Determination of the thermoelectric properties of a skutterudite-based device at practical operating temperatures by impedance spectroscopy

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Abstract
Skutterudite-based thermoelectric materials are promising candidates for waste heat recovery applications at intermediate temperatures (300–500 °C) owing to their high dimensionless figure of merit and power factor. Recently, several researchers have reported the high performance of skutterudite-based thermoelectric devices obtained by optimizing the crystal structure and microstructure of skutterudite materials and developing metallization layers for device fabrication. Despite extensive research efforts toward maximizing the power density and thermoelectric conversion efficiency of skutterudite-based devices, the thermoelectric properties of such devices after fabrication remain largely unknown. Here, we systematically investigated the factors that affect the thermoelectric properties of skutterudite-based devices within the range of practical operating temperatures (23–450 °C). We successfully prepared a two-couple skutterudite-based device with titanium metallization layers on both sides of the thermoelectric legs and characterized it using scanning and transmission electron microscopy and specific contact resistance measurements. Impedance spectroscopy measurements of the two-couple skutterudite-based device revealed the figure of merit of the device and enabled the extraction of three key thermoelectric parameters (Seebeck coefficient, thermal conductivity, and electrical conductivity). The impedance spectra and extracted parameters depended strongly on the measurement temperature and were mainly attributable to the thermoelectric properties of skutterudite materials. These observations demonstrate the interplay between the properties of thermoelectric materials and devices and can aid in directing future research on thermoelectric device fabrication.

Keywords: thermoelectric device; skutterudite; figure of merit; impedance spectroscopy; thermoelectric property
1. Introduction

Owing to the increasing demand for waste heat harvesting technologies, significant advances have been made over the past few decades toward the development of highly efficient bismuth telluride [1–3], germanium telluride [4,5], lead telluride [6,7], skutterudite (SKD) [8–12], half-Heusler [9,13–15], and oxide [9,14] based materials as well as bismuth telluride [16,17], lead telluride [18,19], SKD [20–25], half-Heusler [26–28] based thermoelectric devices. Thermoelectric devices due to their solid-state nature have the advantage of no moving parts and consequently offer compactness, quietness, and high reliability. The thermoelectric conversion efficiency of materials and devices is mainly determined by the figure of merit, \( ZT = T \sigma S^2 / \lambda \), where \( T \) is the absolute temperature, \( \sigma \) the electrical conductivity, \( S \) the Seebeck coefficient, and \( \lambda \) the thermal conductivity. To date, most efforts have focused on enhancing the \( ZT \) values of thermoelectric materials by optimizing the chemical composition [6,8,10] and microstructure [13,15]. However, it is a significant challenge to determine the thermoelectric properties of devices because of their complex structures, which comprise arrays of thermoelectric legs, metallization layers, electrodes, and insulating plates.

To characterize thermoelectric devices, Harman’s method and power output measurements have been widely used to evaluate the \( ZT \) values and thermoelectric power, respectively, but additional extensive measurements are necessary to determine the three key parameters (\( \sigma, S, \) and \( \lambda \)). Recently, several studies have demonstrated that impedance spectroscopy can be used to determine the \( ZT \) values of thermoelectric bismuth telluride [29] and SKD [30,31] materials as well as bismuth telluride [32–38] and SKD [39,40] based devices. In a typical galvanostatic impedance spectroscopy measurement, a sinusoidal current perturbation
drives the thermoelectric system slightly out of temperature equilibrium (appearance of a temperature gradient), inducing reorganization of the carriers owing to the Peltier effect, which allows the Seebeck voltage to be measured as a function of frequency. The impedance value at a given frequency is obtained from the ratio of the measured voltage amplitude to the applied current amplitude together with the phase angle difference. This protocol is repeated over several decades of frequency to obtain the impedance spectrum. $ZT$ and the three key parameters ($\sigma$, $S$, and $\lambda$) can then be extracted using suitable mathematical models (equivalent circuits) [29,37].

Most investigations using impedance spectroscopy for the characterization of bismuth telluride [29,32,34,35,38] and SKD [39,40] based thermoelectric devices have been performed at ambient temperature, with only a few studies reporting the properties of bismuth telluride based devices at operating temperatures up to 150 °C [33,37]. In this work, we employed impedance spectroscopy to elucidate the thermoelectric properties of an in-house-fabricated two-couple SKD-based device within the range of practical operating temperatures (23–450 °C) with the help of state-of-the-art mathematical models along with electrical, thermal, and microstructural characterization. To the best of our knowledge, this is the first detailed and systematic investigation of the factors that affect the properties of SKD-based thermoelectric devices.

2. Materials and methods

2.1 Materials, device fabrication, and electrical/thermal property measurements

Commercial SKD powders (composition $\text{DD}_2\text{Fe}_3\text{CoSb}_{12}$ for p-type SKD and $(\text{Mm},\text{Sm})_2\text{Co}_4\text{Sb}_{12}$ for n-type SKD, where DD and Mm stand for didymium and mischmetal, respectively) were provided by Treibacher Industrie AG [24,41]. The as-received SKD powders
(3 g) were sandwiched between two Ti foils (50 μm thickness, Alfa Aesar, 99.6%) and then cold-pressed using a graphite die with a diameter of 12 mm. SKD discs metallized on both sides with Ti (Ti/SKD/Ti joints) were prepared by spark plasma sintering (SPS) (SPS-211LX, Fuji Electronic Industrial Co., Ltd.) at 620 °C for p-type SKD and 650 °C for n-type SKD at 50 MPa under 1.3 × 10⁻⁴ kPa. To fabricate a two-couple device, cube-shaped legs with sizes of 4 × 4 × 4 mm³ were cut from the sintered discs and polished using 1 μm grade alumina abrasive. The two p-type and n-type SKD legs were integrated in series with Cu electrodes on an Al₂O₃ substrate (thickness 0.76 mm) using an Incusil-series brazing alloy on both sides. Conductive Ag paste was used to connect four copper wires to the SKD device, and then the joint was covered using graphite paste to ensure connection between the SKD device and the wires, as shown in Fig. 1 (a). Fig. 1 (b) shows that similar impedance spectra are obtained for the wireless SKD device suspended in a vacuum chamber and the wired SKD device in a heating block with an insulating cloth. It can been seen from Fig. 1 (b) that the resistance of the wires in the thermoelectric device is less than 10 μΩ; hence, the wiring, insulating cloth, and heating block have insignificant effects on the impedance spectroscopy measurements.
Fig. 1 (a) Photographs of the wireless and wired two-couple SKD device. (b) Impedance spectra of the wireless SKD device suspended in a vacuum chamber and the wired SKD device in a heating block with an insulating cloth.
The interface between Ti and SKD (Ti/SKD) was examined using scanning electron microscopy (SEM) together with energy-dispersive X-ray spectroscopy (EDS) (JEOL JSM-7000F). The details of the Ti/SKD interfacial structure were investigated by high-angle annular dark-field scanning transmission electron microscopy (HAADF-STEM) (JEOL JEM-2100F and JEM-ARM200) together with EDS and selected-area electron diffraction (SAED). The lamellar specimens were prepared using a focused ion beams (FIB) (Helios NanoLab). The SAED patterns were analyzed using the CrysTBox software [42]. The specific contact resistance at the Ti/SKD interface was evaluated using an in-house electrical contact resistivity apparatus at 23 °C [43]. Laser flash analysis (LFA 447, Netzsch) was employed to measure the thermal diffusivity of the SKD materials and Al2O3 substrate. The thermal conductivity was obtained as \( \lambda_{\text{TE}} = D \cdot C_p \cdot \rho \), where \( D \) is the thermal diffusivity, \( C_p \) is the specific heat, and \( \rho \) is the mass density of the specimen. The temperature-dependent \( C_p \) values were measured directly using differential scanning calorimetry (DSC 214 Polyma, Netzsch). The \( \rho \) values were determined from mass and volume measurements for each specimen.

2.2 Impedance spectroscopy measurements and data analysis

The SKD device was suspended in a vacuum chamber (0.13 kPa) and placed in a copper heating block. To avoid direct contact between the thermoelectric device and the copper heating block, the interface was insulated using multiple layers of ceramic insulation cloth (RS-Cloth, Zircar Inc.). A schematic of the experimental setup is shown in Fig. S1 in the Supplementary Material. The impedance spectroscopy data for the SKD device were collected over the temperature range of 23–450 °C in the frequency range of \( 10^{-2} \) to \( 10^3 \) Hz using a
potentiostat/galvanostat (Autolab PGSTAT302N, Metrohm) connected to a 10 A current booster with a sine-wave oscillation current in a four-probe configuration. In a previous study by our group [40], it was found that the oscillation current value corresponding to the Peltier heat power density of 1000 W·m$^{-2}$ for an SKD two-couple thermoelectric device was in good agreement with the results of Beltrán-Pitarch et al. [30]. As the $S$ values of SKD materials increase with temperature, the Peltier power density was employed to optimize the oscillation current amplitude value at each temperature. The optimized current amplitude ($I$) at each measurement temperature was calculated using $I = PP(A/(S_{ave}T))$, where $PP$ is the Peltier power density (1000 W·m$^{-2}$), $S_{ave}$ is the average Seebeck coefficient for p-type and n-type SKD at each temperature [24], $T$ is the absolute temperature, and $A$ is the area of the thermoelectric legs. At each measurement temperature, the isothermal condition was assumed for the impedance spectroscopy measurements when there was no change in the recorded impedance spectra.

The obtained impedance spectra were analyzed using both i) ideal [29] and ii) comprehensive [37] models to characterize the thermoelectric device properties. The ideal model is derived from a one-dimensional heat equation, as described by García-Cañadas and Min [29] under the assumption of adiabatic conditions in all surfaces. It can be written as,

$$Z(j\omega) = R_\Omega + \left(\frac{1}{Z_{WCT}(j\omega)^{-1} + Z_{Wa}(j\omega)^{-1}}\right),$$

(1)

where $R_\Omega$ is the total ohmic resistance, $j = (-1)^{0.5}$, and $\omega$ is the angular frequency. $Z_{WCT}$ and $Z_{Wa}$ are a constant temperature Warburg element and an adiabatic Warburg element, respectively, which are defined as:
\[ Z_{WCT}(j\omega) = R_{TE} \left( \frac{j\omega}{\omega_{TE}} \right)^{-0.5} \tanh \left[ \left( \frac{j\omega}{\omega_{TE}} \right)^{0.5} \right], \]  
\[ Z_{Wa}(j\omega) = R_{C} \left( \frac{j\omega}{\omega_{C}} \right)^{-0.5} \coth \left[ \left( \frac{j\omega}{\omega_{C}} \right)^{0.5} \right], \]

where \( R_{TE} = 2N S^2 T L / (\lambda_{TE} A) \) is the thermoelectric resistance and \( R_{C} = 4N S^2 T L C / (\lambda_{C} A) \) is the thermoelectric resistance induced by the ceramic layer. In addition, \( \omega_{TE} = \alpha_{TE}/(L/2)^2 \) and \( \omega_{C} = \alpha_{C}/L C^2 \) are the characteristic angular frequencies of thermal diffusion for the thermoelectric and ceramic materials, respectively, where \( N \) is the number of thermoelectric couples, \( T \) is the absolute temperature, \( \lambda_{TE} \) and \( \lambda_{C} \) are the thermal conductivities of the thermoelectric element and insulating ceramic plate, respectively, \( L \) and \( L C \) are the lengths of the thermoelectric element and ceramic plate, respectively, and \( \alpha_{TE} \) and \( \alpha_{C} \) are the thermal diffusivities of the thermoelectric element and ceramic plate, respectively.

The \( \sigma, S, \lambda_{TE}, \) and \( ZT \) values can be calculated using the following expressions:

\[ \sigma = 2N \frac{L}{R_{\Omega} A}, \]  
\[ S = \sqrt{\frac{R_{C} \lambda_{C} A}{4NTL_{C}}}, \]  
\[ \lambda_{TE} = \frac{2NS^2 T L}{R_{TE} A}, \]  
\[ ZT = \frac{S^2 \sigma}{\lambda_{TE}} T = \frac{R_{TE}}{R_{\Omega}}, \]

where the parasitic contributions to the ohmic resistance (wires, copper strips, and electrical contacts between them) are neglected.
To estimate the thermoelectric parameters accurately, a more comprehensive model can be derived from a corrected one-dimensional heat equation including thermal convection, radiation, and spreading-constriction thermal impedance corrections, as described by Mesalam et al. [37]:

$$Z_m(j\omega) = R_\Omega + \left(\frac{1}{Z_{WTE,m}(j\omega)^{-1} + Z_{Wa,m}(j\omega)^{-1}}\right),$$

(8)

where the modified constant temperature Warburg element and adiabatic Warburg take the following forms:

$$Z_{WTE,m}(j\omega) = R_{TE} \left(\frac{h_1L^2}{2\lambda_{TE}r_0} + \frac{j\omega}{\omega_{TE}}\right)^{-0.5}\tanh\left[\left(\frac{h_1L^2}{2\lambda_{TE}r_0} + \frac{j\omega}{\omega_{TE}}\right)^{0.5}\right],$$

(9)

$$Z_{Wa,m}(j\omega) = \left\{\frac{4NS^2T}{h_2A} \left(\frac{\eta}{1 - \eta}\right)^{-1} + \frac{4NS^2T}{Y_{SCA}} + \left(\frac{4NS^2T}{h_3A}\right)^{-1} + \left(\eta R_C \left(\frac{j\omega}{\omega_C}\right)^{-0.5}\coth\left[\left(\frac{j\omega}{\omega_C}\right)^{0.5}\right]\right)^{-1}\right\}^{-1}$$

$$+ \left(\eta R_C \left(\frac{j\omega}{\omega_C}\right)^{-0.5}\tanh\left[\left(\frac{j\omega}{\omega_C}\right)^{0.5}\right]\right)^{-1}$$

$$+ \left(4NS^2TL_c^2h_3\omega_C\right)^{-1} \left(\frac{j\omega A\lambda_c^2\eta^2}{j\omega A\lambda_c^2\eta^2}\right)^{-1}\right\}^{-1},$$

(10)
where $\eta$ is the filling factor (0.66), $r_0$ is the thermoelectric leg radius ($\sqrt{A/\pi}$), $Y_{SC}$ is the spreading-constriction impedance between the SKD legs and the Al$_2$O$_3$ plate (given by

$$\frac{1}{Y_{SC}} = \frac{4L_C}{\lambda_C} \sum_{n=1}^{\infty} \left[ \frac{j \delta_n^2 \delta_n \left( \eta \frac{\pi L_C^2}{A} + \frac{j \omega}{\omega_C} \right)^{0.5}}{\delta_n^2 \delta_n^2 \left( \eta \frac{\pi L_C^2}{A} + \frac{j \omega}{\omega_C} \right)^{0.5}} \right],$$

in which $J_0$ and $J_1$ are the zero- and first-order Bessel functions of the first kind, and $\delta_n$ is the $n$th zero root of $J_1$, $\sigma_B$ is the Stefan–Boltzmann constant ($5.67 \times 10^{-8}$ kg·s$^{-3}$·K$^{-4}$), $\varepsilon$ is the emissivity (0.9 for SKD [44] and 0.5 for Al$_2$O$_3$ [37]), $h_1 = 4 \sigma_B \varepsilon_{TE} T^3$ is the radiative heat loss of the thermoelectric legs, $h_2 = 4 \sigma_B \varepsilon_{C} T^3$ is the radiative heat loss at the internal surface of the Al$_2$O$_3$ plate, and $h_3 = h_C + 4 \sigma_B \varepsilon_{C} T^3$ is the radiative and convective heat loss at the external surface of the Al$_2$O$_3$ plate (where $h_C$ is the convective heat transfer coefficient at the external surface of the ceramic plates. The $\sigma$, $S$, $\lambda_{TE}$, and $ZT$ values were calculated using Eq. (4)–(7). For both the models, the complex nonlinear least-squares analysis of the impedance spectra was performed with MATLAB using five free fitting variables ($R_\Omega$, $R_C$, $R_{TE}$, $\omega_C$, and $\omega_{TE}$) simultaneously by minimizing a functional that measures the discrepancies between the obtained data and the models. The fittings of the experimental data to the above equations were carried out using the built-in function “lsqcurvefit” from the MATLAB Optimization Toolbox with a trust-region reflective algorithm and convergence criteria (80,000 iterations, tolerance $10^{-15}$).

3. Results and discussion

The Ti metallization layer has been found to exhibit a low specific contact resistance and excellent bonding characteristics for SKD-based thermoelectric materials (CoSb$_3$ [45–47],
Yb_{0.6}Co_{4}Sb_{12} [48], and Ce_{y}Fe_{x}Co_{4-x}Sb_{12} [49]). Fig. 2 shows the cross-sectional SEM images and EDS elemental line-scan results for the Ti/SKD interfaces, together with the specific contact resistance between Ti and SKD. As shown in the SEM images in Fig. 2a and c, the metallization layers are tightly bonded to the SKD leg surfaces. However, the EDS line-scan results indicate that some intermetallic phases are formed during SPS because of the interfacial reaction at the Ti/SKD interface. To perform a more detailed analysis, HAADF-STEM images and SAED patterns were obtained from the FIB lamella prepared from the Ti/SKD interface. The distinct areas of the HAADF-STEM images obtained by contrast difference are shown in Fig. 3a and g for p-type and n-type SKD, respectively. The SAED patterns along the zone axes of the corresponding areas are shown in Fig. 3b–f and h–l and include representative Miller indices.

Two intermetallic phases with different crystal structures are observed for Ti/p-type SKD, namely, hexagonal TiSb near Ti and orthorhombic (FeCo)Sb$_2$ near p-type SKD. In contrast, three intermetallic phases are observed for Ti/n-type SKD, namely, hexagonal TiSb near Ti, and orthorhombic CoSb$_2$ and cubic TiCoSb near n-type SKD. These observations are consistent with the SEM-EDS results. Moreover, the observation of intermetallic compounds at the Ti/SKD interface is in good agreement with the results reported in the literature [45,46,48–50]. Detailed indexing of the SAED patterns is provided in the Supplementary Material (Fig. S2 and S3). According to previous reports [45,46,48,49], the formation and growth of TiSb and TiCoSb phases are further accelerated upon heat treatment at temperatures above 500 °C, resulting in an increase in the electrical contact resistance and fracture failure of Ti/SKD interfaces. Hence, in this study, the impedance spectroscopy measurements of the SKD device were only performed up to 450 °C to avoid the formation of further TiSb and TiCoSb phases. Even though there is an interfacial reaction between Ti and SKD, the specific contact resistivity at the Ti/SKD interface
was very low (3.06 $\mu\Omega\cdot\text{cm}^2$ for p-type SKD and 4.37 $\mu\Omega\cdot\text{cm}^2$ for n-type SKD), in agreement with previous studies [48,49].

**Fig. 2** Scanning electron microscopy (SEM) images with energy-dispersive X-ray spectroscopy (EDS) line scanning profiles (left) and electrical contact resistivity (right) of Ti metallized (a, b) p-type skutterudite (SKD) joints and (c, d) n-type SKD joints. Insets: photographs of the Ti metallized SKD thermoelectric legs.
**Fig. 3** High-angle annular dark-field scanning transmission electron microscopy (HAADF-STEM) images at the Ti/SKD interface and selected-area electron diffraction (SAED) patterns of the corresponding regions in the HAADF-STEM images along the $[u \ v \ w]$ zone axis for (a–f) p-type SKD and (g–l) n-type SKD.
Prior to the detailed analysis, Kramers–Kronig (KK) transformation tests were performed to evaluate the impedance spectra of the SKD device. This method provides a very effective way to assess the consistency of spectra and to exclude spectra containing non-steady-state behavior in the field of thermoelectrics [40], solid state ionics [51,52], and electrochemistry [53]. As shown in Fig. 4, to validate the measurements, residual plots for the deviations of the real ($\Delta Z_{\text{re}}(\omega)$) and imaginary ($\Delta Z_{\text{im}}(\omega)$) parts can be obtained from the differences between the experimental impedance data ($Z_{\text{re}}$ and $Z_{\text{im}}$) and the KK transformation results ($Z_{\text{re,KK}}$ and $Z_{\text{im,KK}}$). The impedance spectra at frequencies higher than 0.2 Hz are mainly due to the contacts (electrical contacts and Al$_2$O$_3$ plates); hence, the observation of residual errors close to zero at frequencies higher than 0.2 Hz indicates that the in-house-fabricated two-couple SKD device has excellent contact characteristics. At frequencies lower than 0.2 Hz, the residual errors are higher (up to 0.4%) and increase slightly with temperature. The impedance spectra at frequencies lower than 0.2 Hz are related to the Seebeck voltage of both p-type and n-type SKD thermoelectric materials. The high residual errors at frequencies lower than 0.2 Hz could originate from the heat loss of the thermoelectric legs upon Peltier heating/cooling during impedance spectroscopy measurements. Nevertheless, as the residual errors are less than $\pm 0.5\%$ over the entire frequency range [40,53], all of the impedance spectra of the SKD device are a response to the current oscillation applied under the steady-state conditions. Therefore, all of the impedance spectra are KK-transformable and can be further analyzed by nonlinear least-squares analysis to evaluate the thermoelectric properties of the two-couple SKD device.
Fig. 4 Kramers–Kronig (KK) transformation results (empty symbols) of impedance spectra (filled symbols) (left) and residual plots (right) for the two-couple SKD device at (a, b) 150 °C, (c, d) 250 °C, (e, f) 350 °C, and (g, h) 450 °C.
Fig. 5 shows the Nyquist plots of the impedance spectra for the two-couple SKD device over the temperature range 23–450 °C. The high-frequency real axis intercept of each impedance spectrum corresponds to the ohmic electrical resistance of the SKD device, which increases with temperature, mainly because of the increase in the electrical resistance of the SKD thermoelectric materials. From this point towards lower frequencies, a linear trend overlapped with an impedance arc is observed due to the response of the Al₂O₃ insulating plates and thermoelectric materials integrated in the SKD device upon application of current perturbation. The contribution of the arc increases with temperature, mainly owing to the increase in $ZT\sigma^{-1}$ of the SKD thermoelectric materials with temperature.
Fig. 5 Impedance spectra of the two-couple SKD device at various temperatures (filled symbols) and mathematical model fitting results (lines) using the (a) ideal and (b) comprehensive models.
The obtained spectra were analyzed using both the ideal and the comprehensive models. Both models require five fitting parameters ($R_Ω$, $R_C$, $R_{TE}$, $ω_C$, and $ω_{TE}$) to describe the spectrum obtained at each temperature when the thermal conductivity of the Al$_2$O$_3$ plate ($λ_C$) is known at each temperature. Considerable care must be taken to use an appropriate value for $λ_C$, as it is used to obtain the $S$ values and, then, the $λ_{TE}$ values. Fig. 6 shows $λ_C$ as a function of temperature, as determined using differential scanning calorimetry and laser flash analysis results. As shown in Fig. 6, $λ_C$ decreases by 50% with the increase in temperature; hence, it is necessary to measure the thermal conductivity of the insulating plate of the thermoelectric device to realize accurate determination of the thermoelectric properties from the impedance spectra of the two-couple SKD device. For the comprehensive model, the convective heat transfer coefficient ($h_C$) value at the external surface of the Al$_2$O$_3$ plate at each temperature is required. At room temperature, a minimum value of 25 W·m$^{-2}$·K$^{-1}$ obtained a reasonable fitting between the experimental and simulated data, whereas the $R_C$ values were overestimated when the $h_C$ value was less than 25 W·m$^{-2}$·K$^{-1}$. We explored varying the $h_C$ values to investigate the effect of $h_C$ on the $R_C$ values. The $R_C$ values were found to be within the error range given by nonlinear least-squares fitting analysis at other temperatures when $h_C$ was within the range 25 ± 15 W·m$^{-2}$·K$^{-1}$. It has been reported that typical heat transfer coefficient values for natural convection at ambient pressure are in the range of 2–25 W·m$^{-2}$·K$^{-1}$ [54]. The somewhat high heat transfer coefficient value at the external surface of the Al$_2$O$_3$ plate may possibly be explained by convective contributions from other parts of the SKD device. Even though the $h_C$ value is somewhat higher than expected at this experimental pressure, the impact of $h_C$ on the $ZT$ value of the device is insignificant (less than 6%) [54] and within the fitting error in this study. Therefore, we used 25 W·m$^{-2}$·K$^{-1}$ as the $h_C$ value when performing the comprehensive model analysis.
Fig. 6 Thermal conductivity of the Al$_2$O$_3$ insulating plate ($\lambda_C$) in the two-couple SKD device as a function of temperature.
Fig. 7 shows the values obtained for the five fitting parameters ($R_\Omega$, $R_C$, $R_{TE}$, $\omega_C$, and $\omega_{TE}$) as functions of temperature in the range 23–450 °C. The error bars represent the computed standard error of each parameter obtained from the converged fitting procedure. All the resistances ($R_\Omega$, $R_C$, and $R_{TE}$) increased with increasing temperature, whereas the angular frequencies ($\omega_C$ and $\omega_{TE}$) were found to decrease. The increase in $R_\Omega$ is due to the increase in the overall ohmic resistance of the device, whereas the increase in $R_{TE}$ is mainly due to the increase of the Seebeck voltage of the SKD thermoelectric materials. The angular frequencies, $\omega_C$ and $\omega_{TE}$, are directly proportional to the thermal diffusivity of the Al$_2$O$_3$ plate and SKD materials, respectively. The thermal diffusivity of the Al$_2$O$_3$ plate decreased by ~30% with the increase in temperature, whereas that of SKD showed a negligible temperature dependence. Hence, the observed trends for $\omega_C$ and $\omega_{TE}$ reflect the thermal diffusivity trends of the Al$_2$O$_3$ plate and SKD materials. The most pronounced change was observed for the $R_C$ value, which increased from ~0.1 mΩ at 23 °C to ~1.5 mΩ at 450 °C. Using the comprehensive model, the $R_C$ value at 450 °C was approximately 15 times greater than the value at 23 °C; hence, the $R_C$ of the SKD device contributes more than 20% of the overall electrical resistance of the device at 450 °C, but less than 8% at 23 °C.
Fig. 7 Variations in the resistance ($R_\Omega$, $R_C$, and $R_{TE}$) and angular frequency ($\omega_C$ and $\omega_{TE}$) values extracted from the (a, c) ideal and (b, d) comprehensive models as functions of temperature. The error bars represent the standard errors of the complex nonlinear least-squares fitting; where bars are not shown, they are smaller than the symbol size.
To assess the $\lambda_{TE}$, $\sigma$, $S$, and $ZT$ values of the SKD device, the three resistances ($R_\Omega$, $R_C$, and $R_{TE}$) must be accurately obtained from the fitting. As the fitting error of $R_\Omega$ is less than 0.1% of its fitted value, we have constructed error maps in the $R_C$ and $R_{TE}$ plane for the values obtained by fitting the impedance spectra at 150, 250, 350, and 450 °C to the comprehensive model. The goodness-of-fit parameter $\chi^2$ was introduced to assess the quality of fitting: $\chi^2 = \sum_{k=1}^{n} ((g_k - g_{m,k})/g_k)$, where $g_k$ and $g_{m,k}$ represent the experimental data obtained and the fitted values predicted by the model, respectively. $\chi^2$ values were calculated for a 100 × 100 grid around the fitted $R_C$ and $R_{TE}$ values to demonstrate the effect of both $R_C$ and $R_{TE}$ on $\chi^2$. Fig. 8 shows the error maps of the fitting results, confirming that an absolute maximum for reciprocal $\chi^2$ of greater than 500 ($\chi^2 \leq 0.002$) can be obtained for the fitted $R_C$ and $R_{TE}$ values, signifying the reliability of the fitting in this study.
Fig. 8 Fitting error plots showing the values of reciprocal $\chi^2$ calculated for 100 $\times$ 100 grid around the fitted $R_C$ and $R_{TE}$ values. The highest value of $\chi^2$ at the center of the plot indicates the lowest deviation of the fitted values from the values obtained at (a) 150 °C, (b) 250 °C, (c) 350 °C, and (d) 450 °C.
Fig. 9 shows the $S$, $\lambda_{\text{TE}}$, $\sigma$, and $ZT$ values of the SKD device as a function of temperature, as obtained from the impedance spectra using both the ideal and the comprehensive models. The values measured for p-type and n-type SKD materials using ZEM-3 (Ulvac-Riko) and laser flash analysis (LFA 457, Netzsch) [24] are also given for comparison. It can be seen that the $S$ and $\lambda_{\text{TE}}$ values from the comprehensive model show better agreement than those from the ideal model, whereas the $\sigma$ and $ZT$ values from both models are similar. Therefore, it can be concluded that a model that considers thermal convection, radiation, and spreading-constriction thermal impedance corrections will afford more accurate estimations of $S$ and $\lambda_{\text{TE}}$ than the adiabatic ideal model. It was found that $\sigma$ decreases with increasing temperature, whereas $S$ increases with increasing temperature. $\lambda_{\text{TE}}$ increases until 300 °C, and then tends to decrease until 450 °C. The $\sigma$ values obtained for the SKD device are approximately 20% lower than average values for p-type and n-type SKD materials owing to the presence of parasitic electrical resistances between the electrical contacts and SKD and the resistance of the electrical connection wires. The obtained $S$ values for the SKD device are 8–20% higher than the average values for p-type and n-type SKD materials from both the ideal and the comprehensive models. However, the average power factor ($S^2\sigma$) values of the SKD device only deviate by 7% and 17% in comparison with the average values of p-type and n-type SKD materials from the comprehensive and the ideal models, respectively. The $\lambda_{\text{TE}}$ values obtained for the SKD device are 50% and 100% higher than those of p-type and n-type SKD materials from the comprehensive and the ideal models, respectively; hence, the somewhat low $ZT$ values of the SKD device (60% of the average value for p- and n-type SKD) in this study could be due to the high thermal conductivity values from the impedance spectra. This observation may be associated with variations in the thermoelectric properties of the SKD thermoelectric legs during device fabrication. It is necessary to conduct further studies.
to verify the differences between the thermoelectric properties of devices and materials and understand the underlying fundamental properties of thermoelectric devices.
Fig. 9 Variations in (a) electrical conductivity ($\sigma$), (b) thermal conductivity ($\lambda_{\text{TE}}$), (c) Seebeck coefficient ($S$), and (d) figure of merit ($ZT$) of the two-couple SKD device as a function of temperature, as determined from the impedance spectra by employing the ideal and comprehensive models. For comparison, the properties of SKD materials, measured using ZEM-3 (Ulvac-Riko) and laser flash analysis (LFA 457, Netzsch) [24], are also displayed.
4. Conclusions

We have systematically investigated the thermoelectric properties of a skutterudite-based thermoelectric device using impedance spectroscopy in the temperature range 23–450 °C. Prior to the impedance spectroscopy measurements, the interface from the titanium metallized skutterudite joints was characterized by combining scanning and transmission electron microscopy with specific electrical contact resistance measurements. Intermetallic phases (titanium antimony and titanium cobalt antimony compounds), found at the interface between titanium and skutterudite owing to the interdiffusion of titanium and skutterudite during the spark plasma sintering process, exhibited a low specific contact resistance of 3–4.5 μΩ·cm². All impedance spectra of the skutterudite-based thermoelectric device were Kramers–Kronig-transformable, which enabled least-squares analysis using both the ideal and the comprehensive mathematical models. Both models were used to extract the figure of merit and three key parameters (Seebeck coefficient, thermal conductivity, and electrical conductivity) from the impedance spectra as a function of temperature. Both models yielded similar electrical conductivity and figure of merit values, whereas the comprehensive model estimated the Seebeck coefficients and thermal conductivity values more accurately than the ideal model. The figure of merit values for the skutterudite-based device were 60% lower than those of skutterudite materials, mainly because the thermal conductivity of the device was higher than that of the skutterudite materials, whereas the average power factor values of the device were close to those of the materials. These deviations may be associated with variations in the thermoelectric properties of the skutterudite thermoelectric legs during device fabrication. These encouraging results indicate that the impedance spectroscopy is a useful tool for investigating the properties of thermoelectric devices. Furthermore, this study provides an important opportunity
to advance the understanding of thermoelectric device property, which is valuable information for the design and optimization of thermoelectric conversion system to achieve the best performance for the real applications.

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Declarations of interest: none

Glossary

DD, didymium

EDS, energy-dispersive X-ray spectroscopy

FIB, focused ion beam

KK, Kramers–Kronig

HAADF-STEM, high-angle annular dark-field scanning transmission electron microscopy

Mm, mischmetal
SAED, selected-area electron diffraction

SEM, scanning electron microscopy

SKD, skutterudite

SPS, spark plasma sintering

A, area of the thermoelectric legs

$C_p$, specific heat

$D$, thermal diffusivity

$h_1$, radiative heat loss of the thermoelectric legs

$h_2$, radiative heat loss at the internal surface of the $\text{Al}_2\text{O}_3$ plate

$h_3$, radiative and convective heat loss at the external surface of the $\text{Al}_2\text{O}_3$ plate

$h_C$, convective heat transfer coefficient at the external surface of the ceramic plate

$I$, current amplitude

$J_0$, zero-order Bessel function

$J_1$, first-order Bessel function

$L$, length of the thermoelectric element

$L_C$, length of the ceramic plate

$N$, number of thermoelectric couples

$PP$, Peltier power density
$R_C$, thermoelectric resistance induced by the ceramic layer

$R_{TE}$, thermoelectric resistance

$R_\Omega$, ohmic resistance

$r_0$, thermoelectric leg radius

$S$, Seebeck coefficient

$S_{\text{ave}}$, average Seebeck coefficient

$T$, absolute temperature

$Y_{SC}$, spreading-constriction impedance

$Z_{\text{im}}$, imaginary part of the impedance

$Z_{\text{re}}$, real part of the impedance

$Z_{Wa}$, adiabatic Warburg element

$Z_{WCT}$, constant temperature Warburg element

$ZT$, figure of merit

$\alpha_{TE}$, thermal diffusivity of the thermoelectric element

$\alpha_C$, thermal diffusivity of the ceramic plate

$\varepsilon$, emissivity

$\lambda$, thermal conductivity

$\lambda_C$, thermal conductivity of the insulating ceramic plate
\( \lambda_{\text{TE}} \), thermal conductivity of the thermoelectric element

\( \eta \), filling factor

\( \rho \), mass density

\( \sigma \), electrical conductivity

\( \sigma_B \), Stefan–Boltzmann constant

\( \chi^2 \), goodness-of-fit parameter

\( \omega \), angular frequency

\( \omega_C \), characteristic angular frequency of thermal diffusion of ceramic materials

\( \omega_{\text{TE}} \), characteristic angular frequency of thermal diffusion of thermoelectric materials
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Supplementary Material

Determination of the thermoelectric properties of a skutterudite-based device at practical operating temperatures by impedance spectroscopy

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**Fig. S1** Schematic of experimental setup (side view).
**Fig. S2** Indexing of the selected-area electron diffraction (SAED) patterns at the Ti/SKD interface for p-type SKD.
Fig. S3 Indexing of the selected-area electron diffraction (SAED) patterns at the Ti/SKD interface for n-type SKD.