very good bullwhip measure. For the traditional scenario, Chen et al. (2000b) only had a lower bound on variance amplification. From Table III, we see that this lower bound significantly underestimates the variance amplification at higher levels in the chain.

In row d and e, we give the percentage gap between the simulated and the $(\bar{W}_N/\prec)$ predictions. On average, the deviations are quite satisfactory for both scenarios.

4.3 Using 'demand signalling' forecasting in order-up-to policies

Consider the following inventory policy:

$$S_t = S_{t-1} + \gamma (D_t - D_{t-1}), \text{ and } O_t = S_t - \textit{inventory position}_t, \quad (13)$$

where O_t is the ordering decision made at the end of period t, S_t (S_{t-1}) is the order-up-to level at the end of period t (t-1), D_t (D_{t-1}) is the observed demand during period t (t-1) and $\bullet$ is the 'signalling factor', which is a constant between zero and one. We still have an order-up-to policy, but the order-up-to level is updated every period using the most recently observed demand information. Policies of this type are called 'demand signal processing' by Lee et al. (1997a). For $\bullet$ = 1, (20) is quite an intuitive policy, often used by players of the Beer Game. If the retailer experiences a surge of demand in one period, it will be interpreted as a signal of high future demand and a larger order will be placed. The behaviour of this policy in a one level supply chain was examined in Dejonckheere et al. (2001). The policy turned out to create a huge bullwhip effect.

We will repeat again the three-step procedure for the demand signalling policy in a multi-level supply chain. The block diagram can be found in Appendix A. From the block diagram, the multi level transfer functions (step 1) have been derived. They are presented in Appendix B. With the transfer functions, the Frequency Responses can easily be plotted (step 2) and the Noise Bandwidths can be calculated (step 3) for our multi-level supply chain in both the traditional and the information-enriched scenario. This is done in the Figures 6 and 7 below for an illustrative signaling factor $\bullet$ = 0.6.

As can be seen in the Figure 7 above, this policy creates large amounts of bullwhip for all levels and in both scenarios. In all cases, there is an overshoot for all different frequencies and the overshoot increases proportionally with frequency. This is intuitively clear since we only use the two most recent demand observations and these short-run demand fluctuations correspond to high frequency signals.

Sharing information again significantly reduces the increase in bullwhip. In the traditional chain, the variance ratio at level four is up to 172; with information sharing, this is reduced to 17, thus ten times lower. Note that we have a log scale on the Y- axis of Figure 7. With a normal scale, we would again have a geometric versus an approximately linear increase in bullwhip for the two different scenarios.

The predictions are again very close to the simulated results as can be seen from the Table IV below.

	Traditional Supply Chain				**Supply chain with information enrichment**			
Echelon	**1**	**2**	**3**	**4**	**1**	**2**	**3**	**4**
a) Control engineering theory	2.92	10.37	41.044	172.09	2.92	6.28	11.08	17.32
b) Simulation results	2.939	10.498	41.729	175.5	2.939	6.346	11.257	17.748
% GAP a-b	0.66	1.24	1.67	1.98	0.66	1.06	1.6	2.47
Average GAP a-b	**1.39%**				**1.45%**			

Table IV. Verifying the control engineering based predictions for demand signal processing OUT policies through simulation

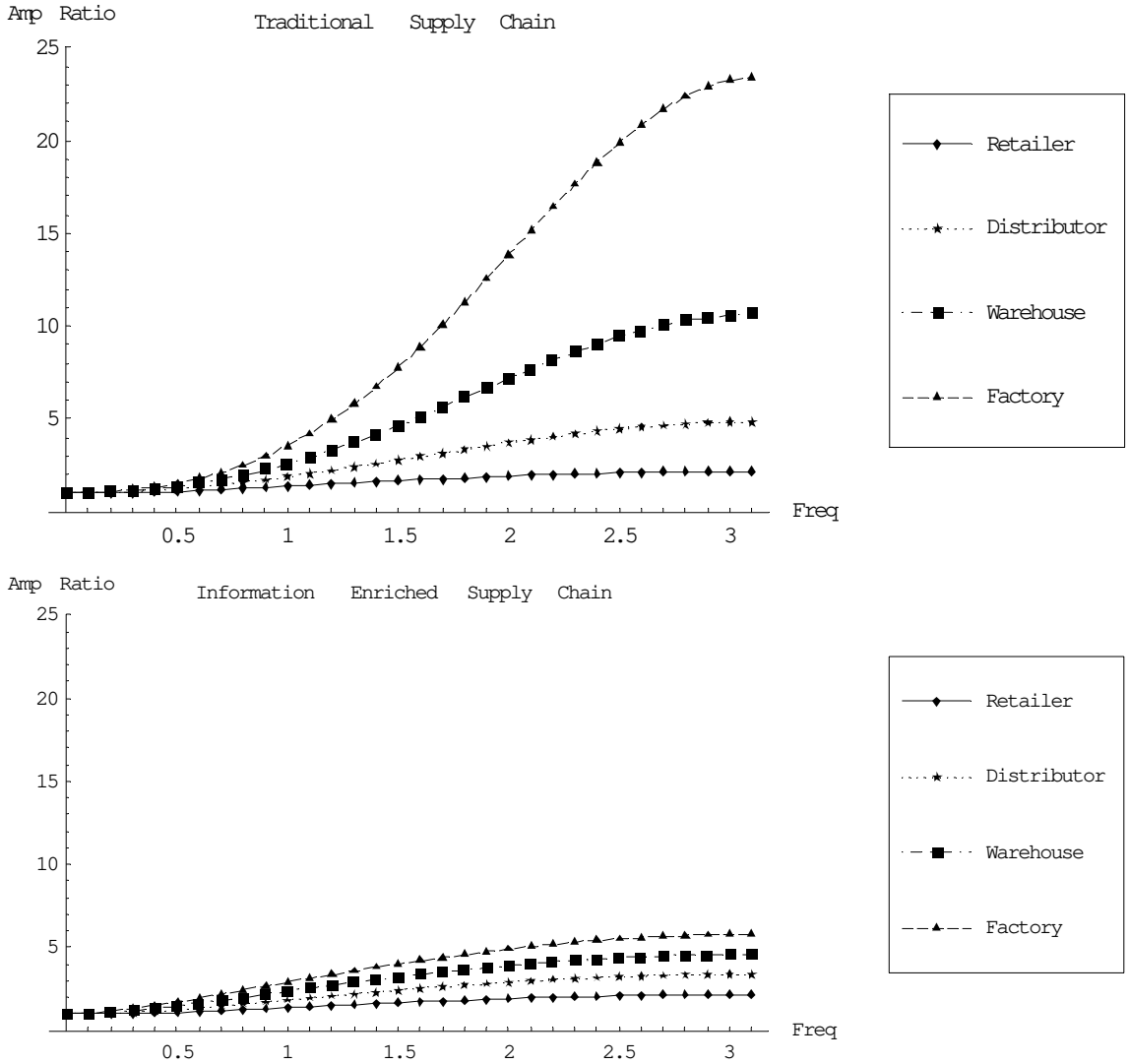


Figure 6. Frequency Responses for the demand signalling policy with moving average forecasts in the traditional supply chain scenario (upper graph) and in the information enriched scenario (lower graph).

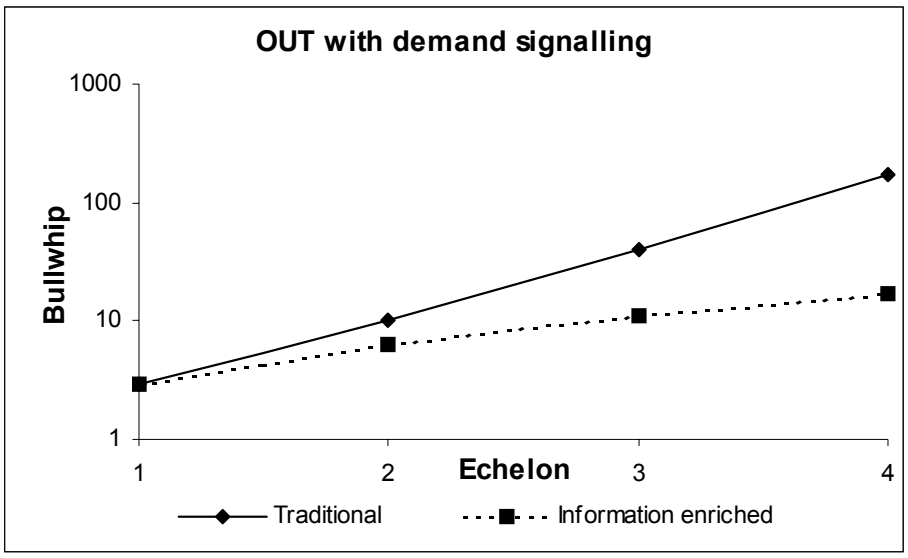


Figure 7. Variance ratios for demand signalling for both scenarios

4.4. Conclusions for order-up-to policies

In this section, we have analysed three types of order-up-to policies: order-up-to policies with exponential smoothing forecasts, order-up-to policies with moving average forecasts, and 'demand signalling' order-up-to policies. For the three policies, we applied the three-step approach explained in section 3. First, for all policies, we were able to derive the multi-level transfer functions for both the traditional supply chain and the information enriched supply chain. The transfer functions were very simple and elegant, even for higher echelons in the supply chain. Second, we plotted FRs and third, we computed variance ratio predictions ($W_N/\prec$) for the three policies under both scenarios. Our control engineering based predictions were verified via simulation and turned out to be very accurate. For the moving average forecasting policy, the predictions were identical with statistical bounds found in literature.

For the three policies, the same conclusions could be made. In a traditional scenario, bullwhip increases geometrically with the level in the chain and gets alarmingly large at higher echelons in the supply chain. Information sharing is able to reduce the increase in bullwhip significantly; the increase in now linear as the orders proceeds up the chain. However, bullwhip is not eliminated with information sharing! There is still bullwhip at all levels in the chain, and the variance ratio still increases with the level in the chain.

Note that we confirm some results found in literature (Chen et al., 2000a and b) and extend others. A first extension is the fact that in our approach, the different members in the chain are allowed to use different forecasting parameters. Secondly, we are able to make interesting conclusions based on the Frequency Responses (step 2) without having to make any assumptions on the demand pattern.

5. Information sharing with a smoothing replenishment rule

Order-up-to policies seem to unavoidably result in a bullwhip effect when demand has to be forecasted. Even with information sharing, bullwhip still increases with the level in the chain. In this section we analyse a general decision rule that does not have this drawback. We first analytically explain the smoothing decision rule, next, we analyse the rule in a multi-level supply chain for both scenarios with the three-step procedure explained in section 3. This decision rule is a natural extension of the Inventory and Order Based Production Control System (IOBPCS) model introduced by Towill, (1982) and Automatic Pipeline, Inventory and Order Based Production Control System (APIOBPCS) due to John, Naim and Towill (1994). Disney, Naim and Towill (1997) and Disney and Towill (2001b) have analysed and optimised this rule for inventory and bullwhip costs. APIOBPCS has been shown to be a general case of the linear production rule due to Deziel and Eilon (1967), which has some very interesting robustness and stability properties that have been identified by Disney (2001) and exploited by Disney and Towill (2001a).

<u>The decision rule</u>

The order quantity in period t, O_t, is given by:

$$O_t = \hat{D}_t^{Ta} + \frac{1}{Tn}(TNS_t - NS_t) + \frac{1}{Tw}(DWIP_t - WIP_t)\ , \tag{14}$$

where $\hat{D}_t^{Ta}$ is the demand forecast using simple exponential smoothing with parameter Ta, TNS_t a target net stock level, NS_t is the current net stock in period t, $DWIP_t$ is the desired WIP level, and WIP_t finally is the current work in process (or on-order) position in period t. TNS_t is the target net stock level, similar to the safety stock in order-up-to policies. It is updated every period according to the new demand forecast and equals $\hat{D}_t^{Ta}$. $DWIP_t$ is updated every period as well, $DWIP_t = (L-1)\hat{D}_t^{Ta}$. Note that we only have L-1 orders in WIP when the lead time is L because of our order of events: the ordering decision is made at the end of the

period and by this time one order has already been received and thus disappeared from the WIP. *Ta*, *Tn* and *Tw* are the key parameters or controllers of the decision rule. The policy can be described in words as 'ordering quantities are set equal to the sum of forecasted demand, a fraction (1/*Tn*) of the discrepancy of finished goods net stock, and a fraction (1/*Tw*) of our on-order position discrepancy.'

- ***Relationship with order-up-to policies***

Before we derive the transfer function, it is important to see the difference between policy (14) and an order-up-to policy. We defined the order-up-to policy as follows:

$$O_t = \hat{D}_t^{\,L} + k\,\hat{\sigma}_t^{\,L} - \textit{inventory position}_t \tag{15}$$

For simplicity, we have set k = 0 and increase the lead time L by one period. Inventory position equals net stock (*NS*) + products on order (*WIP*). We then successively obtain:

$$\begin{aligned} O_t &= (L+1)\,\hat{D}_t^{\,Ta} - NS_t - WIP_t \\ O_t &= \hat{D}_t^{\,Ta} + (\hat{D}_t^{\,Ta} - NS_t) + ((L-1) * \hat{D}_t^{\,Ta} - WIP_t) \\ O_t &= \hat{D}_t^{\,Ta} + (TNS_t - NS_t) + (DWIP_t - WIP_t) \end{aligned} \tag{16}$$

So (16) turns out to be completely analogous to the smoothing rule presented in (14) with parameters $Tn = Tw = 1$. In an order-up-to policy, the order quantity is a summation of the demand forecast, a net stock discrepancy (or error) term and a WIP discrepancy term, but both the net stock and WIP errors are completely taken into account. ***This is the key difference with our decision rule (14) in which the errors are included only fractionally. These fractional adjustments will exactly be the reason why the decision rule (14) will be able to generate smooth ordering patterns.*** Another difference is that in our smoothing decision rule, we have two separate feedback loops (one for the net stock and one for the WIP), whereas in an order-up-to policy, there is only one joint feedback loop for the inventory position. At first sight, these are small differences, but the impact is dramatic. Note that the decision rule presented in (14) is a very general rule. For this paper, we use exponential smoothing to forecast demand, but it is obvious that other forecasting methods can be used in (14). The astute reader will easily observe that order-up-to policies are actually a special case of our general rule, namely the case $Tn = Tw = 1$.

- ***The smoothing behaviour of the rule for a single level supply chain***

Dejonckheere et al. (2001) have shown that for a single level supply chain (only one chain member (e.g. the retailer) has to make ordering decisions) the transfer function equals:

$$F(z) = \frac{O}{D} = \frac{z^{1+Tp}(-(1+Ta)Tw + Tn(Tp+Tw)(-1+z) + (2+Ta)Twz)}{(Ta(-1+z)+z)(Tw + Tn(-1+(1+Tw(-1+z))z^{Tp}))} \tag{17}$$

Dejonckheere et al. (2001) showed that this decision rule (14) is capable of generating smooth ordering patterns. For an illustrative parameter setting *Ta*=8, *Tn*=4, *Tw*=4, (hereafter denoted parameter setting (8/4/4)) and when applied to normally distributed demands, the decision rule will reduce variance of demand down to approximately one third. Although there is still some moderate overshooting at a few low frequencies, this was more than compensated by filtering out the higher range of the frequency spectrum. The goal of this section is to extend the analysis to multi-echelon supply chains for both the traditional and the information enriched supply chain scenario. In the analysis, we will follow the three-step scenario proposed in section 3.

Step 1. Derive the transfer functions

For the traditional supply chain scenario, the general transfer function for level n is then as follows:

$$TF_n = \frac{O_n}{D} = \prod_{j=1}^{n} \frac{z^{1+Tp_j}((-1-Ta_j)Tw_j + z(2+Ta_j)Tw_j + (-1+z)Ti_j(Tp_j + Tw_j))}{(z+(-1+z)Ta_j)(Tw_j + Ti_j(-1+z^{Tp_j}(1+(-1+z)Tw_j)))} \tag{18}$$

When all supply chain members use the same parameter values and the lead-times are the same, then the transfer functions can be simplified to:

$$TF_n = \frac{O_n}{D} = \left(\frac{z^{1+Tp}((-1-Ta)Tw + Ti(Tp+Tw)(-1+z) + (2+Ta)Twz)}{(Ta(-1+z)+z)(Tw + Ti(-1+(1+Tw(-1+z))z^{Tp}))} \right)^n \tag{19}$$

Next, for the <u>information-enriched</u> scenario, the general transfer function (see Appendix C for the block diagram) for level n is then:

$$TF_n = \frac{O_n}{D} = \frac{z^{1+Tp_n}((-1+z)Tn_n(Tp_n + Tw_n) + (Tw_n(-1+z+(z+(-1+z)Ta_n)X_n))}{(z+(-1+z)Ta_n)(Tw_n + Tn_n(-1+z^{Tp_n}(-1+z^{Tp_n}(1+(-1+z)Tw_n)))} \tag{20}$$

where when n=1: $X_1=1$, n=2,…….,^ $X_n=TF_{n-1}$

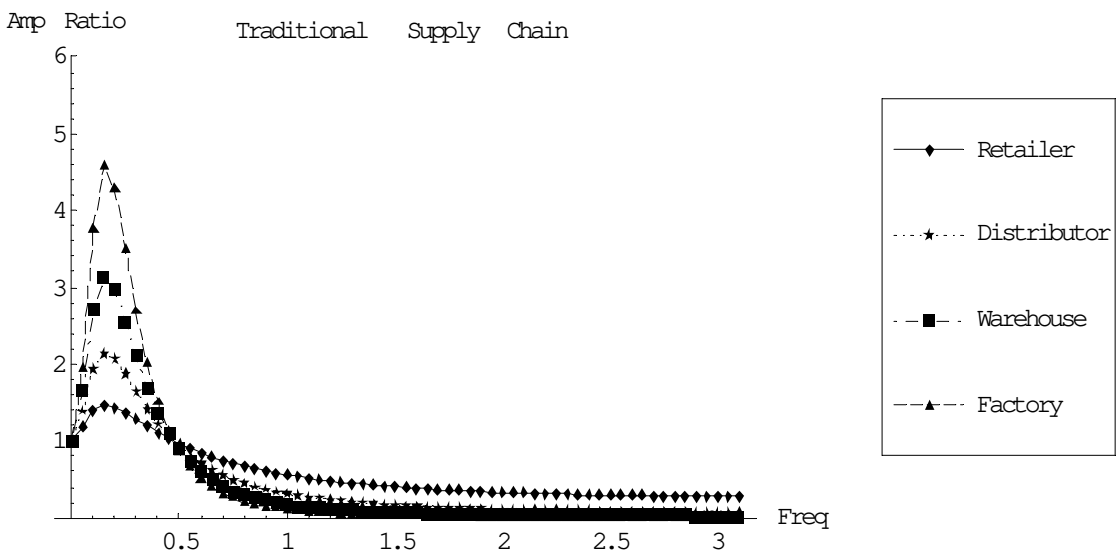
Finally, for the <u>information-enriched</u> scenario, and with all supply chain member using the same parameters and having the same lead-time, we have:

$$TF_n = \frac{O_n}{D} = \frac{z^{1+Tp}((-1+z)Tn(Tp+Tw) + (Tw(-1+z+(z+(-1+z)Ta)X_n))}{(z+(-1+z)Ta)(Tw + Tn(-1+z^{Tp}(-1+z^{Tp}(1+(-1+z)Tw)))} \tag{21}$$

where when n=1: $X_1=1$, n=2,…….,^ $X_n=TF_{n-1}$

<u>Step 2. Plot the frequency response</u>

Using these transfer functions, we plot the FRs for an illustrative case where all lead times are 3 time periods and all supply chain members use the parameter setting (8/4/4).



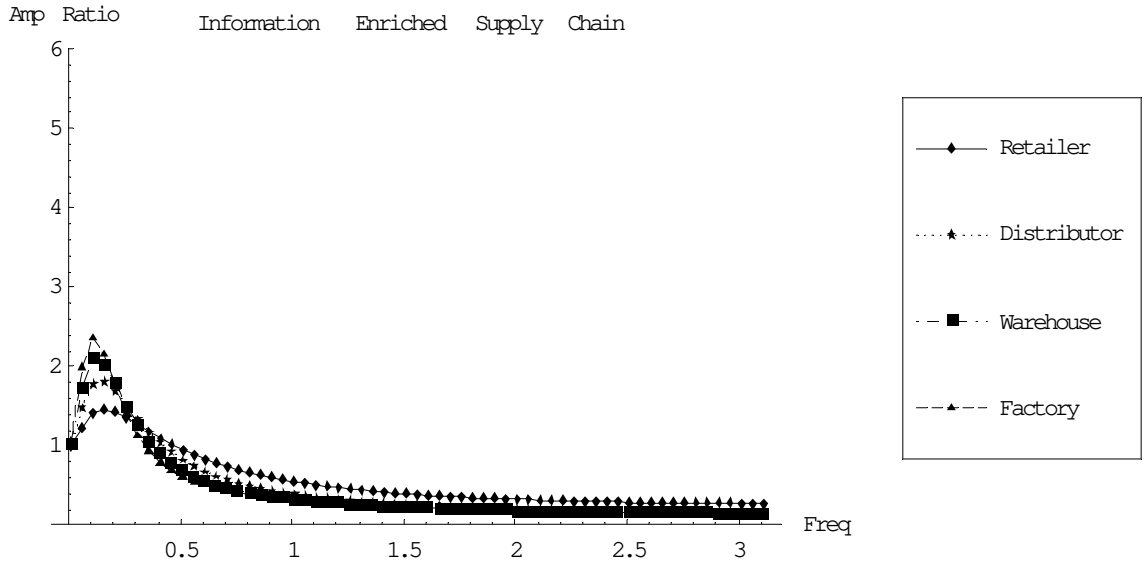


Figure 8. Frequency responses for the smoothing decision rule (14) in the traditional supply chain scenario (upper graph) and in the information enriched scenario (lower graph).

In the upper graph, we see the dynamics of our smoothing rule in the traditional supply chain. The overshooting at the low frequencies increases quite a lot for higher levels in the supply chain. This means that we're not sure anymore whether or not the decision will reduce order variability at those levels. The benefits of information sharing are clearly shown in the lower graph; the frequency response remains more or less identical at higher levels in the chain. This is a very desirable result, because it means that we will be able to keep the order variability acceptably low throughout the chain.

<u>Step 3. The noise bandwidth</u>

Finally, we computed $W_N \ / \prec$ for a four level supply chain in both scenarios. The results are given in the Figure 9 below:

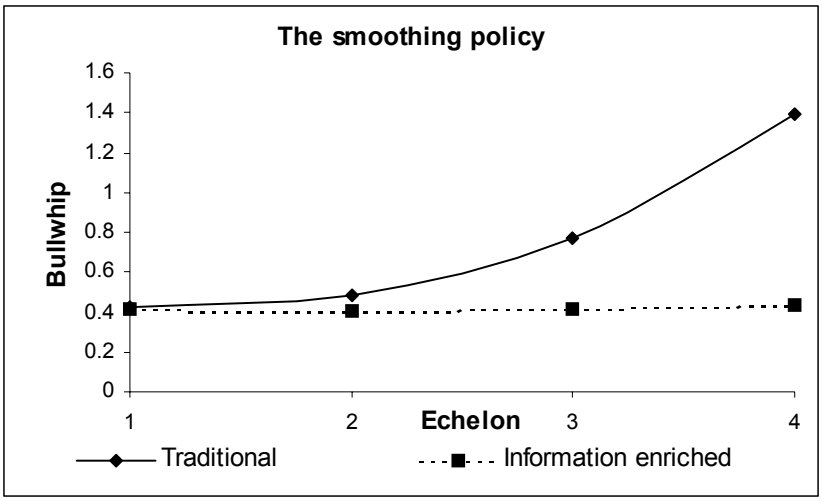


Figure 9. Variance Ratios for the smoothing policy for both scenarios

Figure 9 plots the variance ratios for the parameter setting (8/4/4) at all levels in the chain. We can clearly see that information sharing is essential to ensure smoothed demand signals throughout the chain. In the traditional supply chain, from level four onwards, the decision rule would not reduce variance anymore and will actually create bullwhip.

Note that the selection of the parameters is not the scope of this paper, for this see Disney, Naim and Towill (1997) and Disney and Towill (2001b). Of course, more or less variability reduction could be obtained by selecting other parameters. ***In this paper, we wish to emphasise the dynamics of the decision rule (14) in multi-level supply chains: information sharing helps to avoid variance amplification at higher levels in the chain.***

Finally, also for this decision rule, we cross-checked the control engineering variance ratio predictions with spreadsheet simulation. The results are summarised in Table V below. They confirm again that the W_N $/\prec$ is a very accurate tool to predict variance ratios for normally distributed demands

	Traditional supply chain				**Supply chain with information enrichment**			
Echelon	**1**	**2**	**3**	**4**	**1**	**2**	**3**	**4**
a) Control engineering theory	0.423	0.4828	0.773	1.376	0.423	0.407	0.42	0.434
b) Simulation results	0.432	0.493	0.787	1.398	0.432	0.417	0.433	0.45
% GAP a-b	2.07	2.15	1.84	1.68	2.07	2.48	2.99	3.72
Average GAP a-b	**1.94%**				**2.82%**			

Table V. Verifying the control engineering based predictions through simulation

6. Summary

Where possible, we have found the following closed form expressions for bullwhip that are highlighted in Table VI. Note that completely general closed form expressions have yet to be found in some cases, but amazingly information enriched supply chains are generally tractable, whilst the traditional supply chain closed forms involve transcendental equations and we have yet to find generic solutions. In such cases the graphs and results presented in this paper were produced via enumeration of equation 2.

7. Conclusion

In this paper we have shown the benefits of sharing end customer demand information throughout a multi level supply chain. We have compared a traditional (decentralised) supply chain with a (centralised) supply chain with information enrichment for two classes of replenishment rules: order-up-to policies and smoothing policies. Our methodology was based on control systems engineering. We have used a three-step procedure to analyse all replenishment rules that are treated in the paper: first, the transfer function is derived, second, the Frequency Response is plotted, and finally, the Noise Bandwidth is calculated. Based on the Noise Bandwidth, we are able to predict the magnitude of variance increase (or decrease) for normally distributed demands. The predictions have been verified through simulation, and compared with results from statistical analysis if available in the literature, and they have proved to be very accurate.

Order-up-to policies are often used both in practice and in research because they are known to minimise inventory and shortage costs. However, as shown, they do have some serious drawbacks. We have integrated three types of forecasting in the order-up-to system (exponential smoothing, moving average, and 'demand signal processing') and all of them created variance amplification or bullwhip. In the traditional supply chain, whatever forecasting method was used, the magnitude of bullwhip increases geometrically upstream in the supply chain. Information sharing is very beneficial, if not indispensable, for order-up-to policies, since then the magnitude of bullwhip can be significantly reduced for higher levels in the chain.

	Traditional supply chain	**Supply chain with information enrichment**
OUT Exponential smoothing	Bullwhip for i.i.d demands at each echelon = $\frac{13+2Ta^2+2Tp(5+Tp)+Ta(11+4Tp)}{(1+Ta)(1+2Ta)}$	Bullwhip at each echelon for i.i.d at end consumer with same parameters at each echelon = $\frac{\left[1+3Ta+2(Ta^2+n(1+2Ta)(2+Tp)+n^2(2+Tp)^2)\right]}{(1+Ta)(1+2Ta)}$
OUT Moving average	Bullwhip of i.i.d. demands at each echelon = $\frac{8+4Tm+Tm^2+8Tp+2TmTp+2Tp^2}{Tm^2}$	Bullwhip at each echelon for i.i.d at end consumer with same parameters at each echelon = $\frac{Tm^2+2n(2+Tp)(Tm+n(2+Tp))}{Tm^2}$
OUT Demand signal processing	Bullwhip at each echelon for i.i.d at end consumer with same parameters at each echelon when: n=1, $1+2\gamma(1+\gamma)$ n=2, $1+2\gamma(1+\gamma)(2+3\gamma(1+\gamma))$ n=3, $(1+2\gamma(1+\gamma))(1+2\gamma(1+\gamma)(2+5\gamma(1+\gamma)))$ n=4, $1+2\gamma(1+\gamma)(4+\gamma(1+\gamma)(18+5\gamma(1+\gamma)(8+7\gamma(1+\gamma))))$	Bullwhip at each echelon for i.i.d at end consumer with same parameters at each echelon = $1+2n\gamma(1+n\gamma)$
Smoothing policy	Bullwhip for i.i.d demands at each echelon when Tn=Tw=Ta+1= $\frac{13+10Ta^2+2Tp(5+Tp)+2Ta(11+4Tp)}{(1+2Ta)^3}$	Bullwhip expressions are not obvious. Graphs produced via numerical integration of Noise Bandwidth/$\prec$

Table VI. Summary of closed form expressions for bullwhip

More specifically, for the supply chains with information enrichment, the increase of bullwhip will generally be of a linear nature with the level in the chain instead of geometrical. However, the bullwhip cannot be completely eliminated through information sharing: there is still variance amplification at all levels, and the magnitude of amplification still increases upwards the chain. Hence if production is inflexible and significant costs are involved in following fluctuating order rates, order-up-to policies may not be optimal. In the last section of the paper we have therefore analysed a class of policies that are able to reduce variance of demand and thus have a smoothing or dampening impact. We have also shown that information sharing is very crucial for these policies as well. In the traditional supply chain, such a smoothing policy can lose its dampening abilities at higher levels of the chain, whereas in the information enriched chain, smoothed order rates may be realized by all levels in the chain.

Acknowledgements

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Appendix A: Block diagram for multi-level order-up-to policies in the information enriched scenario

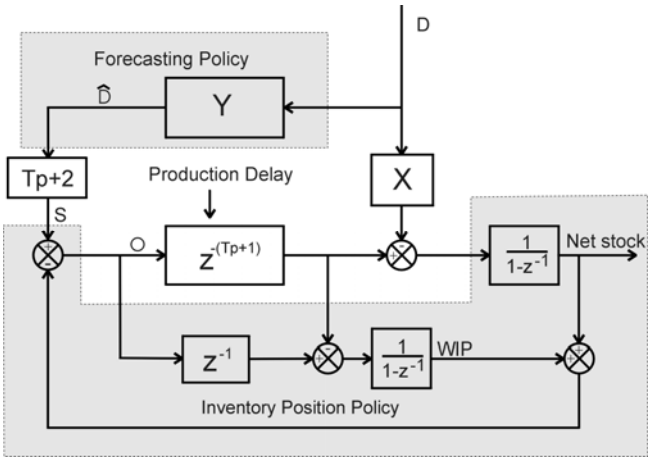

Figure A.1. Block diagram for Order-Up-To supply chains

Where when n=1: $X_1=1$, n=2,…….,^ $X_n=TF_{n-1}$

Inventory Policy	Forecasting Policy, Y
Order-up-to policy with Demand Signal Processing	• with 0<•≤ 1
Order-up-to policy with Exponential Smoothing	$\frac{\alpha}{1-(1-\alpha)z^{-1}}$; $\alpha=1/(1+T_a)$
Order-up-to policy with Moving Average	$\frac{1-z^{-T_m}}{T_m\left(1-\frac{1}{z}\right)}$; T_m ⌒ $2T_a+1$
Pure order-up-to policy	0

Dejonckheere, J., Disney, S.M., Lambrecht, M.R. and Towill, D.R., (2004), "The impact of information enrichment on the bullwhip effect in supply chains: A control engineering perspective", European Journal of Operational Research, Vol. 153, No. 3, pp727–750. ISSN: 0377-2217. DOI: 10.1016/S0377-2217(02)00808-1.

Appendix B: Transfer Functions for order-up-to policies and the smoothing replenishment rule with different forecasting methods in both the traditional and information-enriched scenario

Policy	Focus	Traditional supply chain	Information enriched supply chain
Order-up-to policy with exponential smoothing forecasts	TF_1	$TF_1 = \frac{z(Ta + Tp + 3) - (Tp + 2 + Ta)}{-Ta + z + Taz}$	
	General TFs	$TF_n = \frac{O_n}{D} = \prod_{j=1}^{n}\left[\frac{z(Ta_j + Tp_j + 3) - (Tp_j + 2 + Ta_j)}{-Ta_j + z + Ta_j z}\right]$	$TF_n = \frac{O_n}{D} = TF_1 + \sum_{j=2}^{n}\left[\frac{(2+Tp_j)(z-1)}{-Ta_j + z + Ta_j z}\right]$
	Same Parameter TFs	$TF_n = \frac{O_n}{D} = \left[\frac{z(Ta + Tp + 3) - (Tp + 2 + Ta)}{-Ta + z + Taz}\right]^n$	$TF_n = \frac{O_n}{D} = \frac{z(2n+1+Ta+nTp)-(2n+Ta+nTp)}{-Ta+z+Taz}$
Order-up-to policy with moving average forecasts	TF_1	$TF_1 = \frac{O}{D} = \frac{-2 - Tp + 2z^{Tm} + Tmz^{Tm} + Tpz^{Tm}}{Tmz^{Tm}}$	
	General TFs	$TF_n = \frac{O_n}{D} = \prod_{j=1}^{n}\left[\frac{-2-Tp_j + 2z^{Tm_j} + Tm_j z^{Tm_j} + Tpz^{Tm_j}}{Tm_j z^{Tm_j}}\right]$	$TF_n = \frac{O_n}{D} = TF_1 + \sum_{j=2}^{n}\left[\frac{(1-z^{-Tm_j})(Tp_j + 2)}{Tm_j}\right]$
	Same Parameter TFs	$TF_n = \frac{O_n}{D} = \left[\frac{-2 - Tp + 2z^{Tm} + Tmz^{Tm} + Tpz^{Tm}}{Tmz^{Tm}}\right]^n$	$TF_n = \frac{O_n}{D} = \frac{-2n - nTp + 2nz^{Tm} + Tmz^{Tm} + nTpz^{Tm}}{Tmz^{Tm}}$
Order-up-to policy with demand signalling forecasts	TF_1	$TF_1 = \frac{O}{D} = 1 + \gamma - \frac{\gamma}{z}$	
	General TFs	$TF_n = \frac{O_n}{D} = \prod_{j=1}^{n}\left[1 + \gamma_j - \frac{\gamma_j}{z}\right]$	$TF_n = \frac{O_n}{D} = 1 + \sum_{j=1}^{n}\gamma_j - \sum_{j=1}^{n}\frac{\gamma_j}{z}$
	Same Parameter TFs	$TF_n = \frac{O_n}{D} = \left[1 + \gamma - \frac{\gamma}{z}\right]^n$	$TF_n = \frac{O_n}{D} = 1 + n\gamma - \frac{n\gamma}{z}$
Smoothing replenishment rule with exponential smoothing forecasts when n=1: $X_1=1$, n=2,….,^ $X_n=TF_{n-1}$	TF_1	$TF_1 = \frac{O}{D} = \frac{z^{1+Tp}((-1-Ta)Tw + Ti(Tp+Tw)(-1+z) + (2+Ta)Twz)}{(Ta(-1+z)+z)(Tw + Ti(-1+(1+Tw(-1+z))z^{Tp}))}$	
	General TFs	$TF_n = \frac{O_n}{D} = \prod_{j=1}^{n}\frac{z^{1+Tp_j}((-1-Ta_j)Tw_j + z(2+Ta_j)Tw_j + (-1+z)Ti_j(Tp_j+Tw_j))}{(z+(-1+z)Ta_j)(Tw_j + Ti_j(-1+z^{Tp_j}(1+(-1+z)Tw_j)))}$	$TF_n = \frac{O_n}{D} = \frac{z^{1+Tp_n}((-1+z)Tn_n(Tp_n+Tw_n) + (Tw_n(-1+z+(z+(-1+z)Ta_n)X_n))}{(z+(-1+z)Ta_n)(Tw_n + Tn_n(-1+z^{Tp_n}(-1+z^{Tp_n}(1+(-1+z)Tw_n)))}$
	Same parameter TFs	$TF_n = \frac{O_n}{D} = \left(\frac{z^{1+Tp}((-1-Ta)Tw + Ti(Tp+Tw)(-1+z) + (2+Ta)Twz)}{(Ta(-1+z)+z)(Tw + Ti(-1+(1+Tw(-1+z))z^{Tp}))}\right)^n$	$TF_n = \frac{O_n}{D} = \frac{z^{1+Tp}((-1+z)Tn(Tp+Tw) + (Tw(-1+z+(z+(-1+z)Ta)X_n))}{(z+(-1+z)Ta)(Tw + Tn(-1+z^{Tp}(-1+z^{Tp}(1+(-1+z)Tw)))}$

Appendix C: Block diagram for the multi-level smoothing decision rule in information enriched scenario

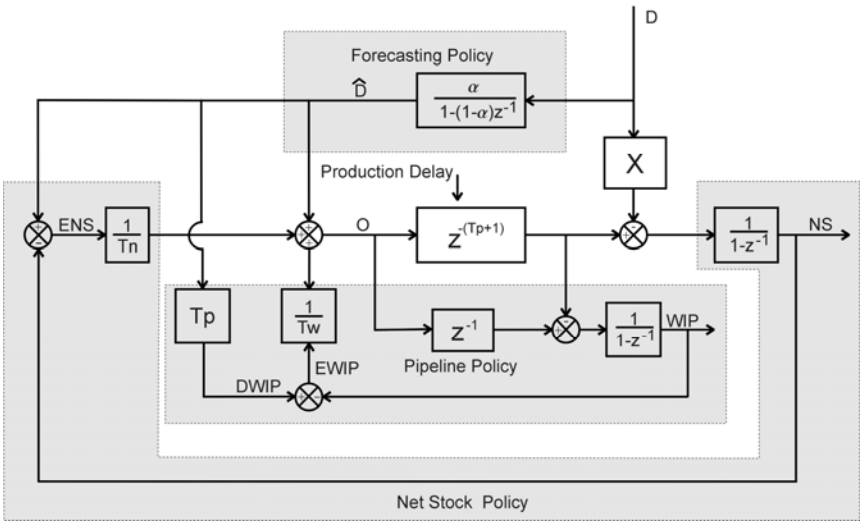

Figure C.1. Block diagram for the smoothing replenishment rule supply chain

when n=1: $X_1=1$, n=2,…….,^ $X_n=TF_{n-1}$

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Footnotes

[1] An independent and identically distributed normal demand pattern with a mean of 100 and a standard deviation of 10, 10000 time-periods long was used. The same demand signal is used throughout this paper.

[2] Chen et al 2000a gives the lower bound for a single level of the supply chain as $\frac{Var(O)}{Var(D)} \geq 1 + 2L\alpha + \frac{2L^2\alpha^2}{2-\alpha}$, which is easily extended for a multi-level traditional supply chain to $\frac{Var(O_n)}{Var(D)} \geq \prod_{j=1}^{n}\left(1 + 2L_j\alpha_j + \frac{2L_j^{\,2}\alpha_j^{\,2}}{2-\alpha_j}\right)$. The statistical lower bounds for the information enriched supply chain is given by $\frac{Var(O_n)}{VarD} = 1 + 2\left(\sum_{j}^{n} L_j\right)\alpha_i + \frac{2\left(\sum_{j=1}^{n} L_j\right)^2 \alpha_n}{2-\alpha_n}$.

This was found by the authors via an analogy with the tight bounds for the moving average forecast presented by Chen et al 2000b.