Late Marriage as a Contributor to the Industrial Revolution in England

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Abstract

Was the European Marriage Pattern an important contributor to England’s precocious economic development? We examine this question by embedding the possibility in a historically substantiated demographic-economic model, supported by both cross-section and long time series evidence. Persistent high mortality and powerful mortality shocks in the fourteenth and fifteenth centuries lowered life expectations. Subsequently increased life expectancy reduced the number of births necessary to achieve a given family size. Fewer births were achieved by a higher age at first marriage of females. Later marriage not only constrained population growth but also provided greater opportunities for female informal learning, especially through ‘service’. In a period when the family was the principal institution for socialising future workers, such learning was a significant contributor to the intergenerational transmission and accumulation of human capital. Our paper shows how, over the centuries, the gradual induced rise of human capital raised productivity and eventually brought about the Industrial Revolution. Without the contribution of late marriage to human capital accumulation broadly interpreted, real wages in England would not have risen strongly in the early nineteenth century and would have been about half the level actually achieved.

Key Words

English Economic Development, Marriage Pattern, Demography, DSGE

JEL Classification

O11, J11, N13

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Hajnal identified a distinctive European marriage pattern: the late age at first marriage of females in western Europe by 1500, and the high proportion of females remaining celibate. This ‘low pressure’ European demographic regime was later linked to precocious European economic growth. The linkage has been questioned recently by Denison and Ogilvie with an analysis of the 2,622 observations of female age at first marriage for 39 European societies over the four centuries before 1900. They show that European countries with the most extreme forms of the marriage pattern did not become the most economically advanced in these years. They go on to draw attention to many other likely causal influences on European economic growth, as well as to the endogeneity of the marriage pattern. This paper demonstrates that these linkages can, to some extent, be quantified in a model of English economic development allowing an important role for the marriage pattern.

Explanations in history or social science can rarely be based plausibly on a single ‘cause’ or ‘reason’. It is unlikely that any single institution or event was responsible for a transformation of society—such as the English Industrial Revolution—especially when so many have previously been accorded this honour. Rather, there must have been an interaction of a considerable number of elements. The approach here is therefore to embed the idea to be tested in a model, a simplification of reality, that conforms with the salient evidence. The central hypothesis is that the late age of female first marriage was a major contributor to England’s Industrial Revolution because of the contribution to human capital accumulation. We adopt the OECD broad definition of human capital; advances in useful knowledge, from schooling, from successful technological innovations, from parenting and from many other sources. The test of the hypothesis is whether, without this nuptiality pattern, the Industrial Revolution, here defined as an eventual strong and sustained rise in real wages, would have occurred when it did.

The timing of the increase in real wages has been controversial. Some ‘pessimists’ defer it until as late as the 1850s whereas optimists identify a rise much earlier. Ashton famously remarked that the great achievement of the Industrial Revolution was to absorb the massive increase in population without a fall in wages. For this reason the modelling of the relation between

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2 Hajnal, ‘European marriage patterns in perspective’.
4 Denison and Ogilvie, ‘Does the European marriage pattern explain growth?’, ‘Institutions, demography, and economic growth’.
5 OECD, ‘Wellbeing of nations’.
6 Feinstein, ‘Pessimism perpetuated’; Lindert and Williamson, ‘English workers' living standards during the industrial revolution’.
7 Ashton, The industrial revolution, p161.
population and economy is critical. Malthus and others point to deferred marriage as a constraint on population growth\(^8\). But this contribution must have been more complex, for population growth in England was faster than elsewhere. Between 1600 and 1820, India’s populations is estimated to have risen 55 per cent, China’s grew by nearly 140 per cent\(^9\) while England’s expanded by almost 180 per cent.\(^10\)

We provide some new historical evidence that later female marriage age was associated with greater female human capital, as measured by literacy. This is a connection not made by Malthus and one that may be expected to have encouraged the transmission of more human capital to children. Baten et al have found a such a significant numeracy effect in Eastern Europe in the seventeenth to nineteenth centuries\(^11\). Acquisition of literacy and numeracy were just two elements of human capital accumulation where parenting mattered. As many innovators of the Industrial Revolution show, general cultural orientation could be as important.

We go on to demonstrate that an identified structural model, consistent with the long but unbalanced or fragmentary demographic and economic time series available for England, is capable of explaining the eventual real wage increase of the Industrial Revolution. The proposed economic growth process is triggered by the high mortality of the fourteenth century and the subsequent re-balancing of power that broke the feudal system. Gradual accumulation of human capital, broadly defined, ultimately launches a strong growth in real wages after a long struggle to offset the consequences of a rising population. A merit of our approach is that it can support counterfactuals implied by our central hypothesis; not only can we show what would have happened without the high marriage age but we can also demonstrate how England’s economic history might have resembled those of other economies if specific conditions had been different. Also, estimation of the model by the simulated method of moments allows a closer connection between theory and data than simple calibration.\(^12\)

First we substantiate the critical elements of the model and discuss the available time series data. The following three sections look at the historical evidence respectively for population, mortality and marriage age, for real wages, labour supply and GDP, and for human capital and the link with female marriage age. Section IV employs international cross-section data at the end of the European Marriage Pattern period to demonstrate a link between late female marriage age and human capital, measured by literacy. The following section outlines the model that utilises the English time series and its essential features. Then we show how well English

\(^8\) Ashraf and Galor, ‘Dynamics and stagnation in the malthusian epoch’.
\(^9\) Maddison, ‘Statistics on world population, GDP and per capita GDP’.
\(^10\) Wrigley and Schofield, The population history of England, Table A3.3.
\(^11\) Baten et al, “Girl power” in Eastern Europe?”
\(^12\) Boucekkine et al, ‘Early literacy achievements, population and the transition to modern growth’; Cervellati and Sunde, ‘Human capital formation, life expectancy, and the process of development’. Our complete model is in an Appendix and the estimation details are available from the authors in a separate paper.
economic growth can be explained by the model, primarily by simulation. We demonstrate that removing the marriage age effect prevents or defers the eventual real wage growth of the English Industrial Revolution. We then place the English experience in a wider context by considering counterfactuals that might explain the different growth outcomes in other historical economies.

I.

A vital characteristic of English economic history is the population collapse in the fourteenth century (Figure 1). Whether precipitated by a Malthusian crisis of over-population or simply a response to an exogenous series of famines and plagues of extraordinary intensity and frequency, the evidence of demographic decline is clear. In counties at opposite ends of England, Durham and Essex, village-level data shows population falling by more than one half. Clark’s calculations from wage data reach a broadly comparable conclusion for aggregate English population. Broadberry et al estimate population in 1290 only 60,000 below the 1348 peak of 4.81 million. Thereafter their benchmarks show population falling to a low of 1.9 million in 1450. English parish registration generally began in 1541. From these series Wrigley and Schofield estimate total English population, which we link to Broadberry et al’s medieval data in Figure 1.

Driving the decline in population we postulate is a ‘high mortality regime’ that we distinguish from ‘temporary mortality shocks’. The Great European famine of 1315-17 and the animal epidemics suggest a beginning of this regime. From a measure of the mortality of creditors, Nightingale concludes that the high mortality regime lasted until at least the middle of the fifteenth century. Wage data that peaks around the middle of the fifteenth century indicates that population decline in the face of the high mortality terminated somewhat earlier (to allow time for increased numbers of new labour force entrants to influence the labour market); in the absence of major changes in birth rates, the high mortality regime then ended about 1420. Cummins’ finding of a structural break in European noble lifespans around 1400, when longevity increased from about 50 to 55, is broadly consistent with this characterisation, bearing

13 Postan and Titow, ‘Heriots and prices on winchester manors’.
14 Hatcher and Bailey, Modeling the Middle Ages, pp. 55-65.
15 Dodds, ‘Estimating arable output’; Poos, ‘The rural population of essex in the later Middle Ages’.
16 Clark, ‘The long march of history’.
17 Broadberry et al, British economic growth.
18 Broadberry et al annual index numbers are in Appendix 5.3 of their book. We adopt the Broadberry data because of the benchmarks derived from samples of manorial populations. Clark, ‘The long march of history’, estimates a peak population of six million but Russell, ‘British medieval population’, p. 280, concluded the figure was just under four million.
19 Lee (‘Inverse projection and back projection: Comparative results and sensitivity tests for England’; ‘Inverse projection and demographic fluctuations: A critical assessment of new methods’) agrees that the estimates are a fair representation of population.
20 Nightingale, ‘Some new evidence of crises and trends of mortality in late medieval England’.
21 Cummins, ‘Longevity and the rise of the west’.
in mind that noble life styles and environments differed substantially from the majority of the population.\textsuperscript{22}

\textbf{Figure 1} Population and Wage (15-Year Average Index)

![Graph showing population and wage data over time.]

Notes: The wage series is computed based on male earnings\textsuperscript{23} and female wage\textsuperscript{24}. Population is based on Broadberry et al\textsuperscript{25} and Wrigley and Schofield\textsuperscript{26}. The original data are either annual or decadal, and the 15-year average here is to be in line with the model period.

High mortality rates are apparent from three longitudinal studies of late medieval monasteries beginning in 1395.\textsuperscript{27} 1430 seems to be a low in the crude death rates of these studies, consistent with our ending of the high mortality regime. Hatcher et al emphasise high levels of monastic mortality towards the end of the fifteenth and into the early sixteenth century. We identify this later fifteenth century mortality rise as a shock or a series of shocks rather than a regime shift.\textsuperscript{28}

The fourteenth century demographic catastrophe precipitated fundamental institutional change. Ensuing labour scarcity undermined the unfree relations of serfdom and permitted less restricted household choices.\textsuperscript{29} One of these, we contend, was the later age of female first marriage.\textsuperscript{30} Family reconstitutions for five Lincolnshire manors between 1272 and 1478 indicate

\textsuperscript{22} De La Croix and Licandro’s ‘famous people’ data are insufficiently accurate over this period to use as mortality change evidence.
\textsuperscript{23} Clark, ‘Average earnings and retail prices, UK, 1209-2010’.
\textsuperscript{24} Humphries and Weisdorf, ‘The wages of women in England 1260-1850’.
\textsuperscript{25} Broadberry et al, \textit{British economic growth, 1270-1870}.
\textsuperscript{26} Wrigley and Schofield, \textit{The population history of England, 1541-1871: A reconstruction}.
\textsuperscript{28} Jonker, ‘Life expectancy’ considers inquisition post mortems for fourteenth century taking into account truncation and censoring of the data and concludes that the Black Death did not reduce life expectancy very much. We are obliged to infer then that the rich tenants in chief were different from the poor. Otherwise Jonkers’ suggestions that fifteenth and fourteenth century mortalities were similar cannot be squared with wage and population movements over these two centuries.
\textsuperscript{29} Bailey, \textit{The decline of serfdom in late Medieval England}.
\textsuperscript{30} Hajnal, ‘European marriage patterns in perspective’; De Moor and Zanden, ‘Girl power’.
that before the Black Death in 1348 the mean age at first marriage for women was 21.4.\textsuperscript{31} Afterwards this age rose to 24.6.\textsuperscript{32} Marriage age was not always clear because medieval marriage in England simply required the consent of both parties and a witness. Solemnisation by the Church could come later. Hence there can be considerable uncertainty about actual marriage age.\textsuperscript{33}

The new marriage pattern has been interpreted as a response to improved female employment prospects.\textsuperscript{34} Greater labour scarcity and higher living standards induced a shift away from grain production towards animal husbandry, in which supposedly there was a female comparative advantage. However, women’s wages in England do not seem to have conformed to this pattern.\textsuperscript{35} Improved employment prospects should have raised female wages as they did wages for males, yet (unmarried) women’s annual contract wages did not increase.\textsuperscript{36} Kussmaul suggests almost the opposite pattern; the sharp drop in population caused by plague encouraged enforcement of labour service.\textsuperscript{37} This was facilitated by keeping young women under the employer’s roof, and this in turn required deferred marriage until the end of service.

Deferred female marriage was consistent with individuals’ objectives because mortality fell while there was greater labour scarcity in the later fifteenth century. Marriage age rose because eventually lower mortality required fewer births to achieve a given surviving number of children. This explanation is compatible with Kussmaul’s interpretation and with Humphries and Weisdorf’s findings. The rise in marriage age was a rational household response to the ending of a high mortality regime. Service and delayed marriage age were a means of achieving a target family size. When the mortality regime shifted in the fifteenth century, if marriage age rose the supply of unmarried female workers will have increased relative to that of married females. Such an increase in the marriage age may have been sufficient to prevent a fall in the supply of this type of labour. By contrast for daily female workers, labour supply would have declined by at least as much as for male daily wage earners and so their wages would have risen correspondingly.

If a small number of births was required (because of relatively low mortality), marriage would be delayed and there would be time for saving to establish separate household. Neolocality,
that facet of the marriage pattern whereby a newly married couple lived separately from both
the husband’s and the wife’s natal household, is a likely consequence of these choices.

Later changes in the average female age at first marriage are doubtful from the available evi-
dence. Weir and Schofield concluded that celibacy movements dominated in explaining gross
reproduction rate before 1700 or 1741, but afterwards the age at marriage mattered more. In
the eighteenth century according to one estimate there was a rise in fertility because marriage
age fell, with little change in the proportion never married. Dennison and Ogilvie find the
opposite trend in marriage age—a rise—in their broad European sample from the sixteenth to
the nineteenth centuries. Nonetheless, the Schofield data probably represents the current con-
sensus for England for marriage age and celibacy.

II.

An essential indicator of the long run growth of wellbeing is the average real wage, even though
at the beginning of the period the majority of workers were not wage earners. Many lived di-
rectly off the produce of the land they worked, sometimes in exchange for labour services on a
manorial demesne. Employment and remuneration changed over the centuries. While attempts
were made to hold back market forces, Clark provides evidence that medieval efforts were
unsuccessful where male wages are concerned, other than in the very short run. Hence it is
reasonable to assume that the marginal productivity of wage labour was closely related to mar-
ginal productivity in self-employment and across trades.

Clark’s series shows that earnings were apparently higher in 1450 than in 1800, an outcome
conventionally attributed to labour scarcity (and land fixity) at the earlier date. Allen’s real
wage measure show broadly the same pattern, but is focussed on specific large towns across
Europe. The Clark data is based upon separate indices of male farm day wages, male coal
mining day wages, and male building wages (to represent both the secondary and tertiary sec-
tors). These are aggregated into a national male wage using estimates of the share of males
employed in each sector.

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38 Weir, ‘Rather never than late’; Schofield, ‘English marriage patterns revisited’.
39 Unregistered and clandestine marriage may have been a significant proportion of the total (Schofield, ‘English
marriage patterns revisited’).
40 Wrigley et al, English population history from family reconstitution 1580-1837.
41 Dennison and Ogilvie, ‘Does the European marriage pattern explain growth?’
42 Schofield, ‘English marriage patterns revisited’. We take the bold ‘observed’ figures in Table 2 from the col-
umns headed AM2 and PEM (marriage age and proportion ever married).
43 Such as the Statute of Labourers 1351.
44 Clark, ‘The condition of the working class in England’.
45 Clark, ‘What were the British earnings and prices then?’: He has more data with a wider coverage than Allen
‘The Great Divergence’.
46 The Allen ‘Great divergence’ real wage series shows broadly the same pattern, but is focussed on specific
large towns such as London and Oxford.
Figure 2 Comparison of Different Wage Series (Annual)

Figure notes: The Clark wage series is available in Measuring Worth website. The Feinstein-Allen wage series is based on Feinstein\textsuperscript{47} and Allen\textsuperscript{48}. The Clark-HW wage series is computed based on Clark’s male wage\textsuperscript{49} and female wage\textsuperscript{50}, weighted by participation rates and working hours of different types of female labour\textsuperscript{51}. Following Clark’s wage series, the real earnings are based on 2010 prices.

Whereas Clark assumes that women’s earnings constituted 25 per cent of men’s in the total of earnings throughout six centuries, instead we aggregate his real male wage/earnings with Humphries and Weisdorf’s\textsuperscript{52} two sets of female wages (annual and casual). This requires a choice of weights to aggregate the two female series and to combine the female series with the male. We assume all single females\textsuperscript{53} were on annual contract wages or the equivalent, including factory work, and use Wrigley and Schofield’s\textsuperscript{54} life table level 3 and level 9 to establish the age distribution of the population. Married women’s participation rates (i.e. whether or not they work for wages at all) are taken from Horrell and Humphries\textsuperscript{55} for 1781-1795 onwards. Before that period we allow a trend increase in participation to allow for the increasing demand for spinning until 1770\textsuperscript{56}. For those married women that did work we took Arthur Young’s (agricultural labour) estimate of 17 weeks work.\textsuperscript{57} Finally, we use Horrell and Humphries\textsuperscript{58}

\textsuperscript{47} Feinstein, ‘Pessimism perpetuated’.
\textsuperscript{48} Allen, ‘Pessimism preserved’.
\textsuperscript{49} Clark, ‘Average earnings and retail prices, UK, 1209-2010’.
\textsuperscript{50} Humphries and Weisdorf, ‘The wages of women in England 1260-1850’.
\textsuperscript{51} Muldrew, ‘Th’ancient distaff’ and ‘whirling spindle’; Allen, ‘The high wage economy and the industrial revolution’; Horrell and Humphries, ‘Women’s labour force participation and the transition to the male breadwinner family 1790-1865’.
\textsuperscript{52} Humphries and Weisdorf, ‘Wages of women’.
\textsuperscript{53} During the high mortality regime (1300-1420), the single female workers are aged 15-20; during the low mortality regime (1421-1870), the single female workers are aged 15-25 due to a higher marriage age.
\textsuperscript{54} Wrigley and Schofield, \textit{The population history of England}.
\textsuperscript{55} Horrell and Humphries, ‘Women’s labour force’.
\textsuperscript{56} Muldrew, ‘Measuring the contribution of spinning’.
\textsuperscript{57} Rogers, \textit{Six centuries}, p. 481.
\textsuperscript{58} Horrell and Humphries, ‘Old questions, new data’.
estimates of the ratio of male earnings to family income to combine male and female wages/earnings. These adjustments make surprisingly little difference to the trajectory of the Clark male series (Figure 2); the principal effect is to reduce earnings in the fifteenth and sixteenth centuries. In other periods movements in component series seem to be offsetting. For instance in the century after 1750 female annual contract wages doubled while day wages hardly improved at all. Male wages rose by perhaps 50 per cent.

The cost of living deflator is a vital contributor to the real wage or earnings. For this reason we also report Allen’s\textsuperscript{59} Industrial Revolution real wage index which, largely because of the deflator, places the sustained rise in wages later than the Clark series (Figure 2).

Without other long-run series of factor rewards, implied GDP per capita must track wages. With our specification, wide long-run swings in real wages need not change labour’s share in GDP, because of factor substitution. Higher wage rates in the fifteenth century, encouraged human capital to be substituted for unskilled labour. This implies smarter farming, services and manufacturing in the fifteenth century—and less smart in the eighteenth century. But human capital was more abundant in the eighteenth century so this effect dominates increased labour supply. The other significant substitution possibility is between land and labour. Although the total agricultural area is broadly fixed for most of the period, shifts from arable to pastoral farming can be interpreted as a substitution of land for labour, as Voth and Voigtlander emphasise\textsuperscript{60}.

Changed factor scarcities may in part have triggered the breakdown of the feudal system itself. Feudal landlords and bailiffs would be incentivised to allow serfs to commute customary labour services for rent when labour was abundant and wages were low. But the peasant would prefer to abandon labour dues when wages were high, the reverse for the landlord. If the system did not break, because of landlord power and solidarity, such commutation when wage rates were high would not have been permitted. Therefore, viewing the Wars of the Roses as competition among landlords would help explain the disintegration of the feudal system. As discussed above, Kussmaul suggests that it did not entirely break; ‘service in husbandry’ was the reaction of small landowners to labour scarcity. They could control labour effort more effectively if the workers were living with their master and mistress. If they lived under the same roof, they could not marry and therefore the service regime, in which 15-25 year olds were the largest group, raised the age at marriage.

\textsuperscript{59} Allen, ‘Pessimism preserved: Real wages in the British industrial revolution’.
\textsuperscript{60} ‘Fertility restriction’
Labour inputs might vary because of changes in work intensity. However, there is no trend in days worked a week by builders since 1250.\textsuperscript{61} This counts against the view that days worked varied substantially over the period and that earnings and GDP per head failed to rise in line with fifteenth century wage rates. Hence we do not require an ‘industrious revolution’ (an increase in work intensity) for the Industrial Revolution. Instead the gradual accumulation (and diffusion) of human capital by the eighteenth century had raised the demand for labour by enhancing its productivity. By contrast an ‘industrious revolution’\textsuperscript{62} in the eighteenth century, increasing work intensity because of the availability of new consumer goods, could have lowered wage rates and reduced the demand for human capital. For these reasons and for simplicity, we assume a fixed labour input as a proportion of the working age population.\textsuperscript{63}

III.

In our interpretation the European Marriage Pattern (EMP) is linked to human capital acquisition and accumulation. Following the OECD,\textsuperscript{64} by human capital we mean ‘the knowledge, skills, competencies and attributes embodied in individuals that facilitate the creation of personal, social and economic well-being’. Hence unlike the demographic data and wages, fundamental measurement problems ensure that complete human capital indices are not readily constructed. Fully measuring human capital over time is problematic; the concept covers a wider range of skills and attributes than literacy, numeracy and schooling. In addition the weighting attached to the various components can be expected to change between different epochs. Nor is the skill premium any help in the English case. This is first because it is a relative measure, not the absolute indicator required, and second because the constancy of the skill ratio after 1450 suggests social regulation of market forces.\textsuperscript{65}

Theoretically, individuals’ human capital is measured as the capitalised value of their expected earnings and therefore depends upon the technology of the time and the competition they face, as well as their skills. Innovations and their diffusion can redistribute human capital. So the nineteenth century handloom weavers found to their costs\textsuperscript{66} but engine-makers, millwrights

\textsuperscript{61} Clark, ‘Average earnings and retail prices’. Obviously this does not rule out seasonal variation over the centuries. Clark contends that the Broadberry et al.\textit{British economic growth} conclusion about variation in weekly labour supply depends on their excessively high assumed share of farming in employment before 1700.

\textsuperscript{62} De Vries, ‘The industrial revolution and the industrious revolution’.

\textsuperscript{63} From the viewpoint of economic development as increased well-being, it should be noted that leisure is a component of well-being that is not measured by GDP per capita on which Broadberry et al are focussed. This is an important reason for the central place of real wages in the present model.

\textsuperscript{64} OECD,\textit{ Well-being of nations}.

\textsuperscript{65} Van Zanden, \textit{The long road to the industrial revolution}; Phelps Brown and Hopkins, ‘Seven centuries of building wages’.

\textsuperscript{66} For example from their peak in 1800-6 to the 1830s, nominal wages fell by about two thirds. Wood \textit{History of wages in the cotton industry}, p. 112.
and machine-makers, increasingly represented as patentees at the same time, were human capital beneficiaries\textsuperscript{67}. Consequently total human capital—expected earning power—in the economy could rise strongly while some workers became worse off.

Leaving aside measurement for the moment, late age at marriage and the associated pattern of service was likely to be a widespread and critical learning experience. In half of the English parishes sampled by Kussmaul, servants (employees living in the master’s house) were at least half of the hired workforce. Female servants were as liable to be employed as male.\textsuperscript{68} From the age of 15 or earlier, young people were likely to leave their home to live and work in several places elsewhere, commonly until marriage around 25 or later. At their new domiciles they could learn different ways of management and discipline from their early upbringing. Female servants did not merely fulfill purely domestic functions, although some of their duties were of a housekeeping nature.\textsuperscript{69} In the urban north the mercantile and victualling trades appear to have created the most employment for female servants and the metal working trade the least. The textile, clothing and leather trades were intermediate categories. In rural employment women generally ran the dairy, milked the cows, cared for small animals, weeded and performed ancillary tasks to agriculture—ale-making and cooking.\textsuperscript{70} They also might work in non-agricultural subsidiary or principal employments. Joseph Idle (sic), weaver of Murton, Westmoreland, in 1787 recorded his ‘family’ as including Betty Robinson, spoolswinder. An essential component of the analysis is that service and late marriage age were educative in a general sense.\textsuperscript{71}

A second element in the interpretation is the transmission of human capital. Informal home education by mothers was an important means of accumulating children’s human capital for the majority of the population for most of this period (we discuss other channels later). A literate mother would find it easier to ensure that her child was literate, and the same applies to work discipline as to literacy. This type of transmission has been widely discussed.\textsuperscript{72} Historical evidence of such a process is found in South Carolina before the Civil War\textsuperscript{73} and in Victorian Britain\textsuperscript{74}. In families with more educated mothers the cost or price of achieving a given ‘child quality’ was lower. So this ‘price’ was a negative function of age at marriage.\textsuperscript{75} It also depended on the likelihood of children surviving (related to the mortality rate) and on the opportunity cost of time the wage rate).

\textsuperscript{67} Macleod, \textit{English patent system}, p. 149.
\textsuperscript{68} Kussmaul, \textit{Servants in husbandry in early modern England}, p. 57.
\textsuperscript{69} Goldberg, ‘Female labour, service and marriage in the late medieval urban north’.
\textsuperscript{70} Kussmaul, \textit{Servants in husbandry in early modern England}, p. 34.
\textsuperscript{71} Froide, \textit{Never married}, p. 89.
\textsuperscript{73} Murray, ‘Family literacy and skill training in the Antebellum south’.
\textsuperscript{74} Mitch, \textit{The rise of popular literacy in Victorian England}, ch. 4.
\textsuperscript{75} The price of child quality includes school and apprenticeship fees in principle as well.
There is present day evidence for the relationship between female later marriage age and greater children’s education. Maternal education and maternal age have positive impacts both on cognitive skills and behavioural problems. Mothers’ age at marriage and education affect child quality in India and offspring adult health in the US. As this research suggests, what was transmitted by the EMP is likely to have been sufficiently general to fall under the description of the ‘culture’ that Clark identifies as largely responsible for international variations in 1910 cotton mill productivity.

A partial measure and proxy for human capital, male and female literacy, was rising gradually and steadily long before the standard date for the Industrial Revolution. Similar results are obtained for numeracy by examining age heaping. As population more than doubled between the 1750s and 1830s the growth of male literacy slowed. Schofield’s marriage register data shows total literacy rising from just under 50% to about 58%. Patent registrations, however, show a quite different, exponentially rising, trajectory. Between the same two decades they increased by a factor of 20.

The acceleration of patenting reflects a speeding up of industrial advances. Some inventive activity was undertaken outside the coverage of patent protection (patenting was expensive); Crompton’s ‘mule’ is a prominent example. Nonetheless, patents remain an approximate summary measure of the growth of innovation during the Industrial Revolution from the mid eighteenth century.

These innovations of the eighteenth and early nineteenth centuries were achieved by individuals using only everyday skills, great drive and entrepreneurial abilities. The ensuing rise in human capital took place despite low growth in literacy and schooling (perhaps following from

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76 Georgiadis and Manning, ‘Change and continuity among minority communities in Britain’; Field and Ambus, ‘Early marriage, age of menarche, and female schooling attainment in Bangladesh’.
79 Clark, ‘Why isn't the whole world developed?’.
80 De Pleijt, ‘Human capital and long run economic growth’.
81 A'Hearn et al, ‘Quantifying quantitative literacy’; De la Croix and Licandro, ‘The longevity of famous people from Hammurabi to Einstein’.
82 Between the decades from 1754 and from 1830 female literacy rose from 37% to just over 50% while for males the rise was from 62% to 66%.
84 Dutton, The patent system and inventive activity, p. 111; Macleod English patent system p. 8.
85 Khan, ‘Knowledge, human capital and economic development’.
the rapid population growth\textsuperscript{86}). While an apprentice, James Brindley (1716-1772) of Bridge-water canal fame, taught himself to write but he never mastered it thoroughly.\textsuperscript{87} The innovator of the water frame, Richard Arkwright (1732-92) ‘was scarcely able to write’. ‘When consider-ably more than fifty years of age, he encroached upon his sleep, in order to gain an hour each day to learn English grammar, and another hour to improve his writing and orthography’.\textsuperscript{88} The ‘spinning jenny’ inventor, James Hargreaves (d 1778), ‘though illiterate and humble, must be regarded as one of the greatest inventors and improvers in the cotton manufacture.’\textsuperscript{89} George Stephenson (1781-1848), whose steam locomotive ‘The Rocket’ began the railway age, was illiterate until aged 18 when he was earning enough money to be able to attend night school.\textsuperscript{90}

Innovators who never, or hardly, attended school as children necessarily acquired their drive and skills at least partly through their parents. Brindley’s mother taught him the little she knew and encouraged his formation of good habits by her hard work.\textsuperscript{91} George Stephenson’s parents ‘set before him an example of sobriety, economy and patient industry’.\textsuperscript{92} With a more prosperous background James Watt, who radically improved the Newcomen steam engine, was taught during his early years entirely at home. There he learned reading, writing and arithmetic from his parents, as well as drawing and how to use carpentry tools.\textsuperscript{93}

Apprenticeship was another means of acquiring and transmitting human capital in England. Wallis points out that the pre-modern apprentice in England was not ‘raw’ labour—they added considerable value to their Masters’ business.\textsuperscript{94} Also since the learning was by watching and by doing, the apprentice’s capacity to absorb was influenced by their prior attainment and orient-ation. Hence the relation of family education to apprenticeship education was sequential and complementary, rather than alternative.\textsuperscript{95}

There was a similar relationship between family education and schooling. Despite attendance being voluntary throughout the period of concern here, schools were continually being founded. In the York diocese Hoeppner Moran shows that 50 song and reading schools were established

\textsuperscript{86} De Pleijt and Weisdorf (‘Human capital formation from occupations’) widen the debate with their job classification exercise. Their probit model for the chances of being unskilled shows large broadly similar negative ef-fects of literacy and of being in an industrial parish, suggesting industrialization allowed alternative approaches to skill acquisition. This is confirmed by their finding of mobility up the skill ladder over individuals’ lifetimes. Possible deskilling is suggested by the positive time trend, accelerating towards the industrial revolution period. But we attribute this to the problems created for job classification by increasing structural change.

\textsuperscript{87} Smiles, \textit{James Brindley and the early engineers}, p. 139.

\textsuperscript{88} Baines, \textit{History of the cotton manufacture in Great Britain}, pp. 148, 195.

\textsuperscript{89} Baines, \textit{History of the cotton manufacture in Great Britain}, p. 156.

\textsuperscript{90} Smiles, \textit{The life of George Stephenson, railway engineer}, pp. 16-19.

\textsuperscript{91} Smiles, \textit{James Brindley and the early engineers}, pp. 129-130.

\textsuperscript{92} Smiles, \textit{The life of George Stephenson, railway engineer}, p. 16.

\textsuperscript{93} Smiles, \textit{Lives of Boulton and Watt}, p. 88.

\textsuperscript{94} Wallis, ‘Apprenticeship and training in premodern England’.

\textsuperscript{95} De la Croix et al, ‘Clans, guilds, and markets’.
between 1400 and 1500, and 54 more between 1500 and 1548.\textsuperscript{96} The dissolution of the monasteries seems to have triggered three unusual decades in which 127 grammar schools were created.\textsuperscript{97} Non-classical schools showed a very different trajectory; foundations rose gradually to a peak of 754 in the first three decades of the eighteenth century.\textsuperscript{98} No doubt these contributed to the rise of the literacy rate from about five per cent in 1500\textsuperscript{99} to around 50 per cent by the 1750s.\textsuperscript{100}

As an illustration of how aggregate human capital might have changed we construct a geometric index of literacy and patent stock. To allow that many patents were of no significance we apply a discount factor to the patent flow. Towards the middle of the eighteenth century we reduce the discount factor to reflect the improvement in the patenting system\textsuperscript{101} From 1852 when the costs of patenting were greatly reduced the discount factor is changed again. The trajectory of this index is shown in Figure 8. Because the weights in combining the component series are inevitably arbitrary\textsuperscript{102} we contend the human capital series generated by the model is more reliable. If the model and the other data are correct then so will be the model-generated human capital series.

IV.

While earlier data to test the hypothesis of a relation between late age at marriage and female human capital is not available, we can find systematic evidence from later in the EMP era. The US census for 1910 includes married female immigrants as well as natives and, in effect, their age at first marriage. These permit the testing of a link between marriage age and literacy. Literacy is a component of human capital and the variation between individuals is unlikely to be offset by other aspects of human capital; the innovators discussed above were innovative despite their virtual illiteracy not because of it. It is therefore reasonable to infer that literacy understates human capital.

The ability to write can be lost if not used and so the likelihood that many women first learned to write before their marriage does not preclude the possibility that maintaining and building on this aspect of human capital was facilitated by a later age at marriage. We test this hypothesis

\textsuperscript{96} Moran, \textit{The growth of English schooling}.
\textsuperscript{97} Boucekkine et al, ‘Early mortality declines at the dawn of modern growth’.
\textsuperscript{98} School attendance was not compulsory until 1880 in England.
\textsuperscript{99} Cressy, \textit{Literacy and the social order}, p. 176.
\textsuperscript{100} Schofield, ‘Dimensions of illiteracy’.
\textsuperscript{101} Macleod, \textit{The English patent system}, p. 7.
\textsuperscript{102} Arbitrary aggregation weights are not always recognised as potentially misleading, as with the UN human development index. The ranking of countries by this index can be changed with different weights attributed to different national values.
while controlling a woman’s being in education,\textsuperscript{103} her age, property ownership and urban residence. In addition we add controls for region of origin. Since our hypothesis is that the process is cumulative and began many centuries before the census date, we expect to find a Western European literacy effect stronger than in any other region (except perhaps North America, largely populated in the previous century from Western Europe).

The data are summarised in \textbf{Table 1}. The sample includes migrants originating from every region of the world. The majority of the 156,986 observations of married women were born in North America (the base case) but 9.4 per cent originated in Central and Eastern Europe and about seven per cent came from Northern and Western Europe (including 1.38 per cent from England). The average age at marriage was about 22 and almost one half owned property and lived in urban area.

\textbf{Table 1} Married females in the US 1910 (IPUMS, 1\% sample)

<table>
<thead>
<tr>
<th>Variables</th>
<th>Obs.</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Marriage Age</td>
<td>156,986</td>
<td>21.730</td>
<td>5.304</td>
<td>1</td>
<td>86</td>
</tr>
<tr>
<td>Age</td>
<td>156,986</td>
<td>37.313</td>
<td>12.939</td>
<td>12</td>
<td>100</td>
</tr>
<tr>
<td>In School</td>
<td>156,986</td>
<td>0.019</td>
<td>0.136</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Urban Resident</td>
<td>156,986</td>
<td>0.460</td>
<td>0.498</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Own a Property</td>
<td>156,986</td>
<td>0.449</td>
<td>0.497</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>N. America</td>
<td>127,403</td>
<td>81.16%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C. &amp; S. America</td>
<td>484</td>
<td>0.31%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N. &amp; W. Europe</td>
<td>10,872</td>
<td>6.93%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>England</td>
<td>2,165</td>
<td>1.38%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S. Europe</td>
<td>3,101</td>
<td>1.98%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C. &amp; E. Europe</td>
<td>14,764</td>
<td>9.40%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E. &amp; S. Asia</td>
<td>195</td>
<td>0.12%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>W. Asia</td>
<td>120</td>
<td>0.08%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Australasia</td>
<td>18</td>
<td>0.01%</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

We run a probit regression with self-assessed ability to write as dependent variable and with quadratic age at marriage and age independent variables (\textbf{Table 2}). Consistent with the hypothesis, England’s literacy effect is significantly stronger than for any other region, including North America. The probability of a married woman being able to write is around 15.6 per cent higher if she originated in Western rather than Eastern Europe and not significantly different from North America if England is excluded. The margin over Asia is far greater; about 40 per cent.

\textsuperscript{103} The Census definition is ambiguous but the proportions are so low they cannot refer to ever been in school.
### Table 2 Marginal Effects of Marriage Age on Literacy

<table>
<thead>
<tr>
<th></th>
<th>Coefficients</th>
<th>Standard Errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Marriage Age (A)</td>
<td>0.0121***</td>
<td>0.0004</td>
</tr>
<tr>
<td>(A^2)</td>
<td>-0.0002***</td>
<td>0.0000</td>
</tr>
<tr>
<td>Age</td>
<td>-0.0012***</td>
<td>0.0003</td>
</tr>
<tr>
<td>(Age^2)</td>
<td>-0.0000**</td>
<td>0.0000</td>
</tr>
<tr>
<td>Currently in School</td>
<td>0.0233***</td>
<td>0.0039</td>
</tr>
<tr>
<td>Urban Resident</td>
<td>0.0701***</td>
<td>0.0014</td>
</tr>
<tr>
<td>Own a Property</td>
<td>0.0642***</td>
<td>0.0013</td>
</tr>
<tr>
<td>C. &amp; S. America</td>
<td>-0.4729***</td>
<td>0.0235</td>
</tr>
<tr>
<td>N. &amp; W. Europe</td>
<td>-0.0009</td>
<td>0.0032</td>
</tr>
<tr>
<td>((England))</td>
<td>0.0528***</td>
<td>0.0032</td>
</tr>
<tr>
<td>S. Europe</td>
<td>-0.4811***</td>
<td>0.0093</td>
</tr>
<tr>
<td>C. &amp; E. Europe</td>
<td>-0.1562***</td>
<td>0.0038</td>
</tr>
<tr>
<td>E. &amp; S. Asia</td>
<td>-0.3796***</td>
<td>0.0363</td>
</tr>
<tr>
<td>W. Asia</td>
<td>-0.4132***</td>
<td>0.0471</td>
</tr>
<tr>
<td>Australasia</td>
<td>-0.0849</td>
<td>0.0936</td>
</tr>
<tr>
<td>Number of Obs.</td>
<td>156986</td>
<td></td>
</tr>
<tr>
<td>Pseudo R2</td>
<td>0.126</td>
<td></td>
</tr>
<tr>
<td>AIC</td>
<td>83921.37</td>
<td></td>
</tr>
<tr>
<td>BIC</td>
<td>84080.79</td>
<td></td>
</tr>
</tbody>
</table>

Notes: Base case is North America, * p<0.05, ** p<0.01, *** p<0.001. Standard errors in parentheses. To test the exogeneity of marriage age, we applied “ivprobit” using “no children after 35 years old” as the instrument. The null hypothesis that marriage age is exogenous cannot be rejected at 5% level. The N. & W. Europe dummy includes England, so the England dummy measures the extra effect of being English. These results are robust to inclusion of interaction terms between countries of origin and other control variables.

The marriage age effect is highly significant statistically. Quantitatively an increase in the marriage age from 18 to 25 is associated with a 3.43 per cent higher chance 104 of being able to write. Passed on over many generations these small increments cumulatively could have given Western Europe the literacy advantage here identified. Less easily measured aspects of human capital, such as work discipline, are likely to have been transmitted in the same way to a similar extent.

V.

We draw inferences about the effects of late marriage from different data, the English time series already discussed, by creating a model in which forward looking individuals can belong to one of four 15-year cohorts. Childhood (0-15) is the first of these life phases. The marriage and child choice is made in the second phase (16-30) while the costs and benefits of these

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104 This effect is calculated using total differentiation: \( d\Pr(\text{literacy}) = dA \times (0.0121 + 2 \times (-0.0002) \times A) = 3.43\% \), where \( A = 18 \) and \( dA = 25 - 18 = 7 \).
choices are incurred in phase three (31-45). Phase four ends with certain death unlike the previous life stages. 15-year life stages constitute the unit period of the model. Choices and mortality rates are specific to each cohort.

Figure 3 An Outline of the Demographic-Economic Model of England

Notes: The grey shapes are exogenous variables/shocks determined outside the model, while the others are endogenously determined within the model. The dotted shapes indicate the variables optimally chosen by a rational household, and the dashed shapes indicate the variables (partially) determined by a rational producer. The two solid circles (wage rate and celibacy rate) are determined by the aggregate forces. The grey arrows indicate the causal effects, of which the black ones are the key mechanisms of our model. \( A \): female marriage age, \( n \): target number of children, \( q \): target quality of children, \( z \): consumption; \( P \): aggregate population, \( H \): aggregate human capital, \( Y \): aggregate output, \( w \): wage/income; \( \mu \): celibacy rate determined by \( A \) and \( w \).

The diagram (Figure 3) gives a simplified representation of the model. People have Constant Elasticity of Substitution (CES) preferences for the surviving number of children \((n)\), their ‘quality’ \((q)\) and consumption goods \((z)\). They choose births to achieve target completed family size and hence, in combination with exogenous mortality, population \((P)\). They also choose child quality, which eventually influences the aggregate human capital \((H)\) as the children grow up. The contribution of child quality to \(H\) is the family education, in contrast to the nonfamily education, such as schooling, apprenticeship and on-the-job training. Both are captured in the human capital formation mechanism in our model. Marriage age \((A)\) and mortality affect the cost or price of ‘child quality’ while the cost or price of a surviving child only depends on mortality. In this way they influence the number \((n)\) and ‘quality’ \((q)\) of children. In particular, the negative effect of the female marriage age on the price of child quality is the key mechanism (the marriage age effect) to be explored in this paper. The proportion remaining unmarried \((\mu)\)
affects the growth of population and varies with the wage rate and the marriage age via a search-matching mechanism.

On the production side of the economy, output \((Y)\) is determined by assumed fixed land and variable labour \((P)\) and human capital \((H)\). These inputs are combined in a Cobb-Douglas constant returns to scale function together with productivity shocks.\(^{105}\) Wages \((w)\), a constraint on household choices, are determined by the marginal productivity of labour.

Households in this model plan their lives in an uncertain environment; they are subject to a variety of unexpected events. Productivity and mortality shocks are distinguished. Runs of poor harvests (such as the Great European Famine of 1315-17) and livestock disease constitute a negative productivity shock. Epidemic diseases such as bubonic plague, typhus and smallpox were mortality shocks. Mortality is autocorrelated and so a shock in one period has repercussions in the next. This implies that periods of high mortality such as occurred in the seventeenth century (and their ending) in principle might be attributed to random events, despite their persistence.\(^{106}\) Shocks can explain but are not explained. A structural break is assumed for the shock structures and the average celibacy rate around 1420, but household preferences and technologies are kept constant across mortality regimes.

The primary source of demographic data is Wrigley and Schofield’s crude mortality and fertility rates derived from 404 Anglican parish registers of baptism, marriage and burial.\(^{107}\) Steady states of mortality rates are set to match the historical life expectancy pattern over the entire period. A life expectancy at birth of 23 (as Hatcher\(^{108}\) calculates for the Canterbury Benedictine priory) is assumed for the ‘high mortality’ regime from about 1300 to 1420 (level 3), and a life expectancy of 38 (level 9) for the subsequent ‘low mortality’ regime of the remainder of the sample period.\(^{109}\)

Most of the model parameters are estimated to minimise the squared distance between the simulated and the observed mean values and standard deviations of data series, using the simulated method of moments. The calibrated exceptions are ‘average stopping births age’ (35 years old)\(^{110}\) and ‘gaps between births’ (2.5 years).\(^{111}\)

\(^{105}\) The impact of physical capital is not separately distinguished. The human capital concept in our model reflects the accompanying accumulation of physical capital because physical capital must be paired with human capital to be effective. Also in production activities investment in physical capital was not a significant proportion of output until the coming of the railways. Feinstein and Pollard Capital formation in the United Kingdom.

\(^{106}\) We have in mind De la Croix and Licandro’s finding, as well as Geoffrey Parker’s account.

\(^{107}\) Wrigley and Schofield, The population history of England.

\(^{108}\) Hatcher, ‘Mortality in the fifteenth century’.


\(^{110}\) Flinn, The European demographic system 1500-1820, p. 335.

\(^{111}\) This assumed constancy is a simplification because birth spacing typically increases with births.
The estimated elasticity of substitution (0.152) implies very little possibility of switching between numbers of children, and child quality, in the preferences. This is an important difference between the present exercise and the post-Malthusian regime of Galor and Weil’s unified growth.\textsuperscript{112} There, technological developments raise the rate of return to human capital and stimulate a substitution of child quality for child quantity. But with the low value estimated here, substitution is unlikely to have mattered historically as much as the continuous gradual accumulation of human capital.\textsuperscript{113}

The contribution of education by the family (0.6) is estimated to be greater than education outside the family in the wider society (0.4). But the family education contribution includes payment of school and apprenticeship fees and other educational demands on the household budget. The elasticity of marriage age is found to be -1.288; if marriage age rises by one per cent relative to age 16, the price of child quality decreases by 1.288 per cent. For example, when the modelled marriage age rises, as it does, from 18 in the high mortality regime to 24 in the low mortality regime (by 33 per cent), child quality is about 42.5 per cent cheaper, \textit{ceteris paribus}.\textsuperscript{114} Consistent with the specification, the celibacy rate increases when marriage age is higher than expected and falls when wages are higher than anticipated.

Using the estimated or calibrated values of the parameters, we can replicate the sample means or theoretical steady states and the standard deviations of the real wage, births, deaths and population\textsuperscript{115}. With these parameters the ‘preventative check’ to population (higher marriage age, lower births) of Malthusian theory and the inverse of the ‘positive check’ (lower deaths, higher growth of population), are apparent between the two mortality regimes in \textbf{Table 3}. Premature deaths probabilistically affect the costs of raising a surviving child. Consequently, after the end of the high mortality regime both target family size (and therefore population) and child quality increase. The price of child quality falls further than the price of child quantity because of the additional marriage age effect shown in \textbf{Figure 3}.

The fall in the price of child quality increases human capital which raises the productivity of labour, offsetting diminishing returns to a rising population. When wages increase, so does target child number, child quality, and consumption. Greater affluence tends to lowers marriage

\textsuperscript{112} Galor and Weil, ‘Population, technology, and growth’.
\textsuperscript{113} A low elasticity indicates gross complementarity. If the price of quality rises, expenditure on quality also rises and switches from the number of children, so that fewer are likely to be demanded.
\textsuperscript{114} This qualification is important because the overall drop in the price of quality is 60%, which also includes the effect of lower mortality rates.
\textsuperscript{115} The simulated periods (580 years, 38 periods) are longer than the sample periods of birth rates and death rates (330 years, 22 periods) but the same as population and wage. The model closely matches the second moments of the observed data, but there are discrepancies between the simulated steady states and sample means (the ‘first moments’). This is because the simulated steady states are not equivalent to sample means. The sample means measure the average levels of the variables in real history, which is never in a steady state because of continual shocks. In contrast, the steady states are defined as the long-run levels when the effects of shocks die away.
age. Pushing in the other direction, higher wages also raise child ‘price’, or opportunity cost, reducing projected family size. But the biggest impact comes from the fall in mortality between the two regimes that requires fewer births to achieve a given target family size. The mortality effect strongly raises the marriage age. Because this last outcome dominates the determination of marriage age, the end of the high mortality regime raises the age, as noted above.

### Table 3 Simulated and Observed Means and Standard Deviations

<table>
<thead>
<tr>
<th>Variables</th>
<th>Regime</th>
<th>Simulated Steady State</th>
<th>Sample Mean</th>
<th>Simulated Std. Dev.</th>
<th>Sample Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Marriage Age</td>
<td>HMR</td>
<td>18.02</td>
<td>[18~21]</td>
<td>3.04</td>
<td></td>
</tr>
<tr>
<td></td>
<td>LMR</td>
<td>24.11</td>
<td>[24~26]</td>
<td>0.34</td>
<td></td>
</tr>
<tr>
<td>Population Growth</td>
<td>HMR</td>
<td>0.000</td>
<td>-0.085</td>
<td>0.141</td>
<td><strong>0.151</strong></td>
</tr>
<tr>
<td></td>
<td>LMR</td>
<td>0.069</td>
<td><strong>0.082</strong></td>
<td>0.079</td>
<td><strong>0.079</strong></td>
</tr>
<tr>
<td>Birth Rate</td>
<td>HMR</td>
<td>1.281</td>
<td></td>
<td>0.778</td>
<td></td>
</tr>
<tr>
<td></td>
<td>LMR</td>
<td>0.612</td>
<td><strong>0.627</strong></td>
<td>0.104</td>
<td><strong>0.104</strong></td>
</tr>
<tr>
<td>Death Rate</td>
<td>HMR</td>
<td>0.562</td>
<td></td>
<td>0.150</td>
<td></td>
</tr>
<tr>
<td></td>
<td>LMR</td>
<td>0.337</td>
<td><strong>0.313</strong></td>
<td>0.039</td>
<td><strong>0.039</strong></td>
</tr>
<tr>
<td>Celibacy Rate</td>
<td>HMR</td>
<td>0.155</td>
<td></td>
<td>0.026</td>
<td></td>
</tr>
<tr>
<td></td>
<td>LMR</td>
<td>0.147</td>
<td><strong>[0.128]</strong></td>
<td>0.031</td>
<td></td>
</tr>
<tr>
<td>Wage Growth</td>
<td>HMR</td>
<td>0.080</td>
<td><strong>0.090</strong></td>
<td>0.110</td>
<td><strong>0.103</strong></td>
</tr>
<tr>
<td></td>
<td>LMR</td>
<td>0.014</td>
<td><strong>0.015</strong></td>
<td>0.079</td>
<td><strong>0.079</strong></td>
</tr>
<tr>
<td>Price of Child Number</td>
<td>HMR</td>
<td>0.141</td>
<td></td>
<td>0.080</td>
<td></td>
</tr>
<tr>
<td></td>
<td>LMR</td>
<td>0.080</td>
<td></td>
<td>0.011</td>
<td></td>
</tr>
<tr>
<td>Price of Child ‘Quality’</td>
<td>HMR</td>
<td>0.113</td>
<td></td>
<td>0.080</td>
<td></td>
</tr>
<tr>
<td></td>
<td>LMR</td>
<td>0.044</td>
<td></td>
<td>0.006</td>
<td></td>
</tr>
</tbody>
</table>

Notes: Fragmentary data based on small sample in square parentheses. HMR: high mortality regime (1300-1420). LMR: low mortality regime (1421-1870).

### VI.

To show how well the model with the estimated parameters ‘retrodicts’, we simulate the six centuries’ demographic and economic history. We go on to show the importance of marriage age effect by a simulation without this effect. Finally we consider what might have happened in England had some of the characteristics of other economies been shared.

In Figure 4-Figure 7 the grey broken lines are the simulated series (the conditional expected values based on individuals’ current information) and the black solid lines are the actual series. Most of the fluctuations in the data can be well matched. One novelty of the simulation method we adopt here is that we can make full use of the unbalanced data; the real wage series\textsuperscript{116} has

\textsuperscript{116} Clark, ‘What were the British earnings and prices then?’. 

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more observations than the mortality and birth rate series and than the celibacy and marriage ages. The broken lines unaccompanied by solid lines (Figure 8 and Figure 9) show the series generated by the model, but the data counterparts are unavailable (or there is only fragmentary evidence).

Both observed and simulated population growth turn upwards after the first quarter of the eighteenth century (Figure 4), whereas real wage growth begins permanently much later (Figure 5). An essential point is that the model, driven by the long and gradual accumulation of human capital, captures the eventual upsurge in real wages of the Industrial Revolution. Interestingly the (model-consistent) turning point is later (the 15-year period 1826-40) than indicated by the modified Clark series (1810-25) and corresponds more closely with the Allen’s real wages (though is still below it). If the model is correct then real wages do begin rising later than indicated by the modified Clark series (though they were used in the model).

The intensity and frequency of mortality crises (shocks) diminish with the success of Western European quarantine regulations from the early eighteenth century (Figure 7). The demographic consequences, more surviving children, are realised more rapidly than the effects operating through human capital accumulation. This is because greater child quality must take longer to be translated into higher wages than the stronger demand for children’s numbers takes to trigger an increase in population (Figure 4). Female age at marriage jumps from around 18 to 24 in the mid-fifteenth century (Figure 8), by which time the high mortality regime is assumed over.

The model’s four 15-year life phases imposes the restriction that everyone must die by the age of 60, which upwardly biases the death rate. Hence the simulated death rate of Figure 7 is higher than the observed. The marriage age pattern does not correspond closely with the simulated series (Figure 8). One reason may be that the ‘observed’ data are a small sample, the mean of which is subject to greater variation than would be the ideal series, derived from a much larger number of parishes. On the other hand, the drop in the celibacy rate between the seventeenth and eighteenth centuries is broadly captured by the simulation (Figure 9). The simulated human capital series accelerates broadly similarly to the constructed series although these ‘data’ were not used in the model estimation. A simple visual test suggests that all the longer simulated series fit the data quite well.

\[\text{References:}\]

117 Wrigley and Schofield, *The population history of England*.
118 Schofield, ‘English marriage patterns revisited’.
119 Allen, ‘Pessimism preserved: Real wages in the British industrial revolution’.
120 Chesnais, *The demographic transition*, p. 141.
121 Data used in the model for female age at first marriage are from Wrigley et al (‘English population history from family reconstitution 1580-1837’), which show a fall in marriage age from the 1730s. These data are collected from only 26 parishes that may not be representative of the entire country, however (Ruggles, ‘The limitations of english family reconstitution’).
Figure 4 Simulated and Observed Population

Figure 5 Simulated and Observed Real Wage
Figure 6 Simulated and Observed Birth Rate

Figure 7 Simulated and Observed Death Rate
The importance of the marriage age for human capital in this model can be demonstrated by an exercise that excludes the effect on the price of child quality. This is achieved by assuming there is no inter-generational transmission of human capital through the family. A lower age of
maternity implies the possibility of more children but also imposes a higher ‘price of quality’—
which we can interpret as stemming from the lower accumulation of personal savings and household goods, as well as being an effect of maternal human capital. The simulation reported by the dashed line in Figure 5 shows the consequences for wages; there is no eventual upturn and wages are about half the level actually achieved.

To consider what English growth experience might have been under different circumstances, and to compare with other economies, we replace hypothesized key features of other economies with those of England. A caveat is due here—the simulations are NOT intended to match exactly other countries growth of real wages. Rather, the purpose is to show how the English economy might have deviated from the actual historical path if some parameters or the shock path were different. That is why the ‘countries’ are in quotation marks; only single possible characteristics of these countries are reflected in the simulations. To allow for delayed take-off timing in these simulations, we extend the simulation horizon another 10 periods (150 years).

**Figure 10** Simulated Wage of Other ‘Countries’

In this way we can show that real wages in England are predicted by the model to go on rising after 1870 (when the data used ends). In Figure 10 the black solid line is the observed (modified Clark’s) wage series, and the dashed line is the extended forecast\(^ {122} \).

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\(^ {122} \) The mortality shocks are randomly sampled from the actual shocks during the entire Low Mortality Regime. The productivity shocks are sampled from the actual shocks during the last 6 periods (1780-1870). The latter is to reflect that there might be a structural break in productivity shock structure.
To capture a non-European environment with a low marriage age (Hajnal, 1965, especially Table 4 and discussion) and low real wages that we term ‘Asia’, we broadly follow Jones\textsuperscript{123} in the model adjustments. We keep the high mortality regime throughout the exercise and increase the utility weight of the number of children by 40\% to capture a possible cultural difference\textsuperscript{124}. Figure 10 shows that under these conditions the simulated model of the English economy yields a real wage that is very low and barely rises at all until the last decade of the twentieth century. Also female marriage age remains below 20 and population oscillates wildly.

In the first half of the seventeenth century the Netherlands was probably the most productive economy in the world. Then the country experienced a series of strong negative productivity surprises from 1650 to around 1820. Dutch supremacy on the high seas and in international markets was undermined by Cromwell’s Navigation Acts of the 1650s and Colbert’s protectionist policies in the following two decades. Dutch trade, shipping and real wages suffered (van Zanden and van Leuween, 2012). To imitate the impact we replace England’s actual shocks in the model during that period with only England’s negative shocks. The results shows that, had the English economy been subject to a sequence of savage blows in the seventeenth century like the Netherlands’, real wages would have fallen as they did in the Netherlands. The importance of the nature and timing of shocks, or surprises, that this experiment demonstrates, offers a development of Crafts’ contention that Britain industrialised before France only by chance\textsuperscript{125}.

Germany’s economy may have been more constrained by guild power than England’s\textsuperscript{126}; if so the production function would have been less efficient. This could explain the lag behind England in the sustained rise of real wages, despite the European Marriage Pattern. To test for this possibility the output elasticity of human capital, is adjusted down by 10\% to mimic a ‘German’ type of stronger guild institutions (on the supply side). The simulation confirms the sensitivity of real wage growth to this type of restriction by showing no increase in England’s real wages until after 1870.

VII.

The complexities of the interaction between demography and economy for several centuries of English history inevitably require simplification if the main elements of the process are to be understood. Our model is a judgment based upon current evidence as to what mattered most for the economic development that culminated in the first Industrial Revolution. Linking demography and economics focuses attention on the great length of time over which the effects

\textsuperscript{123} Jones, ‘The European miracle’.
\textsuperscript{124} The mortality shocks are selected from the actual shocks during the High Mortality Regime, but the productivity shocks after 1945 are sampled from the period 1780-1870.
\textsuperscript{125} Crafts, ‘Why was England first?’
\textsuperscript{126} Ogilvie, ‘Guilds, efficiency and social capital’.
of shocks to the inter-generational accumulation of human capital may be felt, stemming from the longevity of human capital, thanks to the potential length of human life.

The model\(^{127}\) shows how the European Marriage Pattern emerged with the ending of the high mortality of the fourteenth and fifteenth centuries. In the higher mortality regime there is a falling population and a tendency for real wages to rise, because of diminishing returns with a fixed land area. The eventual fall in mortality meant that fewer births were necessary for a given completed family size. A rise in the female age at first marriage was the response, though there is considerably more to be said about the contribution of institutional change with the decline of feudalism. Because the Humphries and Weisdorf\(^{128}\) series show no rise for female contract labour wages in the fifteenth century, both the de Moor and van Zanden hypothesis of the EMP originating from higher female wages, and Voth and Voigtlander’s focus on the shift from arable to more female-intensive pastoral farming, seem less satisfactory explanations\(^{129}\).

In the rest of the period, a constant steady state mortality (albeit subject to shocks) is assumed—the ‘lower mortality regime’, which divides into two phases. The earlier is a Malthusian epoch of falling or stagnant real wages and rising population. The later phase is the Industrial Revolution epoch when real wages rise sustainably along with population.

Because of this lower mortality, there was strong population growth during the English Industrial Revolution period, compared with non-EMP areas. This was a constraint upon the rise in real wages. Yet the EMP, one of Malthus’ preventative checks, might have been expected to hold down population expansion compared with other regions without this institution\(^{130}\). The explanation developed in this paper is that the EMP in England also had another, more important, effect—to increase the growth of human capital (which is consistent with De Moor and Van Zanden\(^{131}\), though a more limited process than their female empowerment or autonomy). Human capital accumulation eventually improved productivity fast enough to overcome the consequences of the population growth.

De Pleijt and Van Zanden recently have confirmed the importance of broad human capital in long term European development\(^{132}\), in effect revising earlier work by Allen\(^{133}\). But the later Allen provides an account entirely consistent with the model estimated here.\(^{134}\) Greater skill

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\(^{127}\) In a four overlapping generations structure. Model available in web appendix.

\(^{128}\) Humphries and Weisdorf ‘Wages of women’ Figure 3.

\(^{129}\) De Moor and Van Zanden ‘Girl power’ p3 ‘relatively high remuneration’; Voigtlander and Voth, ‘Fertility restriction’.

\(^{130}\) Voigtlander and Voth, ‘Fertility restriction’.

\(^{131}\) De Moor and Van Zanden ‘Girl power’.

\(^{132}\) De Pleijt and Van Zanden ‘Accounting for the little divergence’.

\(^{133}\) Allen ‘Progress and poverty’.

\(^{134}\) Allen, *The British industrial revolution in global perspective*. 
and discipline warranted higher wages. These high wages incentivised the search for innovations that would reduce labour costs. Innovation was a trial and error process undertaken by determined and enquiring persons usually lacking formal training but with the work discipline acquired at an early stage in life.

Further research could extend the model to allow for changes in labour intensity to address the divergences between wage rates and the Broadberry et al GDP per capita series. Explaining mortality changes, another extension, is likely to trigger a more extended rise in the marriage age. Distinguishing, rather than aggregating, male and female wages in a later, inevitably much more complex, version of the model may also illuminate movements in marriage ages and celibacy rates.

Other West European economies showed a similar marriage pattern to England, yet did not simultaneously experience a similar economic growth pattern. One reason might be that human capital accumulation induced by the marriage pattern simply began later (as in Southern Europe). Anyway, it is likely that the model parameters of these economies or even their structures differed from England’s. We have given some examples of the consequences of such differences in our simulations.

In conclusion, our claim is not that the marriage pattern was a sufficient condition for the English Industrial Revolution at the beginning of the nineteenth century, or for Industrial Revolutions elsewhere. Rather the marriage pattern was a major contributor, along with many other conditions, to the eventual upsurge in real wages that we consider the defining characteristic of the English Industrial Revolution.

**FOOTNOTE REFERENCES**


Allen, R. C. ‘The Great Divergence in European Wages and Prices from the Middle Ages to the First World War’, *Explorations in Economic History* 38, 411–447 (2001)

_______. ‘Progress and poverty in early modern Europe’, *Economic History Review* 56, pp.403-443


135 Kowaleski, ‘Single women in medieval and early modern Europe’.
The British Industrial Revolution in Global Perspective (Cambridge, 2009).


The Decline of Serfdom in Late Medieval England: From Bondage to Freedom (Woodbridge, 2014).


Goldberg, P. 'Female Labour, Service and Marriage in the Late Medieval Urban North', Northern Studies (1986), pp. 18-38.


Parker, G., *Global Crisis: War, Climate Change and Catastrophe in the Seventeenth Century* (New Haven, CT, 2013).


Smiles, S. *James Brindley and the Early Engineers*, (London, John Murray, 1864) [https://archive.org/stream/jamesbrindleyan01smilgoog#page/n7/mode/2up](https://archive.org/stream/jamesbrindleyan01smilgoog#page/n7/mode/2up)


Wood, G. H. *History of Wages in the Cotton Trade during the Past Hundred Years* (London: Sherratt and Hughes, 1910).


APPENDIX: THE FULL MODEL

The Household. The representative combined male and female agent of the generation born in period \( t - 1 \) maximizes her CES utility \( (U) \) in period \( t \) (the young adulthood):

\[
U(n, q, z_t) = \left[ \alpha \cdot \left( \frac{n_t}{2} \right)^{\frac{s-1}{s}} + \beta \cdot \left( q_t \right)^{\frac{s-1}{s}} + \gamma \cdot \left( \frac{z_t}{z_{t-1}} \right)^{\frac{s-1}{s}} \right]^{\frac{s}{s-1}}
\]

(i) The target number of children \( n \) is divided by 2, because the utility of \( n \) children is shared by the two parents. (ii) The target quality of children \( q \) is defined as the ratio of children’s to parents’ human capital levels. (iii) Consumption flow \( z_t \) enters the utility as a relative ratio rather than an absolute level. This ‘habit persistence’, as described in the macroeconomic literature, in material consumption has a justification from psychology; changes in consumption, not the level, affect utility\(^{136}\). There are four constraints in this maximization problem:

(H1) Budget Constraint: \( \sum_{j=0}^{2} w_{j+t} = \frac{1}{2} \pi_{n,t+j} n_t + \frac{1}{2} \pi_{q,t+j} q_t n_t + \sum_{i=0}^{2} z_{t+i} \);

(H2) Marriage Age Equation: \( A_t = A(n_t, ml_{t+1}, m2_{t+1}; a_0, a_1) \);

(H3) Price of Children Quality Equation: \( \pi_{q,t} = \pi_{q} (w_t, A_{t-1}, ml_t, m2_{t+1}; b_0, b_1, \eta) \);

(H4) Price of Children Quantity Equation: \( \pi_{n,t} = \pi_{n} (w_t, ml_t, m2_{t+1}; c_0) \).

Constraint (H1) specifies that lifetime earnings \( (w) \) are equal to the lifetime expenditures (by ‘earnings’ we also mean earnings in kind from domestic production consumed in the household). Expenditure on children is shared by both husband and wife, so is halved for the representative agent\(^{137}\).

Constraint (H2) imposes a restriction on female’s marriage age \( A_t \), which negatively depends on the total planned births, taking premature deaths in childhood and young adulthood \( (ml, m2) \) into consideration\(^{138}\). \( a_0 \) is the age women usually stop planning for, or expecting, new children (assumed around 35), and \( a_1 \) is the average gap between births (2.5 years).

Constraint (H3) states that the \textit{relative} generalised price of educating each surviving child \( (\pi_{q}) \) depends positively on mortality rates \( (m) \), and negatively on mother’s marriage age \( A \), (which

\(^{136}\) Because the other two utility inputs \( (n \) and \( q \) are stationary, the third utility input must be also.

\(^{137}\) Agents make their choices in the expectation (or hope) of living for the full four periods of the model, even though they may be wrong. Remarriage was a common means of maintaining the household in the event of the premature death of a spouse.

\(^{138}\) Family targeting plans are assumed to be based on survival of children to (approximately) age 30, perhaps to support parents in old age.
is an indicator of the level of the mother’s human capital). The parameter $b_0$ can be interpreted as the average proportion of income spent on $q$, while $b_1$ is the lower bound of marriage age. 

$\eta \equiv -d \ln \pi_q / d \ln A$ measures the elasticity of human capital accumulation with respect to marriage age. The coefficient on wages is expected to be positive reflecting the opportunity cost of parental time. The same applies with H4. The price of child quality includes school and apprenticeship fees (or the availability of apprenticeships) in principle as well.

Constraint (H4) indicates that the relative generalised price of feeding and clothing each surviving child ($\pi_n$) increases with mortality rates, where $c_0$ is the average proportion of income spent on $n$.

The law of motion for population ($P_t$: population stock at time $t$) is:

$$P_t = P_{t-1} - D_t + B_t.$$

Total deaths ($D_t$: death flow in period $t$) are the sum of premature and natural deaths:

$$D_t = m_l \cdot B_t + m_2 \cdot G_{1,t-1} + m_3 \cdot G_{2,t-1} + m_4 \cdot G_{3,t-1}.$$

Here, $G_{i,t-1}$ denotes the population of the generation born in period $t - i$ still living at time $t - 1$ (at the beginning of period $t$):

$$G_{1,t} = (1 - m_l) B_t$$
$$G_{2,t} = (1 - m_l) (1 - m_2) B_{t-1}$$
$$G_{3,t} = (1 - m_l) (1 - m_2) (1 - m_3) B_{t-2}$$

Total births in period $t$ ($B_t$: birth flow in period $t$) can be defined as:

$$B_t = \frac{(1 - \mu_t) G_{2,t-1}}{2} \frac{n_{t-1}}{(1 - m_l) (1 - m_2)_{t+1}}.$$

The variable $\mu_t$ captures the proportion of females without any children. It includes both never-married females and married females with no offspring. The unmatched proportion is a function of search and matching costs which are positively correlated with marriage age ($\tau_1 > 0$) and negatively with wage ($\tau_2 < 0$). The term $\bar{\mu}$ is the average celibacy rate, while $A^*_t$ and $w^*_t$ are the expected level of marriage age and wage in absence of all shocks.

$$\mu_t = \bar{\mu} + \tau_1 \frac{A_t - A^*_t}{A^*_t} + \tau_2 \frac{w_t - w^*_t}{w^*_t}.$$

Based on $B_t$ and $D_t$, we can define birth rate ($b_t$) and death rate ($d_t$) to link with the observable historical data. In particular, the birth rate is the birth flow during period $t$ divided by the population stock at the beginning of period $t$ (or at time $t - 1$). The death rate is the death flow
during period $t$ divided by the sum of the population at $t - 1$ and the birth flow during period $t$, because the death rate takes into account the premature deaths of those in their childhood period.

\[ b_t \equiv \frac{B_t}{P_{t-1}}; \quad d_t \equiv \frac{D_t}{P_{t-1} + B_t}. \]

Lastly, the labour force of period $t$ is the population stock ($P_{t-1}$) excluding the generations in their childhood during period $t$.

**Production.** Where $Y$ is output, $H$ is human capital per capita, $\bar{F}$ is a fixed production factor, especially land,$\Pi$ is profit and $x_t$ a random productivity shock, the representative production unit’s (farm’s) problem is:

\[
\max \Pi_t = Y_t - w_t P_{t-1} - \text{fixed costs}, \text{ subject to:}
\]

(F1) Production Function:

\[ Y_t = e^{x_t} P^\theta_{t-1} H^\theta_{t-1} \bar{F}^{1-\theta - \theta}; \]

(F2) Aggregate Human Capital:

\[ H_t = Q_{t-1} \frac{G_1}{P_t} + Q_{t-2} \frac{G_2}{P_t} + Q_{t-3} \frac{G_3}{P_t}; \]

(F3) Generational Human Capital:

\[ Q_t = e^{x_t} \left( Q_{t-2} q_{t-1} \right)^{1-\alpha}. \]

Constraint (F1) describes the production technology with three separate inputs. $H_{t-1}$ is the human capital per capita influencing period $t$, by which the ‘raw’ labour force ($P_{t-1}$) is augmented. The two variables are lagged because the contemporaneous values would include children. $\bar{F}$ represents the factors that cannot be reproduced such as land. For simplicity, it is set equal to 1 without loss of generality and is equivalent to normalising output by land.

Constraint (F2) defines the aggregate human capital ($H_t$), which is a weighted average of the generational human capital in the labour force (the three working ‘generations’).

Constraint (F3) describes how each generation’s human capital level ($Q_t$) is formed.

Finally, to complete the system of equilibrium conditions, a competitive labour market is postulated, so that the marginal product of labour equals the marginal cost:

\[ MP_t \equiv \frac{\partial Y_t}{\partial P_{t-1}} = \theta_t \frac{Y_t}{P_{t-1}} = w_t = MC_t. \]
We assume the exogenous productivity process is AR(1): $x_t = \rho_x \cdot x_{t-1} + e_t$, where $e_t$ is white noise with standard deviation equal to $\sigma_x$. Similarly, there are three exogenous conditional premature mortality rates:

- $m_{1,t} = \bar{m}_1 \cdot \exp(m_{mm1,t})$, where $m_{mm1,t} = \rho_1 \cdot m_{mm1,t-1} + e_{mm1,t}$;
- $m_{2,t} = \bar{m}_2 \cdot \exp(m_{mm2,t})$, where $m_{mm2,t} = \rho_2 \cdot m_{mm2,t-1} + e_{mm2,t}$;
- $m_{3,t} = \bar{m}_3 \cdot \exp(m_{mm3,t})$, where $m_{mm3,t} = \rho_3 \cdot m_{mm3,t-1} + e_{mm3,t}$.

Here, $\bar{m}$’s ($i = 1, 2, 3$) are the steady state mortality rates and $e_{mmi,t}$’s are white noises with standard deviations equal to $\sigma_i$. $m_{mm1}$ is the death rate of generation 1 (those aged up to 15). We set the steady states of mortality rates to match the historical life expectancy pattern over the six centuries\(^ {139}\).

<table>
<thead>
<tr>
<th>Meaning</th>
<th>Estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_1$ AR coefficient of shock mm1</td>
<td>0.522 0.200</td>
</tr>
<tr>
<td>$\rho_2$ AR coefficient of shock mm2</td>
<td>0.063 -0.431</td>
</tr>
<tr>
<td>$\rho_3$ AR coefficient of shock mm3</td>
<td>0.900 0.021</td>
</tr>
<tr>
<td>$\rho_x$ AR coefficient of shock x</td>
<td>0.825 0.406</td>
</tr>
<tr>
<td>$\sigma_1$ Standard deviation of shock emm1</td>
<td>0.487 0.275</td>
</tr>
<tr>
<td>$\sigma_2$ Standard deviation of shock emm2</td>
<td>0.011 0.414</td>
</tr>
<tr>
<td>$\sigma_3$ Standard deviation of shock emm3</td>
<td>0.011 0.627</td>
</tr>
<tr>
<td>$\sigma_x$ Standard deviation of shock ex</td>
<td>0.010 0.050</td>
</tr>
</tbody>
</table>

\(^{139}\) The calibrated rates should not be interpreted as a strict mapping from the life tables in Wrigley and Schofield (1989, p. 714) or any other sources. Since the model only has four generations with a maximum life of 60 years, the mortality rates in each cohort are inevitably different from the life table which has a maximum life of 90 years.