LOAD ESTIMATION OF DOMESTIC SMART METER MEASUREMENTS USING MOVING AVERAGE AND MOVING MEDIAN TECHNIQUES

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ABSTRACT
In this paper, the performance of the moving average and moving median load estimation techniques is investigated using aggregated measurements of domestic smart meters. The load estimation techniques were tested using forward and backward walk approaches. Forward walk aims to estimate future load measurements using past measurements while backward walk estimates missing past measurements of the load using more recent measurements. Simulation results show that the moving average combined with forward walk produces load estimates with higher accuracies than the moving median and backward walk.

Keywords: Load estimation, moving average, smart meter measurements

NOMENCLATURE

<table>
<thead>
<tr>
<th>Abbreviations</th>
<th>Mean Absolute Percentage Error</th>
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<tr>
<td>MAPE</td>
<td>Moving Average</td>
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<tr>
<td>MA</td>
<td>Moving Median</td>
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<tr>
<td>MM</td>
<td>Half-hourly measurement of an</td>
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<td></td>
<td>individual smart meter</td>
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<td>lpi(t)</td>
<td>Index of smart meters, i = 1,2,...,3600</td>
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<tr>
<td>i</td>
<td>Index of half hours, t = 1,2,...,48</td>
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1. INTRODUCTION
Statistical and time series, computational intelligence and hybrid models have been applied to analyse, estimate and forecast the load in power networks [1], [2]. Time-series models estimate the future load based upon historical measurements obtained from load surveys, customer bills or smart meter measurements. Therefore, the underlying assumption is that the future load will follow the trend of the past load. Time series models include the basic simple Moving Average (MA), Auto Regressive Moving Average (ARMA), Auto Regressive Integrated Moving Average (ARIMA) and seasonal ARIMA (SARIMA) [3]. These methods are easy to implement, require no prior knowledge of any variables other than past load measurements and the simplicity of their mathematical models. Therefore, time series models have been widely used for load estimation and short-term load forecasting at a building and regional/national levels [2], [4].

The simple moving average (MA) has been widely used to estimate and forecast future load measurements. To the best of authors’ knowledge, the application of moving median (MM) in load estimation and forecasting has been disregarded in previous research. Also, the use of present measurements to estimate past (historical) measurements of the load has not been reported in literature in previous research papers.

This paper investigates the applicability of the moving median to estimate future load measurements. Furthermore, the paper investigates the suitability of using present measurements to estimate past measurements of individual and aggregated smart meters of residential customers in a backward walk.

2. DATA STRUCTURE
Domestic load profiles based on smart meter measurements were used to investigate the performance of the developed load estimation algorithms. The load profiles were obtained from the Irish Smart Metering Customer Behaviour Trials (CBT) [5]. The Irish CBT is one of the largest and most statistically robust smart metering trials. The trials were carried out in the course of 18 months – from 1st July 2009 until 31st December 2010. More than 4200 domestic customers and 485 small-and-medium enterprises (SMEs) were covered by these trials. Smart meters – that were installed at the customers’ premises – recorded the consumption data. For an individual customer (smart meter), 48 half-hourly average active power consumption measurements represent any daily
load profile. The first measurement – that was recorded at hour 00:30 – is the average power consumed between hour 00:00:00 and 00:29:59, whereas the last measurement – that was recorded at hour 00:00 – is the average power consumption between hours 23:30:00 and 23:59:59.

The measurements of 3600 randomly selected domestic customers and collected between 1st January and the end of 31st December 2010 were used in this study.

In the load estimation algorithm, the measurements between 12:00 am – 11:30 am were assumed available while the measurements between 12:00 pm – 11:30 pm were assumed to be missing. The missing measurements were estimated using the available set of measurements on a daily basis.

This study investigates the estimation aggregated smart meter measurements. An aggregated daily load profile was created through summing the measurements of the 3600 smart meter at each half hour time step. Eq. (1) illustrates the aggregation of smart meter measurements.

\[ L_{agg.\ daily} = \left\{ \sum_{t=1}^{3600} l_p(t) \right\}, \ldots, \left\{ \sum_{t=1}^{3600} l_p(t) \right\} \]  

(1)

### 3. MOVING AVERAGE, MOVING MEDIAN, FORWARD AND BACKWARD LOAD ESTIMATION

#### 3.1 Moving Average (MA) Model

The simple moving average is a smoothing load estimation technique that estimates missing smart meter measurements based upon the average (mean) values of a number of real measurements. The number of these real measurements is defined as the window width which is used to calculate the average values and estimate the missing measurements.

The window defined by the window width is slid along the load profile so that the average values of the new profile are calculated (yellow boxes with red text in Fig. 1).

#### 3.2 Moving Median (MM) Model

The moving median is a load forecasting technique similar to the moving average technique with the median statistical measure replacing the average (yellow box with black text in Fig. 1).

The median is more robust than the mean when the measurements are contaminated with outliers or when the measurements are highly skewed and asymmetric [6].

#### 3.3 Forward – and Backward – Walk Load Estimation

Forward load estimation uses historical measurements to estimate future load. In contrast, backward load estimation applies the present measurements of the load to estimate missing historical measurements (Fig. 2).

#### 3.4 Calculation of Estimation Error

The estimation errors between the real and estimated half-hourly measurements of the aggregated smart
meters are quantified in terms of the Mean Absolute Percentage Error (MAPE) defined in Eq. 2.

\[ \text{MAPE} = \frac{1}{T} \sum_{t=1}^{T} \left| \frac{l_{p\text{act}}(t) - l_{p\text{est}}(t)}{l_{p\text{act}}(t)} \right| \times 100 \]

(2)

where \( l_{p\text{act}} \) is the real measurement, \( l_{p\text{est}} \) is the estimated measurement, \( T \) is the duration of missing measurements of aggregated smart meters.

4. RESULTS AND DISCUSSION

The measurements of aggregated domestic smart meters were estimated using the moving average and moving median in forward – and backward walk. The developed load estimation algorithms were developed in Python 2.7 [7]. The following steps describe the approach adopted to estimate the missing measurements.

1. The window widths used to calculate the mean and median of available real measurements were varied from 1 to 12 hours in one hour steps.
2. The duration of missing measurements (\( T \)) was set to 12 hours (from 12:00 pm – 11:30 pm as described in Section 2).
3. In forward load estimation, the measurements between 12:00 am – 11:30 am on each day were used to estimate the missing measurements of the same day.
4. In backward load estimation, the measurements between 12:00 am – 11:30 am on each day were used to estimate the missing measurements of the previous day.
5. On the last day (31st December 2010) of the period chosen for the simulation, only forward load estimation was carried out.

In Fig. 3, the solid black profile is the actual profile that was obtained using real aggregated measurements collected from domestic smart meters. The dashed lines are the mean values of all estimated profiles that were produced by the load estimation algorithm using window widths in the range of 1 – 12 hours.

Fig. 3 Real and estimated measurements of aggregated smart meters

Fig. 3 shows the smoothing effects of both the moving average and moving median load estimation techniques. The figure also shows the that both techniques fail to

![Fig. 3](image-url)

**Fig. 3 Real and estimated measurements of aggregated smart meters**

![Fig. 4](image-url)

**Fig. 4 Box-whisker plot of MAPE of estimated measurements**
capture the exact trend of the real profile and that there are significant differences between the real and estimated measurements.

4.1 Comparison between Forward and Backward Walk

Fig. 4 shows a box-whisker plot of the MAPE of the estimated measurements for different window widths. The figure shows that the MAPE values in forward load estimation (Fig. 4 (a) and (c)) are significantly smaller than these errors resulting from backward load estimation (Fig. 4 (b) and (d)). In backward load estimation, the mean values of the MAPE (red lines in Fig. 4) are almost two times their values seen in forward load estimation.

4.2 Comparison between Moving Average and Median

A comparison of the MAPE values in (Fig. 4 (a) and (c)) shows that using the Fig. 4 shows that for a window width of up to 7 hours, the load estimated using moving average and moving median has a similar distribution of the errors. For window widths beyond 7 hours, the load estimated using the moving median in the forward walk has wider error distributions than the moving average. Using the moving average – forward walk, the probability density function (pdf) of the MAPE was found to follow a normal distribution with an approximate mean value (red line in Fig. 5) of 27% and standard deviation (green line) of about 9%.

The yellow shaded areas in Fig. 5 represent the first and third quartiles of the MAPE which are the medians of the lower half and upper half of MAPE. In this manner, 25% of the MAPE values are less than or equal to ($q_1 \sim 21\%$) and 75% of the MAPE values are greater than or equal to ($q_3 \sim 34\%$).

Fig. 5 Probability density distribution of MAPE - Moving Average Forward Walk Load Estimation

The results shown in Figs. 3 – 5 indicate that as the window width increases beyond 7 hours, the moving average produces more accurate load estimates than the moving median. Furthermore, estimating the missing load measurements using historical load measurements (forward walk) results in smaller load estimation errors as compared to using present measurements to estimate missing past measurements (backward walk).

CONCLUSIONS

The application of moving average and moving median load estimation techniques to estimate missing measurements of aggregated smart meters was investigated. The employment of historical load measurements to estimate future load demand and present load measurements to estimate missing past measurements of the load was assessed.

Simulation results showed that when a window width of more than 7 hours of measurements is used to estimate the load, the moving average load estimation technique produces more accurate load estimates than the moving median. Furthermore, the application of past measurements (in a forward walk manner) produces load estimates of a higher accuracy than the use of present measurements to estimate missing past load measurements.

The estimation errors with maximum MAPE between 35 – 50% limit the applications of the techniques tested in this paper. It is therefore necessary to improve these techniques or develop new methods that are capable of producing load estimates with higher accuracies.

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REFERENCE


