Peer-to-Peer Energy Trading in a Prosumer Based Community Microgrid: A Game-Theoretic Model

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Abstract—This paper proposes a novel game-theoretic model for peer-to-peer (P2P) energy trading among the prosumers in a community. The buyers can adjust the energy consumption behavior based on the price and quantity of the energy offered by the sellers. There exist two separate competitions during the trading process: (i) price competition among the sellers and (ii) seller selection competition among the buyers. The price competition among the sellers is modeled as a non-cooperative game. The evolutionary game theory is used to model the dynamics of the buyers for selecting sellers. Moreover, an M-leader and N-follower Stackelberg game approach is used to model the interaction between buyers and sellers. Two iterative algorithms are proposed for the implementation of the games such that an equilibrium state exists in each of the games. The proposed method is applied to a small community microgrid with photo-voltaic (PV) and energy storage systems. Simulation results show the convergence of the algorithms and the effectiveness of the proposed model to handle the P2P energy trading. The results also show that P2P energy trading provides significant financial and technical benefits to the community and it is emerging as an alternative to cost-intensive energy storage systems.

Index Terms—Peer-to-peer energy trading, community microgrid, prosumer, game theory, energy storage.

I. INTRODUCTION

ENVIRONMENTAL benefits, financial incentives, and reduction in electricity bills motivate house owners to install renewable based distributed generators and energy storage units at residential buildings [1]. Increasing deployment of distributed generators and energy storage systems with intelligent infrastructures enables the residential consumers to harness energy and inject into the distribution systems. This advancement changes the residential consumers into prosumers. An entity which has the capability to produce, consume, and possibly also has demand response capacities is known as prosumer [2]. A group of prosumers can be integrated as a prosumer energy community. The small-scale power system in a house is known as a prosumer nanogrid [3]. The terms prosumer nanogrid and prosumer are used interchangeably in this paper. Several nanogrids serving in close proximity can be combined to form a community microgrid.

A community microgrid presents a new basis for distribution grid with various operating constraints and business models. It aims at extending the benefits of the traditional microgrid by sharing the distributed energy resources (DERs) among multiple prosumers. In addition, the on-site use of energy generated from local DERs through energy trading in a community is more attractive than feeding the utility grid. This helps to alleviate the adverse impacts on the power systems [4]. A proper energy trading model is necessary to manage the local energy trading among multiple prosumers.

The peer-to-peer (P2P) network is a widely used model for resource sharing in the field of computer science where resources are located in and provided by computers (i.e., peers) at the edge of the network [5]. Since, a community microgrid consists of several prosumers in close proximity having their own generation and demand, it can be modelled as a P2P network. A P2P energy trading model seems to be suitable for energy trading in the prosumer based community microgrid. P2P energy trading is flexible between peers, where excess energy from various small-scale DERs is traded locally [6]. The P2P model encourages local energy trading and demand response (DR) to the available resources in a community [7].

Since P2P energy trading in a community microgrid is a new concept, a proper modelling framework is required to define the business models, to determine energy prices, as well as to implement DR programs in a community. The business model and energy pricing play a vital role because they determine the suitability of P2P trading in a community microgrid in terms of financial benefits. The business model also forms a basis for the implementation of DR programs.

The remainder of the paper is organized as follows. In Section II, state of the art is presented and major contributions in this paper are highlighted. The proposed model of the community microgrid for P2P energy trading is explained in Section III followed by the detail of evolutionary game among buyers in section IV. In Section V, detail of non-cooperative game among sellers is presented. Stackelberg game between sellers and buyers is explained in Section VI. Detail simulation results are discussed in Section VII, and conclusion in Section VIII.

II. STATE OF THE ART

A. Related Works

In [8], the authors have proposed a P2P energy trading model for smart households. The optimal microgrid energy and price for P2P trading are determined in order to minimize the total energy cost. A customer-to-customer based on the architecture model for P2P energy trading is introduced in [9]. The bidding system called Elecbay is proposed for P2P trading and simulation is done using game theory. An efficient and privacy-preserving P2P energy exchange scheme in a smart grid environment is presented in [10], with the novel optimization approach to increase efficient energy transfer.
without information leakage. In [11], a new algorithm has been proposed for automating electricity trading by prosumers in the P2P electricity market. A framework for performance evaluation using several indices is proposed in [12] to assess the economic performance of P2P energy sharing models. A P2P index is developed to assess the feasibility of P2P energy trading in low voltage distribution system in [7]. In [13], the authors have introduced different P2P market paradigms using bill sharing (BS), mid-market rate (MMR), and auction based pricing strategies but DR is not considered.

In line with above works on P2P energy trading, many researchers have carried out researches on demand response management (DRM), energy sharing management, and real-time pricing using various game theoretical approaches in [4], [14]–[16]. In [15], authors have proposed a competitive market based distributed mechanism for energy trading among microgrids using multileader-multifollower Stackelberg game. A Stackelberg game theory is used to study the DRM from various aspects in [17]–[19]. A hybrid approach using stochastic programming and Stackelberg game is presented in [14] to coordinate energy sharing and energy consumption behavior of prosumers by using internal prices.

In these works [4], [7], [13], [14], [20], a separate entity acted as an energy trading coordinator and was responsible for the execution of energy trading. There was no direct communication between buyers and sellers. However, since prosumers in a community are proactive, it is possible to perform P2P energy trading in a community with less involvement of energy trading coordinator (i.e., using the direct communication or interaction between buyers and sellers). The direct involvement of each prosumer during the trading process is the main feature of P2P energy trading. It poses the challenge of modeling the decision-making process of each participant for the greater benefit of the entire community microgrid while considering human factors such as rationality, motivation, and environmental friendliness [21]. In situations where there are many prosumers with conflicting interests, it would be quite challenging either to capture such conflicting interests in the decision-making process of each participant or to motivate them to cooperate for achieving the goals of P2P trading. Therefore, there should be a proper tool for modeling the decision-making process such as deciding energy price, incorporating DR in P2P energy trading in a community microgrid. The modeling tool should be able to deliver the energy management solution considering a diverse set of constraints in a trading process. In this context, considering the interactive and conflicting nature of energy trading in a community microgrid, game theory is a very effective tool for modeling the decision-making process of prosumers participating in the P2P energy trading.

B. Major Contributions

The main contributions of this paper are summarized as follows:

1) A novel game-theoretic model is proposed for P2P energy trading using direct interactions between buyers and sellers in a community microgrid considering the DR capability and privacy of prosumers.

Fig. 1: Structure of a community microgrid for P2P trading

2) The pricing competition among the sellers is modeled as a non-cooperative game, dynamics of buyers in the process of selecting sellers are modeled as an evolutionary game, and interaction between sellers and buyers is modeled as a Stackelberg game.

3) A novel iterative algorithm is proposed to reach the stable state in the evolutionary game among buyers for the seller selection.

4) A novel distributed algorithm is proposed to obtain the equilibrium in both non-cooperative game among sellers and Stackelberg game between buyers and sellers.

III. SYSTEM MODEL

A. General Structure of Community Microgrid

Fig. 1 shows a structure of a smart community microgrid comprising several prosumers. Each prosumer comprises loads and PV systems. Prosumers may or may not have battery energy storage systems (BESS) since deploying energy storage systems in the residential level is costly. The PV system of a prosumer is connected to the load and AC system through DC/AC converter which is also known as PV inverter. If any prosumer has both a PV system and a battery, they can be connected via a DC coupled or an AC coupled topology. In the DC coupled topology, the battery is connected to the PV system at the DC side of the PV inverter through the DC/DC bi-directional converter as shown in Prosumer 1 in Fig. 1. In the AC coupled topology, the battery is connected to the PV system at the AC side of the PV inverter through DC/AC bi-directional converter as shown Prosumer 2 in Fig. 1.

All the prosumers in a community are connected to each other through the bi-directional power and communication links, and a whole community microgrid is connected to the upstream utility grid via a one grid connection point. Smart meters are installed at each prosumer. Also, each prosumer has a local workstation with an energy management system called prosumer energy management system (P-EMS). P2P energy trading algorithm is integrated with the P-EMS software. The smart meter measures the prosumer’s generation, consumption, and energy transaction with other prosumers or with the grid and sends information to the local workstation for processing.

We assume that there is an agent called P2P market operator (P2PMO) which assists in energy trading in a P2P market in a community. P2PMO is a part of the distribution system
operator and is assumed to serve free of charge to assist the P2P trading because P2P trading helps to reduce the congestion in the distribution system. Since the prosumers are serving nearby and the amount of energy traded in the P2P market is small, we assume that transmission losses and transmission cost are negligible. Smart meters are capable of communicating with other entities and work stations are powerful enough to carry out the computational tasks. During the trading process, all the communication tasks are done through the smart meters, and computations are performed in local workstations.

With the existing infrastructures explained, the main aim of the study is to develop an algorithm for the P2P energy trading. The detailed working processes of smart meters, communication systems and physical infrastructures in the community microgrid are beyond the scope of this paper.

Let \( N = \{1, 2, 3, \ldots, N\} \) denote the set of the prosumers in the community with \( n \in N \) and \( N \triangleq |N| \) gives the total number of prosumers in the community. We assume that the total operation time is divided into different slots of equal interval \( \Delta t \). In this study we have considered \( \Delta t = 1 \) hour. Let \( T = \{1, 2, 3, \ldots, T\} \) denote the set of all operation time slots with \( t \in T \) and \( T \triangleq |T| \) gives the total number of operation time slots.

The PV generation profile of prosumer \( n \) during a day can be defined as:

\[
G_{pv,n} = \{G_{1}^{n}, G_{2}^{n}, \ldots, G_{N}^{n}\}, \quad n \in N
\]

The nominal demand profile of prosumer \( n \) during the same period can be defined as:

\[
D_{n} = \{D_{1}^{n}, D_{2}^{n}, \ldots, D_{N}^{n}\}, \quad n \in N
\]

The nominal demand represents the demand from regular appliances before the implementation of any DR program and the installation of BESS.

### B. Battery Energy Storage Systems

Let \( E_{n}^{0} = E_{n} - \Delta t \) and \( E_{n}^{0} = E_{n} + \Delta t \) denote the battery energy level of prosumer \( n \) at the beginning and end of the time slot \( t \), \( P_{ch,n}^{t} \) denote the battery charging power of prosumer \( n \) at time slot \( t \), and \( P_{dch,n}^{t} \) denote the battery discharging power of prosumer \( n \) at time slot \( t \). Let \( \eta_{ch,n}^{t} \) and \( \eta_{dch,n}^{t} \) denote battery charging and discharging efficiency of prosumer \( n \). Assuming the self-discharge of the battery is negligible and the charge/discharge power remains constant during a time slot, the dynamics of the battery level can be modeled as [22]

\[
E_{n}^{t} = E_{n}^{0} + \chi_{n}^{t} P_{ch,n}^{t} \eta_{ch,n}^{t} - \psi_{n}^{t} P_{dch,n}^{t} \eta_{dch,n}^{t} \times \Delta t
\]

where \( \chi_{n}^{t} \) and \( \psi_{n}^{t} \) are binary variables of prosumer \( n \) related to charge and discharge states in time slot \( t \).

To avoid simultaneous charging and discharging of the battery

\[
\chi_{n}^{t} + \psi_{n}^{t} \leq 1
\]

In practical applications, the energy stored in a battery is restricted within a certain range, i.e.,

\[
E_{n}^{min} \leq E_{n}^{t} \leq E_{n}^{max}
\]

where \( E_{n}^{max} \) is the maximum capacity of the battery and \( E_{n}^{min} \) is the minimum amount of energy that prevents the battery from deep discharge.

The limits of charging and discharging power are determined by the size of the inverter as follows

\[
0 \leq P_{ch,n}^{t} \leq P_{ch,n}^{max}
\]

\[
0 \leq P_{dch,n}^{t} \leq P_{dch,n}^{max}
\]

where \( P_{ch,n}^{max} \) and \( P_{dch,n}^{max} \) are the maximum charging and discharging power.

In the conventional peer-to-grid (P2G) trading, the excess PV energy of the prosumer is used to charge the battery until the battery is fully charged and the remaining is fed back to the grid. The stored energy is used to meet the demand when the PV generation is lower than the demand. The prosumer buys energy from the grid when there is insufficient energy from the PV and battery system.

In P2P energy trading, if any prosumer has excess PV energy, the first priority is to supply to the neighbors who have unmet consumption in the community and if there is any remaining power, this is used to charge the battery. When the demand is higher than the PV generation, some of the unmet demand is satisfied by buying the PV power from the neighbors through the P2P market at first, and the remaining, if there is any, is met by discharging the own battery and buying power from the grid. Furthermore, the stored energy in a particular prosumer’s battery can be used to supply its own demand only when that prosumer participates in the P2P market. This is because if any prosumer can sell stored energy in the P2P market, the prosumer having high storage capacity and high excess generation during certain hours can influence the market in other hours, which discourage other prosumers to participate in the P2P market.

The use of battery systems should be justified by saving due to its use. If \( C_{c} \) is the capital cost of the battery system (i.e., the cost of the battery and converter); \( C_{m} \) is the annual maintenance cost of the battery; and \( N_{days} \) is the number of days in a year, then the equivalent daily cost (EDC) of the battery is given by [23]

\[
EDC = C_{c} \times r(1 + r)^{l} \times \frac{1}{N_{days}} + C_{m} N_{days}
\]

where \( r \) is the discount rate and \( l \) is the lifetime of the battery in years. The use of the battery is beneficial only if the saving in a day is higher than the EDC.

### C. Classification of Prosumers as a Seller and a Buyer

The generation-to-demand ratio (GDR) of a prosumer \( n \in N \) in a given time period \( t \in T \) is defined as

\[
GDR_{n}^{t} = \frac{G_{pv,n}^{t}}{D_{n}^{t}}
\]

Let \( S = \{n \in N \mid GDR_{n}^{t} > 1\} \) represent the set of sellers at time slot \( t \) with index \( j \in S \) and \( S = |S| \) gives the total number of sellers at time \( t \).

Let \( B = \{n \in N \mid GDR_{n}^{t} < 1\} \) represent the set of buyers at time slot \( t \) with index \( i \in B \) and \( B = |B| \) gives the total number of buyers at time \( t \).

The amount of power the prosumer \( j \in S \) can sell (export) at time \( t \) is

\[
P_{ex,j}^{t} = (GDR_{j}^{t} - 1)D_{j}^{t}
\]

The maximum amount of power the prosumer \( i \in B \) can procure at time \( t \) is

\[
P_{im,i}^{t} = (1 - GDR_{i}^{t})D_{i}^{t}
\]
D. Non-flexible and Flexible Demand of Prosumer

Suppose that $x_n^t$, $y_n^t$ and $z_n^t$ represent the total demand, non-flexible demand and flexible demand of the prosumer $n \in \mathcal{N}$ at time $t \in \mathcal{T}$ respectively. The total demand of prosumer $n$ at any time $t$ can be defined as:

$$x_n^t = y_n^t + z_n^t \quad (11)$$

Let $Y_{o,n}$ and $Z_{o,n}$ denote the nominal non-flexible and flexible demand profile of the prosumer $n \in \mathcal{N}$. According to (11), the nominal demand of the prosumer $n$ can be expressed as

$$D_n = Y_{o,n} + Z_{o,n} \quad (12)$$

The critical (non-flexible) loads are fixed and has no contribution to DR. The total demand of the prosumer can be modified by adjusting the flexible demand. The adjusted demand of the prosumer $n \in \mathcal{N}$ at time $t \in \mathcal{T}$ can be defined as

$$a_n^t = y_n^t + z_n^t \quad (13)$$

where, $y_n^t$ is the nominal non-flexible demand of prosumer $n$ at time $t$.

Assume that all the flexible loads are continuous in nature. Their reduction affects the comfort level and reduces the energy cost of the prosumers.

If the amount of flexible load in nominal case at any time $t$ is $\beta D_n^t$, then

$$0 \leq z_n^t \leq \beta D_n^t, \quad 0 \leq \beta \leq 1 \quad (14)$$

where $\beta$ is proportion of flexible demand.

E. Utility Function

The responses of different prosumers to the various scenarios can be modeled by using the concept of utility function [24]. The utility function is a method to quantify the level of satisfaction or happiness that the prosumer receives when it consumes a certain amount of power.

If $x$ is the power consumption level and $\lambda$ is the parameter to distinguish a prosumer from the others, the utility function can be represented by $U(x, \lambda)$. There are two types of utility functions frequently used for modeling the electricity consumers. They are logarithmic utility function [14]–[17], [25] and quadratic utility function [18], [19], [24].

In this paper, we consider a quadratic utility function to quantify the satisfaction of prosumer $n \in \mathcal{N}$ at time $t \in \mathcal{T}$.

$$u_{n}^{i}(x_{n}^{t}) = \lambda_{n}^{i} x_{n}^{t} + \frac{\theta_{n}}{2} (x_{n}^{t} - x_{n,min}^{t})^{2} \quad (15)$$

where $\lambda_{n}^{i}$ is a prosumer preference parameter characterizing the prosumers’ behaviours, which vary from prosumer to prosumer and may also vary along the time, and $\theta_{n} > 0$ is a predetermined constant [24]. The terms $x_{n}^{t}$, $x_{n,min}^{t}$ and $x_{n,max}^{t}$ are the actual power consumption, its lower limit and upper limit for prosumer $n$ at time $t$, respectively.

F. Welfare Function for Buyer and Seller

If any buyer $i \in \mathcal{B}$ chooses seller $j \in \mathcal{S}$, the welfare function of the buyer $i$ can be defined as

$$W_i^{j} = u_{i}^{j}(x_{i}^{t}) - \pi_j^{i} x_{j,i}^{t} \quad (16)$$

For any buyer $i \in \mathcal{B}$,

$$x_{i,min}^{j} \leq x_{i}^{t} \leq x_{i,max}^{j} \quad (17)$$

Based on (17), the condition (18) is always true.

Fig. 2: Two-levels of competition in P2P market

$$x_{j,i,min} = x_{i,min}^{j} - \beta_{j,i} D_{i}^{j}$$

$$x_{j,i,max} = x_{i,max}^{j} - \beta_{j,i} D_{i}^{j}$$

where $\pi_j^{i}, x_{j,i,min}^{j}$ and $x_{j,i,max}^{j}$ are the price decided by seller $j$, actual power the buyer $i$ buys from seller $j$ and its lower and upper power limits buyer $i$ can buy from the seller $j$ at time $t$, respectively. Now, the welfare function of any buyer $i$ given by (16) can be expressed as

$$W_i^{j}(x_{j,i}^{t}) = u_{i}^{j}(x_{j,i}^{t} + G_i^{t}) - \pi_j^{i} x_{j,i}^{t} \quad (19)$$

The welfare function of the seller $j \in \mathcal{S}$ can be defined as

$$W_j^{i} = u_{j}^{i}(x_{j,i}^{t}) + \pi_j^{i} min(P_{ex,j}^{t}, S_{j}^{t})$$

$S_{j}^{t}$ is the total demand of electricity comes to seller $j$ at time $t$ and is defined in (22). For any seller the nominal demand is always satisfied so that $x_{j}^{t} = D_{j}^{t}$.

G. P2P Energy Trading Between Buyers and Sellers

The aim of P2P trading is to maximize the welfare of both buyers and sellers, and reduce the dependency on the upstream grid. P2P energy trading is carried out in the following steps:

1) Prosumers register into a P2P market as a seller or a buyer based on their GDR.

2) After grouping of prosumers as a seller or a buyer, P2PMO assigns a unique and encrypted identity to each buyer and seller, which maintains the anonymity of sellers and buyers ensuring privacy.

3) The P2PMO sends uniquely assigned buyers identities to all sellers and sellers identities to all buyers. This establishes direct communication between sellers and buyers.

4) Each anonymous seller and each anonymous buyer participate in a P2P energy trading game to obtain the stable state.

5) Finally, P2PMO receives information about the final price and amount of energy traded anonymously by different prosumers to settle the financial transactions.

There are multiple sellers and multiple buyers in the community. Two levels of competition exist in this multi-seller and multi-buyer P2P energy trading market as shown in Fig. 2.

The competition at the higher level is among the sellers to sell their excess energy to the group of buyers at the lower level. In this paper, competition among the sellers is modeled as a non-cooperative game. The competition at the lower level
is among multiple buyers to select the sellers to buy energy offered by them. In this paper, an evolutionary game approach is used for competition among the buyers.

In addition to the separate competition at two levels of the P2P market, there is an interaction between the two levels. The main aim of this interaction between two levels is to set the appropriate strategies for both sellers and buyers to maintain the balance between supply and demand in the P2P market. In this paper, interaction between the two levels is modeled as a Stackelberg game.

**IV. EVOLUTIONARY GAME AMONG BUYERS**

In general, the players of the game are arranged in multiple groups. In this paper, without loss of generality, all the buyers are arranged in a single group and there is only one population scenario in the game. The evolutionary game for P2P energy trading in the community can be precisely defined as follows:

- **Players**: buyers \(i \in B\)
- **Population**: set of buyers
- **Strategy**: selection of seller
- **Utility**: welfare function of the buyers

We have arranged all the buyers in a single population group, so all the buyers will show the same behavior, i.e., the strategy of each buyer in the population is identical. Each buyer selects a seller to purchase power when they receive the price announced by sellers. Each buyer then gradually adjusts its selection strategy and acts independently in the selection process [25].

Let \(\gamma^t_j\) be the probability of the buyer \(i \in B\) choosing a seller \(j \in S\) at time \(t\), where \(0 \leq \gamma^t_j \leq 1\) and \(\sum_{j=1}^{S} \gamma^t_j = 1\). Since the strategies of all the population (buyers) are identical, the population state can be denoted as \(\gamma^t = [\gamma^t_1, \gamma^t_2, \gamma^t_3, ..., \gamma^t_S]\).

When buyer \(i\) selects seller \(j\), the optimal amount of power buyer \(i\) buying from seller \(j\) can be achieved by maximizing the welfare given by (19) subject to (18). Mathematically,

\[
x^t_{j,i} = \arg \max_{x^t_{j,i}} W^t_{i,j}
\]

(21)

The total demand of electricity comes to the seller \(j\) at time \(t\) is given by

\[
S^t_j = \gamma^t_j \sum_{i=1}^{B} x^t_{j,i}
\]

(22)

The supply-to-demand ratio (SDR) for seller \(j\) at time \(t\) can be defined as

\[
P^t_{ex,j} = \frac{S^t_j}{\gamma^t_j \sum_{i=1}^{B} x^t_{j,i}}
\]

(23)

where, \(\alpha^t_j = \frac{S^t_j}{\gamma^t_j \sum_{i=1}^{B} x^t_{j,i}}\) and \(\gamma^t_j, P^t_{ex,j}\), and \(\alpha^t_j\) are constant during the evolution process.

The actual amount of power that buyer \(i\) buys from the seller \(j\) at time \(t\) is given as

\[
x^t_{j,i,actual} = \begin{cases} 
x^t_{j,i} & \text{if } x^t_{j,i} \geq 1 \\
x^t_{j,i} & \text{if } x^t_{j,i} < 1
\end{cases}
\]

(24)

Assume that the net utility of the seller \(j\) at time \(t\) can be defined as the accumulated welfare of all buyers obtained from seller \(j\). There are two possibilities:

Case 1: If \(P^t_{ex,j} \geq S^t_j\), then the net utility is

\[
\sigma^t_j = \frac{1}{2} \sum_{i=1}^{B} \theta_i (x^t_{j,i,actual})^2 + C
\]

(25)

Case 2: If \(P^t_{ex,j} < S^t_j\), then the net utility is

\[
\sigma^t_j = \left[\nu^t_j - \frac{(\nu^t_j)^2}{2}\right] \sum_{i=1}^{B} \theta_i (x^t_{j,i,actual})^2 + C
\]

(26)

In both (25) and (26), \(C = \left(\lambda G^t_j - \frac{\nu^t_j}{2} G^t_j\right)^2\).

The replicator dynamics is designed to describe the selection dynamics of the buyers as follows:

\[
\frac{\partial \gamma^t_j}{\partial t} = \gamma^t_j (\sigma^t_j - \bar{\sigma}^t) = \gamma^t_j \left(\sigma^t_j - \frac{1}{S} \sum_{j=1}^{S} \sigma^t_j\right)
\]

(27)

where, \(\bar{\sigma}^t\) denotes the average utility, and it can be calculated as

\[
\bar{\sigma}^t = \sum_{j=1}^{S} \gamma^t_j \sigma^t_j
\]

(28)

A stable condition at which population will not change its selection is referred to as evolutionary stable strategy (ESS). Mathematically,

\[
\frac{\partial \gamma^t_j}{\partial t} = \gamma^t_j \frac{\partial \sigma^t_j}{\partial \gamma^t_j} = 0
\]

(29)

The condition for stable state in evolutionary game can also be written as

\[
\sigma^t_j = \sigma^t_j = \bar{\sigma}^t = \sigma^t_S = \sigma^t
\]

(30)

If we consider the dynamics of \(\sum_{j=1}^{S} \gamma^t_j\), then

\[
\frac{\partial \sum_{j=1}^{S} \gamma^t_j}{\partial t} = \sum_{j=1}^{S} \gamma^t_j (\sigma^t_j - \bar{\sigma}^t) = \sigma^t - \sigma^t \sum_{j=1}^{S} \gamma^t_j
\]

(31)

Using (29) and (31), it can be proved that \(\sum_{j=1}^{S} \gamma^t_j = 1\) is always valid during the game. The equilibrium in the evolutionary game is denoted by \(\gamma^t = [\gamma^t_1, \gamma^t_2, \gamma^t_3, ..., \gamma^t_S]\).

The convergence of the game to the evolutionary equilibrium given in (29) using the replicator dynamics given in (27) can be proved via Lyapunov theory [26]. A concept of multiple populations is used when there is a large number of players [25].

The replicator dynamics can be approximated by using the discrete replicator in iterative way though it is time continuous. The discrete replicator for approximation of replicator dynamics is defined as follows:

\[
\gamma^t_j (k+1) = \gamma^t_j (k) + \eta \gamma^t_j (k) \left(\sigma^t_j (k) - \bar{\sigma}^t (k) \right)
\]

(32)

The termination criterion is defined as

\[
|\sigma^t_j (k) - \bar{\sigma}^t (k)| < \epsilon
\]

(33)

where \(k\) is the iteration number, \(\eta\) is the adjustment parameter, and \(\epsilon\) is a small positive constant.

The detail algorithm to reach the evolutionary equilibrium is given in Algorithm 1.

**V. NON-COOPERATIVE GAME AMONG SELLERS**

Each seller aims at maximizing its own welfare by selling the power to buyers who need it. They are non-cooperative and behave in rational manner. A non-cooperative game is used to model the competition among the sellers as follows:

- **Players**: selling role prosumers or sellers \(j \in S\)
- **Strategy**: the price \(\pi^t_j, \forall j \in S\) and \(\forall t \in T\)
- **Utility**: the welfare of the sellers

The welfare function of the seller \(j \in S\) given in (20) can be re-written as

\[
W^t_j = \begin{cases} 
\nu^t_j (x^t_j) + \pi^t_j S^t_j & P^t_{ex,j} > S^t_j \\
\nu^t_j (x^t_j) + \pi^t_j P^t_{ex,j} & P^t_{ex,j} \leq S^t_j
\end{cases}
\]

(34)
Algorithm 1 Algorithm for evolutionary game among buyers

Input: Price vector $\pi^i = [\pi_1^i, \pi_2^i, \ldots, \pi_S^i]$ from the sellers

Output: Equilibrium state $\gamma^i = [\gamma_1^i, \gamma_2^i, \ldots, \gamma_S^i]$

Arbitrarily assign the initial population state $\gamma^i(1) = [\gamma_1^i(1), \gamma_2^i(1), \ldots, \gamma_S^i(1)]$, such that $\sum_{k=1}^S \gamma_k^i(1) = 1$;

do
$k = k + 1;

for all $j \in S$ do

Compute $x_{j,k}^i$ according to (21), $\forall i \in B$;

Compute the SDR $\nu_j^i(k)$ according to (23);

if $\nu_j^i(k) \geq 1$ then

Compute the net utility $\sigma_j^i(k)$ according to (25);

else if $\nu_j^i(k) < 1$ then

Compute the net utility $\sigma_j^i(k)$ according to (26);

end if

end if

end for

Compute the average value of net utility $\bar{\sigma}_j^i(k)$ according to (28);

Update the replicator dynamics according to (32);

while $(|\sigma_j^i(k) - \bar{\sigma}_j^i(k)| > \epsilon)$;

The solution of the game is called Nash equilibrium (NE) which includes the energy price and amount of energy sold. NE exists in the game if [27]

- The player set is finite.
- The strategy sets are closed, convex, and bounded.
- The utility functions are continuous and quasi-concave in the strategy space.

There are $S$ number of sellers, so the player set is always finite. The game first starts by announcing the price, $\pi_j^i$ by the seller. There are limits on $\pi_j^i$ such that $\pi_j^i \in [\pi_{j,min}^i, \pi_{j,max}^i]$. The lower bound $\pi_{j,min}$ is normally determined by the generation cost and the operating expenses. The upper bound $\pi_{j,max}$ is fixed by the government policies. The sellers must consider the price limits during the game. In this paper, we choose feed-in tariff ($\rho_{sell}$) as the lower limit and the grid buying price ($\rho_{buy}$) as the upper limit such that $\pi_j^i \in [\rho_{sell}, \rho_{buy}]$. Hence, the strategy sets are nonempty, closed, bounded, and convex.

The utility function given in (34) is a continuous function of $\pi_j^i$. Since the welfare $W_j^l$ of seller $j$ increases with amount of power for a given $\pi_j^l$, the seller shall assume that all the available power to export ($P_{ex,j}^l$) can be sold to the buyers. The game will be stopped when all the $P_{ex,j}^l$ is sold or all $S_j^l$ is satisfied.

If $P_{ex,j}^l \leq S_j^l$, then

$$\frac{\partial^2 (W_j^l)}{\partial (\pi_j^l)^2} = 0$$

(35)

If $P_{ex,j}^l > S_j^l$, the second order derivative of $W_j^l$ with respect to $\pi_j^l$ is given by

$$\frac{\partial^2 (W_j^l)}{\partial (\pi_j^l)^2} = -2\lambda_j^l \sum_{i=1}^B \frac{1}{\theta_i} < 0$$

(36)

Combining (35) and (36), we get $\frac{\partial^2 (W_j^l)}{\partial (\pi_j^l)^2} \leq 0$, so that $W_j^l$ is always quasi-concave in $\pi_j^l$. Therefore, we can conclude that NE exists in the non-cooperative game among the sellers.

VI. STACKELBERG GAME BETWEEN SELLERS AND BUYERS

The interaction behaviors between the sellers and buyers can be modeled as an M-leader and N-follower Stackelberg game. The sellers are the multiple leaders and the buyers are the multiple followers. Stackelberg game establishes a relationship the evolutionary game and the non-cooperative game. The output of the non-cooperative game, i.e., price vector, is used as an input to the evolutionary game to update the seller selection strategy. The output of the evolutionary game, i.e., evolutionary stable strategy or seller selection probability, is used as an input to the non-cooperative game to update the price vector. So, all the three games are related to each other. All the buyers receive the announced price vector from the sellers and participate in the evolutionary game. Once the evolutionary stable strategy is obtained in the evolutionary game, sellers update their price to obtain a NE, i.e., price.

The sellers are involved in a non-cooperative game and each seller does not know the information of other sellers. In this scenario, it is not possible to obtain the NE by analytical method and iterative approach must be used. An iterative distributed algorithm is designed to obtain the NE among the sellers such that the Stackelberg equilibrium (SE) is reached in the Stackelberg game between sellers and buyers. The price updating strategy of the seller $j$ is designed as follows:

$$\pi_j^l(t + 1) = \pi_j^l(t) + \eta_2 (S_j^l(t) - P_{ex,j}^l)$$

(37)

The termination criterion is defined as

$$|\pi_j^l(t + 1) - \pi_j^l(t)| < \epsilon$$

(38)

Alternatively, the termination criteria in (38) can be expressed as

$$|S_j^l(t) - P_{ex,j}^l| < \epsilon$$

(39)

where $l$ is the iteration counter, $\eta_2$ is the speed adjustment parameter, and $\epsilon$ is a small positive number.

The energy trading game starts with an announcement of the price vector from the sellers. The buyers then receive the announced price vector and play the evolutionary game. When the buyers obtain equilibrium for a given price vector, they send the information of ESS back to the sellers. Once the sellers receive the updated strategy from the buyers, they participate in the non-cooperative game and update their price, and the process is repeated until the final equilibrium state is reached. The detail process is given in Algorithm 2.

P2P energy trading algorithm proposed in this paper uses an iterative pricing mechanism. The demand from the buyers would change in response to the prices provided by the sellers, and this change will, in turn, affects the sellers’ prices. If there are large numbers of prosumers, change in the energy demand may be too high so that the prices may oscillate heavily. This fact may lead to an outcome where the seller selection and pricing iteration never ends, i.e., the algorithm does not converge. The convergence of the algorithm is essential for its implementation. If the convergence cannot be guaranteed, control measures should be taken to enhance the convergence of the algorithm.
The step length control method [28] is used to deal with the effect of random exploration for better convergence of the algorithm. The step length control at each iteration of price update can be achieved by

\[
\max \left( \pi_j^l(l) - \Delta, \pi_j^{l,m_{\text{min}}} \right) \leq \pi_j^l(l + 1) \\
\leq \min \left( \pi_j^l(l) + \Delta, \pi_j^{l,m_{\text{max}}} \right), \quad \Delta = \zeta |\pi_j^l(l)|
\]

(40)

where \( \zeta \) is a non-negative ramping rate.

This technique limits the ramping rate of the price in the iterative process. The buyers reject any price that exceeds the prescribed ramping rate so that all the sellers have to obey the ramping rate limit. Since the buyers’ strategies are the response to the sellers’ price, step length control method for price update in Algorithm 2 ultimately improves the convergence of Algorithm 1.

VII. SIMULATION RESULTS

This section presents the results of simulation studies to assess the performance of proposed game-theoretic model for P2P energy trading in a prosumer-based community microgrid. We consider a community microgrid consists of five prosumers. The community microgrid is connected to the utility grid, thus the prosumers can trade with each other as well as the the retailer. Each prosumer has solar PV system. Each prosumer has both the flexible and non-flexible demand. The generation and load profiles of each prosumers in the community are taken from [20]. For one day study \( \mathcal{T} = \{1, 2, 3, \ldots, 24\} \).

The value of \( N_i \) is selected randomly from [5,10], and \( \theta_i \) is taken as 0.5. Based on the actual electricity price of Singapore, the buying price of electricity from the grid (\( \rho_{\text{buy}} \)) is taken as 20 cents/kWh. The selling price to the grid (\( \rho_{\text{sell}} \)) is assumed 2 cents/kWh. Since the solar PV is used as a local generation, we assume that the P2P market is active only during the day time from 9:00 to 18:00. Prosumer 2 and Prosumer 5 have installed the battery of 20 kWh capacity. Lithium-ion batteries are considered is this work. The minimum battery capacity \( E_{i,m\text{in}} \) is 4 kWh, the charging/discharging efficiency is taken as 90%, and the charging and discharging capacity assumed to be 3 kW. The capital cost (\( C_c \)) of 20 kWh Tesla Powerwall is 7,800 SGD [29]. The discount rate \( r \) is taken 5% in this paper and the lifetime is set to 15 years and maintenance cost (\( C_m \)) for given size of the battery is 150 SGD.

Based on the value of GDR, the prosumers can behave as sellers or buyers during the day time. The GDR of the prosumers at each time slot is shown in Fig. 3. We choose time slot 11 to demonstrate the performance of the proposed method. At time slot 11, Prosumer 1 and Prosumer 5 are treated as sellers and the remaining three prosumers are buyers.

A. Convergence of Evolutionary Game Among Buyers

Each buyer participates in evolutionary game to find the probability of buying power from a particular seller, which is an evolutionary stable strategy. Fig. 4 shows the convergence characteristics of the evolutionary game and it depicts that buyers converge to the stable strategy by executing Algorithm 1. The convergence characteristics of the average net utility is shown in Fig. 5. The average net utility is settled at a certain value after it reaches to a maximum value. The convergence process of the mismatch between the individual net utility and average net utility at \( t = 11 \) is shown in Fig. 6. Fig. 5 and Fig. 6 ensure that buyers get better welfare at the evolutionary stable strategy.
We proposed Algorithm 2 for two purposes: i) to perform the price competition among the sellers using non-cooperative game, and ii) to carry out the negotiation between the buyers and the sellers using Stackelberg game. Since the sellers are leaders of the game; the solution of the Stackelberg game is an optimal response for the price announced by the sellers.

Fig. 7 shows the convergence characteristics of sellers price to the NE. As the seller price approaches NE, the power demand from the buyers to the seller also converges to a certain value and ensures the existence of the Stackelberg equilibrium in the trading process. The convergence characteristics of power demand of sellers are shown in Fig. 8. The welfare of the sellers gradually increase and settles at a certain maximum value as shown in Fig. 9. The convergence characteristics of SDR is given in Fig. 10 and they ensures that power demand and supply match at the end of the game.

The amount of power (in kW) the buyer \( i \) can buy from the seller \( j \) at the equilibrium state is given as

\[
\text{Power price (cents/kWh)} = \text{Price at NE}
\]

The total power procured by Prosumer 2, Prosumer 3, and Prosumer 4 from the P2P market is 3.31 kW, 84.43 kW, and 43.56 kW respectively, which satisfies the power requirement of the buying role prosumers.

C. Comparison of Results

In this section, we have analyzed and compared the results of proposed method with state of art from various aspects to evaluate its performance. As first, We have considered three cases: peer-to-grid (P2G) trading without BESS, P2P without both DR and BESS, and P2P with DR but without BESS using proposed game theoretic model. The total power imported from the grid and the total power exported to the grid in above three cases are shown in Fig.11 and Fig.12 respectively. The effect of P2P trading and DR application can be observed from \( t = 9 \) to \( t = 18 \) in Fig. 11 and Fig. 12 which is the assumed operation time of the P2P market. In Fig. 11, the power imported from grid reduces with the application of P2P method and reduces further with the application of P2P method with DR using proposed method. Similarly, power exported from the grid is reduced with the application of P2P with DR using proposed method compared to P2G but can be seen nearly equal when compared with P2P without DR. Hence, Fig. 11 and Fig. 12 highlight that proposed methodology reduces the dependency on the grid by efficiently allocating all the available energy within the community microgrid.

The overall cost of each prosumer using various approaches of P2P trading is given in Table 1. The total cost of the community microgrid without BESS is shown in Fig. 13. It consists of P2G trading, P2P trading with and without DR using the proposed game-theoretic model, P2P without DR using the BS method [13], P2P without DR using the MMR method [13], and P2P without DR using the SDR method [20]. It can be observed that the total cost from P2G trading is highest among all methods and P2P trading with DR using the proposed method is the lowest. Column 3 of Fig. 13 indicates that the total cost from proposed method is less than the cost obtained by the existing methods even if P2P trading is implemented without DR. Furthermore, In Fig. 13 the cost of the microgrid with DR using the proposed method can be seen significantly less compared to the cost of microgrid using P2G method. The cost of the microgrid with DR using the proposed is only 88.13% of the cost in P2G trading. Therefore, we can say that the proposed game-theoretic model is effective to handle the P2P trading in a prosumer based community microgrid on a daily basis.
TABLE I: Overall daily cost of prosumers in various scenarios/methods

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Overall cost without BESS (SGD)</th>
<th>Overall cost with BESS (SGD)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$P_1$</td>
<td>$P_2$</td>
</tr>
<tr>
<td>P2G trading (Base case)</td>
<td>371.25</td>
<td>466.75</td>
</tr>
<tr>
<td>P2P with DR (proposed method)</td>
<td>314.63</td>
<td>390.71</td>
</tr>
<tr>
<td>P2P without DR (proposed method)</td>
<td>348.17</td>
<td>433.36</td>
</tr>
<tr>
<td>P2P without DR using BS method</td>
<td>350.43</td>
<td>435.12</td>
</tr>
<tr>
<td>P2P without DR using MMR method</td>
<td>343.64</td>
<td>444.92</td>
</tr>
<tr>
<td>P2P without DR using SDR method</td>
<td>337.35</td>
<td>455.50</td>
</tr>
</tbody>
</table>

The EDC of the battery is 2.45 SGD as given by (7). The deployment of BESS is feasible only if the cost saving due to the use of BESS is higher than the EDC. Compared to base case (i.e., P2G trading without BESS), Prosumer 2 and Prosumer 5 can save 0.08 SGD and 0.12 SGD respectively in P2G trading with BESS. When all the prosumers in the community participate in P2P trading using the proposed method without both DR and BESS, Prosumer 2 and Prosumer 5 can save 27.39 SGD and 23.14 SGD respectively. If they participate in P2P without DR but with BESS, Prosumer 2 can save 25.67 SGD and Prosumer 5 can save 21.97 SGD. It can be seen that the saving for the prosumers who installed BESS is higher when they participate in P2P trading without BESS. This is because when they participate in the P2P market, the priority is to sell the excess energy in the P2P market. They cannot fully utilize the battery capacity. It is not necessary that all the prosumers have flexible loads, so the case without DR is chosen for analysis. This study shows that participating in the P2P energy trading in a community could be an alternative to installing the cost-intensive energy storage systems. But, the final conclusion cannot be drawn from the single case study because the P2P market is not matured yet. Several intensive studies and modifications regarding energy trading structures, current policies, and legal obligations are needed before it comes in wide applications.

VIII. CONCLUSION

In this paper, we have presented a game-theoretic model for real-time P2P energy trading in a prosumer based community microgrid. Prosumers in a community involve in a P2P trading is either a seller or a buyer. The interaction between the sellers and buyers is modeled as a Stackelberg game, where sellers are leaders and buyers are followers. The seller selection competition among buyers is modeled as an evolutionary game and an iterative algorithm is proposed to reach the stable state in a game. Also, the price competition among sellers is modeled as a non-cooperative game. A distributed iterative algorithm is used to reach the equilibrium states in a non-cooperative game and Stackelberg game. The proposed method is applied to a small community microgrid with PV and energy storage systems. Simulation results show that each game converges to the stable state using the proposed algorithms. Simulation results also show that the proposed model is effective to handle the P2P energy trading in a community microgrid. The cost of the microgrid is significantly reduced with P2P energy trading in both cases: (i) with BESS and (ii) without BESS, as compared to the cost in conventional P2G trading. This work can be extended by considering the P2P network of several community microgrids as well as the stochastic nature of prosumers.

REFERENCES
