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Manuscript title: Influence of Grain Size Distribution of Inclusions on the Mechanical Behaviors of Track-bed Materials

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Abstract

In the French conventional railway track-bed, an interlayer was created, mainly from the interpenetration of ballast and subgrade. Field investigation showed that the ballast content decreases over depth. In addition, depending on the locations the grain size distribution of ballast grains can be quite different, defining different values of coefficient of uniformity $C_u$. In this study, triaxial tests were carried out on the interlayer soil with 3 $C_u$ values and 5 volumetric inclusion contents $f_v$. X-ray μCT scans were also conducted on representative samples to visualize the corresponding inclusion distributions. Results show that i) at all 3 $C_u$ values, two soil fabrics can be identified, namely grain-grain contact structure for large inclusion contents ($f_v = 35\%$ and $45\%$) and fine-fine contact structure for small inclusion contents ($f_v = 5\%, 10\%$ and $20\%$), and the characteristic volumetric inclusion content $f_{v, cha}$ separating these two categories was larger for smaller $C_u$ since in this case less inclusion contacts were expected to be developed at a given $f_v$; ii) for the grain-grain contact structure, a smaller $C_u$ caused larger maximum deviator stress $q_{max}$, larger friction angle, larger Poisson’s ratio and more dilatancy due to the involvement of more large grains at smaller $C_u$; iii) for the fine-fine contact structure, an opposite trend was observed: the smaller the $C_u$, the smaller the $q_{max}$ the friction angle, the Poisson’s ratio and the dilatancy due to the less quantity of inclusion grains; iv) the change patterns of Young’s modulus and cohesion with increasing $C_u$ were the same for the two soil fabrics - Young’s modulus and cohesion decreased with the increase of $C_u$.

**Keywords**: fabric/structure of soils; gravels; laboratory tests; particle-scale behaviour; shear strength; deformation
Introduction

In France, 94% of the total railway network is constituted by the ancient railway lines, which were built by directly placing the ballast onto the subgrade without separation layer between them. Under years of traffic loadings, a layer namely interlayer was formed, mainly through the interpenetration of fines and ballast. According to the in-situ investigation (Trinh 2011), the content of inclusions (ballast grains) decreases over depth, and roughly two distinct soil fabrics are identified: a ballast grain skeleton with fine particles contained in the pores of grain skeleton for the upper part and a fine matrix with ballast grains floating in it for the lower part. In the French program of conventional railway lines renewal, this interlayer with a dry unit mass as high as 2.4 Mg/m$^3$ (Trinh 2011) was maintained due to its favorable mechanical behavior. Thus, a series of studies have been conducted, especially on the upper part of the interlayer, aiming to clarify the effect of fines and water contents on the mechanical behavior by both monotonic and cyclic triaxial tests (Trinh et al. 2012; Cui et al. 2013; Duong et al. 2013; Duong et al. 2016; Lamas-Lopez 2016). The results showed that the increase of water content decreased the shear strength and enlarged the permanent deformation. In addition, the effect of fines content was closely related to the degree of saturation of the interlayer: when the interlayer was in saturated state, adding fines increased the permanent deformation and decreased the resilient modulus, whereas opposite changes occurred in unsaturated state. Moreover, the static and dynamic responses of the interlayer with different inclusion contents were also studied (Wang et al. 2017, Wang et al. 2018a,
Wang et al. 2018b). The results showed that there was a characteristic volumetric inclusion content \( f_{v-cha} \); the shear strength and dynamic properties (resilient modulus; damping ratio) changed slightly when \( f_v < f_{v-cha} \) and this change became significant when \( f_v > f_{v-cha} \). The \( f_{v-cha} \) corresponded to the transition of soil fabrics from a grain dominated skeleton to a fine dominated skeleton.

Because the interlayer is naturally formed at different locations along the railway lines, the grain size distribution of the inclusions characterized by \( C_u = d_{60}/d_{10} \) (Coefficient of Uniformity) inevitably varies. As a result, the mechanical behavior of the interlayer soils can change. From a practical point of view, this aspect needs to be studied in depth. Numerous studies were undertaken, dealing with the effect of \( C_u \) on the mechanical behavior of sands in terms of small strain shear modulus, shear strength, etc. (Menq 2003; Kokusho et al. 2004; Igwe et al. 2007; Wichtmann and Triantafyllidis 2009; Monkul et al. 2016). To date, the results obtained have not been conclusive. Moreover, to the authors’ knowledge, there are a few studies focusing on soils containing coarse grains. Seif EI Dine et al. (2010) studied a soil mixture of sand and coarse grains at two volumetric inclusion contents and reported that more uniform inclusions with smaller \( C_u \) value led to larger maximum deviator stress and more significant dilatancy. Their work was limited to one kind of soil fabric in which coarse inclusions were not in contact with each other. Moreover, the matrix was constituted of a cohesionless soil – a sand, which does not represent the case of interlayer soils which involve a large amount of fine-grained soils.
In this study, the mechanical behaviors of a soil mixture were investigated with 3 $C_u$ values at 5 inclusion contents by conducting monotonic triaxial tests. The fines part of the mixture was prepared by simulating the fines of “Sénissiat site” (Trinh et al. 2011) in terms of grain size distribution and plasticity index. The coarse inclusions were prepared by applying a similitude method (Lowe 1964) based on the grain size distribution curve of real ballast. This method of materials preparation allowed overcoming the difficulty in taking natural interlayer soils on the one hand, and avoiding to use large-scale experimental devices on the other hand. The samples were prepared by dynamic compaction and the fines in all samples were kept at the optimum water content and the maximum dry density. X-ray μCT scans were performed to identify the structure of soils, in particular the distribution of coarse inclusions inside the soil samples. It is worth noting that this experimental study was limited to triaxial tests on scaled-down ballast inclusions mixed with one soil and at a single water content. Thus, the results obtained cannot be directly generalised to predict/analyse the interlayer mechanical behaviour in the field complex conditions. However, they can greatly help well understand the mechanisms related to the effects of $C_u$ at different inclusion contents.

**Materials and methods**

**Materials**

Since obtaining large quantity of in-situ interlayer soils was difficult, fines and inclusions constituting such soils were fabricated in the laboratory. The fines part was prepared...
following a method proposed by Lamas-Lopez (2016) and used later by Wang et al. (2018a) to simulate the case of “Sénissiat site” (Trinh 2011). In this method, nine commercial soils were mixed together with the mass proportions calculated according to the target grain size distribution (see Table 1). The grain size distribution of the fabricated fines (Fig. 1), along with the liquid limit \( w_L = 32.0\% \) and plasticity index \( I_p = 20\% \), agreed well with those of the in-situ ones (Wang et al. 2018a).

The maximum ballast grain diameter at “Sénissiat site” (Trinh 2011) can reach 63 mm (Fig. 2). Since the diameter of the sample should be larger than 5 times the maximum grain diameter to minimize the sample size effect (Fagnoul and Bonnechere 1969; Nitchiporovitch 1969; Trinh et al. 2012; Duong et al. 2013; Lamas-Lopez 2016; Wang et al. 2018a), in order to use a triaxial cell of 100 mm diameter, micro-ballast with a maximum grain size \( d_{\text{max}} = 20 \) mm (Fig. 2) and with a grain size distribution curve determined through the similitude method (Lowe 1964) was used. Details about this method can be found in Wang et al. (2018a). To prepare the micro-ballast, three kinds of commercial soils (G10-20, G4-10 and HN2-4) were mixed. Their grains size distributions were also shown in Fig. 2.

The above-mentioned micro-ballast fabricated by Wang et al. (2018a) was termed as S1 and the corresponding experimental results were taken for comparison in this study. In order to study the effect of \( C_u \) of inclusions, other two inclusions termed as S2 and S3 were designed and fabricated by reducing the proportion of the smaller grains as compared with S1. For S2, soil HN2-4 was not employed and the fraction of G4-10 was decreased. For S3,
only soil grains from G10-20 with sizes ranging from 16 mm to 20 mm were used after removing the grains smaller than 16 mm by sieving. Table 2 compares the proportions of the soils constituting S1, S2 and S3. The designed grain size distributions of S1, S2 and S3 were put together and represented in Fig.3, with their designed $C_u$ values equal to 2.72, 2.01 and 1.12, respectively. As can be seen from Fig. 3, a smaller $C_u$ value corresponded to a steeper curve and a narrow grain size range, indicating a more uniform pattern. Fig. 3 compares the measured and calculated grain size distributions of S1, S2 and S3. A good agreement can be observed.

**Methods**

To characterize the amount of inclusions in a sample, a parameter namely volumetric inclusion content $f_v$ (Pedro 2004; Seif El Dine et al. 2010; Wang et al. 2017) was adopted:

$$f_v = \frac{V_{in}}{V_{total}}$$ (1)

where $V_{in}$ and $V_{total}$ are the total inclusion volume and the total sample volume, respectively.

The total sample volume $V_{total}$ consists of two parts: the inclusion volume $V_{in}$ and the fines volume $V_{fines}$. The inclusion volume $V_{in}$ corresponds to the volume of the solid micro-ballast grains without pores voids. All pores are assumed to be contained in the fines volume $V_{fines}$.

The $f_v$ value of the natural interlayer soils at “Sénissiat site” (Trinh 2011) was 52.9%. In this study, five $f_v$ values (namely, 5%, 10%, 20%, 35%, 45%) and three $C_u$ values (2.72, 2.01 and 1.12) were considered. Note that the case of $C_u = 2.72$ was already studied by Wang et al. (2018a). For a given $f_v$, the inclusion volume $V_{in}$ and the fines volume $V_{fines} = V_{total} - V_{in}$ can be easily determined when the total sample volume is known. Then, the total mass of the
inclusions can be calculated with the dry unit mass of inclusion $\rho_{\text{s-in}} = 2.68 \text{ Mg/m}^3$ (Trinh, 2011). It should be noted that the fines in all samples were kept at the same state defined by the two compaction parameters ($w_{\text{opt-f}} = 13.7\%$; $\rho_{\text{max-f}} = 1.82 \text{ Mg/m}^3$) (Wang et al. 2017; Wang et al. 2018a; Wang et al. 2018b). The optimum water content was selected for the fines in order to reach the maximum compaction dry density representing the compact state in the field condition (Trinh et al. 2012; Duong et al. 2013; Duong et al. 2016; Lamas-Lopez 2016). The mass of the fines and the water can be obtained accordingly.

For sample preparation, firstly, water was added into the fines to obtain the optimum water content $w = 13.7\%$. Then, the fines soil was stored in a hermetic container for 24 h for water homogenization. Afterwards, the homogenized fines soil was thoroughly mixed with the inclusions previously washed and dried. Finally, the mixed soil was dynamically compacted in a mould in three layers to reach the dimensions of diameter $D = 100 \text{ mm}$ and height $H = 200 \text{ mm}$. It should be noted that under a given compaction energy, the dry density of the fines varies with the change of inclusion content (Houston and Walsh 1993; Torrey and Donaghe 1994). In this study, in order to control the dry densities of the fines in all samples at the same value, higher compaction energies were applied for higher inclusion content. As a result, higher global dry densities were obtained with higher inclusion content, as shown in Table 3.

Monotonic triaxial drained tests were performed at three confining pressures ($\sigma_3 = 30$ kPa, 60 kPa and 120 kPa). Note that no saturation procedure was applied and the sample
water content was kept constant during a test. The shear tests were carried out at a rate of 0.5 mm/min and ended after a peak deviator stress appeared (for samples with large $f_v$ values) or after an axial strain $\varepsilon_1 = 15\%$ was reached (for samples with small $f_v$ values). Note also that for $C_u = 2.01$ at $f_v = 5\%$ and $10\%$ under $\sigma_3 = 60$ kPa, the results are not used here because of a technique problem. Detailed information of these tests is summarized in Table 3.

To investigate the soil fabric, X-ray $\mu$CT scans were carried out on as-compacted samples. The middle part (100 mm diameter and 100 mm height) was scanned for each. The “Ultratom” device designed and constructed by RXsolutions (France) was used with its X-ray source operated at a voltage of 170 kV and 10 $\mu$A. A voxel size of 58.4 $\mu$m can be detected. Note that representative samples were selected for this operation (see Table 3).

**Experimental results**

**Changes of shear behaviour with $f_v$**

The variations of deviator stress $q$ and volumetric stain $\varepsilon_v$ with axial strain $\varepsilon_1$ for soils of $C_u = 1.12$ at five $f_v$ values are presented in Fig. 4 for the three confining pressures. For the deviator stress $q$, as expected, a higher confining pressure led to a larger $q$. Under each confining pressure, the increase of maximum deviator pressure $q_{\text{max}}$ was relatively small with the increase of $f_v$ from $5\%$ to $20\%$. When $f_v$ was larger than $35\%$, $q_{\text{max}}$ increased significantly with $f_v$. As far as the volumetric stain $\varepsilon_v$ is concerned, as expected, the increase of $\sigma_3$ favored the contraction behavior. However, the dilatancy behavior was enhanced by the increase of $f_v$. Similar phenomena were observed for $C_u = 2.72$ (Wang et al. 2018a) and $C_u = 2.01$. 
Fig. 5 shows the peak deviator stress $q_{\text{max}}$ against $f_v$ for different confining pressures. It can be observed that for all the three $C_u$ values the increasing trend of $q_{\text{max}}$ followed a bi-linear pattern with the distinct slopes for $f_v \leq 20\%$ and for $f_v \geq 35\%$. Moreover, a characteristic volumetric inclusion content $f_{v-\text{cha}}$ could be identified at the intersection point of the two fitting lines. It could be suspected that when $f_v \leq f_{v-\text{cha}}$ the soil fabric was controlled by a fine-fine contact structure, while when $f_v \geq f_{v-\text{cha}}$ the soil fabric was controlled by a grain-grain contact structure.

To verify this point, the 3D inclusion distributions in the as-compact ed samples with different $f_v$ values were determined based on the 2D slices obtained via X-ray $\mu$CT scans, using ImageJ software (version 1.6.0_24, 2013, U.S. National Institutes of Health, Bethesda, Maryland). The procedures of histogram thresholding and outlier removal were applied on the 2D slices to eliminate the pixels representing the fines (diameter < 2 mm). Then, only inclusion grains were retained. The 3D structures of the inclusions were established by stacking all the processed slices. Fig. 6 shows the case of $C_u = 1.12$. It can be observed that when $f_v$ was smaller than $f_{v-\text{cha}}$ (5% and 20%), the inclusion grains were floated in the fines matrix. On the contrary, when $f_v$ was larger than $f_{v-\text{cha}}$ (35% and 45%), most of the inclusions were in contact with each other and constituted the soil skeleton. For the sample with $C_u = 2.72$, Wang et al. (2018a) also observed these two distinct soil fabrics.

To further clarify this aspect, quantitative characterization of the inclusion contacts was conducted by determining the average coordination number $N_c$. Twenty 2D cross section
slices were used for this purpose. The coordination number $N_{ci}$ for a certain inclusion grain was defined as the total number of its neighboring grains (Jouannot-Chesney et al. 2006). For each slice, $N_c$ was calculated by averaging the coordination number of all the inclusion grains:

$$N_c = \frac{\sum N_{ci}}{Q}$$  \hspace{1cm} (2)

where $N_{ci}$ is the coordination number of each inclusion grain; $Q$ is the total number of the inclusion grains in a slice.

The obtained coordination numbers for $C_u = 2.01$ and 1.12 are shown in Fig. 7 as a function of $f_v$, along with those determined by Wang et al. (2018b) for $C_u = 2.72$. It can be observed that $N_c$ varied in a narrow range. At each $C_u$ value, $N_c$ increased with increasing $f_v$ and a parabolic function could be used to well describe this increasing trend. Furthermore, the $N_c$ values were small and they were almost the same for $f_v$ ranging from 5% to 20% (smaller than $f_{v-cha}$), indicating that in this range the inclusion grains were rarely in contact and were dispersed in the fines matrix. However, when $f_v$ reached 35% (larger than $f_{v-cha}$), the $N_c$ value increased drastically, suggesting that the contacts between the grains were well developed and the grain-grain contact structure was formed in this $f_v$ range.

**Effect of $C_u$ on the characteristic volumetric inclusion content $f_{v-cha}$**

From Fig. 5 it can be observed that the volumetric inclusion content $f_{v-cha}$ shifted rightwards with the decrease of $C_u$: the $f_{v-cha}$ values are 27.0%, 28.8% and 30.5% for $C_u = 2.72$, 2.01 and 1.12, respectively. This suggests that for a smaller $C_u$, more inclusions were needed to form a grain-grain contact structure. This phenomenon can be explained as follows: as shown in Fig.
7, the data of smaller $C_u$ values lay below those of larger $C_u$ values, indicating that less contacts were developed at smaller $C_u$ value. To some extent, $f_{v,cha}$ could be regarded as a value at which a certain number of contacts was reached. Thereby, with the decrease of $C_u$, the grains number and the contacts number were both expected to decrease. As a result, more grains were needed to form the grains skeleton, leading to a larger $f_{v,cha}$.

Changes of shear behavior with $C_u$ in the case of grain-grain contact structure

To study the effect of $C_u$ in the case of grain-grain contact structure, the samples of $f_v = 35\%$ sheared under $\sigma_3 = 60$ kPa and 120 kPa were considered (Fig. 8). It can be observed that the $q_{\text{max}}$ of smaller $C_u$ was larger. For $f_v = 35\%$ under $\sigma_3 = 30$ kPa, and for $f_v = 45\%$ under $\sigma_3 = 30$ kPa, 60 kPa and 120 kPa, the similar phenomenon was observed, which could be explained as follows: as the maximum grain diameters for the three $C_u$ values were fixed, equal to 20 mm (Fig. 3), a smaller $C_u$ (more uniform pattern) implied more large inclusions. As it was harder for the large inclusions to rearrange during shear, a larger $q_{\text{max}}$ can be expected. This mechanism can be termed as “large or small” of the inclusion in terms of size. The study of Oda et al. (1982) on gravel soils also showed that the difficulty of rearrangement of gravel grains led to a larger shear strength. Regarding the volumetric strain $\varepsilon_v$, a smaller $C_u$ gave rise to a larger dilatancy. This can also be explained by the more large inclusions involved in the case of smaller $C_u$, which favored the dilatancy.
Changes of shear behavior with \( C_u \) in the case of fine-fine contact structure

For the fine-fine contact structure where the inclusions were dispersed in the fines matrix\( (f_v = 5\% \text{ and } 10\%) \), it appears that on the whole \( q_{\text{max}} \) decreased with the decrease of \( C_u \) under \( \sigma_3 = 60 \text{ kPa} \) (Fig. 9b with 2 \( C_u \) values) and \( \sigma_3 = 120 \text{ kPa} \) (Fig. 9c with 3 \( C_u \) values). However, the changes of \( q_{\text{max}} \) with \( C_u \) under \( \sigma_3 = 30 \text{ kPa} \) does not appear significant (Fig. 9a with 3 \( C_u \) values). One possible explanation is that \( q_{\text{max}} \) reflects the friction at failure, and this friction is basically governed by both the soil property (mainly the amount of coarse grains involved in the friction process) and the confining pressure (corresponding to the normal stress in the friction process). Perhaps 30 kPa confining pressure was not high enough to reveal the \( C_u \) effect on \( q_{\text{max}} \). The decreasing trend of \( q_{\text{max}} \) with decreasing \( C_u \) in the cases of 60 kPa and 120 kPa could be explained by the fact that the amount of inclusion grains decreased with the decreasing \( C_u \) and therefore less inclusion grains were expect to be involved in the friction process (in the shear band zone for instance), giving rise to a smaller \( q_{\text{max}} \). The 3D views of the inclusions in the sample of \( f_v = 5\% \) with \( C_u = 2.01 \) and \( C_u = 1.12 \) came to support this reasoning (Fig. 10). This mechanism can be termed as “more or less” of the inclusions in terms of amount.

As far as the volumetric strain \( \varepsilon_v \) is concerned, it can be observed that when \( C_u \) increased, the dilatancy became more significant. This is because larger amount of grains was involved with larger \( C_u \), which increased the grain-grain contacts, leading to larger dilatancy.
At $f_v = 20\%$, the results were a little different from those at $f_v = 5\%$ and 10\%. Under $\sigma_3 = 60$ kPa and 120 kPa (Fig. 11a; Fig. 11b), different $C_u$ values gave similar $q_{\text{max}}$ and similar $\varepsilon_v$. This could be explained by the fact that in that case the inclusions started to be in contact with each other without developing a well-defined skeleton. Thereby, both mechanisms of “large or small” and “more or less” were involved. As the effects of the two mechanisms were opposite, the global mechanical response of soil showed independent feature with respect to $C_u$.

**Variation of Young’s modulus with $C_u$**

The Young’ modulus $E_0$ characterizes the stiffness of the track-bed materials (Lamas-Lopez 2016) and it is often used in analysing the dynamic response of the rail platform (Zhang et al. 2016). The results obtained allowed the effect of $C_u$ of inclusions on this modulus to be assessed. For simplify, Young’s modulus $E_0$ was defined here as the ratio of $q$ to $\varepsilon_1$ at $\varepsilon_1 = 1\%$. The variations of $E_0$ with $C_u$ are depicted in Fig. 12. It appears that when $f_v$ increased, $E_0$ increased and the influence of $C_u$ became more pronounced. At each $f_v$ value, a smaller $C_u$ led to a larger $E_0$, which could also be explained by relatively more large inclusions.

**Variation of Poisson’s ratio and dilatancy angle with $C_u$**

Based on the volumetric strain-axial strain curves, Poisson’s ratio $\nu$ and the dilatancy angle $\psi$ can be determined following the equations proposed by Vermeer and Borst (1984):

$$\nu = \frac{1 - k_c}{2}$$

$$\sin \psi = \frac{k_D}{-2 + k_D}$$

where $k_c$ and $k_D$ are soil constants.
where $k_c$ and $k_d$ are the slopes of the contraction phase and dilatancy phase of the curves, respectively (see Fig. 13).

Fig. 14 depicts the variations of $v$ with $C_u$ at different $f_v$ values. It is observed that at $f_v = 5\%$ and $10\%$ (fine-fine contact structures), $v$ decreased with the decrease of $C_u$, indicating that the lateral strain was decreased when more grains were dispersed in the fines matrix. Note that though $f_v = 20\%$ was categorized in the group of fine-fine contact fabric, the effect of $C_u$ on Poisson’s ratio followed the same pattern of grain-grain contact fabric: the smaller the $C_u$ value, the larger the $v$. In this case, the mechanism of “large or small” of the inclusions might play a dominant role. This increase of $v$ was normal since at a given axial strain the lateral strain was expected to be larger when larger grains are involved.

The variations of dilation angle $\psi$ with $C_u$ are shown in Fig. 15. For the samples with small $f_v$ values (e.g. $f_v = 5\%$) and large confining pressures, it appears difficult to establish any correlations. In addition, the grain-grain contact structure corresponding to smaller $C_u$ values led to a larger $\psi$, while in the case of fine-fine contact structure, $\psi$ seems to decrease with the decrease of $C_u$.

**Variations of cohesion and friction angle with $C_u$**

The values of friction angle $\varphi$ and cohesion $c$ are determined using the following equations (Wood 1990):

$$\sin \varphi = \frac{3M}{6 + M} \quad (4)$$

$$c = \frac{S(3 - \sin \varphi)}{6 \cos \varphi} \quad (5)$$
where $M$ and $S$ are the slope and intercept of the failure line in $p-q$ plane. It is worth noting that in this study, the saturation degrees of all samples at different $f_v$ values were strictly the same (see Table 3), because all water was expected to be contained in the fines soils and both water content and dry density were kept the same for all samples. As suction was developed inside the fines, the initial suctions of all samples were also the same. The obtained values of friction angle $\varphi$ and cohesion $c$ include the contribution of this suction. But of course, suction would change during shearing and this was ignored in the analysis.

Fig. 16 shows the variations of cohesion with $C_u$. On the whole, the cohesion decreased with increasing $C_u$. A possible explanation is as follows: when $C_u$ increased, more small grains were involved and thus more grain-grain contacts were created. As a result, less fine-fine contacts were expected and thus lower cohesion was obtained.

When it comes to the friction angle (Fig. 17), for grain-grain contact structure ($f_v = 35\%$ and $45\%$), a smaller $C_u$ value led to a larger friction angle. Note that the case of $f_v = 20\%$ obeyed the same rule. This can be explained by the more difficult rearrangement of grains in the case of smaller $C_u$ (more larger grains). Oda et al. (1982) and Skinner (1969) also reported the correlation between the difficulty of grains rearrangement and the friction angle for granular materials. As opposed to the grain-grain structure, the fine-fine contact structure ($f_v = 5\%$ and $10\%$) exhibited a smaller friction angle at smaller $C_u$ value. This can be attributed to the presence of less coarse grains along the shear band because the friction angle $\varphi$ is mainly controlled by coarse grains.
Conclusions

Monotonic triaxial tests and X-ray μCT scans were performed on a material simulating the interlayer soils, with inclusions and fines. The effect of coefficient of uniformity $C_u$ of inclusions on the mechanical behaviors was investigated with five volumetric inclusion contents ($f_v = 5\%, 10\%, 20\%, 35\%$ and $45\%$). Three $C_u$ values, $2.72$, $2.01$ and $1.12$, were considered. The obtained results allowed the effect of $C_u$ on the mechanical behavior to be analyzed.

For a given $C_u$, from the variation pattern of peak deviator stress $q_{\text{max}}$ with the volumetric inclusion content $f_v$, a characteristic volumetric inclusion content $f_{v-\text{cha}}$ was identified, defining two soil fabrics: a fine-fine contact structure and a grain-grain contact structure. The $f_{v-\text{cha}}$ increased with the decrease of $C_u$ because in this case more inclusions were needed to develop the grain-grain contact structure. This point was verified by the μCT scans.

In the case of grain-grain contact structure, the number of large inclusion grains is a key parameter controlling different mechanical characteristics. The decrease of $C_u$ was found to increase the friction angle as well as the peak deviator stress $q_{\text{max}}$, because more large inclusions were involved in this case and it was more difficult for large inclusions to rearrange during shearing. A larger Poisson’s ratio and a larger dilatancy angle were observed at smaller $C_u$. This can also be explained by the involvement of more large grains in that case,
leading to a larger lateral strain at a given axial strain in the contraction phase and a larger
volume change in the dilatancy phase.

By contrast, in the case of fine-fine contact structure, it is the amount of all inclusion
grains that governs the overall soil behaviour. It was observed that a smaller $C_u$
corresponding to a smaller inclusion grain quantity led to a smaller friction angle and a
smaller $q_{max}$, due to less inclusion grains that could be expected in the shear band zone. In
addition, the Poisson’s ratio decreased with decreasing $C_u$ because the horizontal restriction
was expected to be weakened with a lower quantity of inclusions dispersed in the fines
matrix. The dilatancy behavior was less pronounced at smaller $C_u$ because less grain
contacts were developed in case of smaller quantity of inclusions.

The variations of Young’s modulus with increasing $C_u$ were the same for the two soil
fabrics: Young’s modulus decreased with the increase of $C_u$ due to relatively less grain-grain
contacts. In addition, the two soil fabrics also shared the same varying pattern in terms of
cohesion: the cohesion decreased with increasing $C_u$. This is because only the fine-fine
contacts were expected to contribute to the cohesion and such contacts decrease with the
increase of inclusions quantity.

The findings from this study help assess the mechanical behavior of interlayer soils
with inclusions of different $C_u$ values. It can be deduced that an interlayer soil with a large $C_u$
value exhibits a low shear strength and thus a low mechanical performance under the effect
of train loading. Therefore, if the $C_u$ values are determined over a rail network, the
mechanical performance related to $C_u$ can be assessed over such a whole network. However, it is worth noting that the test conditions considered in this study are quite limited (triaxial tests on a mixture at a fixed water content and with scale-down inclusions). More comprehensive studies are needed to approach the complex field conditions.

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NOTATION

\( C_u \) \hspace{1cm} \text{coefficient of uniformity}

\( d_{\text{max}} \) \hspace{1cm} \text{maximum grain diameter of micro-ballast}

\( d_{60} \) \hspace{1cm} \text{grain diameter at 60\% passing}

\( d_{10} \) \hspace{1cm} \text{grain diameter at 10\% passing}

\( c \) \hspace{1cm} \text{cohesion}

\( D \) \hspace{1cm} \text{diameter of sample}

\( e \) \hspace{1cm} \text{void ratio of sample}

\( E_0 \) \hspace{1cm} \text{Young's modulus}

\( f_{v-\text{cha}} \) \hspace{1cm} \text{characteristic volumetric inclusion content}

\( f_v \) \hspace{1cm} \text{volumetric inclusion content}

\( H \) \hspace{1cm} \text{height of sample}

\( I_p \) \hspace{1cm} \text{plasticity index}

\( M \) \hspace{1cm} \text{slope of the failure line in } p-q \text{ plane}

\( k_C \) \hspace{1cm} \text{slope of volumetric strain versus axial strain in the contraction phase}

\( k_D \) \hspace{1cm} \text{slope of volumetric strain versus axial strain in the dilatancy phase}

\( N_{ci} \) \hspace{1cm} \text{coordination number for a certain inclusion grain in a slice}

\( N_c \) \hspace{1cm} \text{average coordination number of inclusion grains in a slice}
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q$</td>
<td>total number of inclusion grains in a slice</td>
</tr>
<tr>
<td>$q$</td>
<td>deviator stress</td>
</tr>
<tr>
<td>$q_{\text{max}}$</td>
<td>maximum deviator stress</td>
</tr>
<tr>
<td>$S$</td>
<td>intercept of the failure line in $p$-$q$ plane</td>
</tr>
<tr>
<td>$S_r$</td>
<td>saturation degree of sample</td>
</tr>
<tr>
<td>$V_{\text{in}}$</td>
<td>volume of total inclusion grains</td>
</tr>
<tr>
<td>$V_{\text{total}}$</td>
<td>volume of total sample</td>
</tr>
<tr>
<td>$V_{\text{fines}}$</td>
<td>volume of fines</td>
</tr>
<tr>
<td>$w$</td>
<td>water content of fines</td>
</tr>
<tr>
<td>$w_L$</td>
<td>liquid limit</td>
</tr>
<tr>
<td>$w_{\text{opt-f}}$</td>
<td>optimum water content of fines</td>
</tr>
<tr>
<td>$\rho_d$</td>
<td>dry unit mass of sample</td>
</tr>
<tr>
<td>$\rho_{\text{in-f}}$</td>
<td>dry unit mass of inclusion grains</td>
</tr>
<tr>
<td>$\rho_{\text{max-f}}$</td>
<td>maximum dry unit mass of fines</td>
</tr>
<tr>
<td>$\sigma_3$</td>
<td>confining pressure</td>
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<td>$\varepsilon_1$</td>
<td>axial strain</td>
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<td>$\varepsilon_v$</td>
<td>volumetric strain</td>
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<tr>
<td>$\nu$</td>
<td>Poisson’s ratio</td>
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<tr>
<td>$\psi$</td>
<td>dilatancy angle</td>
</tr>
<tr>
<td>$\phi$</td>
<td>friction angle</td>
</tr>
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</table>
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Table 1. Proportions of the nine commercial soils

<table>
<thead>
<tr>
<th>Soil Symbol</th>
<th>Mass proportion (%)</th>
<th>Grain size range (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bentonite</td>
<td>6.7</td>
<td>0.001 - 0.01 (20% of the particles)</td>
</tr>
<tr>
<td>Speswhite</td>
<td>23.3</td>
<td>0.0003 - 0.01 (80% of the particles)</td>
</tr>
<tr>
<td>C-10</td>
<td>20.0</td>
<td>0.0009 - 0.25</td>
</tr>
<tr>
<td>C-4</td>
<td>16.7</td>
<td>0.0009 - 0.50</td>
</tr>
<tr>
<td>HN 34</td>
<td>3.3</td>
<td>0.063 - 0.50</td>
</tr>
<tr>
<td>HN 31</td>
<td>3.3</td>
<td>0.16 - 0.63</td>
</tr>
<tr>
<td>HN 0.4-0.8</td>
<td>6.7</td>
<td>0.25 - 1</td>
</tr>
<tr>
<td>HN 0.6-1.6</td>
<td>6.7</td>
<td>0.32 - 2</td>
</tr>
<tr>
<td>HN 1-2.5</td>
<td>13.3</td>
<td>0.32 - 3.20</td>
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</tbody>
</table>
Table 2. Basic properties of micro-ballast with 3 different $C_u$ values

<table>
<thead>
<tr>
<th>Sample name</th>
<th>Proportions of the constitutive soils (%)</th>
<th>$C_u$ (designed)</th>
<th>$C_u$ (measured)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>G10-20</td>
<td>G4-10</td>
<td>HN2-4</td>
</tr>
<tr>
<td>S1</td>
<td>34</td>
<td>58</td>
<td>8</td>
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<tr>
<td>S2</td>
<td>85</td>
<td>15</td>
<td>0</td>
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<tr>
<td>S3</td>
<td>16-20 mm</td>
<td>0</td>
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Table 3. Experimental program

<table>
<thead>
<tr>
<th>$f_v$ (%)</th>
<th>$\rho_d$ (Mg/m$^3$)</th>
<th>$e$</th>
<th>$S_r$ (%)</th>
<th>$C_u$</th>
<th>Monotonic Triaxial Test</th>
<th>$\mu$CT Test</th>
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<tbody>
<tr>
<td></td>
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<td></td>
<td></td>
<td></td>
<td>$\sigma_3$ (kPa)</td>
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<td>5</td>
<td>1.86</td>
<td>0.437</td>
<td>78</td>
<td></td>
<td>2.72</td>
<td>30, 60, 120</td>
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<tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>2.01</td>
<td>30, 120</td>
</tr>
<tr>
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<td></td>
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<td></td>
<td>1.12</td>
<td>30, 60, 120</td>
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<tr>
<td>10</td>
<td>1.91</td>
<td>0.404</td>
<td>78</td>
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<td>2.72</td>
<td>30, 60, 120</td>
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<td>2.01</td>
<td>30, 120</td>
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<td>0.343</td>
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<td>30, 60, 120</td>
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<tr>
<td>35</td>
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<td>0.262</td>
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<td>2.72</td>
<td>30, 60, 120</td>
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<td>45</td>
<td>2.21</td>
<td>0.212</td>
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<td>2.01</td>
<td>30, 60, 120</td>
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<td></td>
<td></td>
<td></td>
<td>1.12</td>
<td>30, 60, 120</td>
</tr>
</tbody>
</table>

Note: $\rho_d$, $e$ and $S_r$ are the sample’ dry density, void ratio and degree of saturation, respectively.
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![Graph showing grain size distribution with various data points and labels.](image)

**Fig. 1**

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