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Citation: *Appl. Phys. Lett.* **91**, 172512 (2007); doi: 10.1063/1.2802556

View online: <http://dx.doi.org/10.1063/1.2802556>

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Including effects of microstructure and anisotropy in theoretical models describing hysteresis of ferromagnetic materials

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(Received 10 July 2007; accepted 5 October 2007; published online 24 October 2007)

Two recent theoretical hysteresis models (Jiles-Atherton model and energetic model) are examined with respect to their capability to describe the dependence of the magnetization on magnetic field, microstructure, and anisotropy. It is shown that the classical Rayleigh law for the behavior of magnetization at low fields and the Stoner-Wohlfarth theory of domain magnetization rotation in noninteracting magnetic single domain particles can be considered as limiting cases of a more general theoretical treatment of hysteresis in ferromagnetism. © 2007 American Institute of Physics. [DOI: 10.1063/1.2802556]

The theoretical description of nonlinear, hysteretic processes in general, and ferromagnetic hysteresis in particular, is known to be a difficult problem. In the case of ferromagnetism, this is due to the multiplicity of processes, both reversible and irreversible, that take place simultaneously within a ferromagnet under the action of a magnetic field. Successful descriptions of the behavior of magnetic materials under the influence of the applied field and its history by hysteresis modeling have great impact on the field of magnetism.¹⁻⁴ In addition, it would be very useful to predict the changes of the magnetic properties due to other physical effects, such as applied and residual stresses, fatigue, temperature, or irradiation. Moreover, applications require integration of the model into system design software with sufficient simplicity that will allow fast computation and efficient parameter identification strategies.

In this work two recent hysteresis models, Jiles-Atherton model (JAM) by Jiles and Atherton⁵ and energetic model (EM) by Hauser,⁶ were studied in order to relate their model parameters to the microstructure and anisotropy, as well as to study these models in some limiting cases. The detailed descriptions of these two models can be found elsewhere.^{5,6} For both models, we used simplified anhysteretic functions: either a linear or hyperbolic tangent, which is well known in situations where the magnetic moments are constrained to lie along a single axis (“spin up” or “spin down”). This is referred to as the one dimensional case. Also, working with two and three dimensional cases is possible: the former leading to a series solution for the anhysteretic function, the latter leading to the Langevin function.⁷ The indices *J* and *H* are to distinguish between the *k* coefficients in the JAM and EM, respectively.

The law of Lord Rayleigh describes the magnetization curve at low fields up to coercivity H_c as a parabolic function:⁸

$$M = \chi_0 H + \nu_R H^2, \quad (1)$$

where $\chi_0 = dM/dH$ at $M=H=0$ is the initial susceptibility and ν_R is the Rayleigh constant. If we use a second order

Taylor series of the initial magnetization of the JAM and EM, we can find the relationship between Rayleigh coefficients χ_0 and ν_R and the coefficients of the JAM and EM.

In case of the JAM,⁵ these coefficients are found to be

$$\chi_0 = \frac{cM_s}{a - c\alpha M_s}, \quad (2)$$

$$\nu_R = \frac{a^2 M_s (1 - c)}{2k_J (a - c\alpha M_s)^3}, \quad (3)$$

where M_s is the spontaneous magnetization, c is the reversibility coefficient, k_J is the loss coefficient, α represents the domain coupling, and the geometric demagnetizing factor N_d is neglected.⁵ In the following considerations, we use a linear approximation of an anhysteretic function which is valid for one dimensional case (a hyperbolic tangent) at the origin.

The EM model⁶ with linear anhysteretic magnetization gives the following expressions for the Rayleigh coefficients:

$$\chi_0 = \frac{\mu_0 M_s^2}{k_H q + \mu_0 M_s^2 N_i}, \quad (4)$$

$$\nu_R = \frac{\mu_0^2 M_s^3 k_H q^2}{2(k_H q + \mu_0 M_s^2 N_i)^3}, \quad (5)$$

where N_i is the inner demagnetizing factor (the geometrical demagnetizing factor N_d is neglected), k_H is the loss coefficient, and q is a coefficient in the probability function of irreversible domain wall displacements.^{6,9}

To consider the correlation of these results with micromagnetics, we recall that in a statistical theory of a domain wall pinning by Kronmüller¹⁰ the relationships between defect density N and the geometry of domains and domain walls are established, resulting in $H_c \propto \sqrt{N}$, $\chi_0 \propto 1/\sqrt{N}$, and $\nu_R \propto 1/N$.

Assuming that $N \propto p_0 + p_C$, it is possible to find the initial defect density p_0 if we have H_c , χ_0 , or ν_R for at least two measurements for different amounts of induced defects p_C , due to external influence, such as that caused by irradiation or fatigue:

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TABLE I. JAM and EM coefficients of Fe–C series (Ref. 11).

wt %C	0.00	0.24	0.47	0.74
M_s (MA/m)	1.700	1.655	1.609	1.555
a (kA/m)	2.623	2.427	2.306	1.429
α (10^{-3})	4.280	3.901	3.447	1.849
c (10^{-1})	2.002	1.213	0.743	0.509
k_J (A/m)	245	532	827	958
k_H (J/m ³)	422	980	1560	1810
$q(1)$	13.7	16.9	22.6	29.5
N_i (10^{-4})	1.70	4.99	8.54	9.08

$$\frac{H_{c,2}}{H_{c,1}} = \sqrt{\frac{\nu_{R,1}}{\nu_{R,2}}} = \frac{\chi_{0,1}}{\chi_{0,2}} = \sqrt{\frac{p_0 + p_{C,2}}{p_0 + p_{C,1}}}, \quad (6)$$

where the indices 1 and 2 stand for different amounts of induced defects.

As an example, let us consider the recently studied case of low carbon Fe–C steel samples with different percentages of carbon, which were analyzed by both models.¹¹ The model coefficients are summarized in Table I. Evaluating Eq. (6) for χ_0 and ν_R for different wt % C and averaging them over all combinations of p_C , we find that $p_0=0.0269\%$ for the EM and $p_0=0.0388\%$ for the JAM. Figures 1 and 2 compare functions of $\sqrt{N}=\sqrt{p_0+p_C}$ with the normalized H_c , $1/\chi_0$, and $\sqrt{1/\nu_R}$ for the EM and JAM, respectively. The values of H_c of the major loop, which were computed using the expressions $H_c=k_H/\mu_0M_s$ and $H_c=k_J(1-c)$ for both EM and JAM,¹¹ show the best agreement with \sqrt{N} . However, due to the fact that χ_0 is not directly comparable between JAM and EM,¹¹ we cannot use χ_0 reliably in this case. All these considerations indicate that both models are capable in predicting the initial percentage of impurities or defects in magnetic materials, if measurements with different densities (e.g., caused by fatigue or irradiation) are known.

Another consideration is to relate the coefficients of the JAM and EM to the well-known approach of Stoner and Wohlfarth (SW) of coherent rotation.¹² In order to achieve the directional dependence we use the uniaxial anisotropy energy density

$$w_k = K_u \sin^2 \varphi, \quad (7)$$

where φ is the angle between M and the easy axis. The ideal SW behavior is characterized by a rectangular loop in the

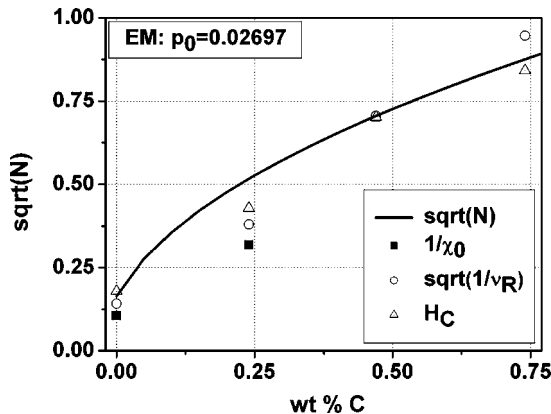


FIG. 1. Normalized dependences of $1/\chi_0$, $\sqrt{1/\nu_R}$, H_c calculated from EM and $\sqrt{N}=\sqrt{p_0+p_C}$ as a function of wt % C of Fe–C series steels.

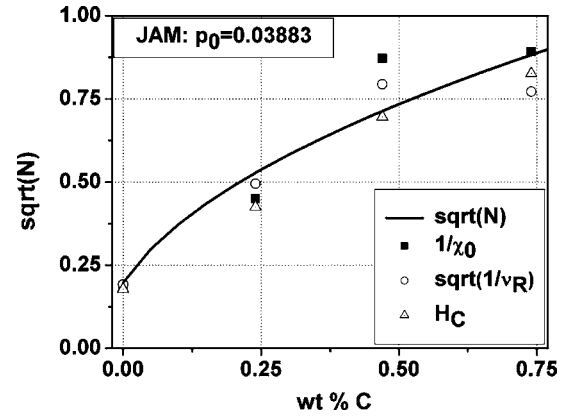


FIG. 2. Normalized dependences of $1/\chi_0$, $\sqrt{1/\nu_R}$, H_c calculated from JAM and $\sqrt{N}=\sqrt{p_0+p_C}$ as a function of wt % C of Fe–C series steels.

$\varphi=0$ direction and $M=M_s H/H_k$ in the $\varphi=\pi/2$ direction.¹²

In order to identify the values of the JAM parameters, which will be used to describe an equivalent Stoner–Wohlfarth system, we consider their linear anisotropy dependence as proposed earlier:¹³

$$a = a_c + c_a \frac{2K_u}{\mu_0 M_s} \sin^2 \varphi, \quad (8)$$

$$\alpha = \alpha_c + c_\alpha \frac{2K_u}{\mu_0 M_s^2} \sin^2 \varphi, \quad (9)$$

$$N_i = \frac{2K_u}{\mu_0 M_s^2} \sin^2 \varphi \quad (10)$$

with $N_i = a/M_s - \alpha$, and where a_c and α_c are the corresponding values of a and α at $\varphi=0$. If the normalized magnetization $m = m_m = 1 - \delta$ with $0 < \delta \ll 1$ at $H = H_k$ for both $\varphi=0$ and $\varphi=\pi/2$, then we can identify¹¹

$$a_c = \frac{H_k}{\operatorname{arctanh} m_m - m_m}, \quad (11)$$

$$\alpha_c = \frac{H_k}{M_s (\operatorname{arctanh} m_m - m_m)}, \quad (12)$$

$$c_\alpha = \frac{H_k - (a_c + H_k) \operatorname{arctanh} m_m + \alpha_c M_s m_m}{H_k (\operatorname{arctanh} m_m - m_m)}, \quad (13)$$

$$c_a = 1 + c_\alpha. \quad (14)$$

For the case of $\varphi=0$, we have to set $c=0$ (which means irreversible switching processes only) and $k_J=H_k$, which leads to $H_c=H_k$. For the case $\varphi=\pi/2$, we have to set $c=1$ (which corresponds to reversible processes only) and $k_J=0$, $H_c=0$. The SW behavior is then found when $\delta \rightarrow 0$, but this may cause numerical problems because of the large numbers involved. Lower m_m at H_k will result in a smooth transition into saturation, as often observed in real materials. Figure 3 shows the JAM results for $\mu_0 M_s = 1$ T and $H_k = 10$ kA/m. Both simulations $\varphi=0$ and $\varphi=\pi/2$ behave as expected. In the case of $\varphi=0$, the modeled coercivity was slightly larger than expected $H_c = 1.15 H_k$.

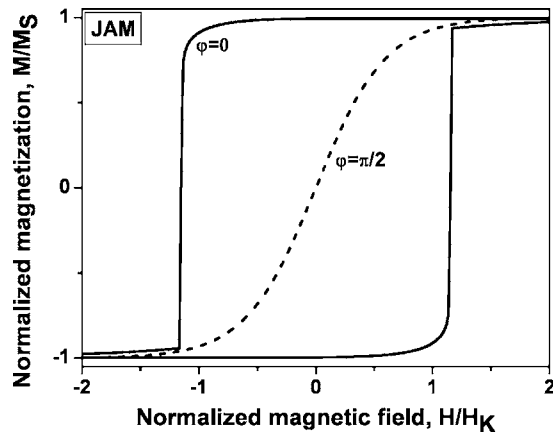


FIG. 3. SW-JAM hysteresis simulations of $\varphi=0$ (with $M_s=800$ kA/m, $a=6$ kA/m, $k_j=10$ kA/m, $\alpha=0.02$, $c=0$, and $N_d=0$) and $\varphi=\pi/2$ (with $M_s=800$ kA/m, $a=6$ kA/m, $k_j=0.01$ A/m, $\alpha=0.0$, $c=1$, and $N_d=0$) with $H_k=10$ kA/m.

Similar considerations can be made for the EM. Here, $k_H=\mu_0 M_s H_k$ and a variation of q will be considered: $q=0$ in the $\varphi=\pi/2$ direction and $q\gg 1$ in the $\varphi=0$ direction. In order to obtain the SW behavior, we set

$$N_i = \frac{2K_u \sin^2 \varphi}{\mu_0 M_s^2}, \quad (15)$$

$$2^g h = \frac{2K_u}{\mu_0 M_s}. \quad (16)$$

This results in $H_s=H_k$ both in the $\varphi=0$ and $\varphi=\pi/2$ directions. Figure 4 shows the EM result for $\mu_0 M_s=1$ T, $H_k=10$ kA/m, $g=120$, $h=10^{-33}$, $k_H=10$ kJ/m³, and $q_0=60$.

The results of this investigation have shown that the Rayleigh law at low fields and Stoner-Wohlfarth coherent rotation can both be described as limiting cases of a more general hysteresis model which encompasses both low and high field behaviors. Through the more general hysteresis model it is possible to include additional features, for example, a smooth approach to saturation at $\varphi=0$ and $\varphi=\pi/2$, $H_c > 0$ in the hard axis direction, $H_c < H_k$ in the easy axis direction, etc. Furthermore, a comparison with micromagnetic modeling showed the dependence of the model coefficients on the defect density and thus the ability to describe the effects of degradation by hysteresis measurements. The predictions of the Rayleigh law and Stoner-Wohlfarth

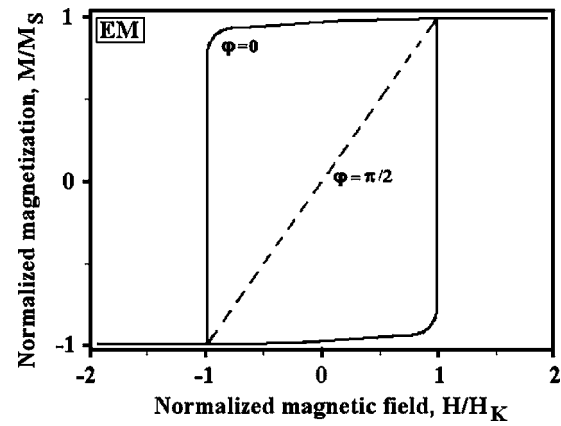


FIG. 4. SW-EM hysteresis simulations of $\varphi=0$ (with $M_s=800$ kA/m, $g=120$, $h=10^{-33}$, $k_H=10$ kJ/m³, and $q\gg 1$) and $\varphi=\pi/2$ (with $M_s=800$ kA/m, $g=120$, $h=10^{-33}$, $k_H=10$ kJ/m³, and $q=0$) with $H_k=10$ kA/m.

magnetization curves may, therefore, be considered as a special case of a more generalized hysteresis model that can be described by the JAM and EM. The analysis also revealed that the coefficients of the anhysteretic functions of the JAM and EM depend approximately linearly on the effective anisotropy. This can be utilized to build relationships between these model coefficients and with the physical properties such as spontaneous magnetization and the coefficients of anisotropy and magnetostriction.

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