All-sky search for continuous gravitational waves from isolated neutron stars using Advanced LIGO O2 data

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We present results of an all-sky search for continuous gravitational waves (CWs), which can be produced by fast spinning neutron stars with an asymmetry around their rotation axis, using data from the second observing run of the Advanced LIGO detectors. Three different semicoherent methods are used to search in a gravitational-wave frequency band from 20 to 1922 Hz and a first frequency derivative from \(-1 \times 10^{-8}\) to \(2 \times 10^{-9}\) Hz/s. None of these searches has found clear evidence for a CW signal, so upper limits on the gravitational-wave strain amplitude are calculated, which for this broad range in parameter space are the most sensitive ever achieved.

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I. INTRODUCTION

Eleven detections of gravitational waves from black hole binaries and from a neutron star binary have been reported in [1]. One of the characteristics of the signals detected so far is that their duration ranges from a fraction of a second to tens of seconds in the detector sensitive frequency band. Other mechanisms, however, can produce gravitational waves with longer durations, not yet detected. In this paper we describe an all-sky search for continuous gravitational waves (CWs), almost monochromatic signals which are present at the detectors during all the observing time. The principal sources for CW emission (see [2] for a review) are spinning neutron stars. If a spinning neutron star (NS) has an asymmetry with respect to its rotation axis, it will emit CWs at twice the rotation frequency.

Fast-spinning neutron stars in the Milky Way can generate continuous gravitational waves via various processes which produce an asymmetry. Crustal distortions from cooling or from binary accretion, or magnetic field energy buried below the crust could lead to the nonaxisymmetry necessary for detectable emission. The excitation of r-modes in a newborn or accreting NS is another promising mechanism for the emission of CWs. Recently, some evidence for a limiting minimum ellipticity was discussed in [3]. A comprehensive review of continuous gravitational wave emission mechanisms from neutron stars can be found in [4]. The detection of a CW, possibly combined with electromagnetic observations of the same source, could yield insight into the structure of neutron stars and into the equation of state of matter under extreme conditions.

Searches for continuous waves are usually split in three different domains: targeted searches look for signals from known pulsars [5–15]; directed searches look for signals from known sky locations like the Galactic Center, supernova remnants and low-mass x-ray binaries such as Sco-X1 [16–26]; all-sky searches look for signals from unknown sources [27–40]. Since all-sky searches need to cover a large parameter space, they are the most computationally expensive. For this reason, the most sensitive coherent search methods (e.g., matched filtering for the full observing run) cannot be used and semicoherent methods which split the full observation time in shorter chunks need to be used.

The interest in all-sky searches stems from the fact that they inspect regions of parameter space that no other searches look at. Although the targeted searches are more sensitive, they are limited to search for known pulsars which are in the sensitive frequency band of the detectors, while all-sky searches look for neutron stars with no electromagnetic counterpart, which could have different or more extreme properties than the observed pulsars.

In this paper we present the results of an all-sky search of CWs by three different pipelines (FrequencyHough [41], SkyHough [42], Time-Domain F-statistic [43]) using O2 data from the Advanced LIGO detectors. Each pipeline uses different data analysis methods and covers different regions of the parameter space, although there exists some overlap between them. Overall, we search the whole sky for gravitational wave frequencies from 20 to 1922 Hz (this number was chosen in order to avoid the violin modes of the test masses found at higher frequencies) and a first frequency derivative from \(-1 \times 10^{-8}\) to \(2 \times 10^{-9}\) Hz/s (positive frequency derivatives are possible for neutron stars which are spun up by accretion from a companion).

No detection has been made, and upper limits on the gravitational wave amplitude are presented.

The outline of the paper is the following: in Sec. II, we summarize the second observing run and give some details...
about the data that is used; in Sec. III, we describe the model of the signal that we want to detect; in Sec. IV, we present the different pipelines which are used; in Sec. V, we describe the results obtained by each pipeline; in Sec. VI, we give some final remarks.

II. LIGO O2 OBSERVING RUN

The LIGO second observing run (called O2) started on November 30, 2016 and finished on August 25, 2017. During this time, three different gravitational-wave detectors of second generation were active and producing data: Advanced LIGO [44], consisting of two detectors with 4-km arm lengths situated in Hanford, Washington (H1) and Livingston, Louisiana (L1), and Advanced Virgo [45], a 3-km detector located in Cascina, Pisa. Advanced Virgo first joined the run at the beginning of August 2017, with less sensitivity than the LIGO detectors, so we have not considered its data for the search described in this paper.

A representative noise curve from O2 for each LIGO detector and a comparison to O1 is shown in Fig. 1. We can observe an improvement of the amplitude spectral density, and we can also observe that the spectra features a number (greatly reduced as compared to O1) of narrow lines and combs affecting several frequency bands, which contaminate the data and complicate the analysis often raising outliers which look like the searched CW signals [46].

A cleaning procedure was applied to H1 data during postprocessing in order to remove jitter noise and some noise lines (more details are given in [47]). All of the searches of this paper used this cleaned dataset. The calibration of this dataset and its uncertainties on amplitude and phase are described in [48]. These searches do not use all the data from the observing run, since times where the detectors are known to be poorly behaving are removed from the analysis. This means that the effective amount of data used is smaller than the full duration of the run. As in previous observing runs, several artificial signals (called hardware injections) have been physically injected in the detectors in order to test their response and to validate the different pipelines. These hardware injections are described in Secs. III, V and in the Appendix.

III. SIGNAL MODEL

An asymmetric neutron star spinning around one of its principal axes of inertia emits a CW signal at twice its rotation frequency. This emission is circularly polarized along the rotation axis and linearly polarized in the directions perpendicular to the rotation axis. The gravitational-wave signal in the detector frame is given by [43]

\[
h(t) = h_0 \left[ \frac{1}{2} \cos \phi(t) + F_\times(t) \cos \sin \phi(t) \right],
\]

where \(F_\times(t)\) and \(F_\times(t)\) are the antenna patterns of the detectors (which can be found in [43]), \(h_0\) is the amplitude of the signal, \(\iota\) is the inclination of the neutron star angular momentum vector with respect to the observer’s sky plane, and \(\phi(t)\) is the phase of the signal. The amplitude of the signal is given by

\[
h_0 = \frac{4\pi^2 G c^4 f^2}{\epsilon d^3},
\]

where \(d\) is the distance from the detector to the source, \(f\) is the gravitational-wave frequency, \(\epsilon\) is the ellipticity or asymmetry of the star, given by \((I_{xx} - I_{yy})/I_{zz}\), and \(I_{zz}\) is the moment of inertia of the star with respect to the principal axis aligned with the rotation axis. These two last quantities are related to the mass quadrupole moment \(Q_{22}\) of the star:

![Amplitude spectral density (ASD) plots for the L1 (left panel) and H1 (right panel) detectors during O1 (blue trace) and O2 (orange trace). The ASD is obtained by averaging over FFTs of 1800 s obtained for the entire run.](image-url)
\[ e = \sqrt{\frac{8\pi Q_{zz}}{15 I_{zz}}}. \]  

(3)

We assume that the phase evolution of the gravitational-wave signal, which is locked to the evolution of the rotational frequency, can be approximated by a Taylor expansion (assumption taken from electromagnetic observations of pulsars) around a fiducial reference time \( \tau_r \):

\[ \phi(\tau) = \phi_0 + 2\pi \left[ f_0(\tau - \tau_r) + \frac{\dot{f}}{2!}(\tau - \tau_r)^2 + \cdots \right]. \]  

(4)

where \( \phi_0 \) is an initial phase and \( f_0 \) and \( \dot{f} \) are the frequency and first frequency derivative at the reference time. The relation between the time at the source \( \tau \) and the time at the detector \( t \) is given by (neglecting relativistic effects like the Einstein and Shapiro delays)

\[ \tau(t) = t + \frac{\bar{r}(t) \cdot \hat{n}}{c}, \]  

(5)

where \( \bar{r}(t) \) is the vector which joins the Solar System barycenter (SSB) and the detector, and \( \hat{n} \) is the vector identifying the star’s position in the SSB. From the previous formula the frequency evolution of the signal can be derived as

\[ f(t) = \frac{1}{2\pi} \frac{d\phi}{dt} \approx f_0 + f_0 \frac{\bar{v}(t) \cdot \hat{n}}{c} + \dot{f} t. \]  

(6)

The second term in the right-hand side of this equation describes the frequency modulation due to the Doppler effect produced by Earth’s rotation and translation around the SSB. This term, together with the spin-down/up of the source, must be properly taken into account when carrying out the search.

Equations (4) through (6) assume that the neutron star is isolated. In case it is part of a binary system, the frequency evolution is complicated by the binary system orbital motion, which introduces an additional frequency modulation. Such modulation, on a signal of frequency \( f \) and neglecting the binary system ellipticity, is given by (see [49])

\[ \Delta f_{\text{orb}} \approx \frac{2\pi}{P} a_p f, \]  

(7)

where \( P \) is the binary orbital period and \( a_p \) is the projected orbital semimajor axis (in light-seconds). By imposing that the orbital frequency modulation is contained into a frequency bin \( \delta f = 1/T_{\text{FFT}} \), where \( T_{\text{FFT}} \) is the duration of the data chunks which are incoherently combined in the analysis (see Sec. IV), we find that two of the search pipelines (FrequencyHough and SkyHough) used in this paper would be fully sensitive to a CW signal from a NS in a binary system if

\[ a_p \ll 0.076 \left( \frac{P}{1 \text{ day}} \right) \left( \frac{f}{100 \text{ Hz}} \right)^{-1} \left( \frac{T_{\text{FFT}}}{1800 \text{ s}} \right)^{-1} \text{ s}. \]  

(8)

For larger orbital frequency modulations the pipelines would start to lose signal-to-noise ratio but a detailed study of this issue is outside the scope of the paper. Out of 259 pulsars in binary systems from the Australia Telescope National Facility catalogue, only six of them have such characteristics, although many undiscovered neutron stars in binary systems could also be part of systems with these properties.

IV. DESCRIPTION OF THE SEARCH METHODS

In this section we introduce and summarize the three different pipelines which have been used in this work.

A. FrequencyHough

The FrequencyHough pipeline consists of an initial multistep phase, in which interesting points (i.e., candidates) are selected in the signal parameter space, and of a subsequent follow-up stage to confirm or reject the candidates. A complete description of the method and of the fundamental implementation features are given in [41,50]. Upper limits are computed with a frequentist approach, by injecting a large number of simulated signals into the data. The pipeline has been previously used in all-sky searches of Virgo VSR2, VSR4 [36] and LIGO O1 Science Runs data [38].

1. Initial analysis steps

The starting point of the analysis is calibrated detector data, used to create “short duration” fast Fourier transforms (FFTs) with coherence time depending on the frequency band being considered, according to Table I. Short time-domain disturbances are removed from the data before constructing the FFTs [51]. Next, a time-frequency map, called peakmap, is built by identifying local maxima (called peaks) above a dimensionless threshold \( \theta_{\text{thr}} = 1.58 \) on the

<table>
<thead>
<tr>
<th>Band [Hz]</th>
<th>( T_{\text{FFT}} ) [s]</th>
<th>( \delta f ) [Hz]</th>
<th>( \delta f ) [Hz/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>10–128</td>
<td>8912</td>
<td>( 1.22 \times 10^{-4} )</td>
<td>( 5.26 \times 10^{-12} )</td>
</tr>
<tr>
<td>128–512</td>
<td>4096</td>
<td>( 2.44 \times 10^{-4} )</td>
<td>( 1.05 \times 10^{-11} )</td>
</tr>
<tr>
<td>512–1024</td>
<td>2048</td>
<td>( 4.88 \times 10^{-4} )</td>
<td>( 2.10 \times 10^{-11} )</td>
</tr>
<tr>
<td>1024–2048</td>
<td>1024</td>
<td>( 9.76 \times 10^{-4} )</td>
<td>( 4.20 \times 10^{-11} )</td>
</tr>
</tbody>
</table>
square root of the equalized power$^1$ of the data [51]. The
peakmap is cleaned using a line persistence veto [41],
based on the projection of the peakmap onto the frequency
axis and on the removal of the frequency bins in which the
projection is higher than a given threshold.

After defining a grid in the sky, with bin size depending
on the frequency and sky location as detailed in [41], the
time-frequency peaks are properly shifted, for each sky
position, to compensate the Doppler effect due to the
detector motion, see Eq. (6). They are then processed by
the FrequencyHough algorithm [41,50], which transforms
each peak to the frequency and spin-down/up plane of
the source. The frequency and spin-down/up bins (which we
will refer to as coarse bins in the following) depend on the
frequency band, as indicated in Table I, and are defined,
respectively, as $\delta f = \frac{1}{f_{\text{obs}}} \delta \lambda$ and $\delta f = \frac{\delta f}{T_{\text{obs}}}$, where $T_{\text{obs}} = 268.37$ days is the total run duration. In practice, as the
transformation from the peakmap to the Hough plane is not
computationally bounded by the width of the frequency
bin, we have increased the nominal frequency resolution by
a factor of 10 [41]. The algorithm, moreover, properly
weights any noise nonstationarity and the time-varying
detector response [52].

The FrequencyHough transform is computationally
very demanding, so the analysis is split into tens of
thousands of independent jobs, each of which computes a
FrequencyHough transform covering a small portion of the
parameter space. The output of a FrequencyHough trans-
form is a 2D histogram in the frequency/spin-down plane
of the source. Candidates for each sky location are selec-
ted from each detector are clustered and then coincident
candidates among the clusters of the two detectors are
sorted using a line persistence veto [41]. Coincident candi-
dates are subject to a ranking procedure, based on the value of a statistic built using the
distance and the FrequencyHough histogram weighted
number count of the coincident candidates, as described
in [41]. In this analysis, after the ranking the eight candidates in each 0.1-Hz band with the highest values of
the statistic have been selected.

2. Candidate follow-up

Candidates passing the ranking selection are followed up
in order to confirm them as potential CW signals or to
discard them, if due to noise fluctuations or detector
disturbances. The follow-up consists of several steps, as
described in [36]. An important implementation novelty we
have introduced in the O2 analysis is the use of the Band
Sampled Data framework (BSD) [53], which allows a
flexible and computationally efficient management of the
data. For each of the $N$ candidates selected by the ranking
procedure, a fully coherent search is done using down-
sampled data from both detectors, covering a band of the
order of 0.2 Hz around the candidate frequency. The
coherent search is done by applying a Doppler and spin-
down/up correction based on the parameters of the candi-
date. Although the coherent search corrects exactly for the
Doppler and spin-down/up effect at a particular point in the
parameter space, corresponding to the candidate, the cor-
rection is extended by linear interpolation to the neighbors of
the candidate itself. In practice, this means that from the
corrected and down-sampled time series, a new set of FFTs

\[
d_{\text{FH}} = \sqrt{\left(\frac{\Delta f}{\delta f}\right)^2 + \left(\frac{\Delta \dot{f}}{\delta \dot{f}}\right)^2 + \left(\frac{\Delta \lambda}{\delta \lambda}\right)^2 + \left(\frac{\Delta \beta}{\delta \beta}\right)^2},
\]

where $\Delta f$, $\Delta \dot{f}$, $\Delta \lambda$, and $\Delta \beta$ are the differences, for each
parameter, among pairs of candidates of the two detectors,
and $\delta f$, $\delta \dot{f}$, $\delta \lambda$, and $\delta \beta$ are the corresponding bin widths.

Pairs of candidates with distance $d_{\text{FH}} < 3$ are considered
coincident. This value was chosen based on a study with
software simulated signals and allows, on one hand, to
reduce the false alarm probability and, on the other, to be
sufficiently robust with respect to the fact that a signal can
be found with slightly different parameters in the two
detectors. Coincident candidates are subject to a ranking
procedure, based on the value of a statistic built using the
distance and the FrequencyHough histogram weighted
number count of the coincident candidates, as described
in [41]. In this analysis, after the ranking the eight
candidates in each 0.1-Hz band with the highest values of
the statistic have been selected.

<table>
<thead>
<tr>
<th>Band [Hz]</th>
<th>FFT duration [s]</th>
<th>$\mathcal{E}$</th>
<th>$\delta f$ [Hz]</th>
<th>$\delta \dot{f}$ [Hz/s]</th>
<th>$\Delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10–128</td>
<td>24600</td>
<td>3</td>
<td>$4.07 \times 10^{-5}$</td>
<td>1.75 $\times 10^{-2}$</td>
<td>1.39</td>
</tr>
<tr>
<td>128–512</td>
<td>24576</td>
<td>6</td>
<td>$4.07 \times 10^{-5}$</td>
<td>1.75 $\times 10^{-12}$</td>
<td>1.65</td>
</tr>
<tr>
<td>512–1024</td>
<td>8192</td>
<td>4</td>
<td>$1.22 \times 10^{-4}$</td>
<td>5.26 $\times 10^{-12}$</td>
<td>1.49</td>
</tr>
</tbody>
</table>

\(^1\)Defined as the ratio of the squared modulus of the FFT of
the data and an autoregressive estimation of the power spectrum, see
[51] for more details.
is built, with a longer duration (by a varying factor $E$, depending on the frequency band, see Table II), as well as the corresponding peakmap. Peaks are selected using a threshold $\theta_{th} = 1.87$, bigger than the initial one (see Sec. IVA). As explained in [36] this is a conservative choice which provides a sensitivity gain and, at the same time, reduces the computational cost of the follow-up. As a result of the FFT length increase and of the new threshold for the selection of the peaks, by using Eq. (67) of [41], which is valid under the assumption of Gaussian noise, we estimate a sensitivity gain $G$ for the detectable $h_0$ in the follow-up, shown in the last column of Table II. A small area, centered around the candidate position, is considered for the follow-up. It covers $\pm 3$ coarse bins, which amounts to seven bins in each dimension, and thus 49 coarse sky positions for each candidate. A refined sky grid is built over this area, with an actual number of points which depends on the frequency band and is, on the average, given by $49E^2$. For each sky position in this refined grid, we evaluate the residual Doppler modulation (with respect to the center of the grid), which is corrected in the peakmap by properly shifting each peak. The FrequencyHough of the resulting ensemble of corrected peakmaps is computed over a frequency and spin-down/up ranges covering $\pm 3$ coarse bins around the candidate values. As before, an over-resolution factor of 10 is used for the frequency. The absolute maximum identified over all the Hough maps provides the refined parameters of the candidate we are considering. Next, to each pair of candidates from the two detectors we apply a series of vetoes, as detailed in the following.

First, we remove the candidates whose median value of the frequency (computed over the full observing time), after the removal of the Doppler and spin-down/up correction, overlaps a known noise line frequency band (i.e., a line due to a detector disturbance, whose instrumental origin has been understood). Second, for each detector a new peakmap is computed using the data coherently corrected with the refined parameters of the candidate and then projected on the frequency axis. We take the maximum $A_p$ of this projection in a range of $\pm 2$ coarse bins around the candidate frequency. We divide the rest of the 0.2 Hz band (which we consider the “off-source” region) into $\sim 250$ intervals of the same width, take the maximum of the peakmap projection in each of these intervals and sort in decreasing order all these maxima. We tag the candidate as “interesting” and keep it if it ranks first or second in this list for both detectors. Surviving candidates are then subject to a consistency test based on their critical ratio, defined as $CR = (A_p - \mu_p)/\sigma_p$, where $\mu_p$ and $\sigma_p$ are the mean and standard deviation of the peakmap projection on the off-source region. Pairs of coincident candidates are removed if their CRs, properly weighted by the detector noise level at the candidate frequency, differ by more than a factor of 5. Further details on O2 outlier selection and properties are given in Sec. V A. 

3. Upper limit computation

Upper limits are computed in each 1-Hz band between 20 and 1000 Hz by injecting software simulated signals, with the same procedure used in [36]. For each 1 Hz band 20 sets of 100 signals each are generated, with fixed amplitude within each set and random parameters (sky location, frequency, spin-down/up, and polarization parameters). These are generated in time domain and then added to the data of both detectors in the frequency domain. For each injected signal in a set of 100, an analysis is done using the FrequencyHough pipeline over a frequency band of 0.1 Hz around the injection frequency, the full spin-down/up range used in the real analysis, and nine sky points around the injection position [36]. Candidates are selected as in the real analysis, except that no clustering is applied, as it would have been affected by the presence of too many signals. We note, however, that clustering is used in the analysis only to reduce the computational cost and does not affect the subsequent steps. After coincidences and ranking, candidates also coincident with the injected signal parameters, within the follow-up volume discussed in Sec. IVA 2, are selected. Those having a critical ratio larger than the largest critical ratio found in the real analysis in the same 1 Hz are counted as detections. For each 1 Hz band, we build the detection efficiency curve, defined as the fraction of detected signals as a function of their amplitude. The upper limit is given by the signal amplitude such that the detection efficiency is 95%. In practice, a fit is used in order to interpolate the detection efficiency curve, as described in [38].

B. SkyHough

The SkyHough method has been used in other searches using data from the Initial LIGO S2, S4 and S5 and Advanced LIGO O1 observing runs [27,28,32,38,40]. Its main description is given in [42]. Here we summarize its main characteristics and the new features that have been implemented in this search. The code for the main part of the search is called lalapps_DriveHoughMulti and is part of the publicly available LALSuite package [55].

1. Initial analysis steps

This pipeline uses short Fourier transforms (SFTs) of the time domain $h(t)$ as its input data, with a coherent duration of each chunk varying as a function of the searched frequency (as shown in Table III). It creates peak-grams, which are spectrograms with the normalized power substituted by $1s$ (if the power is above a certain threshold $\rho_1 = 1.6$) and $0$s, where the normalized power in a frequency bin is defined as

$$\rho_k = \frac{|x_k|^2}{\langle n_k \rangle^2},$$  \hspace{1cm} (10)
where \( \langle n_k \rangle^2 \) is estimated with a running median of 101 frequency bins.

We use the Hough transform to track the time-frequency evolution of the signal including the Doppler modulation of the signal at the detector. In the first stage the pipeline employs a look-up Table (LUT) approach, taking into account that at a given time the same Doppler modulation is produced by an annulus of sky positions (given by \( \Delta \theta \)), which correspond to the width of a frequency bin \( \Delta f \):

\[
\cos \Delta \theta = \frac{c}{v(t)} \frac{f(t) - \hat{f}(t)}{\hat{f}(t)} = \frac{c}{v(t)} \Delta f, \tag{11}
\]

where \( f(t) \) is the observed frequency at the detector and \( \hat{f} = f_0 + f_1 t \) is the searched frequency. The algorithm tracks the sky positions which produce observed frequencies with powers above the threshold. It then stacks these sky positions by following the evolution of the source frequency given by the spin-down/up term at different time stamps and produces a final histogram. The LUT approach reuses the same Doppler modulation pattern for different search frequencies (more details in [42]), which produces computational savings in exchange for not following the exact frequency-time pattern.

For each template (described by \( f_0, \hat{f}, \alpha, \delta \)) being searched, a detection statistic called number count significance is calculated:

\[
s_n = \frac{n - \langle n \rangle}{\sigma_n}, \tag{12}
\]

where \( \langle n \rangle \) and \( \sigma_n \) are the expected mean and standard deviation of the Hough number count \( n \) when only noise is present. The number count \( n \) is the weighted sum of 1s and 0s, where the weights (which are proportional to the antenna pattern functions and inversely proportional to the power spectral density) were derived in [56].

The parameter space is separated in 0.1 Hz frequency bands and in small sky-patches. A toplist is calculated for each of these regions, which has the top templates ordered by the number count significance. For the top templates, a second step is performed where instead of using the look-up Table approach the exact frequency path is tracked. At this second step the power significance is also calculated, which is defined as

\[
s_p = \frac{P - \langle P \rangle}{\sigma_P}, \tag{13}
\]

where instead of summing weighted 1s and 0s the weighted normalized powers given by Eq. (10) are summed. This detection statistic improves the sensitivity of SkyHough with a very small increase of computational cost. The best 5000 templates per sky-patch and 0.1 Hz band are passed to the second step, and only the best 1000 candidates per sky-patch and 0.1 Hz band are used for the postprocessing. Furthermore, at the second step more SFTs are used than in the first step. This is achieved by sliding the initial times of each SFT that was used at the first step, obtaining more SFTs (approximately twice the previous amount), all of them of \( T_c \) contiguous seconds.

The grid resolution was obtained in [42] and it is given by

\[
\delta f = \frac{1}{T_c}, \tag{14}
\]

\[
\delta \hat{f} = \frac{1}{T_c T_{o b s}}, \tag{15}
\]

\[
\delta \theta = \frac{c}{v T_c f P_F}, \tag{16}
\]

where \( P_F \) is a parameter which controls the sky resolution grid. In this search we have set \( P_F = 2 \) for all frequencies.

2. Postprocessing

The postprocessing consists of several steps:

(1) The output of the main SkyHough search is one toplist for each dataset (there are two datasets, each one with data from two detectors, detailed in Table V) and each region in parameter space. We search for coincidental pairs between these toplists, by calculating the distance in parameter space and selecting the pairs which are closer than a certain threshold called \( d_{SH} \). For the coincidental pairs the centers (average locations in parameter space weighted by significance) are calculated. The parameter space distance is calculated as

\[
d_{SH} = \sqrt{\left(\frac{\Delta f}{\delta f}\right)^2 + \left(\frac{\Delta \hat{f}}{\delta \hat{f}}\right)^2 + \left(\frac{\Delta x}{\delta x}\right)^2 + \left(\frac{\Delta y}{\delta y}\right)^2}, \tag{17}
\]

where the numbers in the numerators represent the difference between to templates and the numbers in the denominators represent the parameter resolution (this distance is unitless and is given as a number of bins). The parameters \( x \) and \( y \) are the Cartesian ecliptic coordinates projected in the ecliptic plane.

(2) Search for clusters in the obtained list of centers. This will group different outliers which can be ascribed to a unique physical cause, and will reduce
the size of the final toplist. Again, we set a threshold in parameter space distance (called \(d_3\)) and we find candidates which are closer than this distance.

3. Follow-up

We describe a follow-up method based on the \(F\)-statistic (described in more detail in Sec. IV C) and the GCT metric method [57]. This method uses the *lalapps_HierarchSearchGCT* code, part of the publicly available LALSuite [55], and it is similar in spirit to the multistep follow-up methods described in [58] or [59].

The goal is to compare the \(F\)-statistic values obtained from software injected signals to the \(F\)-statistic values obtained from the outliers. We want to compare the \(F\)-statistic obtained at different stages which scale the coherent time. It is expected that for an astrophysical signal the \(F\)-statistic value should increase if the coherent time increases.

The resolution in parameter space is given by [57]

\[
\delta f = \frac{\sqrt{12m}}{\pi T_c}
\]

\[
\delta \dot{f} = \frac{\sqrt{720m}}{\pi T_c^2 \dot{f}}
\]

\[
\delta \theta = \frac{\sqrt{m_{sky}}}{\pi f \tau_c}
\]

where \(m\) and \(m_{sky}\) are mismatch parameters, \(\gamma\) is a parameter which gives the refinement between the coherent and semicoherent stages and \(\tau_c = 0.021\) s represents the light time travel from the detector to the center of the Earth.

We now enumerate the different steps of the procedure:

1. Calculate the semicoherent \(F\)-statistic of outliers with \(T_c = 7200\) s in a cluster box.

2. Add injections to the original data using a sensitivity depth \((\sqrt{S_n}/h_0)\) value which returns \(F\)-statistic values similar to the values obtained with the outliers in order to compare similar signals. We inject signals in eight different frequency bands with 200 injections per band, with a sensitivity depth of 42 Hz\(^{-1/2}\). Then, search in a small region (around ten bins in each dimension) around the true parameters of the injections with \(T_c = 7200\) s. Finally, analyze the distances in parameter space from the top candidates to the injections.

3. Repeat the previous step increasing the coherent time to \(T_c = 72000\) s.

4. Calculate \(F_{72000\,s}/F_{7200\,s}\) for each of the 1600 injections using the top candidates. The threshold will be the minimum value.

5. Calculate the \(F\)-statistic values of outliers with \(T_c = 72000\) s around the top candidate from the first stage. The size of the window to be searched is estimated from the distances found in step 2.

6. Calculate \(F_{72000\,s}/F_{7200\,s}\) using the top candidate for all outliers. Outliers with values higher than the threshold obtained in step 4 go to the next comparison, and the process is repeated from step 2 increasing the coherent time.

Table IV summarizes the parameters that we have chosen at each different stage. As we will see in the Results section, only two comparisons between three different stages were needed.

### C. Time-domain \(F\)-statistic

The *Time-domain \(F\)-statistic* search method uses the algorithms described in [33,43,60,61] and has been applied to an all-sky search of VSR1 data [33] and an all-sky search of the LIGO O1 data [38,40]. The main tool is the \(F\)-statistic [43] by which one can search coherently the data over a reduced parameter space consisting of signal frequency, its derivatives, and the sky position of the source. The \(F\)-statistic eliminates the need for a grid search over remaining parameters [see Eqs. (1) and (4)]. In particular, the inclination angle \(i\) and polarization \(\psi\). Once a signal is identified the estimates of those four parameters are obtained from analytic formulas.

However, a coherent search over the whole LIGO O2 dataset is computationally prohibitive and we need to apply a semicoherent method, which consists of dividing the data into shorter time domain segments. The short time domain data are analyzed coherently with the \(F\)-statistic. Then the output from the coherent search from time domain segments is analyzed by a different, computationally manageable method. Moreover, to reduce the computer memory required to do the search, the data are divided into narrow-band segments that are analyzed separately. Thus our search method consists primarily of two parts. The first part is the coherent search of narrow-band, time-domain segments. The second part is the search for coincidences among the candidates obtained from the coherent search.
The pipeline is described in Sec. IV of [38] (see also Fig. 13 of [38] for the flow chart of the pipeline). The same pipeline is used for the search of LIGO O2 data presented here except that a number of parameters of the search are different. The choice of parameters was motivated by the requirement to make the search computationally manageable.

As in our O1 searches, the data are divided into overlapping frequency subbands of 0.25 Hz. We analyze three frequency bands: [20–100], [100–434], and [1518–1922] Hz. As a result, the three bands have 332, 1379, and 1669 frequency subbands respectively.

The time series is divided into segments, called frames, of 24 sidereal days long each, six days long, and two days long respectively for the three bands. Consequently in each band we have 11, 44, and 134 time frames, respectively. The O2 data has a number of non-science data segments. The values of these bad data are set to zero. For this analysis, we choose only segments that have a fraction of bad data less than 1/2 both in H1 and L1 data. This requirement results in eight 24-day-long, twenty-six six-day-long, seventy-nine two-day-long data segments for each band respectively. These segments are analyzed coherently using the \( F \)-statistic defined by Eq. (9) of [33]. We set a fixed threshold for the \( F \)-statistic of \( F_0 = 16 \) and record the parameters of all threshold crossings, together with the corresponding values of the signal-to-noise ratio \( \rho \),

\[
\rho = \sqrt{2(F - 2)}. \tag{21}
\]

Parameters of the threshold crossing constitute a candidate signal. At this first stage we also veto candidate signals overlapping with the instrumental lines identified by independent analysis of the detector data.

For the search we use a four-dimensional grid of templates (parametrized by frequency, spin-down/up, and two more parameters related to the position of the source in the sky) constructed in Sec. IV of [61]. For the low frequency band [20–434] Hz we choose the grid’s minimal match MM = \( \sqrt{3}/2 \) whereas for the high frequency band [1518–1922] Hz we choose a looser grid with MM = 1/2.

In the second stage of the analysis we search for coincidences among the candidates obtained in the coherent part of the analysis. We use exactly the same coincidence search algorithm as in the analysis of VSR1 data and described in detail in Sec. VIII of [33]. We search for coincidences in each of the subbands analyzed. To estimate the significance of a given coincidence, we use the formula for the false alarm probability derived in the Appendix of [33]. Sufficiently significant coincidences are called outliers and subjected to further investigation.

The sensitivity of the search is estimated by the same procedure as in O1 data analysis (38, Sec. IV). The sensitivity is taken to be the amplitude \( h_0 \) of the gravitational wave signal that can be confidently detected. We perform the following Monte Carlo simulations. For a given amplitude \( h_0 \), we randomly select the other seven parameters of the signal: \( f, \dot{f}, \alpha, \delta, \phi_0, \theta \) and \( \psi \). We choose frequency and spin-down/up parameters uniformly over their range, and source positions uniformly over the sky. We choose angles \( \phi_0 \) and \( \psi \) uniformly over the interval \([0, 2\pi]\) and cos \( \iota \) uniformly over the interval \([-1, 1]\). We add the signal with selected parameters to the O1 data. Then the data are processed through our pipeline. First, we perform a coherent \( F \)-statistic search of each of the data segments where the signal was added. Then the coincidence analysis of the candidates is performed. The signal is considered to be detected, if it is coincident in more than five of the eight time frames analyzed, 14 out of 26, and 40 out of 79 for the three bands respectively. We repeat the simulations 100 times. The ratio of numbers of cases in which the signal is detected to the 100 simulations performed for a given \( h_0 \) determines the frequentist sensitivity upper limits. We determine the sensitivity of the search in each 0.25 Hz frequency subband separately. The 95% confidence upper limits for the whole range of frequencies are given in Fig. 12; they follow very well the noise curves of the O2 data that were analyzed. The sensitivity of search decreases with decreasing coherence time we use for the three bands of our \( F \)-statistic search. Additionally it decreases in our high-frequency band because of the looser grid used than in low frequency bands.

V. RESULTS

In this section we detail the results obtained. The region in frequency and first frequency derivative searched by each of the three different pipelines is shown in Fig. 2.

Although no detections have been made, we give details on the different procedures and outliers which were found, and we also present 95% confidence level (C.L.) upper limits on the strain \( h_0 \) given by Eq. (2), shown in Fig. 3.
The best upper limit is $\approx 1.7 \times 10^{-25}$ at around 120 Hz. These results are significantly better (of a factor of about 1.4) than those obtained on O1 data with the same pipelines [38,40], thanks to improvements in the pipelines themselves, to the better sensitivity of the detectors and to the longer duration of the observing run. These upper limits do not take into account the calibration uncertainty on amplitude, which over the run was no larger than 5% and 10% for H1 and L1 respectively [48].

Our O2 results are comparable with the upper limits obtained in O1 by the Einstein@Home project [39] over the range 20–100 Hz. Note, however, that the Einstein@Home search covered a spin-down/up range smaller by almost 1 order of magnitude. Moreover, while the Einstein@Home search is, in principle, more sensitive due to the use of much longer data segments (compared to the FrequencyHough and SkyHough pipelines), with 210 hr duration, it is also less robust in case of deviations from the assumed signal model described in Sec. III. At frequencies higher than 100 Hz, the previous best upper limits were obtained in [62] using O1 data. Our results improve on those upper limits by approximately 17%.

The 95% C.L. upper limits on $h_0$ can be converted to upper limits on ellipticity $e$ by using Eq. (2) with a canonical value for the moment of inertia of $10^{38}$ kg m$^2$ and by using different distances:

$$e = \frac{c^4}{4\pi^2 G I_{zz} f^2} h_0 d.$$

These results are shown in the left panel of Fig. 4. This has been obtained by using the best $h_0$ upper limits between the three pipelines: from 20 to 1000 Hz, the FrequencyHough results have been used; from 1000 to 1500 Hz, the results from SkyHough have been used; from 1518 to 1922 Hz the results from Time-domain F-statistic have been used. For sources at 1 kpc emitting CWs at 500 Hz, we can constrain the ellipticity at $\approx 10^{-6}$, while for sources at 10 kpc emitting at the same frequency we can constrain the ellipticity at $10^{-5}$.

A complementary way of interpreting the limits on ellipticity is shown in the right panel of Fig. 4. The various set of points give the relation between the absolute value of the signal frequency time derivative (spin-down) and the signal frequency for sources detectable at various distances, assuming their spin-down is only due to the emission of gravitational waves. They have been computed by means of the following relation obtained inverting the equation for the so-called spin-down limit amplitude $h_{sd}^0$, which is a function of the source distance $d$, frequency $f$ and spin-down $\dot{f}$, see e.g., Eq. (A7) in [63]:

$$|\dot{f}| = 1.54 \times 10^{-10} \left( \frac{I_{zz}}{10^{38} \text{ kg m}^2} \right)^{-1} \left( \frac{h_{sd}^0}{10^{-25}} \right)$$
$$\times \left( \frac{f}{100 \text{ Hz}} \right) \left( \frac{d}{1 \text{ kpc}} \right)^2 \text{ [Hz/s]},$$

where we have replaced the spin-down limit amplitude with the 95% upper limits shown in Fig. 3. The dashed lines are constant ellipticity curves obtained from Eq. (A9) of [63]:

$$|\dot{f}| = 1.72 \times 10^{-14} \left( \frac{I_{zz}}{10^{38} \text{ kg m}^2} \right) \left( \frac{f}{100 \text{ Hz}} \right)^{1/2}$$
$$\times \left( \frac{e}{10^{-6}} \right)^2 \text{ [Hz/s]},$$

FIG. 3. Upper limits on the strain amplitude $h_{0}^{95\%}$ for the three pipelines.
For a signal to be detectable, its spin-down/up would need to be equal or above the given traces (notice that, as shown in Fig. 2, the maximum absolute spin-down searched is $10^{-8}$ Hz/s, which marks a limit to the signals we are sensitive to). For example, a source emitting a signal with frequency higher than about 500 Hz and ellipticity equal or greater than $10^{-6}$ would be detectable up to a distance of about 1 kpc if its spin-down is, in modulus, larger than $10^{-10}$ Hz/s.

The three searches carried out by the different pipelines have different computational costs: FrequencyHough spent 9 MSU; SkyHough spent 2.5 MSU; Time-domain F-statistic spent 24.2 MSU, where 1 MSU hour corresponds to 1 million Intel E5-2670 core-hour to perform a SPECfp computation. We remind the reader that each of these pipelines covered different search bands.

**A. FrequencyHough**

In this section we report the main results of the O2 all-sky search using the FrequencyHough pipeline. The spin-down range covered by the analysis is $(+2 \times 10^{-9} \text{ Hz/s}, -10^{-8} \text{ Hz/s})$ up to 512 Hz and $(+2 \times 10^{-9} \text{ Hz/s}, -2 \times 10^{-9} \text{ Hz/s})$ from 512 Hz up to 1024 Hz.

The number of initial candidates produced by the FrequencyHough transform stage was about $5 \times 10^9$ (of which about $7 \times 10^7$ belong to the band 20–128 Hz, about $1.1 \times 10^9$ to the band 128–512 Hz and about $3.8 \times 10^9$ to the band 512–1024 Hz, for both Hanford and Livingston detectors. As the total number of coincident candidates remained too large, $1.09 \times 10^9$, we reduced it with the ranking procedure described in Sec. IV A. The total number of candidates selected after the ranking was 59025. Each of these candidates was subject to a multistage follow-up procedure, described in Sec. IV A 2. The total number of candidates passing the follow-up and all the veto steps was 154, after removing the candidates due to the hardware injections. Among these, only 27 were found in coincidence between the two detectors [within a distance $d_{FH} < 3$ as defined in (9)]. From these surviving candidates we selected the outliers less consistent with noise fluctuations. In particular, we choose those for which the final peakmap projections have an average (over the two detectors) critical ratio (see Sec. IV A 2) $CR > 7.42$. This is the threshold corresponding, under the assumption of Gaussian noise, to a false alarm probability of 1% after having taken into account the look-elsewhere effect (on the follow-up stage) [38]. We found only one candidate an average CR above the threshold. It was at a frequency of about 440.4 Hz, and occurred due to a high CR value in the LIGO Hanford detector. For this candidate, we have looked at the starting peakmaps, without Doppler correction, around its frequency, which clearly show the presence of a transient line of duration $\approx 2$ days at the candidate frequency in Hanford data, see Fig. 5. We have then discarded this candidate as a possible CW signal. The remaining 26 subthreshold candidates, which will be further analyzed in a forthcoming work, are listed in VI. The analysis was run on distributed computational resources accessed through the EGI grid middleware [64].

As we did not find any significant candidate, we have computed upper limits. They have been evaluated in 1-Hz bands, as described in IV A 3, and are shown in Fig. 3. The total amount of frequency bands vetoed by the persistency veto is negligible, as it amounts to less than 0.55% and 0.45% respectively for LIGO Hanford and Livingston. There are a few 1 Hz bands where we have not evaluated the upper limit, due to the fact that we do not have...
candidates or, due to disturbances, we have not been able to recover the 95% of the injections, or more, at any amplitude. These bands are those with the following initial frequencies: \(f = 22, 23, 24, 26, 35, 36, 37, 39, 42, 51, 56, 65, 71, 73, 79, 120, 763, 995, 996, 998\) Hz. Comparing upper limits with O1 results [38], see Fig. 6, we notice an improvement of \(\sim 30\% - 40\%\) at frequencies between \(\sim 150\) and 500 Hz, while the gain is significantly bigger, up to a factor of \(\sim 2\), at lower frequencies. This is the first time we have extended the FrequencyHough analysis above \(\sim 500\) Hz. The statistical uncertainty on the upper limit is lower than about 5%, because of the amplitude step used for the injections, which amounts to \(2 \times 10^{-26}\).

As a test of the capabilities of the pipeline to recover signals, we report in Appendix A1 the parameters of the recovered hardware injections, together with the error with respect to the injected signals. We note that we were able to recover, with very good accuracy, the parameters of all 12 hardware injections with frequency in the analyzed band.

B. SkyHough

SkyHough has analyzed frequencies from 50 to 1500 Hz and spin-down/up values from \(-10^{-8}\) to \(10^{-9}\) Hz/s as shown in Fig. 2. The four different coherent times that have been used are shown in Table III. This analysis uses the C02 cleaned dataset [48], and splits the data from H1 and L1 in two datasets, divided by time as shown in Table V, where the start and stop times for each dataset are indicated. The main search generates a toplist per dataset per 0.1 Hz band of 10000 candidates with a maximum of 1000 per sky-patch. The number of sky-patches depends on the frequency: to minimize the computational cost of the search, we try to minimize the number of sky-patches for a limited amount of random access memory. From 50 to 850 Hz, there are 28 sky-patches; from 850 to 1000 Hz, 31 sky-patches; from 1000 to 1150 Hz, 38 sky-patches; from 1150 to 1250 Hz, 45 sky-patches; from 1300 to 1500 Hz, 28 sky-patches. After applying the postprocessing stage previously described (with distance thresholds of \(d_{\text{co}} = 3\) and \(d_{\text{cl}} = \sqrt{14}\)), we are left with 4548 0.1 Hz bands (from a total of 14500) having coincidental pairs.

<table>
<thead>
<tr>
<th>Dataset 1</th>
<th>H1</th>
<th>116754839/1174691692</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>L1</td>
<td>1167546403/1174688389</td>
</tr>
<tr>
<td>Dataset 2</td>
<td>H1</td>
<td>1180982628/1187731792</td>
</tr>
<tr>
<td></td>
<td>L1</td>
<td>1179816663/1187731695</td>
</tr>
</tbody>
</table>

Table V. Start/stop times in GPS units of each dataset used by the SkyHough pipeline. The observation time parameter used for the spin-down resolution given by Eq. (16) is \(T_{\text{obs}} = 7915032\) s, the maximum span of these datasets.
We apply the population veto, used in many past searches, which demands that each dataset contributes to each cluster with at least two different templates. After applying this veto, only 1539 outliers remain.

The next step is to apply the $F$-statistic follow-up method described in Sec. IV B 3 to these 1539 outliers. The thresholds obtained are 1.47 and 3.66 for the first and second comparison respectively, as shown in Fig. 7. Only 17 outliers are above the threshold at 3.66, as shown in Fig. 8. All of the outliers which are above the final threshold correspond to one of the hardware injections listed in Table VII or to one known source of instrumental noise, listed in [46]. The 17 surviving outliers and their parameters are listed in Table VIII, with comments about their likely origin.

We recover six of the nine hardware injections that are in the SkyHough searched parameter space. We lose the other three mainly for two reasons: there were brief periods when the hardware injections were not active, which causes the increase of the $F$-statistic to not be as high as it should be (this happens to two of the three lost hardware injections, which are present in our initial list of 1539 outliers); we only select one cluster per 0.1 Hz band, and if in that band there is a more significant cluster due to a noise disturbance the signal cluster will not be followed (this happens to one of the three lost hardware injections, which forms a cluster but a more significant noise disturbance is present in that 0.1 Hz band).

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TABLE VII. Parameters of the hardware-injected simulated continuous-wave signals during the O2 data run (parameters given at epoch GPS 1130529362).

<table>
<thead>
<tr>
<th>Label</th>
<th>Frequency [Hz]</th>
<th>Spin-down [nHz/s]</th>
<th>(\alpha) [deg]</th>
<th>(\delta) [deg]</th>
</tr>
</thead>
<tbody>
<tr>
<td>ip0</td>
<td>265.575533</td>
<td>(-1.45 \times 10^{-3})</td>
<td>71.55193</td>
<td>-56.21749</td>
</tr>
<tr>
<td>ip1</td>
<td>848.969641</td>
<td>(-3.00 \times 10^{-1})</td>
<td>37.39385</td>
<td>-29.45246</td>
</tr>
<tr>
<td>ip2</td>
<td>575.163521</td>
<td>(-1.37 \times 10^{-4})</td>
<td>215.25617</td>
<td>3.44399</td>
</tr>
<tr>
<td>ip3</td>
<td>108.857159</td>
<td>(-1.46 \times 10^{-8})</td>
<td>178.37257</td>
<td>-33.4366</td>
</tr>
<tr>
<td>ip4</td>
<td>1393.540559</td>
<td>(-2.54 \times 10^{-1})</td>
<td>279.98768</td>
<td>-12.4666</td>
</tr>
<tr>
<td>ip5</td>
<td>52.808324</td>
<td>(-4.03 \times 10^{-9})</td>
<td>302.62664</td>
<td>83.83914</td>
</tr>
<tr>
<td>ip6</td>
<td>146.19370</td>
<td>(-6.73 \times 10^{0})</td>
<td>358.75095</td>
<td>-65.42262</td>
</tr>
<tr>
<td>ip7</td>
<td>1220.555270</td>
<td>(-1.12 \times 10^{-2})</td>
<td>223.42562</td>
<td>-20.45063</td>
</tr>
<tr>
<td>ip8</td>
<td>191.031272</td>
<td>(-8.65 \times 10^{1})</td>
<td>351.38958</td>
<td>-33.41852</td>
</tr>
<tr>
<td>ip9</td>
<td>763.847316</td>
<td>(-1.45 \times 10^{-8})</td>
<td>198.88558</td>
<td>75.68959</td>
</tr>
<tr>
<td>ip10</td>
<td>26.341917</td>
<td>(-8.50 \times 10^{-2})</td>
<td>221.55565</td>
<td>42.87730</td>
</tr>
<tr>
<td>ip11</td>
<td>31.427458</td>
<td>(-5.07 \times 10^{-4})</td>
<td>285.09733</td>
<td>-58.27209</td>
</tr>
<tr>
<td>ip12</td>
<td>38.477939</td>
<td>(-6.25 \times 10^{0})</td>
<td>331.85267</td>
<td>-16.97288</td>
</tr>
<tr>
<td>ip13</td>
<td>12.428001</td>
<td>(-1.00 \times 10^{-2})</td>
<td>14.32394</td>
<td>-14.32394</td>
</tr>
<tr>
<td>ip14</td>
<td>1991.092401</td>
<td>(-1.00 \times 10^{3})</td>
<td>300.80284</td>
<td>-14.32394</td>
</tr>
</tbody>
</table>

Although no detections were made, we produce all-sky averaged upper limits on the strain of the signal \(h_0\) (these upper limits are valid for all the frequency bands except the ones which have one of the 1539 outliers). We add software simulated signals to the original data by using lalapps_Makefakedata_v5. We have injected signals at ten different 0.1 Hz bands for each of the four coherent times (a total of 40 bands), which can be seen in Table VI. These are bands which do not have outliers or instrumental known sources of lines or combs.

We have used five different sensitivity depths at each coherent time, with 400 signals per sensitivity depth. The sensitivity depth is given by

\[
D = \sqrt{S_n/\bar{h}_0},
\]

where \(S_n\) is the one-sided power spectral density. We inject signals at random positions in the sky, covering the full spin-down/up range and with random polarization, inclination and initial phase. For each band and depth, we calculate the efficiency (number of detected signals divided by total number of signals). We follow the same procedure as in the all-sky search: we run the main search and then apply coincidences, clustering and the population veto. We assume that a signal is detected if the total distance from the recovered cluster to the actual injection is less than 13 bins.

At each of the 40 frequency bands, we fit a sigmoid given by

\[
s = 1 - \frac{1}{1 + e^{b(x-a)}}.
\]

An example of this fitting can be seen in Fig. 9. From the estimated coefficients \(a\) and \(b\) along with the covariance
matrix $C_{ab}$, we calculate the 1-sigma envelope (the error) on the fit, which is given by

$$\sigma_S = \pm \sqrt{\left(\frac{\delta S}{\delta a}\right)^2 C_{aa} + \left(\frac{\delta S}{\delta b}\right)^2 C_{bb} + 2 \frac{\delta S}{\delta a} \frac{\delta S}{\delta b} C_{ab}}.$$  \hspace{1cm} (27)

After finding the 95% efficiency sensitivity depth at each of the 40 frequency bands (which can be seen in Fig. 10), we calculate a mean sensitivity depth for each of the four different frequency regions. The results are given in Table VI. From these results and by using Eq. (25), we calculate the upper limits on $h_0$, which are shown in Fig. 11. The trace has a shadow enclosing a 7.5% error, which we obtain by estimating the maximum difference in each of the four frequency regions shown in Fig. 10 between the ten different points and the mean sensitivity depth. Figure 11 also shows a comparison with the results obtained in the previous search with O1 data.

C. Time-domain $F$-statistic results

In the [20–100], [100–434], and [1518–1922] Hz bandwidth ranges under study, 3380 0.25-Hz wide subbands in total were analyzed. As a result of vetoing candidates around the known interference lines, a certain fraction of the bandwidth was not analyzed. As a result 16.0% of the band under study was vetoed.

In Figs. 13–15 the results of the coincidence search are presented for bandwidth ranges [20–100], [100–434], and [1518–1922] Hz respectively. The top panels show the maximum coincidence multiplicity for each of the subbands analyzed. The maximum multiplicity is an integer.
that varies from 4 to the number of time frames in each of the bandwidth analyzed. This is because we record coincidences of multiplicity of at least 4.

The bottom panel of Figs. 13–15 shows the results for the false alarm probability of coincidence for the coincidence with the maximum multiplicity. This is the probability that a coincidence among candidates from all the time-domain segments in a given frequency subband occurs by chance. This false alarm probability is calculated using formula (A. 6) in the Appendix of [33].

We define outliers as those coincidences with false alarm probabilities less than 0.1%. This criterion was adopted in our Virgo data search [33] and also in one of the Einstein@Home searches [28]. As a result we obtained 30 outliers. Among the 30 outliers, eight are identified with the hardware injections. Table XII presents the estimated parameters obtained for these hardware injections, along with the absolute errors of the reconstructed parameters (the differences with respect to the injected parameters).

The remaining 23 outliers are listed in Table XI. They include 16 that are seen only in H1 data and three in only the L1 data. In the case of the remaining outliers their amplitude is stronger in H1 than in L1 whereas the noise level in H1 is lower than in L1. Consequently no credible gravitational wave candidates were found.

VI. CONCLUSIONS

In this paper we have presented the first results of an all-sky search for CW signals using Advanced LIGO O2 data with three different pipelines, covering a frequency range from 20 to 1922 Hz and a first frequency derivative from $-1 \times 10^{-8}$ to $2 \times 10^{-9}$ Hz/s. For this broad range in parameter space, this is the most sensitive search up to 1500 Hz. Each search found many outliers which were followed up but none of them resulted in a credible astrophysical CW signal. On the contrary, they were ascribable to noise disturbances, to hardware injections, or consistent with noise fluctuations.

Although no detections have been made, we have placed interesting 95% C.L. upper limits on the gravitational wave strain amplitude $h_0$, the most sensitive being $\simeq 1.7 \times 10^{-25}$ in the 123–124 Hz region, as shown in Fig. 3. The improved results over the O1 search are due to the better sensitivity of the detectors, the use of a longer dataset and improvements of the pipelines. For the semicoherent methods used in this analysis, the strain sensitivity is proportional to [65–67]

$$h_0 \propto \sqrt{\frac{S_n}{T_{coh}}}N^{-1/4}. \quad (28)$$
For the same observing time, an improvement of 1.15 in the noise floor of both detectors translates to an expected improvement of 1.15 of the minimum detectable strain (looking at Fig. 1, we see that H1 has improved by $\sim$1.15, while L1 has improved less, so at most a factor of 1.15 is what we expect from the improved noise floors). To ascertain the influence of the run duration (assuming that the same coherent time has been used), we compare the different number of segments used. Comparing O1 [40] where $\sim$6600 segments were used by the SkyHough pipeline (similar numbers apply also for the other pipelines), with O2 where $\sim$10600 segments have been used, the expected improvement is of $\sim$1.12, which multiplied by 1.15 results in a $\sim$1.28 factor. The difference between this and the 1.4 observed improvement is due to the enhancement of the pipelines.

By converting the upper limits to an astrophysical reach, as shown in Fig. 4, we see that the searches presented in this paper provide already astrophysical interesting results. For instance, in the “bucket” region (around $\approx$150 Hz), we would be able to detect a CW signal from a neutron star within a distance of 100 pc if its ellipticity were at least $10^{-6}$. Similarly, in the middle frequency range, around $\approx$500 Hz, we would be able to detect the CW signal up to a distance of 1 kpc, with $\epsilon > 10^{-6}$. Finally at higher frequencies, around $\approx$1500 Hz, the same signal would be detectable up to a distance of 10 kpc if $\epsilon > 10^{-6}$ and 1 kpc if $\epsilon > 10^{-7}$. Such levels of ellipticity are comparable or below the maximum value we may expect for neutron stars described by a standard equation of state [68]. Although approximately 2500 neutron stars have been observed through their electromagnetic emission, a much larger number of undiscovered neutron stars is expected to exist in our Galaxy, a small fraction of which could be nearby.

Further all-sky analyses are planned on O2 data, by extending the parameter space and looking at subthreshold candidates. The O3 observing run has started in April 2019 and will last for approximately 1 year. The full network of LIGO and Virgo detectors is being upgraded and improved, and we expect that the noise floor in O3 run will be significantly better than for O2. This, and the foreseen longer run duration, will make future searches more sensitive (if the noise floor improves by 1.5 and the run is 1.5 times longer than O2, we expect an improvement of $\sim$1.66 on the strain upper limits), increasing the chances of a CW detection or allowing us to place tighter constraints on the nonasymmetries of neutron stars in our Galaxy and to put constraints on the unseen neutron star population.

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APPENDIX A

1. FrequencyHough hardware injections recovery

Table IX shows the parameters of the recovered signals, together with the error with respect to the injected signals. As shown in the Table, we have been able to detect all 13 injections done in the analyzed frequency band and the estimated parameters do show a very good agreement with the injected ones.

<table>
<thead>
<tr>
<th>Label</th>
<th>CR</th>
<th>Frequency [Hz]</th>
<th>Spin-down [nHz/s]</th>
<th>α [deg]</th>
<th>δ [deg]</th>
</tr>
</thead>
<tbody>
<tr>
<td>ip13</td>
<td>27.8</td>
<td>12.427537 (−0.000008)</td>
<td>−0.0070 (0.0030)</td>
<td>11.14 (−0.23)</td>
<td>17.78 (2.29)</td>
</tr>
<tr>
<td>ip10</td>
<td>147.8</td>
<td>26.338028 (−0.000012)</td>
<td>−0.08423 (0.00077)</td>
<td>221.14 (0.043)</td>
<td>42.79 (−0.15)</td>
</tr>
<tr>
<td>ip11</td>
<td>130.8</td>
<td>31.424762 (0.000027)</td>
<td>−0.0009 (0.0014)</td>
<td>284.52 (0.28)</td>
<td>−58.27 (0.04)</td>
</tr>
<tr>
<td>ip12</td>
<td>78.7</td>
<td>38.192818 (−0.000003)</td>
<td>−6.2542 (−0.0042)</td>
<td>336.06 (2.24)</td>
<td>−25.74 (4.35)</td>
</tr>
<tr>
<td>ip5</td>
<td>161.9</td>
<td>52.808286 (−0.000038)</td>
<td>0.00000 (4 × 10^−8)</td>
<td>301.87 (−0.37)</td>
<td>−83.641 (0.099)</td>
</tr>
<tr>
<td>ip3</td>
<td>67.42</td>
<td>108.85166 (0.000007)</td>
<td>−0.00088 (−0.00088)</td>
<td>178.29 (−0.02)</td>
<td>−33.59 (−0.09)</td>
</tr>
<tr>
<td>ip6</td>
<td>76.4</td>
<td>145.862390 (0.000036)</td>
<td>−6.7245 (0.0055)</td>
<td>358.85 (0.27)</td>
<td>−65.63 (−0.01)</td>
</tr>
<tr>
<td>ip8</td>
<td>72.7</td>
<td>190.636613 (−0.000055)</td>
<td>−8.661 (−0.011)</td>
<td>351.20 (−0.08)</td>
<td>−33.04 (0.19)</td>
</tr>
<tr>
<td>ip0</td>
<td>293.6</td>
<td>265.575312 (−0.000032)</td>
<td>0.0000 (0.0041)</td>
<td>71.64 (0.05)</td>
<td>−56.29 (−0.03)</td>
</tr>
<tr>
<td>ip2</td>
<td>297.3</td>
<td>575.163534 (0.000019)</td>
<td>0.00000 (0.00014)</td>
<td>215.18 (0.03)</td>
<td>3.44 (−0.02)</td>
</tr>
<tr>
<td>ip9</td>
<td>394.2</td>
<td>763.847307 (−0.000010)</td>
<td>0.0053 (0.0053)</td>
<td>198.86 (−0.01)</td>
<td>75.65 (−0.02)</td>
</tr>
<tr>
<td>ip1</td>
<td>408.4</td>
<td>848.955908 (−0.000048)</td>
<td>−0.2974 (0.0026)</td>
<td>37.43 (0.02)</td>
<td>−29.49 (0.02)</td>
</tr>
</tbody>
</table>

**Table IX.** Hardware injection recovery with the FrequencyHough pipeline. The reported values have been obtained at the end of the full analysis, including the follow-up. The values in parentheses are the absolute errors, that is the difference with respect to the injection parameters. The reference time is MJD 57856.826840. The sky position is given in equatorial coordinates.

**Table X.** List of subthreshold candidates in the FrequencyHough analysis. Reference time is MJD 57856.826840. All parameters are averages of the values in each detector.
2. Frequency Hough selected subthreshold candidates

Table X shows the parameters of the candidates which have been excluded because, after the follow-up and the verification stages, are in coincidence (within the standard distance, equal to 3) but below the CR threshold value CR\text{the} = 7.42.

3. \( F \)-statistic outliers

Table XI presents the parameters of the final 23 outliers from the \textit{Time-domain F-statistic} pipeline, along with comments on their likely causes. None is a credible gravitational wave signal.

\begin{table}[h]
\centering
\caption{\textit{Time-domain F-statistic} pipeline outliers in the frequencies ranges [20–100], [100–434], and [1518–1922] Hz. The columns provide outliers false alarm probability (FAP) as well as the nominal frequencies and frequency derivatives, right ascensions and declinations found for the outliers, along with comments indicating the likely sources of the outliers. Frequencies are converted to epoch GPS 1174899130.}
\begin{tabular}{llllll}
\hline
Index & FAP & Frequency [Hz] & Spin-down [nHz/s] & \( \alpha \) [deg] & \( \delta \) [deg] & Description \\
\hline
1 & \( 2.4 \times 10^{-4} \) & 21.428 & 0.014 & -63.801 & 100.39 & Present only in H1 \\
2 & \( 1.1 \times 10^{-4} \) & 27.481 & 0.027 & -63.159 & 272.42 & Present only in L1 \\
3 & \( 1.5 \times 10^{-5} \) & 29.970 & -0.003 & -66.303 & 89.028 & Present only in H1 \\
4 & \( 1.3 \times 10^{-5} \) & 31.764 & 0.030 & -54.723 & 111.99 & Interference much stronger in H1 than in L1 \\
5 & \( 6.3 \times 10^{-5} \) & 39.763 & 0.015 & -63.094 & 103.49 & Interference much stronger in H1 than in L1 \\
6 & \( 3.2 \times 10^{-4} \) & 42.945 & 0.015 & -65.709 & 94.921 & Present only in H1 \\
7 & \( 4.5 \times 10^{-5} \) & 59.951 & -0.026 & -64.569 & 86.213 & Present only in H1 \\
8 & \(< 10^{-8} \) & 82.753 & -4.229 & -52.811 & 294.54 & Interference stronger in H1 than in L1 \\
9 & \( 1.3 \times 10^{-4} \) & 83.447 & 0.011 & -65.403 & 94.618 & Interference much stronger in H1 than in L1 \\
10 & \( 2.6 \times 10^{-4} \) & 85.714 & -0.003 & -65.907 & 90.414 & Present only in H1 \\
11 & \( 1.6 \times 10^{-4} \) & 85.830 & -0.004 & -63.420 & 88.385 & Present only in H1 \\
12 & \(< 10^{-8} \) & 107.11 & 0.195 & -6.0084 & 209.42 & Present only in H1 \\
13 & \(< 10^{-8} \) & 107.14 & 0.032 & -6.4358 & 9.2631 & Present only in H1 \\
14 & \( 3.8 \times 10^{-5} \) & 119.90 & 0.046 & -6.8506 & 9.3225 & Interference stronger in H1 than in L1 \\
15 & \(< 10^{-8} \) & 128.57 & 0.362 & -7.0981 & 155.81 & Present only in H1 \\
16 & \(< 10^{-8} \) & 130.93 & 0.122 & -5.2438 & 9.0533 & Present only in H1 \\
17 & \( 7.9 \times 10^{-8} \) & 199.86 & -0.041 & -8.1570 & 138.45 & Present only in L1 \\
18 & \( 1.6 \times 10^{-8} \) & 299.41 & 0.132 & -8.2426 & 135.07 & Present only in H1 \\
19 & \(< 10^{-8} \) & 299.54 & 0.331 & -7.0534 & 109.15 & Present only in H1 \\
20 & \(< 10^{-8} \) & 299.71 & -0.229 & -6.1941 & 7.7833 & Present only in H1 \\
21 & \( 1.3 \times 10^{-7} \) & 303.27 & 0.034 & -3.4995 & 235.37 & Present only in H1 \\
22 & \(< 10^{-8} \) & 314.91 & -0.145 & -6.7862 & 7.9226 & Present only in L1 \\
23 & \(< 10^{-8} \) & 1800.1 & -1.700 & -68.404 & 75.855 & Present only in L1 \\
\hline
\end{tabular}
\end{table}

\begin{table}[h]
\centering
\caption{Hardware injection recovery with the \textit{Time-domain F-statistic} pipeline. The values in parentheses are the absolute errors, that is, the difference with respect to the injection parameters. Frequencies are converted to epoch GPS 1174899130.}
\begin{tabular}{llllll}
\hline
Label & FA & Frequency [Hz] & Spin-down [nHz/s] & \( \alpha \) [deg] & \( \delta \) [deg] \\
\hline
ip0 & \(< 10^{-8} \) & 265.5746 & (0.0007) & -0.0466 & (0.0425) & 69.42 (2.12) & -57.21 (0.99) \\
ip3 & \(< 10^{-8} \) & 108.8573 & (-0.0001) & -0.1386 & (0.1386) & 174.96 (3.42) & -34.81 (-1.37) \\
ip5 & \( 2.9 \times 10^{-4} \) & 52.8085 & (0.0002) & -0.1865 & (0.1865) & 236.90 (65.72) & -74.18 (9.66) \\
ip6 & \(< 10^{-8} \) & 145.8721 & (0.0048) & -6.0512 & (0.6788) & 358.75 (58.00) & -68.01 (2.58) \\
ip8 & \(< 10^{-8} \) & 190.6248 & (0.0002) & -8.5306 & (0.1193) & 331.41 (19.98) & -35.87 (2.45) \\
ip10 & \( 2.2 \times 10^{-4} \) & 26.3380 & (0.0001) & -0.0683 & (0.0167) & 223.10 (1.54) & 37.23 (5.64) \\
ip11 & \( 4.0 \times 10^{-4} \) & 31.42475 & (0.00001) & -0.0056 & (0.0051) & 286.25 (1.10) & -58.39 (0.11) \\
ip12 & \( 5.7 \times 10^{-2} \) & 38.2005 & (0.0001) & -6.1165 & (0.1335) & 326.26 (5.60) & -33.71 (16.73) \\
\hline
\end{tabular}
\end{table}


[34] J. Aasi et al. (LIGO Scientific and Virgo Collaborations), First all-sky search for continuous gravitational waves from


[38] B. P. Abbott et al. (LIGO Scientific and Virgo Collaborations), All-sky search for periodic gravitational waves in the O1 LIGO data, Phys. Rev. D 96, 062002 (2017).


[58] M. Alessandra Papa et al., Hierarchical follow-up of subthreshold candidates of an all-sky Einstein@Home search for continuous gravitational waves on LIGO sixth science run data, Phys. Rev. D 94, 122006 (2016).


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